Chaotic to periodic spontaneous pulsing in current driven silicon p-i-n structures

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Abstract

We report the experimental observations and modeling results confirming the transition to periodicity from chaotic (although still deterministic) spontaneous firing patterns for silicon p-i-n structures at 4.2K controlled by a constant current source. These patterns are determined by measuring the interpulse time interval (IPTI) between pulses. In contrast to a naive view, which suggests periodic patterns under current conservation, we see a rich spectrum of behavior due to the space charge effects, when the control parameter (drive current) is varied. In particular, a bi-modal pattern is found in which the IPTIs occur in two distinct groups rather than as a single continuous distribution. Nonlinear dynamics analysis tools as well as a statistical approach (i.e. Markov process) confirm the change from chaotic to periodic behavior as the control parameter increases. A theoretical model based on device physics for the buildup of space charge in the i-region of the diode giving insight into semiconductor device fabrication is developed to explain the bi-modal pulsing behavior observed. This can be used to enhance our understanding of various pulsing device applications, including IR detectors, parallel processors and neural networks.

Spontaneous pulsing has been observed at cryogenic temperatures in circuits containing silicon p-i-n diodes driven by a constant voltage source [1] and a constant current source [2]. This pulsing phenomenon has been studied in connection with a number of applications, including infrared detectors [3], where the variations in the periodicity and mode locking of firing patterns are used [4]. In artificial neural networks [5], the response is similar to the action potential for a biological neuron, exhibiting summation over inputs (from multiple sources and/or the same source). The output can be distributed to multiple outputs through a filter circuit [6], allowing complex interconnectivity. Combining two pulsing diodes to form a single channel, and with suitable interconnections of these channels, a parallel processor [7] could be achieved. Although the devices were driven under a constant bias as infrared detectors, other situations, such as a parallel processor, require current driven structures. The models explaining constant voltage driven diodes [3] are capable of predicting the IR responses. However, the only current driven model in literature [2] deals with a pure periodic firing pattern. In order to fully understand the current driven situation, we have studied the temporal patterns associated with circuits driven by a constant current source. This does not imply that no new interesting phenomena exist in the domains previously studied, since all indications are the system under study exhibits a rich variety of deterministic nonlinear dynamic effects. Rather than fully exploring
those effects, we are interested in exploring the situation where a different constraint is imposed in order to see if new insights arise. A naive view might suggest that constant input current would lead to simple periodic spiketrains. However, this need not be true, because different degrees of space charge neutralization affect the initial conditions for the next interpulse time interval (IPTI). Although the time averaged output current must equal the constant input current, there may be pulse-to-pulse variations in charge stored in the diode, and related temporal variations in IPTIs. By changing the circuit parameters, this variation can be maximized or minimized.

Extensive theoretical predictions on transition to chaos, by quasiperiodicity [8], through the overlap of mode locked resonances for Josephson junctions and charge density waves [9,10], and experimental studies on hydrodynamic systems [11–14] in various states of Rayleigh–Bénard convection have been carried out. However, semiconductor systems in Si [1,15] and Ge [16,17] with spontaneous pulse outputs have not been studied extensively. Dramatic effects have been observed in temporal patterns of spiketrains for diodes controlled by constant voltage sources [1] and by infrared sources [4] observing, 115 Farey fraction frequencies out of a possible 129 (up to order 20) in the power spectra [1], and period-8 and -9 modelocking in temporal patterns for certain ranges of IR source temperature [4].

