P2211K

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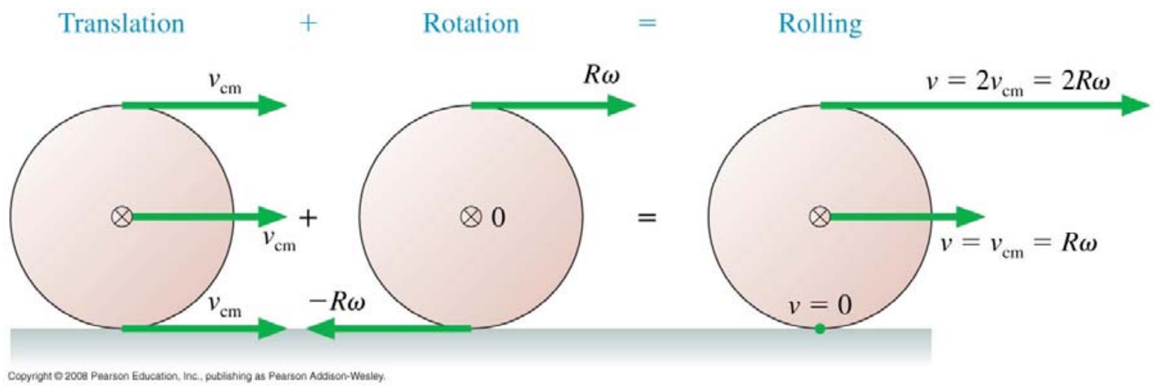


Rolling Motion: Rolling without slipping

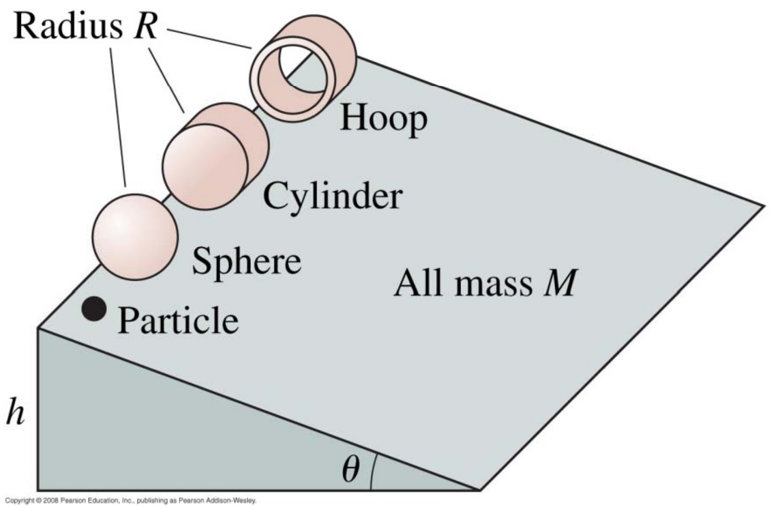
- When an object ***rolls without slipping***, the axle translates by one circumference when the object has made one revolution. Thus, the angular velocity and linear velocity are related according to:

$$v_{center} = \omega R$$

- Note also that ***no slipping*** means that the point of the rolling object in contact with the surface has zero linear velocity; otherwise the object would be slipping (spinning or skidding).
- The ***kinetic energy*** of an object rolling without slipping is partly in the linear motion of the center of mass and partly in the rotational motion about the center:

$$K = ½mv^2 + ½\ I\omega^2$$

- On pages 366 & 367, the book discusses a downhill ***race*** between objects of the same radius and the same mass but with different mass distributions (hollow vs. solid, etc.). The point is that different distributions of the same mass lead to different values of I and thus different amounts of the total energy in rotation vs. linear. The more in the linear motion, the faster the object moves.,



Consider the objects to the left:

- For the hoop, I = MR^2
- For the cylinder, I = ½MR^2
- For the solid sphere I = $^2/_5MR^2$
- For a hollow sphere, I = ⅔MR^2
- etc. all have a number times MR^2 so the values can be summarized as I = cMR^2, where c is a fraction reflecting the values for the specific shapes.

