P2211K

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Overview of course so far:

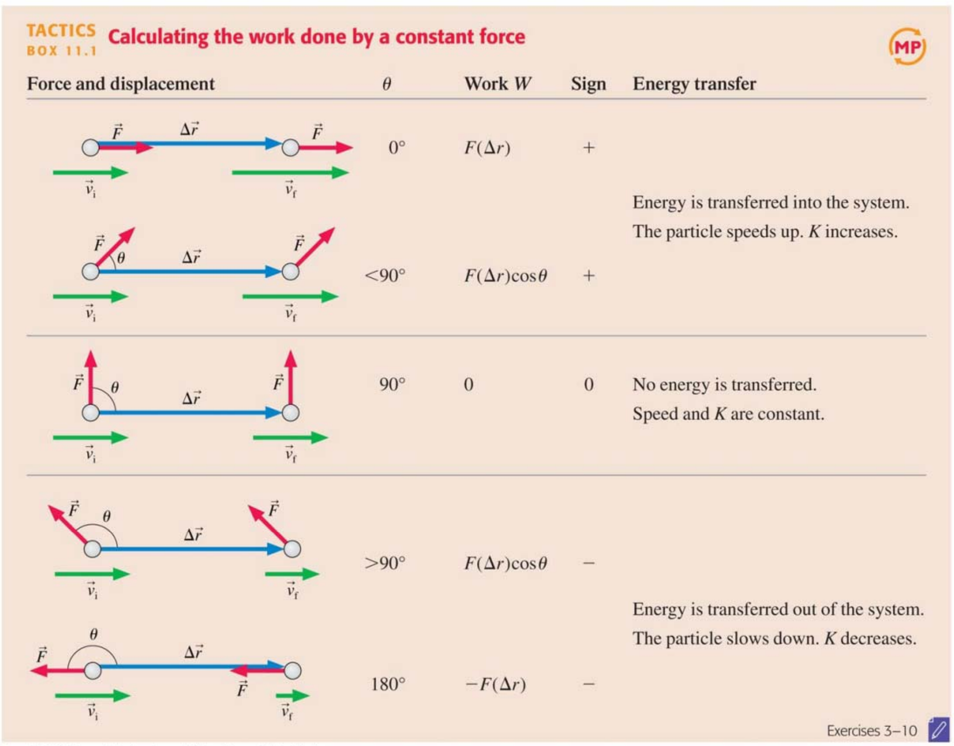
- Objects move (velocity, acceleration, etc.)
- Forces cause acceleration ($F = ma$)
- Moving objects have kinetic energy ($K = \frac{1}{2}mv^2$)
- Forces cause changes in kinetic energy
- Some forces can be associated with potential energy (gravity, springs, etc., but NOT friction)
- Forces do WORK when they cause an object to change position;
- Force, WORK, potential energy, and kinetic energy are all inter-related;
- Rate of doing WORK: Power

Chapters 10 & 11:

- ***Work, the basic idea:*** A force does work when it causes an object to move parallel to its line of action. If the object's movement is not parallel to the force, then only the component of the force along the motion is "effective" in doing work.

  Work = ($F_{effective}$)×(distance)= (Fcosθ)d = Fdcosθ

  Units of work = (Newton) (meter) = Joules



TACTICS BOX 11.1 Calculating the work done by a constant force

Force and displacement	θ	Work W	Sign	Energy transfer
	0°	$F(\Delta r)$	+	Energy is transferred into the system. The particle speeds up. K increases.
	<90°	$F(\Delta r)\cos\theta$	+	
	90°	0	0	No energy is transferred. Speed and K are constant.
	>90°	$F(\Delta r)\cos\theta$	−	Energy is transferred out of the system. The particle slows down. K decreases.
	180°	$-F(\Delta r)$	−	

Exercises 3–10

- Work can be ***positive*** (+) or ***negative*** (-). Positive work is when the effective force (Fcosθ) is parallel to the movement (d), and negative work is when the effective force is opposite to the movement.

- Work can be viewed as the result of the ***net force***, and it also can be viewed as the result of the contributions from each individual force acting on the object.

$$W = (F_{net}\cos\theta)d = (F_1\cos\theta_1)d + (F_2\cos\theta_2)d + \ldots$$
$$= W_1 + W_2 + \ldots$$

Work and kinetic energy:

- **Previously,** we found that manipulation of a kinematics relation led to the connection between force × distance and kinetic energy: $\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = (F)d \Rightarrow (K_f - K_i) = \Delta K = Fd$

$$\textit{Thus, } \Delta K\textit{, the change in kinetic energy, equals (is the result of) } F \times d \textit{ !!}$$

- We can adapt this to the definition of work by noting that F is $F_{effective}$ in this relation because it is one-dimensional. Thus, we have that $\Delta K = F_{effective}d = F\cos\theta d = Fd\cos\theta = W$

$$\Delta K = W$$

- This is the ***Work – Kinetic Energy*** theorem, that the change in K of an object equals the net work done on it.

Prob. 11-45. Susan's 11.0 kg baby brother Paul sits on a mat. Susan pulls the mat across the floor using a rope that is angled 30° above the floor. The tension is a constant 30.0 N and the coefficient of friction is 0.20.