In this paper, we report the experimental observation of new effects associated with deterministic nonlinear dynamics in circuits containing silicon p-i-n diodes at 4.2K. The circuits generate spiketrains and are qualitatively similar to neuron equivalent circuits [1]. We have measured a large number of IPTIs for various driving currents. The nonlinear dynamics are associated with an integrate-and-fire mechanism in the diode [1]. Here we give a qualitative description of the pulsing method, more quantitative details will be given later and elsewhere [2]. The sample consists of a $D = 100 \ \mu$m thick i-region, lightly doped ($n = 10^{14}$ cm$^{-3}$) with phosphorus sandwiched between 1 $\mu$m thick p and n regions, doped with boron and phosphorus ($\sim 10^{19}$ cm$^{-3}$) respectively, with an $A = 1$ mm$^2$ cross sectional area. A band diagram for the diode at various times in the pulsing cycle is shown in Fig. 1. Initially the diode has an extremely thin layer of space charge at the n-i interface with a small potential drop (Fig. 1a). Under a forward biased condition, electrons are injected into the i-region at the n-i interface, accelerated by the electric field and collected in the p-region. The input to the circuit, as seen in Fig. 2, in this case will be a constant current. Part of this current will go to the input capacitor charging it, and part will pass through the diode. The current density passing through the diode (injection current) is found from the Richardson–Dushman equation for the thermionic current density at an interface. Since the interface is not abrupt but rather has some finite thickness, the barrier at the n-i interface will be lowered by the electric field. This effect has been included in calculating the current. The current passing through the diode will create space charge by impact ionization of the impurities in the i-region. This space charge, in combination with the charging of the input capacitor, will lead to an increased field at the n-i interface (Fig. 1b). Because of the strong nonlinear dependence of the injection current density on the field strength, the current will increase rapidly, and a pulse will occur. At this point the input capacitor is discharged, and the output capacitor is charged. When this occurs, the field at the p-i interface becomes zero, then negative, (Figs. 1c,d) and instead of impact ionization producing more space charge, recombination of free electrons with the ionized impurities leads to a decrease in the space charge, resetting the diode for a new pulse cycle. The output capacitor discharges through the load resistor, and the buildup of space charge for the next pulse begins. The pulses are detected by monitoring the voltage across the output capacitor. When the current pulse passes through the diode, the output voltage rises rapidly ($t \sim 90 \ \text{ns}$) to a peak value of 1–3 V followed by an exponential decay with time constant $\tau = R_L C_L$ where $R_L$ and $C_L$ denote the load resistor and load capacitor. The diode design parameters are extremely critical to this spontaneous pulsing, and have been analyzed for purely periodic firing patterns [2]. The IPTIs $t_1, t_2, t_3, \ldots$, which are the experimentally measured quantities, define a time series which can be accurately determined because of the high slew rate
Fig. 1. Band diagram showing the different stages for a pulsing p-i-n diode. (a) Initial condition with low bias and thin space charge layer. (b) Some later time with increased bias due to charge on input capacitor and increased space charge from impact ionization. (c) Field at p-i interface has now gone to zero. This is the critical case between impact ionization and capture. (d) Field at p-i interface is negative. Space charge is now being removed by capture of electrons at ionized impurities.

Fig. 2. The simple circuit used to obtain the spontaneous pulsing. The liquid helium cooled diode is driven by a dc current. $C_l$ and $C \approx 100 \, \text{pF}$ are due to the cables and $R_L \approx 300 \, \text{k}\Omega$ is the load resistance. Interpulse time intervals (IPTIs) are denoted by $t_1$, $t_2$, etc.

associated with the fast risetime. We use a latching scalar to count pulses from a fast clock ($\leq 10 \, \text{MHz}$) and to latch counts associated with every pulse in a spiketrain.

