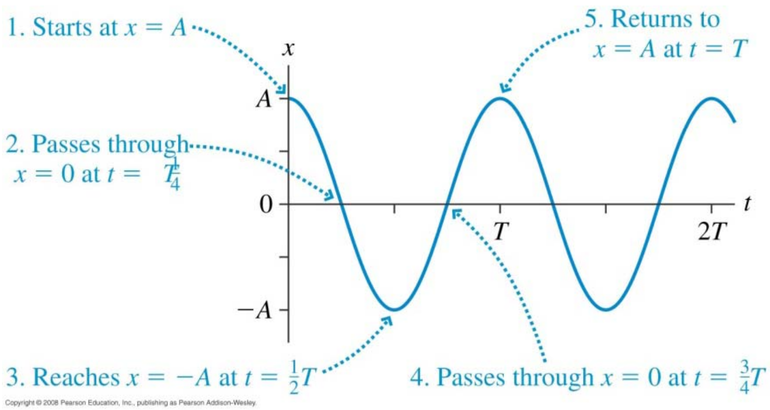
P2211K

11 / 16 / 2010

Oscillations, vibrations, and simple harmonic motion (SHM)

Definition of SHM: Sinusoidal (or cosine) oscillations



Functional form: x(t) = Acos(ωt + φ), where

x(t) = position

ω = angular frequency (radians / s) = 2πf

f = frequency in Hertz (Hz)

A = amplitude

φ = phase (to come later)

Other relations:

f = frequency = repetitions / time

T = Period = time / repetition

T = 1/f

- Note that the motion has gone through one complete cycle when ωt = 2π. Thus the functional form can be rewritten in many forms:

$$x(t) = A\cos(\omega t + \phi) = A\cos(2\pi ft + \phi) = A\cos(\frac{2\pi t}{T} + \phi)$$

- Also, note that velocity and acceleration are related to position by

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

So how do we get SHM?

- ***Basic answer***: spring-like forces, *i.e.*, ones described by a linear restoring behavior--- F = -kx.
- ***Why is this important***?
  1. SHM gives the most basic version of a ***periodic*** motion;
  2. The characteristic of a linear restoring force is a good approximation of more complex forces (one example is the simple pendulum);
  3. Chemical bonds, and thus their vibration behaviors, can be approximated by this type of force;
  4. The deflection (or bending) behavior of structural units (beams, etc.) can be described to a good approximation by these forces;
  5. etc.
- How do we know spring-like forces produce SHM? Look at the "F = ma" relation:

$$ma(t) = -kx(t) \Rightarrow m\frac{d^2x(t)}{dt} = -kx(t) \Rightarrow \frac{d^2x(t)}{dt} + \left(\frac{k}{m}\right)x(t) = 0$$

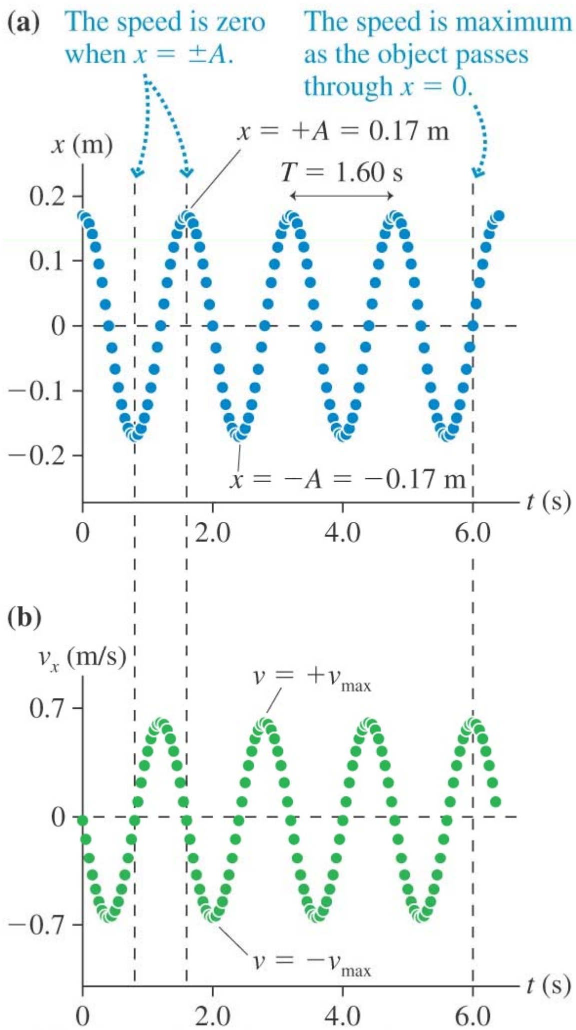
For this type of differential equation, the general solution is

