P2211K

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Chapters 10 & 11:

Work and energy: Still another way to describe things.

- **Basic Idea**: When an object is acted on by forces, the acceleration (or deceleration) leads to gain (or loss) of ***kinetic energy*** as its speed changes.
- ***Definition:*** Kinetic energy $K = \frac{1}{2}mv^2$. (K is a ***scalar*** and ***not a vector*** as it depends only on the object's speed.)
- ***Example*** of energy and force relations for cases of constant acceleration (as is appropriate for gravity near the Earth's surface).
  - ***An object of mass m*** is acted on by the force ***F = ma*** as it travels the **distance *d***. Over this distance, the ***speed changes from its initial value v_i to its final value v_f***:

From the set of kinematic relations (Chapters 1 - 4), we know that

$$v_f^2 = v_i^2 + 2ad.$$

The connection between K and F is revealed by the following algebraic steps :

1. Divide both sides by 2 and multiply both sides by the mass m :

$$\frac{m}{2}\left(v_f^2\right) = \frac{m}{2}\left(v_i^2 + 2ad\right) \Rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + \left(ma\right)d$$

2. Rearrange algebraically and recognize that ma = F :

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + \left(ma\right)d \Rightarrow \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) = \left(F\right)d$$

3. Recognize that $\frac{1}{2}mv^2 = K$:

$$\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) = \left(F\right)d \Rightarrow \left(K_f - K_i\right) = \Delta K = Fd$$

Thus, ΔK, the change in kinetic energy, equals (is the result of) $F \times d$!!

The idea of ***Potential Energy***

- The observation that ***$\Delta K = F \times d$*** for the case of constant acceleration (constant force) will be developed more fully in Chapter 11 when the concept of ***work*** will be introduced along with the work-energy theorem.
- For now, we will restrict our discussion to the effects of gravity near the surface of the earth. This will allow us to introduce the concept of ***potential energy.***
- For this discussion, we need to recognize two things: gravity acts only in the negative vertical direction (-***y***), and the distance over which the object travels while being acted on by the force is related to the coordinates by ***$d = y_f - y_i$***.
- With this recognition, we have:

$$\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) = (F)d = (F_G)(y_f - y_i) = m(-g)(y_f - y_i) = mg(y_i - y_f)$$

or,

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = mgy_i - mgy_f$$

- Note that the final speed above is greater when the object moves from higher positions to lower ones (i.e., $v_f > v_i$ if $y_i > y_f$) . This corresponds to our experience that an object picks up speed (and thus kinetic energy) as it moves from higher to lower locations. With this recognition, we have the idea that the object has the ***potential*** "to acquire kinetic energy" depending on its height.
- Thus, the expression ***mgy*** is referred to as the ***gravitational potential energy (GPE, or U_g) of the object with mass m at height y.***

Potential and Kinetic Energy

- The relations above:

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = mgy_i - mgy_f$$

can be rewritten as

$$K_f - K_i = U_{gi} - U_{gf} \Rightarrow \Delta K = -\Delta U_g \Rightarrow \Delta K + \Delta U_g = 0$$

