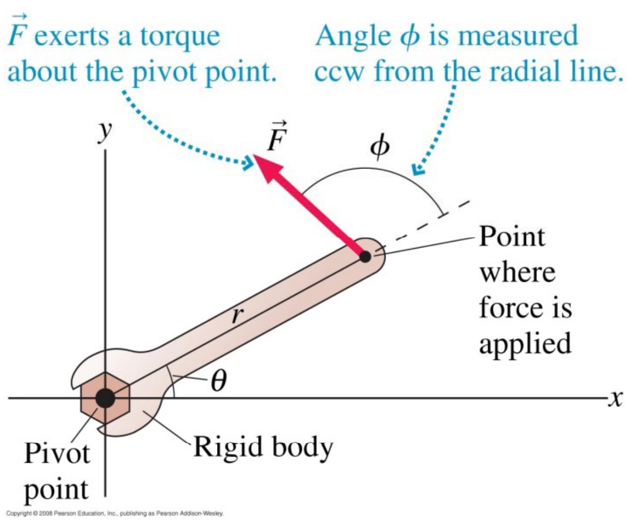
P2211K

10 / 28 / 2010

Torque

- ***Basic idea:*** torque is the application of force in a way that generates a tendency to rotate. As is sketched below, it incorporates the idea of a lever arm and leverage (or ***moment arm and moment)***.
- ***Definition:*** $\tau = F_{effective} r$
- ***Important Note on the difference between torque and work***: Previously, we defined Work as ($F_{effective}$)(distance), and these can look confusingly similar. However, for Work, $F_{effective}$ is ***parallel to the direction of motion*** and causes the speed to change (increase or decrease). In contrast, for torque, $F_{effective}$ is ***perpendicular to the lever arm*** and causes the tendency to rotate.



Work = ($F_{effective}$) (distance) = $F_{//}d$= $Fd\cos\theta$

Torque = ($F_{effective}$) (lever arm) = $F_{\perp}d = Fd\sin\varphi$

- ***Units:*** the units of torque are the newton meter (Nm). This combination is the same as for work, which are re-labeled Joules. Torque retains Nm as its units to maintain its distinction from work---they are **NOT** the same.

Torque and Rotational Dynamics:

- For rigid bodies constrained to rotate about an axis, applied forces cause torque only through their components perpendicular to the line between the point of application and the rotation axis (r in the torque relation). (The force component along r has the tendency "rip apart" the object, and that destructive situation is not the case under consideration.
- From this perspective, and the discussion in the book related to equations 12.28 – 12.30 (p.355), the effect of torque on a rigid body is to cause angular acceleration (α) as described by the relation

$$\tau = I\,\alpha \quad \text{(rearrangement of eq. 12.31, p. 356)}$$

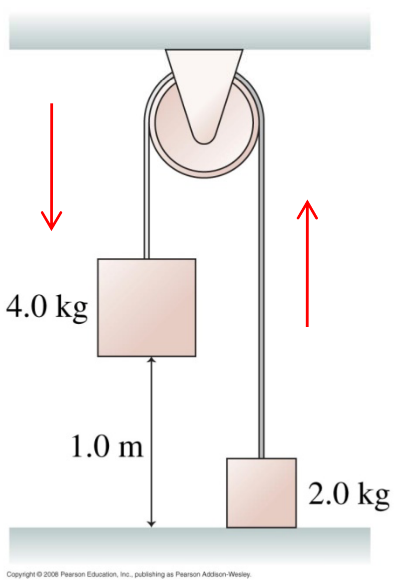
- Correlations between rotational and linear (translational) dynamics:

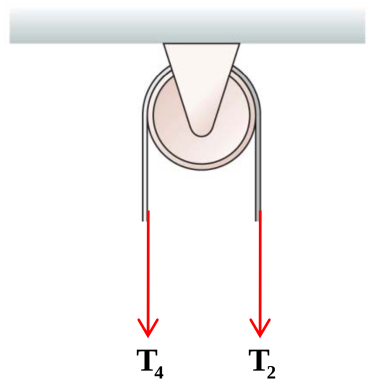
TABLE 12.3 Rotational and linear dynamics

Rotational dynamics		Linear dynamics	
torque	τ_{net}	force	$\vec{F}_{net}$
moment of inertia	I	mass	m
angular acceleration	α	acceleration	$\vec{a}$
second law	$\alpha = \tau_{net}/I$	second law	$\vec{a} = \vec{F}_{net}/m$

Revisiting Prob 12-70 (sort of) from above: The two blocks in the figure are connected by a massless rope that passes over a pulley. The pulley is 12 cm in diameter, has a mass of 2.0 kg, and has rotational inertia like a solid disk. The rope does not slip on the pulley and the pulley rotates on its axle without friction.

