Complexity in Nanoplasmonics: Linear, Nonlinear, and Quantum

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• Introduction
• Applications of nanoplasmonics
• Localization of optical energy in time and space: Hot spots and ultrafast plasmonics
• Adiabatic compression and high-harmonic generation
• Spaser as a quantum generator
• Spaser in stationary (CW) mode
• Spaser as a quantum nanoamplifier
• Experimental observation of the spaser
• Conclusions on spasers
• Spasing and loss compensation in plasmonic systems with gain
• Conclusions on loss compensation
Nanoplasmonics is about nanolocalization of optical energy

Concentration of optical energy on the nanoscale

Photon: Quantum of electromagnetic field

Surface Plasmon: Quantum of electromechanical oscillator
Nanoplasmonic colors are very bright. Scattering and absorption of light by them are very strong. This is due to the fact that all of the millions of electrons move in unison in plasmonic oscillations.

Nanoplasmonic colors are also eternal: metal nanoparticles are stable in glass: they do not bleach and do not blink. Gold is stable under biological conditions and is not toxic in vivo.


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Applications of Nanoplasmonics:

1. Ultrasensitive and express sensing and detection using both SPPs and SPs (LSPRs): see, e.g., J. N. Anker, W. P. Hall, O. Lyandres, N. C. Shah, J. Zhao, and R. P. Van Duyne, *Biosensing with Plasmonic Nanosensors*, Nature Materials 7, 442-453 (2008);

2. Near-filed scanning microscopy (or, nanoscopy): NSOM (SNOM)


4. Photo- and chemically stable labels and probes for biomedical research and medicine

5. Nanoplasmonic-based immunoassays and tests. Home pregnancy test (dominating the market), PSA test (clinic), troponin heart-attack test, and HIV tests (in trials)


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Plasmonic Near-Field **Hot Spots:** Happy 17th Anniversary!


\[ R_{\text{Speckle}} \sim \frac{\lambda}{A} L \]

- \( R_{\text{Speckle}} \) is speckle size
- \( \lambda \sim 100 \text{ nm} \) is reduced wave length
- \( A \) is laser spot size,
- \( L \) is distance to the screen


Nanoplasmonics is intrinsically ultrafast:

Surface plasmon relaxation times are in \( \sim 10\text{-}100 \text{ fs} \) range

<table>
<thead>
<tr>
<th>( \tau_n ) (fs)</th>
<th>( \omega_n ) (eV)</th>
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<td>25</td>
<td>1.0</td>
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Best area for plasmonics

Spectrally, surface plasmon resonances in complex systems occupy a very wide frequency band; for gold and silver:

\[
\Delta \omega \approx \omega_p / \sqrt{2} \approx 4 \text{ eV}
\]

Including aluminum with plasmon responses in the ultraviolet, this spectral width increases to \( \sim 10 \text{ eV} \).

Corresponding rise time of plasmonic responses \( \sim 100 \text{ as} \)
Localized SP hot spots and SPPs coexist in space and time on nanostructured surfaces


30 femtoseconds from life of a nanoplasmonic system
Localized SP hot spots are deeply subwavelength as seen in PEEM (photoemission electron microscope)
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Adiabatic Compression

\[ \text{Field enhancement:} \]
\[ \sim \frac{L_s}{R} \quad \text{(for 2d compression),} \quad L_s \approx 25 \text{ nm} \]
\[ \sim \left( \frac{L_s}{R} \right)^{3/2} \quad \text{(for 3d compression)} \]

Plasmonic generation of ultrashort extreme-ultraviolet light pulses

In-Yong Park\textsuperscript{t,}, Seungchul Kim\textsuperscript{t,}, Joonhee Choi\textsuperscript{t,}, Dong-Hyub Lee\textsuperscript{1}, Young-Jin Kim\textsuperscript{1}, Matthias F. Kling\textsuperscript{2}, Mark I. Stockman\textsuperscript{3} and Seung-Woo Kim\textsuperscript{1}\textsuperscript{*}