or,

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow TE_f = TE_i$$

if the "total energy" is defined as $TE = K + U_g$

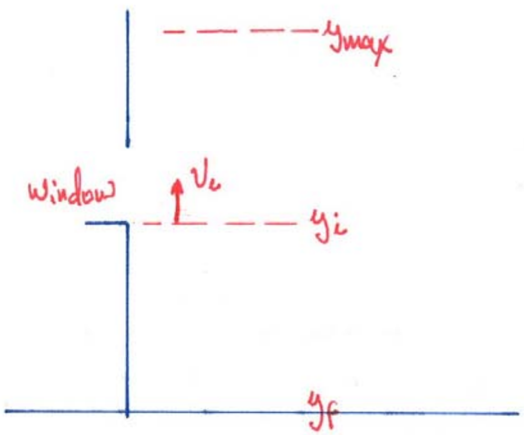
- Interpretation of these relations:
  - o $\Delta K = -\Delta U_g$ indicates that the ***changes in K and U_g are opposite***; that is, when ***one increases*** the ***other decreases***;
  - o $\Delta K + \Delta U_g = 0$ indicates that ***the increase in one is exactly the same amount as the decrease in the other;***
  - o The relation $TE_i = TE_f$; indicates that the ***total energy*** of the system remains constant during the motion. That is, the ***total energy*** is ***conserved***.
- ***Very important point---Only CHANGES in U_g are important:***
  - o Obviously, the value of U_g depends on the origin chosen for the coordinate system: ***for one choice it might be +20 m and for another it might be 0***.
  - o Thus, TE and U_g are useful only as concepts for working purposes while obtaining TE_i and TE_f; their absolute values are not important.

Working with Kinetic and Potential Energy

Units of energy: 1 kg (m^2 / s^2) = 1 Nm = 1 Joule

Problem 10-5. A boy reaches out of a window and tosses a ball straight up with a speed of 10 m/s. The ball is 20 m above the ground as he releases it. Use energy to find:

a. the ball's maximum height above the ground;
b. the ball's speed as it passes the window on its way down;
c. the speed of impact on the ground.



Principle: TE is the same at all locations.

$$TE = \tfrac{1}{2}mv_i^2 + mgy_i = m\left(\tfrac{1}{2}v_i^2 + gy_i\right) = m(250\ m^2/s^2)$$

a. the ball's maximum height above the ground;

$$TE\big|_{@max} = TE = m(250\ m^2/s^2) = \tfrac{1}{2}mv_{@max}^2 + mgy_{@max} = m\left(10\ m/s^2\right)y_{@max}$$

$$because\ v_{@max} = 0$$

$$\therefore\ \ y_{@max} = 25\ m$$

b. the ball's speed as it passes the window on its way down;

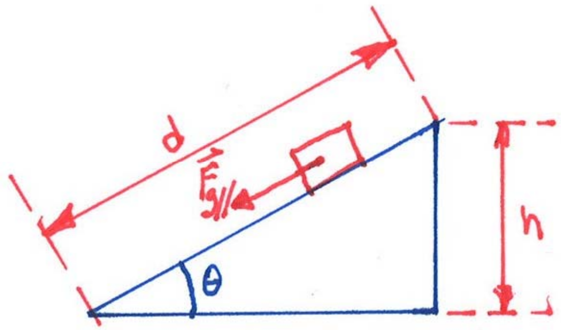
$$This\ is\ @\ y_i\ again, and\ TE = TE_i, so\ v = v_i.$$

c. the speed of impact on the ground.

$$TE = m(250\ m^2/s^2) = \tfrac{1}{2}mv_f^2 + mgy_f = m\left(\tfrac{1}{2}v_f^2 + 0\right)$$