$$x(t) = A\sin(\omega t) + B\cos(\omega t) \qquad \text{with } \omega^2 = \frac{k}{m}$$

(A and B are amplitude parameters determined by the initial conditions.)

- Thus, this demonstrates that SHM is the result of the linear restoring force.

Dynamics of SHM



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

- From the relations for ***x(t), v(t), and a(t),*** we see that the ***maximum displacement is A***, ***the maximum speed is ωA***, and ***the maximum acceleration is $\omega^2 A$***. Furthermore, from the relations between sin and cos functions, ***the maximum displacement occurs when the speed is zero, and the maximum speed occurs when the displacement is zero***.
- Also, we know from previous discussions that the spring-mass system is ***conservative*** so that

$$\Delta K + \Delta U = 0 \Rightarrow E_T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

and that

$$E_T = \tfrac{1}{2}mx^2_{max} = \tfrac{1}{2}kA^2$$

or

$$E_T = \tfrac{1}{2}mv^2_{max} = \tfrac{1}{2}m\omega^2 A^2$$

Problem 14-13. A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At *t* = 0, the mass is at x = 5.0 cm and has v_x = -30.0 cm/s. Determine:

a. The period. $T = 1/f = 0.5$ sec
b. The angular frequency. $\omega = 2\pi f = 12.57$ rad/s
c. The amplitude. $E_T = ½kA^2$; Need $k = m\omega^2 = 31.6$ N/m &
   $E_T = ½kx^2 + ½mv^2 = 0.0485$ J, so
   $A = (x^2 + mv^2/k)^{½} = (x^2 + v^2/\omega^2)^{½} = 5.54$ cm
d. The phase constant. $x(t) = A\cos(\omega t + \varphi)$, so $x(0) = A\cos(\varphi)$ &
   $\varphi = \cos^{-1}[x(0)/A] = \cos^{-1}[5/5.54] = 25.5° = 0.445$ rad
e. The maximum speed. $E_T = ½m(v_{max})^2$, so $v_{max} = (2E_T/m)^{½} = 69.6$ cm/s
   also, $v_{max} = A\,\omega$
f. The maximum acceleration. $a_{max} = \omega^2 A = 875.3$ cm/s^2
g. The total energy. $E_T = 0.0485$ J
h. The position at *t* = 0.40 s. $x(0.40) = 5.54\cos(12.57 * 0.04 + 0.445)$ cm
   $= 3.23$ cm

Energy in Simple Harmonic motion (SHM):

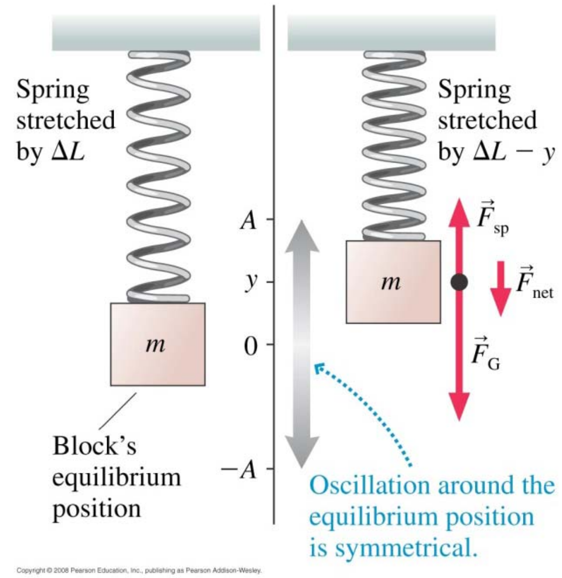
- Review Example 14.5, p. 419

Dynamics of SHM:

- Review Example 14.6, pp. 422-423

Vertical Oscillations: How does g affect things?