- If the blocks are released from rest, what is the speed of the 4.0 kg block as it reaches the floor?
- How long does it take to reach the floor?





Tensions in the rope

- Basic principle: $F = ma$ and $\tau = I\,\alpha$
- $M_4 a = M_4 g - T_4$; $M_2 a = T_2 - M_2 g$
- String doesn't slip $\rightarrow \tau = (T_4 - T_2) r_{pulley} = I\,\alpha$
- ($T_2 \neq T_4$; otherwise the net torque = 0!!)
- String doesn't slip $\rightarrow a = \alpha\, r_{pulley}$
- Solid disk $\rightarrow I = \frac{1}{2} M_{pulley} (r_{pulley})^2$
- Putting it together: $a = (20/7)\ m/s^2$
- $(v_f)^2 = 0 + 2\,[(20/7)\ m/s^2]\ 1m = [(40/7)\ (m/s)^2 \rightarrow v_f = 2.39\ m/s$
- $t = (2h/a)^{1/2} = 0.84\ s$
- **Same Answers as above!!!**

Static equilibrium

- ***Basic idea:*** Static equilibrium of a rigid body requires not only that ***the net force is zero***, it also requires that ***the net torque on the object is zero***.
- ***Operational procedure***:
  1. Use free body diagrams to identify **ALL** the forces acting on the object, and use them to write the force equilibrium equations:

$$ma_x = 0 = \sum_{All\ Forces} F_{ix}$$

$$ma_y = 0 = \sum_{All\ Forces} F_{iy}$$

  2. Identify a ***"center of rotation***" and use it along with the free-body diagrams to identify **ALL** forces causing clockwise rotations and ***ALL*** forces causing counterclockwise rotations about the axis and write the torque equilibrium equation:

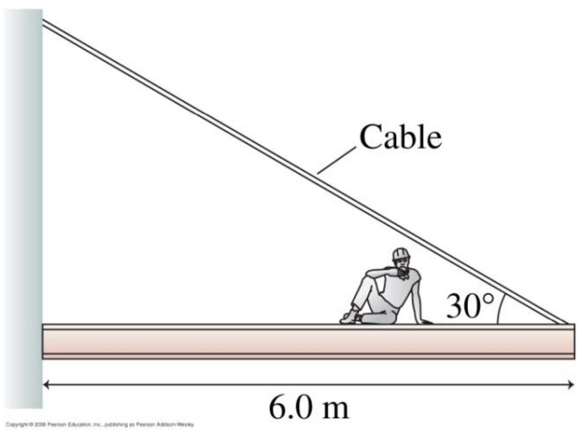
$$\tau_{clockwise} = \tau_{counterclockwise}$$

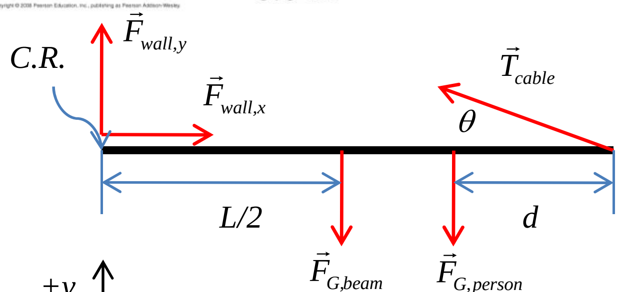
$$\sum_{All\ CW\ Forces} F_{\perp} d = \sum_{All\ CCW\ Forces} F_{\perp} d$$

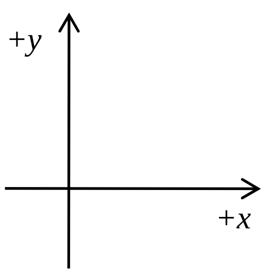
- These three relations permit the computation of up to three unknown parameters.

Problem 12-63. An 80 kg construction worker sits down 2.0 m from the end of a 1450 kg steel beam (L = 6.0 m) to eat his lunch. The cable supporting the beam is rated at 15000 N.

- Should the worker be worried?







- Basic question: need to know if the tension in the cable T > 15,000 N.