Here our focus is on the results obtained from a single diode in the circuit shown in Fig. 2 under different constant current values. The data consisted of sets of 2100 IPTIs taken for drive currents between 25 and 72 nA at 1 or 2 nA intervals. The patterns formed by the IPTIs fell into 3 general groups. Examples of these groups with successive IPTIs are shown in Fig. 3. For driving current less than $\sim 33 \, \text{nA}$ the IPTIs take on all values between their extremes. In Fig. 3b 100 IPTIs for 28 nA driving current are shown with every second point connected. The two groups of points (labeled odd and even) show a similar pattern and give no clear indication of periodicity. However, it will be shown later that these are not random fluctuations. Above 33 nA, there is a forbidden region in the IPTI distribution allowing us to define the IPTIs as short (S) or long (L) according to whether they are longer or shorter than the forbidden values. The most striking feature of this segment is the strong period 2 component in the series. This is shown in Figs. 3c and d where there is a strong preference for a given IPTI to be of the same type as the IPTI two intervals before it. The period 2 nature is broken intermittently by an extra pulse, producing 2 successive pulses of the same type. Below 50 nA, these are almost exclusively LL groupings that occur at what appear to be random intervals. For 50 nA and above, the extra pulse is always part of an SS pattern. These SS groupings also occur much more periodically than the LL patterns at lower currents do. At 68 nA, the intermittency has developed into a fairly regular pattern of pulses occurring with roughly equal frequency after 6 or 7 LS patterns.

Fig. 4 shows the return maps for the first 2048 IPTIs shown in Fig. 3. At 28 nA driving current, the pattern fills the space almost completely inside a well-defined area, though even here some structure is visible. In particular, the gap (shown with an arrow) is a region where an IPTI can occur only if the previous IPTI is near the minimum, this is an indication of the pulse separation into long and short interval groups, and shows that the pattern is not random. At 48 nA, the spread is reduced and the points are moved toward the boundary giving well defined groups. This can be
interpreted as 3 types of IPTI patterns, one which has a strong period 2 firing pattern, a noisy period one pattern caused by an LL pattern, and a single point near the origin which indicates an SS pattern. As the driving current is increased to 68 nA, the group which gives the noisy period 1 pattern disappears, and the spread (and modulation) of the linear period 2 pattern gets reduced. At the same time, the number of SS patterns increases forming a period 1 pattern that is much less noisy. This reduction of the spread gives rise to a finer grouping in the linear segment near the Y axis associated with the fact that the second interval in an SS pattern is shorter than the average for S intervals. The number of LSL (845) and SSL patterns (119) indicates that the extra S IPTI occurs after an average of 7 LS patterns or every 14 pulses. This growth in the period one pattern at 68 nA indicates that the single SS at 48 nA is not random, but may be due to higher order periodic pattern. However, at 48 nA there is no apparent shortening of the L following the SS pattern so that this pattern may have different causes at the two currents.

The autocorrelation (see Fig. 5) also shows the trend towards periodic patterns as the current increases. At 28 nA, no significant correlation is seen. As the current increases a number of distinctive features appear, the first being an alternating correlation and anticorrelation that indicates the presence of a strong period 2 component in the time series. On top of this period 2 oscillation, there is an envelope that modulates the function which is determined by the pattern of intermittencies [18]. At 48 nA this envelope decays rapidly indicating the LL patterns occur at an average interval of 14 pulses but with significant deviations. At 68 nA the envelope has developed a periodic pattern indicating a periodicity of ~ 14 for the SS patterns. This is an average of the actual observed values of 13 and 15.