- No effect on frequency
- Shifts x = 0 position by x = mg / k



The simple pendulum: how does it execute SHM?

- For SHM, $ma = -kx = md^2x/dt^2$
- This has the form: $d^2\boldsymbol{g}/dt^2 = -(k/m)\boldsymbol{g}$, where ***g*** represents a coordinate [such as linear position (x), arc length (s), angular position (θ), etc.] and leads to $\omega^2 = k/m$.
- For the simple pendulum (as discussed in section 14.5, pp. 425-426), the force law, with $s = L\theta$ and the small-angle approximation that $\sin\theta \sim \theta$, becomes $mL(d^2\theta/dt^2) = \sim -mg\theta$, which has the form $d^2\theta/dt^2 = \sim -(g/L)\theta$.
- Thus, by comparison with the spring-mass system, $\omega^2 = g/L$.
- Alternately, we can note that the potential energy for the exact SHM system has the form $U = ½kx^2$, which is a parabola. For the pendulum, $U_g = mgL(1-\cos\theta)$, which is a circle. A circle ~ a parabola for small θ.

Damped harmonic motion:

- Basic idea: when set into "motion," the oscillations will continue until the energy is removed (or dissipated).
- Typical damping is proportional to speed (or the 1st derivative of the generalized coordinate ***g*** mentioned before).
- "Damping" removes the vibrational energy.

http://www.lon-capa.org/~mmp/applist/damped/d.htm

Driven oscillations:

- Basic idea: all objects have a "natural frequency" and a damping factor
- Energy can be put into the system @ the natural frequency.
- If the damping is low enough that the energy put in is more than that taken out by damping, the vibrational amplitude will build up, possibly to catastrophic levels.

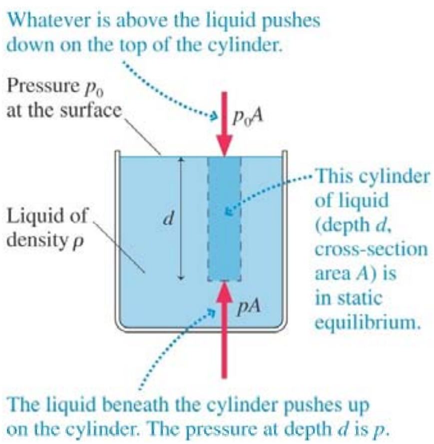
http://www.walter-fendt.de/ph14e/resonance.htm

Driven harmonic motion—voice breaking glass:
http://www.youtube.com/watch?v=eTWDEsGlPO8&NR=1&feature=fvwp 1:30
http://www.youtube.com/watch?v=IZD8ffPwXRo&feature=related 1:42

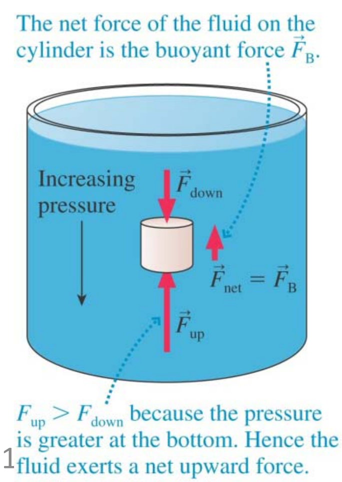
Driven harmonic motion—Tacoma Narrows bridge:
http://www.youtube.com/watch?v=j-zczJXSxnw