$$\therefore\ v_f = \sqrt{500}\ m/s = 22.4\ m/s$$

Conservation of energy, continued

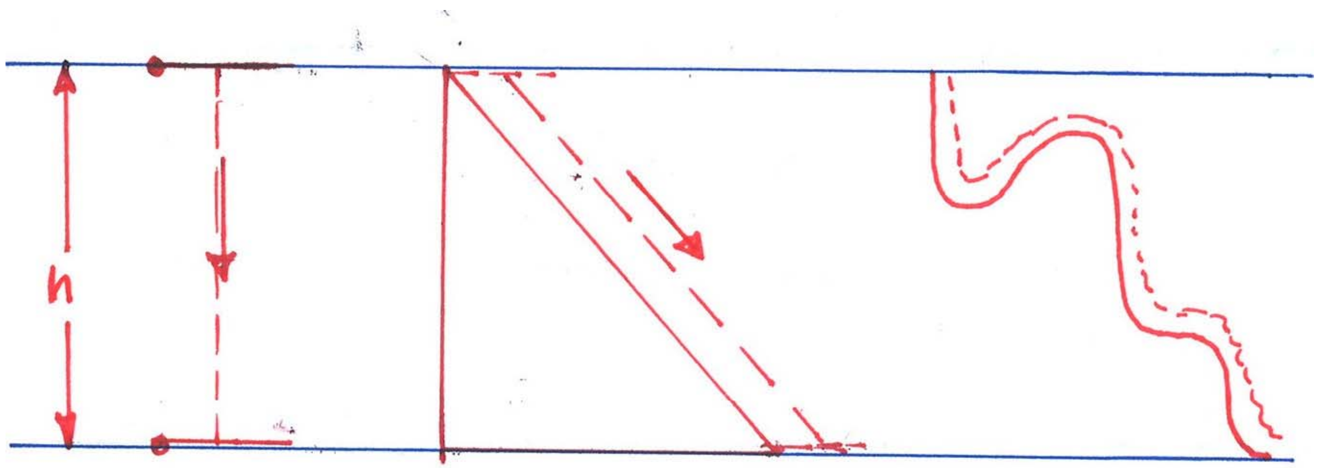


- ***What if the motion is not strictly vertical and is along a sloping (or curved) path?***
- The object to the left is on a frictionless incline and slides along it the distance d. It slides because its weight (F_G) has a component along the incline given by $F_{G//} = mg(\sin\theta)$.
- In addition, the distance d is related o tthe height of the incline by $h/d = \sin\theta$, or $d = h/\sin\theta$.

- Above, we found that $\Delta K = F\times d$, so in this case, $\Delta K = F\times d = mg(\sin\theta) \times h/\sin\theta = mgh$.
- Since $h = y_i - y_f$, this evaluates to $\Delta K = mg(y_i - y_f) = -\Delta U_G$.
- Thus, for the incline, the change in kinetic energy due to gravity, and the object's change in speed, depends only on how far it moves in the vertical direction---h is the important distance, and not d.
- On pp. 274-275, the book uses this model to generalize this result for motion on curved paths (the "roller-coaster").

Question: How long does it take an object to fall the distance h directly vs. sliding down the incline the distance d?

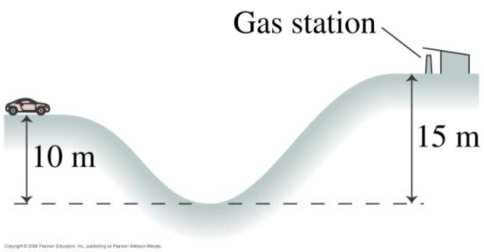
A general principle for gravitational potential energy (U_g): The change in potential energy depends only on the change in height and not on the path actually traveled (if no other forces, such as friction, act on the object).



Conservation of energy, continued: example of a curved path

Problem 10-13. A 1500 kg car traveling at 16 m/s suddenly runs out of gas while approaching the valley shown in the figure.

- What will be the car's speed as it coasts into the gas station on the other side of the valley?



- Analysis: (Friction is not a factor.) TE for the car is the same at all points along the path.
- Thus, TE @ the start = TE @ gas station.

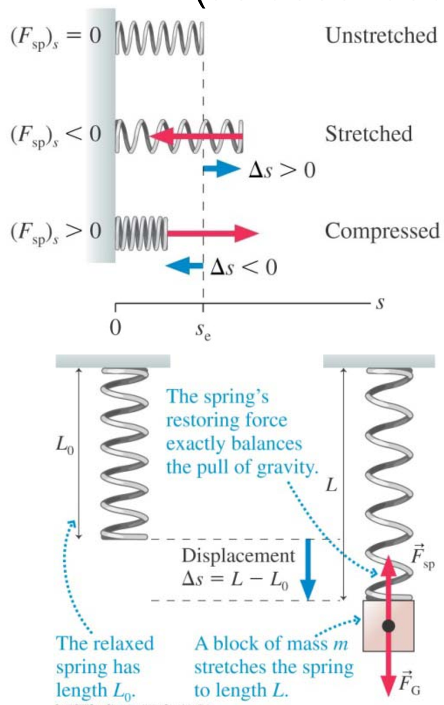
$$TE = \tfrac{1}{2}Mv_i^2 + Mgy_i = M\left[\tfrac{1}{2}\left(16\ m/s\right)^2 + \left(10\ m/s^2\right)\left(10\ m\right)\right] = M(228\ m^2/s^2)$$