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This invention changed civilization as we know it
This invention is used many more times than all others combined
This is the most valuable element of nanotechnology: nanoamplifier, whose pairs in c-MOS technology form digital bistable amplifiers and logical gates for information processing

**MOSFET US Patent**

Aug. 27, 1963
DAWON KAHNG
3,102,230
ELECTRIC FIELD CONTROLLED SEMICONDUCTOR DEVICE
Filed May 31, 1960

The FET transistor is extremely vulnerable to ionizing radiation damage of the gate oxide, catastrophic degradation
Speed of a processor ~ 3 GHz is determined by electric interconnects

Bandwidth ~ 10-100 GHz
Low resistance to ionizing radiation

Goal of plasmonics is to keep, amplify, and manipulate optical energy on nanoscale, just like the transistor does with electric energy
Quantum Nanoplasmonics: Surface Plasmon Amplification by Stimulated Emission of Radiation (SPASER)


The original spaser geometry


Spaser field per one plasmon in the core

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Erice, Italy    p.17
11/11/2011 4:30 AM
Spaser is the ultimately smallest quantum nano-generator

For small nanoparticles, radiative loss is negligible. Spaser is fully scalable.
Quantization of the SP system, valid in the quasistatic regime for times shorter than the SP lifetime $\tau_n = 1/\gamma_n$, is carried out by using the following approximate expression for the energy $H$ of an electric field $E(r, t)$, which is obtained for a dispersive system by following Ref. [13],

$$H = \frac{1}{4\pi T} \int_{-\infty}^{\infty} \frac{d[\omega e(r, \omega)]}{d\omega} E(r, \omega)E(r, -\omega) \frac{d\omega}{2\pi} d^3r.$$  

(2)

The electric field operator\(^4\) of the quantized SPs is\(^4\)

$$E(r) = -\sum_n A_n \nabla \varphi_n(r) (\hat{a}_n + \hat{a}^\dagger_n), \quad A_n = \left( \frac{4\pi \hbar s_n}{\varepsilon d s'_n} \right)^{1/2}$$

$s(\omega) = \varepsilon_d/ [\varepsilon_d - \varepsilon_m(\omega)]$ is Bergman’s spectral parameter, $\varepsilon_d$ is the permittivity of the ambient dielectric, and $\varepsilon_m(\omega)$ is the metal permittivity.

The spaser Hamiltonian has the form

$$H = H_g + \hbar \sum_n \omega_n \hat{a}_n \hat{a}^\dagger_n - \sum_p E(r_p) d^{(p)}$$

where $H_g$ is the Hamiltonian of the gain medium.

These equations of spaser theory are nonlinear describing a non-equilibrium second-order phase transition to spasing.

Quantum Theory of SPASER

Nondiagonal element of density matrix (polarization):

$$\hbar \rho_{12}^{(p)} = -\left[i(\hbar \omega - \varepsilon_{12}) + \hbar \Gamma_{12}\right] \rho_{12}^{(p)} + i \hbar n_{12}^{(p)} a_n \tilde{\Omega}_{12}^{(p)},$$

$$\tilde{\Omega}_{12}^{(p)} = -A_n d_{12}^{(p)} \nabla \varphi_n(r_p) / \hbar$$

Diagonal elements of density matrix (inversion):

$$\dot{n}_{12}^{(p)} = -4 \text{Im}[a_n \tilde{\Omega}_{12}^{(p)} \rho_{12}^{(p)}] - \gamma_2 (1 + n_{12}^{(p)}) + g (1 - n_{12}^{(p)})$$

SP field amplitude (semiclassical approximation):

$$\dot{a}_n = [i(\omega - \omega_n) - \gamma_n] a_n + i a_n \sum_p \tilde{\Omega}_{12}^{(p)} \rho_{12}^{(p)*}$$