Ch. 15: Fluids & elasticity

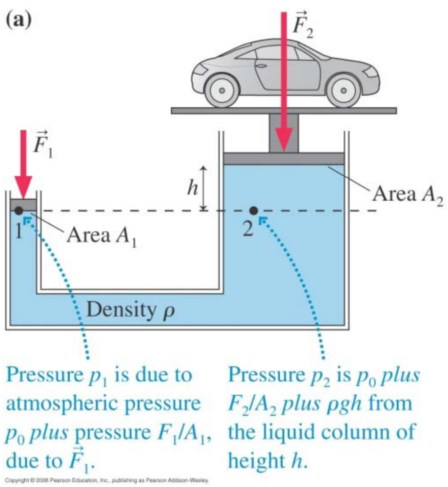
- ***Fluids***: fluids are not rigid and "can take on the shape of their container." Gases & liquids are obvious examples. An important characteristic of many liquids is that they are (nearly) incompressible—examples are water and hydraulic fluids.



- ***Pressure:*** P = force / area = F/A; Units = N/m^2 (1 N/m^2 = 1 Pa); lb/in^2; mm Hg (1 mm Hg = 1 Torr); cm H_2O; etc
- ***(mass) Density***: ρ = mass / volume = M / V; units = kg/m^2; g/cm^3, kg/liter; g/ml, etc. (1 ml = $1cm^3$ = 1cc)
- ***Pressure at a depth in a fluid***—is due to the weight of the fluid above it: Above the space of area A is a column of fluid with height d (the depth); the weight of this column is Mg = ρ(Ad)g, which leads to P = F/A = Mg/A = ρgd
- ***Archimedes's principle, buoyancy, and*** floating—arise from the pressure-at-a-depth result: for any 3-dimensional object, the pressure at its bottom, which is at a greater depth, is larger than the pressure at its top.
- ***The expression for the buoyant force***: At the bottom, the pressure is $\rho g d_{bottom}$, at the top it is $\rho g d_{top}$, and their difference is $\Delta P = \rho g(d_{bottom} - d_{top})$. $(d_{bottom} - d_{top})$ = height of the object. Multiplying both sides by A gives the net force $F_{net} = \Delta P(A)$ = ρgA height. Because A*height is the volume of the object, this becomes $F_{buoyant} = \rho g V_{obj} = \rho g V_{disp}$, where v_{disp} is the volume of fluid displaced.



- **Pressure in fluids & <u>Pascal's Principle</u>**: ***"A change in pressure at one point in an incompressible fluid appears undiminished at all points in the fluid."***
- ***Hydraulics:*** come from application of Pascal's principle



  - In the situation sketched to the side, applying force F_1 to the piston of area A_1 creates pressure $P = F_1/A_1$. This is "transmitted" to the other piston of area A_2 to create force $F_2 = PA_2 = (F_1/A_1)A_2 = F_1(A_2/A_1)$. Obviously, if $(A_2/A_1) > 1$, then $F_2 > F_1$. ($F_2/F_1 = A_2/A_1$ is the "mechanical advantage" of the system.)
  - However, to accomplish this, the smaller piston must "pump" a volume of fluid into the larger cylinder $V = d_2A_2$. To do so, the smaller piston must move $d_1 = V/A_1 = d_2(A_2/A_1)$, which is ***greater than*** d_2 by the same ratio that F_2 ***is greater than*** F_1.
  - If the two pistons are not at the same level ($h_2 \neq h_1$), and this naturally occurs when they move, then there is a pressure difference on both sides due to the "pressure-at-a-depth" result: $\Delta P = \rho_{fluid}(h_2 - h_1)g$. Holding an object with the pistons at different heights requires adding this pressure to the lower piston.
- ***Work through example 15.7 on page 455.***

Assignment:

- **Review Chapter 13 with a focus on the more general form for gravitational potential energy and the "escape velocity," section 13-5.**
- **Read Chapters 14 & 15**
- **Begin Reading Chapter 16**