$$TE = M(228\ m^2/s^2) = \tfrac{1}{2}Mv_f^2 + Mgy_f = M\left[\left(\tfrac{1}{2}v_f^2\right) + \left(10\ m/s^2\right)\left(15\ m\right)\right]$$

$$v_f^2 = 156\ m^2/s^2 \Rightarrow v_f = \sqrt{156}\ m/s = 12.5\ m/s$$

A variable force: springs, elastic forces, and Hooke's "Law"

- ***Description:*** **F = -kΔs**. The force is proportionally greater for greater stretch / compression, it is opposite to the direction of stretch / compression (as described by the "-"), and "k" is the "force constant.".



- This type of force plays a very important role in modeling physical behaviors of complex systems.
- For example, the stretching behavior of chemical bonds often is described using the terminology of "force constant." (This type of force also leads to oscillations described by "simple harmonic motion" and is basic to the way infrared spectroscopy is used to describe molecular vibrations.)
- In addition, Hooke's law plays a major role in the way structural engineers describe the behavior of buildings, bridges, etc. when they are "under load." This is an important component of structural design and predicting structural stability—how much they will "deflect" and how they will vibrate.

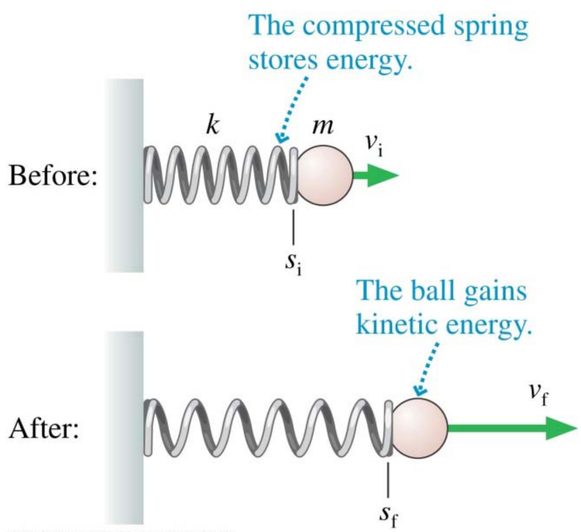
- ***A simple example—the spring scale***: A 6 kg object is attached to the end of a spring characterized by k = 20×10^2 N/m. How much does the spring stretch?

$\Delta s = F / k = \text{Weight} / k = (6\text{ kg})(10\text{ m/s}^2) / 20\times10^2\text{ N/m} = 3\times10^{-2}\text{ m} = 3\text{ cm}$

Potential energy associated with stretched / compressed springs:

- In equations 10.27 through 10.37, the book uses a calculus procedure based on the chain rule to calculate the relation between the kinetic energy an object gains and the “un-stretching” of a spring with the result that $U_s = \frac{1}{2}k(\Delta s)^2$.