Spectral width of spaser emission (Schawlow-type formula)

$$\gamma_s = \frac{\Gamma_0}{2N_p + 1}$$
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Theory of Spaser in Stationary Regime

Physically, the spaser action is a result of spontaneous symmetry breaking when the phase of the coherent SP field is established from the spontaneous noise. Mathematically, the spaser is described by homogeneous differential Eqs. (4)-(6) derived and solved in Sec. II.B. These equations become homogeneous algebraic equations for the stationary (CW) case. These equations always have a trivial, zero solution. However, when their determinant vanishes, they also possess a nontrivial solution describing spasing, whose condition is

\[
(\omega_{s} - \omega_{n} + i \gamma_{n})^{-1} \times \left(\omega_{s} - \omega_{21} + i \Gamma_{12}\right)^{-1} \sum_{p} \left| \tilde{\Omega}_{12}^{(p)} \right|^{2} n_{21}^{(p)} = -1 ,
\]

where \( \omega_{s} \) is the spasing frequency, \( \tilde{\Omega}_{12}^{(p)} = -\frac{A_{m} d_{12}^{(p)} \nabla \varphi_{n}(r_{p})}{\hbar} \) is the single-plasmon Rabi frequency, \( d_{12}^{(p)} \) is the transition dipole moment of a \( p \)th chromophore, \( \varphi_{n}(r_{p}) \) is the electric potential of the spasing mode at the position this chromophore, \( \gamma_{n} \)

\[
n_{21}^{(p)} = (g - \gamma_{2}) \times \left\{ g + \gamma_{2} + 4 \left| \Omega_{12}^{(p)} \right|^{2} / \left[ (\omega_{s} - \omega_{21})^{2} + \Gamma_{12}^{2} \right] \right\}^{-1} ,
\]

From the imaginary part of Eq. (10) we immediately find the spasing frequency

\[
\omega_{s} = (\gamma_{n} \omega_{21} + \Gamma_{12} \omega_{n}) / (\gamma_{n} + \Gamma_{12}) ,
\]

which generally does not coincide with either the gain transition frequency \( \omega_{21} \) or the SP frequency \( \omega_{n} \), but is between them (this is a frequency walk-off phenomenon similar to that of laser physics). Substituting Eq. (11) back to Eqs. (10)-(11), we obtain a system of equations

\[
\frac{(\gamma_{n} + \Gamma_{12})^{2}}{\gamma_{n} \Gamma_{12} \left[ (\omega_{21} - \omega_{n})^{2} + (\Gamma_{12} + \gamma_{n})^{2} \right]} \times \sum_{p} \left| \tilde{\Omega}_{12}^{(p)} \right|^{2} n_{21}^{(p)} = 1 ,
\]

\[
n_{21}^{(p)} = (g - \gamma_{2}) \times \left[ g + \gamma_{2} + \frac{4 N_{n} \left| \tilde{\Omega}_{12}^{(p)} \right|^{2} (\Gamma_{12} + \gamma_{n})}{(\omega_{12} - \omega_{n})^{2} + (\Gamma_{12} + \gamma_{n})^{2}} \right]^{-1} .
\]

This system defines the stationary (CW) number of SPs per spasing mode \( N_{n} \).
SPASER Threshold Condition [Consistent with original PRL 90, 027402-1-4 (2003)]:

Since $n_{21}^{(p)} \leq 1$, from Eqs. (12), (13) we immediately obtain a necessary condition of the existence of spasing,

$$\frac{(\gamma_n + \Gamma_{12})^2}{\gamma_n \Gamma_{12} \left[ (\omega_{21} - \omega_n)^2 + (\Gamma_{12} + \gamma_n)^2 \right]} \sum_p \left| \tilde{\Omega}_{12}^{(p)} \right|^2 \geq 1.$$  

(14)

This expression is fully consistent with [4]. The following order of magnitude estimate of this spasing condition has a transparent physical meaning and is of heuristic value:

$$\frac{d_{12}^2 Q N_c}{h \Gamma_{12} V_n} \gtrsim 1,$$

(15)

where $Q = \omega / \gamma_n$ is the quality factor of SPs, $V_n$ is the volume of the spasing SP mode, and $N_c$ is the number of gain medium chromophores within this volume. Deriving this estimate, we have neglected the detuning, i.e., set $\omega_{21} - \omega_n = 0$.