- Use work and energy to find Paul's speed after being pulled 3.50 m.

$$W_T = W_F + W_f = F\cos\theta d - \mu_k nd$$

$$n = m_{Paul}g - F\sin\theta = 110\ N - 15\ N = 95\ N$$

$$W_T = (30\ N)(\cos 30°)(3.5\ m) - (0.2)(95\ N)(3.5\ m) = 24.4\ J$$

$$W_T = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2(24.4\ J)}{11\ kg}} = 2.12\ m/s$$

Multiplying vectors: The dot product, a convenient way to get Fdcosθ

- ***Definition:*** The "dot" (or scalar) product of vectors A and B is the product of the two magnitudes and the cosine of the angle between them: $\vec{A}\bullet\vec{B} = ABcos\theta_{AB}$
- ***Note that*** the result is AB when the angle is 0°, that the result is 0 when the angle is 90°, and that the result is –AB when the angle is 180°.
- Algebraic properties of the dot product: $It\ is\ commutative: \vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A}$

  $It\ is\ distributive: \vec{A}\bullet\left(\vec{B}+\vec{C}\right) = \vec{A}\bullet\vec{B} + \vec{A}\bullet\vec{C}$
- The distributive property makes it convenient to perform the dot product on vectors expressed in the unit vector format: $Let\ \vec{A} = A_x\hat{i} + A_y\hat{j}\ and\ \vec{B} = B_x\hat{i} + B_y\hat{j}$

  $then\ \vec{A}\bullet\vec{B} = A_xB_x\hat{i}\bullet\hat{i} + A_xB_y\hat{i}\bullet\hat{j} + A_yB_x\hat{j}\bullet\hat{i} + A_yB_y\hat{j}\bullet\hat{j}$

  $and\ \vec{A}\bullet\vec{B} = A_xB_x + A_yB_y\ \ (because\ \hat{i}\bullet\hat{i} = \hat{j}\bullet\hat{j} = 1\ and\ \hat{i}\bullet\hat{j} = \hat{j}\bullet\hat{i} = 0)$
- Note also that this gives a convenient way of calculating the angle between two vectors because

$$\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y = AB\cos\theta,\ so$$

$$\cos\theta = \frac{A_xB_x + A_yB_y}{AB}$$

- Result: $Work = W = \vec{F}\bullet\vec{d}$

Work when the force depends on the position: F = F(s)

- **Basic Idea**: over the interval Δs where Δs approaches, but does not reach zero, F(s)~F(s+ Δs). Thus, ΔW = F Δs over this interval, and for Δs → 0 we can write it as the differential dW =Fds.

$$dW = \vec{F} \bullet d\vec{s} \Rightarrow W = \int dW = \int_{start}^{finish} \vec{F} \bullet d\vec{s}.$$

- ***Example of work done by the spring force*** when the spring is compressed (or stretched) to s and allowed to return to the "relaxed" position, 0:

$$F = -ks \Rightarrow dW = -ksds \Rightarrow W = \int dW = -k\int_{s}^{0} sds = \tfrac{1}{2}ks^2$$

- For future reference, notice the similarity of the spring's work relation above to that of its potential energy given earlier.

Force, Work, and potential energy:

- **Observation**: In some cases, forces have the effect of "storing" energy. For example, when a spring is compressed (or stretched), it is necessary for an external force (a person's hand perhaps) to ***do work*** against the spring's force. However, when the spring is released, its internal force will do work that exactly equals the work by the external force in the beginning. Gravity is another example of this effect: raising an object with increases its height, and gravity will do work on an the object as it falls back to the original height.
- Forces that have this capacity to store energy are called ***conservative forces***, and it is possible to associate a ***potential energy*** relation with them (Gravity and springs are examples.) It turns out that a general property of conservative forces is that the work they do, and the potential energy associated with them, depend only on the end points of the path taken by an object under their influence. (Our earlier examples with gravity and the "roller-coaster" type of path illustrate this property.) ***{Re. pp. 313-315 in the book}***
- ***Relation between Work & Potential Energy for a conservative force:***

$$\Delta U_F = - \Delta W_F$$

- For example, when an object is raised the distance ***h*** against gravity, the force of gravity does negative work (-mgh) because F_G is directed ***down*** and the change in position is ***up***. That is, they are opposite so the angle between them is 180°: ***ΔW_G is –mgh but ΔU_G is +mgh.***

- ***Relation between Potential energy and Force:*** Since $\Delta U_F = - \Delta W_F$, and $\Delta W_F = F\Delta s$, it follows that $\Delta U_F = - F\Delta s$. Converting these to differentials (the case where the Δ's approach, but do not reach, zero) gives:

$$dU_F = -Fds, \text{and } F = -\frac{dU_F}{ds}$$

$$\text{(for one - dimensionsal cases. The more general case requires higher - level math)}$$

Power, work, and energy:

- **Basic Idea**: The time over which work is done, or energy is expended, is an important parameter. For example it is one thing to do 500 Joules of work in 5 seconds, and another to do it in 2 years.
- ***Definition:*** Power = Work (or energy) / time
- ***Units:*** 1 joule in 1 second is 1 Watt.

$$P = \frac{work\ (or\ energy)}{time} = \frac{dW}{dt}$$

$$1\,Watt = 1W = \frac{1\ J}{1\ s}$$

- Note that we can relate this back to the definition of work in terms of force:

$$P = \frac{dW}{dt} = \frac{\vec{F} \bullet d\vec{s}}{dt} = \vec{F} \bullet \frac{d\vec{s}}{dt} = \vec{F} \bullet \vec{v}$$

- Thus, when an force acts on an object moving at v, it is doing work at the rate P.

Assignment:

Continue reading and working on Chapters 10 & 11; begin reading Chapter 12.