- Thus, if all have the same mass and radius, then the hoop has the largest value of I and the solid sphere has the lowest. As a consequence, when rolling without slipping with the same kinetic energy, the solid sphere will have the highest linear speed and the hoop will have the lowest.

$$\Delta K_T = -\Delta U_g$$

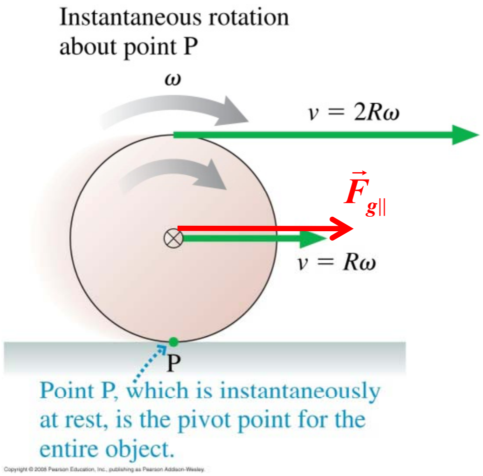
$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}cM\left(\frac{R^2}{R^2}\right)v^2 = Mgh$$

$$\tfrac{1}{2}M(1+c)v^2 = Mgh \Rightarrow v^2 = \frac{2gh}{(1+c)}$$

$$v = \sqrt{\frac{2gh}{(1+c)}}$$

- (The particle has no shape and therefore no rotational kinetic energy.)
- If the objects begin from rest and roll without slipping down the incline of height h, then they arrive at the bottom with speed as shown to the left.

Example: analyze the previous case using torque methods



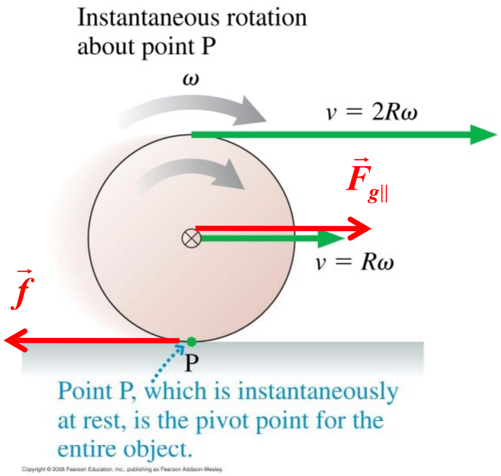
Torque method 1, basic points:

- On the incline, the object rotates about the point of contact (P in the diagram). As a result, it is necessary to use the rotational inertia about that point in the analysis. The adjustment requires application of the parallel axis theorem: $I_P = cMR^2 + MR^2 = (c + 1)MR^2$
- The torque acting on the object comes from the component of its weight down the incline ***(Mgsinθ)*** acting about the point of contact with the lever arm R: ***torque*** $= (Mg\sin\theta)R = I_P\alpha$.
- Because ***acceleration*** $= R\alpha$, we have

$$a = R\alpha = (Mg\sin\theta)R^2 / I_P = g\sin\theta / (c + 1)$$

- Finally, because ***a*** is constant and the objects move the distance $d = h/\sin\theta$ along the incline as they drop by h (the hypotenuse), the final speed is given by $(v_f)^2 = 0 + 2ad = 2gh / (c + 1)$, which is the same result obtained from the energy approach.

Example: analyze the previous case using torque methods



Torque method 2, basic points:

- On the incline, the object rotates about the center of mass, and it translates down the incline with acceleration a (at the instantaneous speed v). In this case, the appropriate rotational inertia is that about the cm (the center of the object). $\boldsymbol{I_{cm} = cMR^2}$
- The torque about the center of the object comes from the force of friction acting up the incline at point P. (The component of its weight acts at the center and thus creates no torque about this point.) The frictional force creates torque about the center with lever arm R: ***torque = fR =*** $\boldsymbol{I_{cm}\alpha}$.
- Because ***acceleration =*** $\boldsymbol{R\alpha}$, for the torque relation, we have