The spasing is essentially a quantum effect.

It is non-relativistic: does not depend on $c$

The spasing condition does not directly contain gain per cm and the Purcell factor

[E. M. Purcell, Phys Rev 69, 681 (1946)]

but is related to them.
Stationary (CW) spaser regime

This quasilinear dependence \( N_n(g) \) is a result of the very strong feedback in spaser due to the small modal volume.

Mark I. Stockman,
Spasing-Required Gain of Bulk Gain Medium

\[ g \geq g_{th}, \quad g_{th} = \frac{\omega}{c \sqrt{\varepsilon_d}} \frac{\text{Re} \, s(\omega) \, \text{Im} \, \varepsilon_m(\omega)}{1 - \text{Re} \, s(\omega)} \]

\[ s(\omega) = \frac{\varepsilon_d}{\varepsilon_d - \varepsilon_m(\omega)}; \quad 1 > \text{Re} \, s(\omega) > 0 \]

Realistic gain for direct band-gap semiconductors
Scaling of Spaser

Field in spaser: \( E \sim \frac{\hbar \omega}{R^{3/2}} \sqrt{N_p} \sim 1 \ \text{MV} \left( \frac{R}{10 \ \text{nm}} \right)^{-3/2} \sqrt{N_p} \)

Heat per flop: \( H = \hbar \omega N_p \)

Threshold: \( g \geq g_{th}, \quad g_{th} = \frac{\omega}{c \sqrt{\varepsilon_d}} \frac{\text{Re} s(\omega) \text{Im} \varepsilon_m(\omega)}{1 - \text{Re} s(\omega)}, \quad s(\omega) = \frac{\varepsilon_d}{\varepsilon_d - \varepsilon_m(\omega)} \)

Switching time: \( \tau \sim \frac{1}{\gamma_p N_p} \sim 100 \ \text{fs} \)

Quantum limit: \( \omega \tau \sim \frac{1}{\gamma_p N_p} \sim \frac{Q}{N_p} \sim 100 \geq 1 \)

Conclusion: Spaser is orders of magnitude more efficient (less heat per flop) than transistor. It can operate at the quantum limit.
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The spaser as a Nanoamplifier

Strategic goal is all-optical ultrafast (multi-THz) processors of signals and information, which are also radiation hardened

Major problem: any quantum amplifier (laser and spaser) in a CW regime possesses exactly zero amplification (it is actually a condition for the CW operation).

We have proposed to set the spaser as a nanoamplifier in three ways:

1. In transient mode (before reaching the CW regime), the spaser still possesses non-zero amplification

2. With a saturable absorber, the spaser can be bistable. There are two stable states: with the zero coherent SP population ("logical zero") and with a high SP population that saturates the absorber ("logical one" state). Such a spaser will function as a threshold (digital) amplifier

3. Removing or reducing feedback, polaritonic spaser can function just like an optical amplifier but with a nanoscale size
Amplification in Spaser with a Saturable Absorber (1/3 of the gain chromophores)

Stationary pumping


Pulse pumping
Spaser Nanoamplifier in Direct Bandgap Semiconductors

InGaNa doped

Experimental Observations of Spaser


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Demonstration of a spaser-based nanolaser

M. A. Noginov\(^1\), G. Zhu\(^1\), A. M. Belgrave\(^1\), R. Bakker\(^2\), V. M. Shalaev\(^2\), E. E. Narimanov\(^2\), S. Stout\(^1,3\), E. Herz\(^3\), T. Suteewong\(^3\) & U. Wiesner\(^3\)