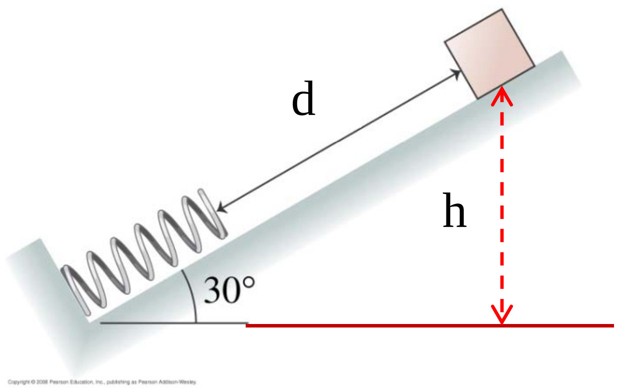
- (This same result can be obtained from Work concepts and we’ll see that in Chapter 11.)



$$K_f + U_{sf} = K_i + U_{si}$$

Problem 10-41. A 50.0 g ice cube can slide without friction up and down a 30.0° slope. The ice cube is pressed against a spring at the bottom of the slope, compressing the spring 10.0 cm The spring constant is 25.0 N/m.

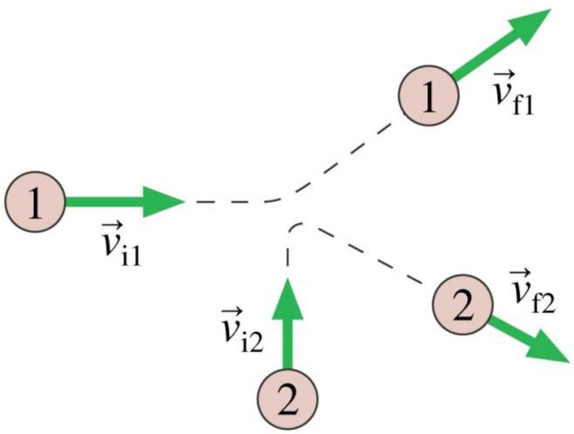
- When the ice cube is released, what distance will it travel up the slope before reversing direction?



- Compression = Δs = 10 cm
- $h = d(\sin\theta) = d(\sin 30°)$
- At start, v =0 & at max height v = 0
- So, max height occurs when $U_g = U_s$
- $\frac{1}{2}k(\Delta s)^2 = mgh = mgd(\sin 30°)$
- And $d = [\frac{1}{2}k(\Delta s)^2] / [mg(\sin 30°)] = 0.5$ m (using $g=10 m/s^2$)

Perfectly elastic collisions: Collisions in which the kinetic energy is conserved (as well as the momentum)

- In section 10.6, the book discusses and solves the case of perfectly elastic collisions when all the motion (initial & final) is in a straight line.
- However, the more general case is when the objects go off at angles after the collision (think about a game of pool).



- Conceptually, the two dimensional problem is the same as that in one dimension: momentum is conserved and kinetic energy is conserved. However, the geometry and trigonometry introduce more complexity in actually arriving at an analysis.
- Here is an assessment of the relations available an the possible known / unknown parameters:
- $\Delta p_x = 0$ --- $p_{ix} = p_{fx}$
- $\Delta p_y = 0$ --- $p_{iy} = p_{fy}$
- $\Delta K = 0$ --- $K_i = K_f$
- Parameters: m_1, m_2, v_{1ix}, v_{1iy}, v_{2ix}, v_{2iy}, v_{1fx}, v_{1fy}, v_{2fx}, v_{2fy}
- Thus, have 3 equations and 10 parameters; need to know at least 7 to get a solution.
- Common case; ***know m's, initial v's, and one final v; find the other final v and the two final directions (θ's).***

Perfectly elastic collisions: Collisions in which the kinetic energy is conserved (as well as the momentum)

- Simplest case: that where one object is at rest initially and the other travels toward it along +x.

$$x\ momentum:\ m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta_{1f} + m_2 v_{2f} \cos\theta_{2f}$$

$$y\ momentum:\ 0 + 0 = m_1 v_{1f} \sin\theta_{1f} + m_2 v_{2f} \sin\theta_{2f}$$

$$K:\ \tfrac{1}{2} m_1 v_{1i}^2 + 0 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Assignment:

Continue reading and working on Chapter 10, and begin reading Chapter 11.