$$a = R\alpha = fR^2 / I_{cm} = f / cM$$

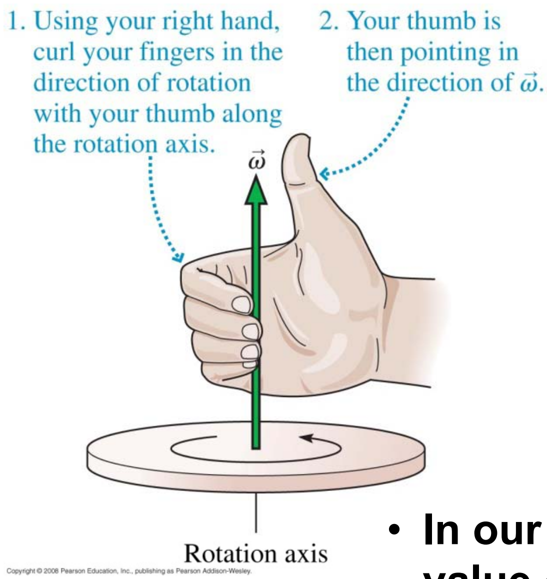
- For this case, it also is necessary to use the force relations. Down the incline,

$$F_{net} = F_{g//} - f = Mg\sin\theta - f = Ma \rightarrow f = M(g\sin\theta - a).$$

- Combining these two yields:

$$a = g\sin\theta \ / (c + 1)$$

- Finally, because ***a*** is constant and the objects move the distance $\boldsymbol{d = h/\sin\theta}$ along the incline as they drop by h (the hypotenuse), the final speed is given by $\boldsymbol{(v_f)^2 = 0 + 2ad = 2gh / (c + 1)}$**,** which is the same result obtained from the energy approach.



USING VECTORS TO DESCRIBE ROTATIONAL KINEMATICS AND DYNAMICS:

- **"Clockwise" and "counterclockwise" are useful for describing rotational motion (ω for example);**
- **Unfortunately, these terms are difficult to use computationally—consequently, for that purpose, it is useful to define vector representations of these quantities with the approach shown to the left.**

- **In our discussion of angular momentum below, we will see the value of systematic descriptions based on this approach.**
- **This also leads to use of the vector cross product.**

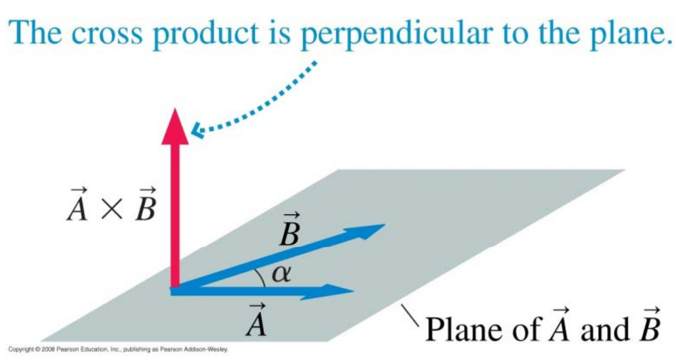
THE CROSS PRODUCT OF VECTORS:

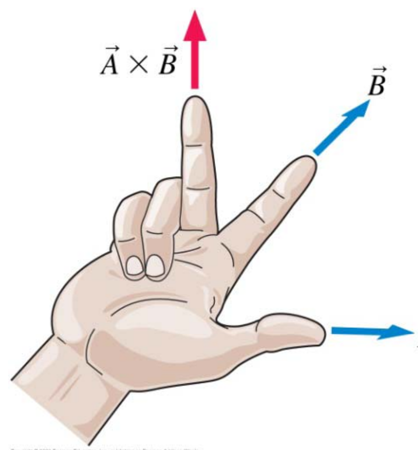
In connection with Work, we introduced the dot, or scalar, product of vectors:

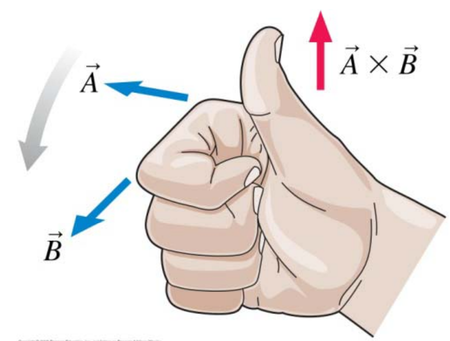
$$\vec{A} \bullet \vec{B} = ABcos\theta_{AB}$$

A second method of multiplying vectors is the ***cross product***, the result of which also is a vector: $\vec{A} \times \vec{B} = \vec{C}$, *where*

$$C = ABsin\theta_{AB}\text{, perpendicular to both } \vec{A} \text{ and } \vec{B}$$







TORQUE & CROSS PRODUCTS:

- Torque now can be re-stated using the cross-product formalism:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