**Figure 1** | **Spaser design.** a, Diagram of the hybrid nanoparticle architecture (not to scale), indicating dye molecules throughout the silica shell. b, Transmission electron microscope image of Au core. c, Scanning electron microscope image of Au/silica/dye core–shell nanoparticles. d, Spaser mode (in false colour), with \(\lambda = 5\) circles represent the 14-nm strength colour scheme is s
Figure 2 | Spectroscopic results. Normalized extinction (1), excitation (2), spontaneous emission (3), and stimulated emission (4) spectra of Au/silica/dye nanoparticles. The peak extinction cross-section of the nanoparticles is $1.1 \times 10^{-12}$ cm$^2$. The emission and excitation spectra were measured in a spectrofluorometer at low fluence.
Figure 4 | Stimulated emission. a, Main panel, stimulated emission spectra of the nanoparticle sample pumped with 22.5 mJ (1), 9 mJ (2), 4.5 mJ (3), 2 mJ (4) and 1.25 mJ (5) 5-ns optical parametric oscillator pulses at λ = 488 nm. b, Main panel, corresponding input–output curve (lower axis, total launched pumping energy; upper axis, absorbed pumping energy per nanoparticle); for most experimental points, ~5% error bars (determined by the noise of the photodetector and the instability of the pumping laser) do not exceed the size of the symbol. Inset of a, stimulated emission spectrum at more than 100-fold dilution of the sample. Inset of b, the ratio of the stimulated emission intensity (integrated between 526 nm and 537 nm) to the spontaneous emission background (integrated at <526 nm and >537 nm).
Lasing in metal-insulator-metal sub-wavelength plasmonic waveguides

Martin T. Hill, Milan Marell, Eunice S. P. Leong, Barry Smaalbrugge, Youcai Zhu, Minghua Sun, Peter J. van Veldhoven, Erik Jan Geluk, Fouad Karouta, Yok-Siang Oel, Richard Nötzel, Cun-Zheng Ning, and Meint K. Smit

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Fig. 1. Structure of cavity formed by a rectangular semiconductor pillar encapsulated in Silver. (a) Schematic showing the device layer structure. (b) Scanning electron microscope image showing the semiconductor core of one of the devices. The scale bar is 1 micron.
Fig. 2. Spectra and near field patterns showing lasing in devices. (a) Above threshold emission spectrum for 3 micron long device with semiconductor core width d~130nm (±20nm), with pump current 180 μA at 78K. Inset: emission spectra for 20 (green), 40 (blue) and 60 (red) μA, all at 78K. (b) Lasing mode light output (red crosses), integrated luminescence (blue circles), versus pump current for 78K. (c) Actual near field pattern (in x-y plane) for 6 micron (d = 130nm) device captured with 100x, 0.7 NA long working distance microscope objective and infrared camera, the scale bar is 2 micron, for below threshold 30 μA, and (d) above threshold 320 μA. (e) Simulated vertical (z) component of the Poynting vector taken at 0.7 microns below the pillar base, shows most emitted light at ends of device. (f) Spectra for a 6 micron long device with d~310nm at 298K, pulsed operation (28 ns wide pulses, 1MHz repetition). Spectra for peak currents of 5.2mA (red), 5.9mA (green) and 7.4mA (blue), (currents were estimated from the applied voltage pulse amplitude). The spectra for 5.9 and 7.4 mA are offset from 0 for clarity. Inset shows the total light collected by the spectrometer from the device for currents ranging from 0 to 10mA.
LETTERS

Plasmon lasers at deep subwavelength scale

Rupert F. Oulton\textsuperscript{1,*}, Volker J. Sorger\textsuperscript{1,*}, Thomas Zentgraf\textsuperscript{1,*}, Ren-Min Ma\textsuperscript{3}, Christopher Gladden\textsuperscript{1}, Lun Dai\textsuperscript{3}, Guy Bartal\textsuperscript{1} & Xiang Zhang\textsuperscript{1,2}