An attempt was made to obtain correlation dimensions for the time series by a Grassberger–Procaccia algorithm [19]. Even though the system might be expected to have a relatively low dimension, the correlation dimension calculation failed to converge.
This occurred for all drive currents. As seen in Fig. 6 at 28nA the dimension is large ($\sim 10$). For the 68 nA case the short separation portion shows this high dimension, while the large separation portion shows a smaller dimension of 2.5–3.0 associated with the intermittencies. There was evidence from a false nearest neighbor approach [20] that the dimension of the attractor was high. Also, an approach using the distance to the $i$th nearest neighbor [21] led to the same conclusion. Since the limited number of data points (in a discontinuous data set) does not allow us to obtain correlation dimensions accurately, we consider a mapping into symbolic dynamics [22]. This approach is suggested by the natural division into S and L groupings at higher current. To start, the IPTI data at a given current was divided into a fixed number of bins. Above 33 nA, driving current where the gap between S and L IPTIs appears, the division was into 2 bins one each above and below the gap labeled L and S respectively. For the 28 nA case, division into 2 bins with S $\leq 35$ ms and L $> 35$ ms, and a second division into 3 bins with S $< 20$ ms, 20 $\leq$ M $\leq 35$ ms, and L $> 35$ ms were both tried. We then determined the number of occurrences of all possible words of length $N \leq 17$ symbols. From these counts the conditional probabilities $P(x_N | x_1 x_2 x_3 \ldots x_N)$
were determined. A word of length \( N \) is a \( k \)th order Markov process if 
\[ P(x_N|x_{N-k}x_{N-k+1} \ldots x_N) = P(x_N|x_{N-k}x_{N-k+1} \ldots x_{N-k+2} \ldots x_N) \].
Representing the lefthand side by \( P_n \) and the righthand side by \( P_k \), then 
\[ \chi^2 = \frac{1}{n} \sum_{1}^{n} \frac{(P_n - P_k)^2}{P_n} \] should go to zero when \( k \) is greater than or equal to the order of the Markov process. There are two considerations in performing this calculation. First, to obtain the order the maximum word length must be larger than the order of the process. Second, there must be sufficient occurrences of the various words to accurately determine the conditional probabilities. These two conditions tend to conflict with each other, sometimes making it impossible to obtain results from a small data set. This occurred in the 48 nA case where a maximum word length sufficiently short to allow determination of the probabilities was shorter than the order, making it impossible to assign an order to the process. This is probably related to the nonperiodicity of the intermittencies. The calculated order is an indicator of the periodicity of the process. A determination of order 0 can be the result of one of three cases. The first is the simple period one case, for example, SSS, ..., which can be readily identified from the data. The second case is a random distribution with no memory of previous events. The third case is a chaotic process for which the relevant period is much longer than the maximum word length. It is not possible to distinguish between the last two cases on the basis of a single choice of symbol assignment. However, the random case will remain 0 for any choice of symbol assignment while the chaotic case may show changes as the bins used in assigning symbols are changed. This is the case at 28 nA where the 2 symbol result gave an order of 0 while the three symbol result did not fall to zero for any word length that could be used, indicating an order too large to be determined. This confirms that the pattern at 28 nA is chaotic.

When the order is nonzero the period can be any value greater than the order. For example, the pattern SLSL has order 1 and period 2, while the pattern SLSMSLSLM has order 2 and period 4. In addition there can be several drops in \( \chi^2 \) corresponding to different periodicities present in the data. Our 68 nA case is of this type. As seen in Fig. 7 there is a large drop at order 1 which is associated with the strong period 2 component of the pattern. However, there is also a second drop at order 12. This second drop is due to the extra short interval at the odd/even interchanges which occur most commonly after 12 IPTIs. This produces a period 13 pattern. There is no drop at 14 corresponding to the period 15 pattern since the elimination of the shorter period patterns leaves only the long period possibility. When this test is run on a data set modified by dropping the extra S IPTI the drop at order 12 vanishes. (See inset in Fig. 7.)

Now we turn our attention to a simple model to explain this behavior. Since the approach is based on our previous work [2], only the major equations with brief descriptions are given here. However, as we expand the model with new ideas, more details will be given. Our aim here is to get an expression for current density with time.

The voltage \( V \) across the diode in Fig. 2 is given by

\[ V = \frac{A}{C} \int_{t_i}^{t_f} (i_t - j) \, dt - \frac{A}{C_L} \int_{t_i}^{t_f} j \, dt, \] (1)
with $i_T$ the input current density, $j$ the injection current density at the n-i interface, $A$ the diode area, and $C_L$ and $C$ the load and input capacitances. (While $i_T$ is not a directly measurable quantity, since it uses the diode area rather than the area of the wire to define the density, the combination $Ai_T$ is the total input current to the circuit.) Integrals over time are required by the transient nature of the impulses. Another expression for the voltage across the diode is $V - V_0 = ED + V_{bi}$ where $E$ is the field at the p-i interface, $D$ is the i-region thickness, $V_{bi}$ is the built-in potential [23] due to the i-region space charge and $V_0 = 1.1$ V is the flatband voltage for silicon.