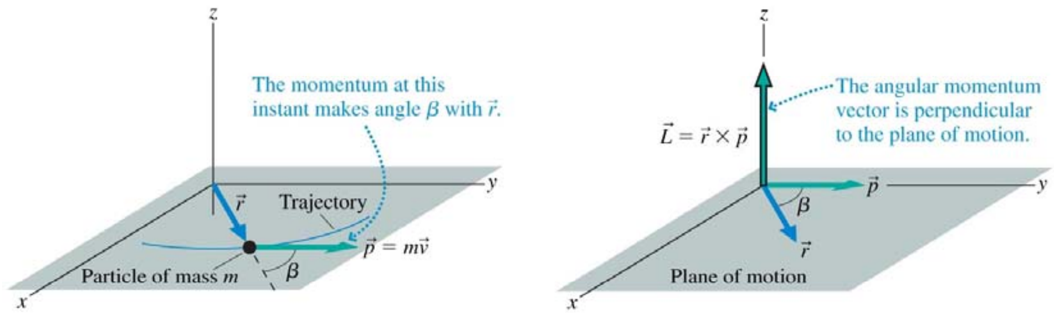
- This allows combining multiple sources of torque into a net vector using standard vector addition methods:

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots = \sum \vec{\tau}_i$$

ANGULAR MOMENTUM:

The fundamental definition of angular momentum (***L***) is as follows:

$$\vec{L} = \vec{r} \times \vec{p}$$



The particle is moving along a trajectory.

The vector tails are placed together to determine the cross product.

For the motion of rigid bodies, this translates into the relations:

$$\vec{L} = I\vec{\omega} \ \& \ \ \vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

TABLE 12.4 Angular and linear momentum and energy

Angular momentum	Linear momentum
$K_{rot} = \frac{1}{2}I\omega^2$	$K_{cm} = \frac{1}{2}Mv_{cm}{}^2$
$\vec{L} = I\vec{\omega}$ *	$\vec{P} = M\vec{v}_{cm}$
$d\vec{L}/dt = \vec{\tau}_{net}$	$d\vec{P}/dt = \vec{F}_{net}$
The angular momentum of a system is conserved if there is no net torque.	The linear momentum of a system is conserved if there is no net force.

*Rotation about an axis of symmetry.

CONSERVATION OF ANGULAR MOMENTUM:

The relation is like that for linear momentum:

$$\vec{L}_f = \vec{L}_i \;\Rightarrow\; I_f\vec{\omega}_f = I_i\vec{\omega}_i$$

Example: A 2.0 kg, 20-cm-diameter turntable rotates at 100 rpm on frictionless bearings. Two 500 g blocks fall from above, hit the turntable simultaneously at opposite ends of a diagonal, and stick.

- What is the turntable's angular velocity, in rpm, just after this event?

This event amounts to a completely inelastic "rotational collision."

Initially, the two blocks have no angular momentum. However, adding them to the rotating turntable (a disk) increases its Rotational Inertia.

Initial $I = I_{disk} = \frac{1}{2}M_{disk}R^2_{disk}$

Each block adds $I = M_{block}R^2_{disk}$ ***because they attach at the edge of the disk.***

Final $I = I_{disk} + I_{blocks} = \frac{1}{2}M_{disk}R^2_{disk} + 2M_{block}R^2_{disk}$

Thus, conservation of angular momentum and this information lead to:

$$I_{initial}\omega_{initial} = \left(\tfrac{1}{2}M_{disk}R^2_{disk}\right)\omega_{initial} = \left(\tfrac{1}{2}M_{disk}R^2_{disk} + 2M_{block}R^2_{disk}\right)\omega_{final} = I_{final}\omega_{final}$$

and

$$\omega_{final} = \omega_{initial}\left[\frac{\left(\tfrac{1}{2}M_{disk}R^2_{disk}\right)}{\left(\tfrac{1}{2}M_{disk}R^2_{disk} + 2M_{block}R^2_{disk}\right)}\right] = \omega_{initial}\left[\frac{\left(\tfrac{1}{2}M_{disk}\right)}{\left(\tfrac{1}{2}M_{disk} + 2M_{block}\right)}\right] = 50\ rpm$$

Assignment:

- **Continue reading and working on Chapter 12; especially you should review the book's worked-out examples.**
- **Begin reading Chapter 13 with a focus on the more general form for gravitational potential energy and the "escape velocity," section 13-5.**