2d plasmonic field confinement

Complexity in Nanoplasmonics 2011

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Erice, Italy p.37

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Room-temperature sub-diffraction-limited plasmon laser by total internal reflection

Ren-Min Ma††, Rupert F. Oulton††, Volker J. Sorger†, Guy Bartal† and Xiang Zhang†‡∗

1d +2d plasmonic field confinement
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BRIEF CONCLUSIONS

1. Spaser is a nanoscopic quantum generator of coherent and intense local optical fields

2. Spaser can also serve as a nanoscale ultrafast quantum amplifier with a switch time \( \sim 100 \text{ fs} \) for silver and \( \sim 10 \text{ fs} \) for gold. It has the same size as MOSFET and can perform the same functions but is \( \sim 1000 \) times faster.

3. Spaser has been experimentally observed recently. This experiment is in an excellent qualitative agreement with theory. The observed spaser is single mode. Its pumping curve is linear with a threshold. Its linewidth is inversely proportional to pumping rate.

4. Numerous plasmon-polariton spasers (plasmonic nanolasers) have been designed. In contrast to spaser, their length is on the order of micron (transverse mode size is nanometric).

5. The most promising applications of the spaser are an ultrafast nanoamplifier, local optical energy source, active nano-label, and an element of metamaterials with compensated loss.
This device is more like a spaser than an effective low-loss medium.
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Spaser Action, Loss Compensation, and Stability in Plasmonic Systems with Gain

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(Received 16 November 2010; revised manuscript received 28 February 2011; published 11 April 2011)

We demonstrate that the conditions of spaser generation and the full loss compensation in a dense resonant plasmonic-gain medium (metamaterial) are identical. Consequently, attempting the full compensation or overcompensation of losses by gain will lead to instability and a transition to a spaser state. This will limit (clamp) the inversion and lead to the limitation on the maximum loss compensation achievable. The criterion of the loss overcompensation, leading to the instability and spasing, is given in an analytical and universal (independent from system’s geometry) form.

DOI: 10.1103/PhysRevLett.106.156802

PACS numbers: 73.20.Mf, 42.50.Nn, 78.67.Pt, 81.05.Xj
Consider an isotropic metamaterial that can be described by complex permittivity and permeability. A known homogenization procedure leads to an exact result for the (effective) permittivity of the composite

\[ \bar{\varepsilon}(\omega) = \frac{1}{V |E(\omega)|^2} \int_V \varepsilon(r, \omega) |e(r, \omega)|^2 dV \]

Here \( E \) is the macroscopic field and \( e(r) \) is the (mesoscopic) local field inside the metamaterial. This local field is expressed as an eigenmode expansion

\[ e(r, \omega) = E(\omega) - \sum_n \frac{a_n}{s(\omega) - s_n} E_n(r), \quad s(\omega) = \frac{\varepsilon_h(\omega)}{\varepsilon_h(\omega) - \varepsilon_m(\omega)} \]

where \( E_n(r) \) is the eigenmode field. Assume that: there is a resonance with an \( n \)-th eigenmode, the metal has a high quality factor, \( Q \gg 1 \), and the metal’s fill factor \( f \) is not too small, so \( Qf \gg 1 \). Then the local field is

\[ e(r, \omega) = i \frac{a_n}{\text{Im} s(\omega_n)} E_n(r) \]
Then the effective permittivity becomes (where \( b_n > 0 \) is a coefficient):

\[
\bar{\varepsilon}(\omega) = b_n \left[ s_n \varepsilon_m(\omega) + (1 - s_n)\varepsilon_h(\omega) \right]
\]

\[
b_n = \frac{1}{V} \left( \frac{Q \int_V \theta(r)E_n(r)dV}{s_n (1 - s_n)} \right)^2
\]

In the case of the full inversion (maximum gain) and in exact resonance, the imaginary part of the host-medium permittivity describes stimulated emission as given by the standard expression

\[
\varepsilon_h(\omega) = \varepsilon_d - i \frac{4\pi |d_{12}|^2 n_c}{3\hbar \Gamma_{12}}
\]

where \( \varepsilon_d = \text{Re}\varepsilon_h \), \( d_{12} \) is the dipole matrix element of the gain transition in a chromophore center of the gain medium, \( \Gamma_{12} \) is a spectral width of this transition, and \( n_c \) is the concentration of these centers.