The space charge buildup is modeled in terms of a depletion width $w$ [23]

$$\frac{dw}{dt} = \frac{D - w}{e} \sigma j,$$  \hspace{1cm} (2)

where $e$ is the electron charge and $\sigma$ is the impact ionization cross section [24]. After using Eq. (2) to change the integration variable from $r$ to $w$ in Eq. (1) we integrate Eq. (1). Then using $V_{bi} = w^2 en/2e$ and the expression for the voltage across the diode with the approximation $w - w_i \ll w_i$ we obtain the following equation for $i_T$ and $t$:

$$\frac{Ai_T}{C} - \frac{Ae(w - w_i)(C + C_L)}{(D - w_i)\sigma CC_L} = ED + V_0 + \frac{w^2 en}{2e},$$  \hspace{1cm} (3)

with $w_i$ the initial depletion width, $n$ the impurity density in the i-region and $\epsilon$ the permittivity in the i-region.

The injection current density is given by a modified [1] Richardson–Dushman equation

$$j = A^* T^2 \exp \left( \frac{\Delta - edF}{kT} \right).$$  \hspace{1cm} (4)

where $\Delta$ is the interfacial work function at the n-i interface [1,25], $d$ is an interface thickness parameter [1], $A^*$ is the Richardson–Dushman constant, $k$ is Boltzmann’s constant and $T$ is the temperature.

From Gauss’s law and integration by parts, the field at the n-i interface for a space charge distribution $\rho$ can be shown to be

$$F = \frac{V - V_0}{D} + \frac{1}{De} \int_{D - w}^D \rho(x) \, dx.$$

Assuming uniform distribution of impurities $\rho = ne$, and substituting for $V$ and making a Taylor series expansion of $w^2$ around $w_i$ gives the following expression for $F$:

$$F = \frac{1}{D} \left[ \frac{Ai_T}{C} - \frac{Ae(w - w_i)}{C(D - w_i)\sigma} - V_0 + \frac{ne(D - w_i)w}{\epsilon} \right] + \frac{nw_i^2}{2e}.$$

Substituting the above expression and Eq. (4) into Eq. (2) we find a relationship which describes the temporal dependence of the depletion width $w$ as

$$\exp \left[ (\beta - \gamma)w \right] = \theta + R \frac{\beta - \gamma}{\alpha} \exp (\alpha t)$$

in terms of the constants

$$\alpha = \frac{AedT}{CDkT},$$

$$\beta = \frac{Ae^2 d}{CDkT(D - w_i)\sigma},$$

$$\gamma = \frac{ne^2 d(D - w_i)}{De kT},$$

$$\theta = \exp \left[ (\beta - \gamma)w_i \right] - \frac{R(\beta - \gamma)}{\alpha},$$

$$R = \exp (\beta w_i) \exp \left[ \frac{ne^2 dw_i^2}{2De kT} \right] \eta,$$

where

$$\eta = \frac{(D - w_i)}{e} \sigma A^* T^2 \exp \left[ -\frac{\Delta + edV_0}{kT} \right].$$

Depletion will continue efficiently as long as the electric field sweeps carriers out of the n-region, i.e. until $E = 0$. Substituting the above expression in the Eq. (3) and setting the field at the p-i interface $E = 0$, (see Fig. 1c) we obtain an expression which describes the relationship between the firing time interval and input current density with the device parameters.
\[
\frac{A_{\text{ip}} t}{C} = (G + M) \ln \left[ \theta + \frac{R (\beta - \gamma)}{\alpha} \exp(\alpha t) \right] \\
+ G_1 = 0,
\]

where \( G = \frac{\Delta e (C + C_L)}{\epsilon (\beta - \gamma)} \), \( M = \frac{enw_i}{2e(\beta - \gamma)} \), and \( G_1 = G(\beta - \gamma) w_i - V_0 + \frac{enw_i^2}{2e} \).