Forces :

$$a_x = 0 \Rightarrow 0 = F_{wall,x} - T\cos\theta \Rightarrow F_{wall,x} = T\cos\theta$$

$$a_y = 0 \Rightarrow 0 = F_{wall,y} - M_{beam}g - F_{person}g + T\sin\theta$$

Torques :

$$\tau_{cw} = \tau_{ccw} \quad \text{Choose C.R. @ wall contact}$$

$$\frac{L}{2}\left(F_{G,beam}\right) + \left(L - d\right)\left(F_{G,person}\right) = L\left(T\right)\sin\theta$$

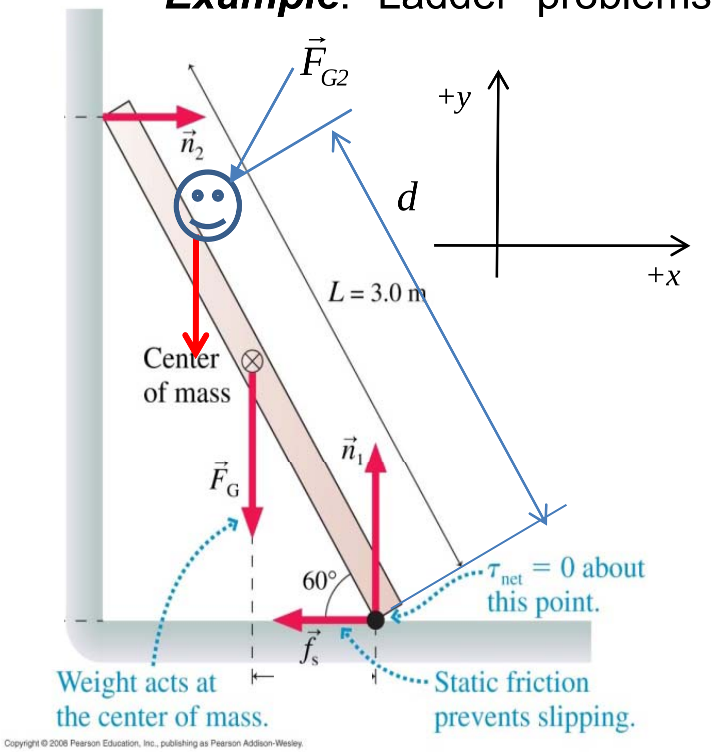
$$T = \frac{\frac{L}{2}\left(F_{G,beam}\right) + \left(L - d\right)\left(F_{G,person}\right)}{L\sin\theta}$$

$$T = 15{,}567\ N\ > 15{,}000\ N$$

$\therefore$ ***He should worry!!!***

- Note that only the torque relation was needed with the chosen C. R.

Example: "Ladder" problems



As sketched below, a ladder (***L = 3.0 m & mass = 15 kg***) leans against a frictionless wall at the angle ***θ=60°***. If the coefficient of static friction between the ladder and the floor is $\mu_k = 0.4$, how far up the ladder (d) can a 75 kg person climb before it begins to slip on the floor?

Basic points:

- $F_{G,Ladder} = M_L g$
- $F_{G,person} = M_p g$
- **Center-of-mass of ladder is @ L/2**
- $f_{s,max} = \mu_k n_1$

Forces:

$$a_x = 0 \Rightarrow 0 = n_2 - f_{max} = n_2 - \mu_k n_1 \Rightarrow n_2 = \mu_k \left(M_L + M_P\right) g$$

$$a_y = 0 \Rightarrow 0 = n_1 - M_L g - M_P g \Rightarrow n_1 = \left(M_L + M_P\right) g$$

Thus,

$$f_{max} = \mu_k \left(M_L + M_P\right) g$$

Torques :

$$\tau_{ccw} = \tau_{cw}$$

(Choose CR @ contact point of ladder & floor)

$$\frac{L}{2}\left(F_{g,L}\right)cos\theta + d\left(F_{g,P}\right)cos\theta = L\left(n_2\right)sin\theta$$

$$\frac{L}{2}\left(M_L g\right)cos\theta + d\left(M_P g\right)cos\theta = L\left[\mu_k\left(M_L + M_P\right)g\right]sin\theta$$

$$d = \frac{L\left[\mu_k\left(M_L + M_P\right)g\right]sin\theta - \frac{L}{2}\left(M_L g\right)cos\theta}{\left(M_P g\right)cos\theta}$$

$$d = L\left[\frac{\mu_k\left(M_L + M_P\right)tan\theta}{M_P} - \frac{M_L}{2M_P}\right]$$

$\therefore$ *d = 2.2 m (from bottom)*

Assignment:

- **Continue reading and working on Chapter 12; especially you should review the book's worked-out examples.**