The condition for the full electric loss (over)compensation at the resonant frequency \( \omega = \omega_n \) is \( \text{Im}\bar{\varepsilon}(\omega) \leq 0 \), which reduces to...
\[ \frac{4\pi}{3} \left| d_{12} \right|^2 n_c \left[ 1 - \text{Re} s(\omega) \right] \geq 1 \quad \text{or} \quad g \geq g_{th}, \quad g_{th} = \frac{\omega}{c \sqrt{\varepsilon_d}} \frac{\text{Re} s(\omega) \text{Im} \varepsilon_m(\omega)}{1 - \text{Re} s(\omega)}, \] where \( g \) is the required gain.

- This is a criterion for both the loss compensation and spasing, the latter obtained previously in: M. I. Stockman, *The Spaser as a Nanoscale Quantum Generator and Ultrafast Amplifier*, Journal of Optics 12, 024004-1-13 (2010).
- This criterion is analytical and exact, provided that the metamaterials is resonant and dense, and that its eigenmodes are non-uniform in space -- hot spots or reflection from facets -- create a feedback.
- Thus, an attempt at a full compensation of losses will cause spasing instead, which will saturate the gain transition, eliminate the net gain, clamp the inversion, and make the complete loss compensation impossible.
- This criterion does not depend on the geometry of the system or any specific hot spots of local fields, predicated on the gain medium filling all the space left by the metal.
Spasing criterion as a function of optical frequency. The straight line (red on line) represents the threshold for the spasing and full loss compensation, which take place for the curve segments above it. (a) Computations for silver. The chromophore concentration is $n_c = 6 \times 10^{18}$ cm$^{-3}$ for the lower curve (black) and $n_c = 3 \times 10^{19}$ cm$^{-3}$ for the upper curve (blue). The magenta solid circle and black diamond show the values of the spasing criterion for the conditions of Refs. 2 and 3, respectively. (b) Computations for gold. The chromophore concentration is $n_c = 3 \times 10^{19}$ cm$^{-3}$ for the lower curve (black) and $n_c = 2 \times 10^{20}$ cm$^{-3}$ for the upper curve (blue).


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BRIEF CONCLUSIONS

• The same criterion is obtained for both the loss compensation and spasing
• This criterion is analytical and exact, provided that the metamaterials is resonant and dense, and that its eigenmodes are non-uniform in space (contain “hot spots”), which creates an inherent feedback
• Thus, an attempt at a full compensation of losses will cause spasing instead, which will saturate the gain transition, eliminate the net gain, clamp the inversion, and make the complete loss compensation impossible
• This criterion does not depend on the geometry of the system or any specific hot spots of local fields, predicated on the gain medium filling all the space left by the metal
Breaking the Cloak: Relativistic Causality

For CW radiation, the ray that bends around the cloak carries radiation with higher than $c$ phase velocity, which is possible

$$v_p = (\pi - 1)c > 2c$$

For pulse radiation, the ray that bends around the cloak carries radiation with group velocity than must be less than $c$ (relativistic causality). Thus, it arrives with a delay,

$$\Delta t = \left( \frac{\pi}{v_g} - \frac{1}{c} \right) D > \left( \frac{\pi - 1}{c} \right) D = \left( \frac{\pi - 1}{c} \right) \frac{D}{\lambda} \approx \frac{\pi - 1}{c} \frac{D}{\lambda} > \frac{c}{\lambda} > T$$

which for a macroscopic cloak is much larger than the period $T$ (typically, $>10^6 T$).