In the limit \( edJ/DCKT \gg f \) this can be solved numerically for the IPTIs. When the initial depletion width and interface thickness are varied, it was found that the intervals are very sensitive to both \( w_i \) and \( d \), with a 10% change in \( w_i \) producing a 40% change in the interpulse time interval, and a 10% change in \( d \) producing a 50% change in the interval. In addition, the circuit and device parameters are extremely critical for this spontaneous pulsing. For instance, a thinner i-region, or a lower i-region concentration will hamper the space charge buildup which will lead to breakdown without spontaneous pulsing. Also the interface thickness parameter \( (d) \) \([2,1]\), which is a measure of the sharpness of the doping profiles, plays a key role in the firing patterns \([2]\). Unlike the voltage driven case, the cross sectional area of the diode affects the firing pattern \([2]\) in addition to controlling the pulse height. However, this effect is small until some limiting value associated with the other parameters is approached where the IPTIs rapidly increase. See Fig. 9. This limit is associated with the inability to charge the input capacitor, and can be increased by increasing the drive current or reducing the input capacitor. The variation of the IPTIs with the diode and circuit parameters is shown in Figs. 8 and 9.

The initial depletion width \( w_i \) and the interface thickness \( d \) were varied to fit the experimental data. The known diode and circuit parameters used were \( C = C_L = 100 \text{ pF}, R = 300 \text{ k}\Omega, D = 150 \text{ \mu m}, \) and \( n = 10^{14}/\text{cm}^3 \). The mean values of the S and L IPTIs at varying currents are shown in Fig. 10. Below 33 nA, the grouping into S and L IPTIs is not clear and could have some mixing of the S and L types, causing variations in the results. Because of this mixing, fitting was not carried out in this region. The data was split into three regions based on the current: below 46 nA, between 46 and 50 nA, and above 50 nA. The region between 46 and 50 nA is a transition region with only three points available, so no attempt was made to fit the data here. This division was chosen because the intermittencies were primarily LL below 46 nA and primarily SS above 50 nA. The initial depletion width and interface thickness were modeled by \( w_i = w_{i0} + (\Delta w_i/\Delta t) t \) and \( d = d_0 + (\Delta d/\Delta t) t \) with the zero current values \( w_{i0} \) and \( d_0 \), and the slopes \( \Delta w_i/\Delta t \) and \( \Delta d/\Delta t \) determined by a least squares fit to the data in each segment. The S IPTIs were fit first since they were much less noisy than the L IPTIs. When this was done we found \( d(\text{nm}) = 70 - 10t \) agrees with the data in both regions. This results in an interface thickness \( (d) \) that varies from 69.7 to 69.3
Fig. 9. The variation of IPTI with (a) diode area and (b) load capacitance for the same parameters as in Fig. 8. The large rise in the IPTIs for (a) is caused by the discharge of the input capacitor at low currents. Decreasing the input capacitor decreases the area at which this rise occurs.

nm over the current region considered. The decrease in the interface thickness with increasing current can be understood as an effect of the increasing field at the n-i interface due to the space charge buildup. As the current goes up the mean space charge increases, producing a larger field at the n-i interface. This larger field causes a small change in the location of the barrier peak leading to a reduction of the interface thickness. The initial depletion width was \( w_{IS} (\mu\text{m}) = 1.721 + 2.15i_T \) in the 35–46 nA range and \( w_{IS} (\mu\text{m}) = 1.786 + 1.18i_T \) in the 50–72 nA range. There is no reason to expect the interface thickness to change for the S and L pulses at the same current. However, the initial depletion width can be different for 2 cycles at a given current. Hence, the same interface thickness values were used to obtain the fit for the L IPTIs giving, \( w_{IL} (\mu\text{m}) = 1.62 - 8.2i_T \) in the 35–46 nA range and \( w_{IL} (\mu\text{m}) = 2.06 - 22i_T \) in the 50–72 nA range. The resultant parameters for \( w_i \) are shown in Table 1. The IPTI fit is shown by the lines in Fig. 10 with the parameters used shown in Fig. 11. The values between 46 and 50 nA were found by connecting the ends of the fits to \( w_i \) and \( d \).

Table 1

<table>
<thead>
<tr>
<th>Current Region</th>
<th>( w_{IS} ) (( \mu\text{m} ))</th>
<th>( \frac{\Delta w}{\Delta i_T} ) (m/( A ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I \leq 46 \text{ nA} )</td>
<td>1.721</td>
<td>2.15 \times 10^{-6}</td>
</tr>
<tr>
<td>( I \geq 50 \text{ nA} )</td>
<td>1.786</td>
<td>1.18 \times 10^{-6}</td>
</tr>
</tbody>
</table>

Fig. 10. Plot of both L and S IPTI vs. driving current. The points were determined from averaging the diode data. The theoretical fit was obtained by a least squares fit in the low and high current regions. The dashed lines indicate the regions used to obtain the fits. The center region values were determined by connecting the endpoints of the \( d \) and \( w_i \) values determined for the other two regions.

Fig. 11. The parameters \( w_i \) and \( d \) used to fit the data in Fig. 10. \( w_{IS} \) and \( w_{IL} \) stand for short and long IPTIs respectively. The dashed lines indicate the three regions used. The same value of \( d \) was used in both cases.
value. The initial depletion width needing to be less than zero can be considered as a constraint against the existence of a type of IPTI. The difference in the variation of the $S$ and $L$ IPTIs can be explained by the much smaller initial depletion width in the $L$ case. This makes variations of the same absolute size in the depletion width much more significant for the $L$ case than for the $S$ case.

The model has been applied to determine the parameters critical for pulsing to occur [26]. When this was done the most critical parameters were $D$, $d$, and $n$. Both $n$ and $D$ showed minimum values which are probably associated with the need for sufficient impurities to achieve the space charge needed for a pulse. They all showed maximum values which were probably related to the existence of a steady state for the current. The values for $n$ and $d$ for our diode are very near the pulsing limit. In Fig. 12 we show the dependence of the IPTIs on both $D$ and $d$ for 20 and 100 nA drive currents. Here we can see that for the larger current pulsing occurs over almost all of the parameter space shown, while at the lower drive current, more of the parameter space no longer supports pulsing. The large variations in the pulsing behavior of diodes are probably related to the variation in the maximum values for pulsing. The use of fabrication techniques such as MBE which provide careful control of both $n$ and $d$ will be necessary for the fabrication of uniform arrays.

In conclusion, we have observed a route to periodicity which might be a combination of the theoretically predicted routes such as period doubling bifurcations, and overlapping of different modelocking regions. Our modeling results based on the circuit and device physics explain the phenomena observed. In the time series shown in Fig. 3 the variations get reduced by introducing an extra pulse which is also periodic. This can be interpreted as a period increasing situation (as opposed to period doubling) if the extra short interval is neglected, which is the main route to periodicity in this system. A route to periodicity can be converted mathematically into a route to chaos by inverting the control parameter. However, in our case, the control parameter that is naturally measured increases as the system becomes periodic as opposed to the many systems which move towards chaos when the control parameter is increased.

In addition to showing interesting nonlinear phenomena, this work is interesting from a pure physics standpoint. Also, the current driven diode model will help advance the understanding of the collective behavior of these devices which were used in parallel processor circuit channels [7]. Further, this can be used as a guide towards fabricating arrays of these devices with required pulsing rates and patterns for different applications such as infrared detectors [4], intensity to frequency [3] or current-to-frequency converters.

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