P2211K

10 / 07 / 2010

Impulse & Momentum: ***(Chapter 9)***

- Momentum: another way of thinking about the nature of motion;
  - o **Definition**: $\vec{p} = m\vec{v}$
  - o **Concept**: the full description of an object's motion involves more than just its velocity. For example, there's "something" different about a ping-pong ball and a baseball moving at the same speed (or a bicycle and a Mack truck…). The mass of the object is important and the definition of momentum captures that aspect.
  - o **Relation to force**: $Force = mass \times acceleration \Rightarrow \vec{F} = m\vec{a}$

$$but\ \vec{a} = \frac{d\vec{v}}{dt},\ so$$

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt},\ when\ m\ is\ constant$$

This leads to Newton's more general definition of Force :

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\vec{a} + \vec{v}\frac{dm}{dt}$$

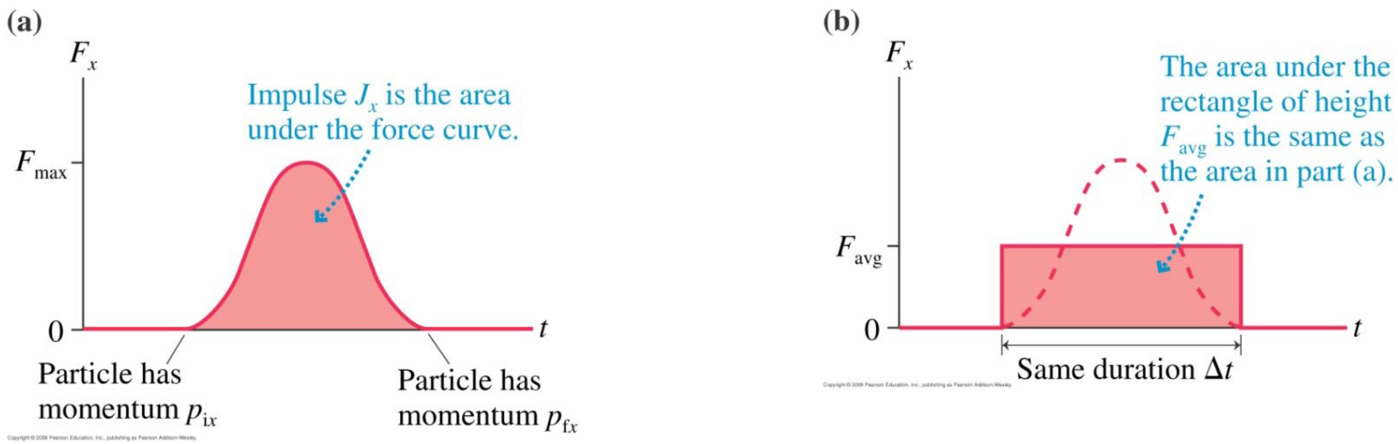
This allows for the possibility that mass might change.

For example, this could describe a rocket as its fuel is depleted.

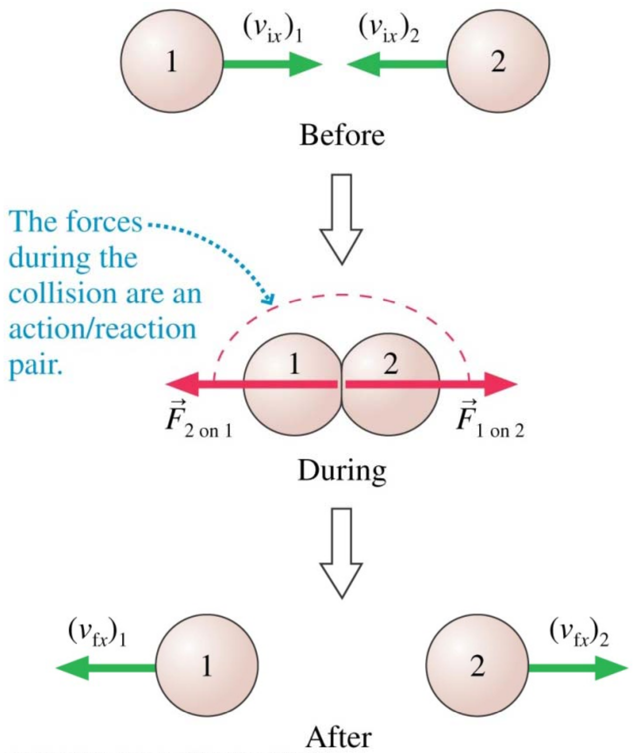
- Update to what it is that force does: ***Force changes momentum***

Momentum, “force of impact,” and Impulse (pp. 242-243)

- ***Basic point:*** During a collision, the force of impact is time dependent, perhaps in a complicated fashion, but the net result is a change in momentum:



- ***Definition of Impulse:*** $\vec{J} = \vec{F}_{av}\Delta t = \Delta\vec{p}$
- ***Summary:*** **the effect of forces during a collision is a change in momentum**



Momentum & Collisions

- During a collision between two objects, each exerts a "force of impact" on the other, and these forces are equal (but opposite) according to Newton's 3rd law. As well, the duration of the impact is same for both.
- Thus, the change in momentum for one object is equal but opposite to the change in momentum for the other:

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2 \Rightarrow \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0.$$

- The net result is that the total momentum of the objects together after the collision is the same as it was before. (Of course, the momentum of each object will not be the same before & after the collision; it's their sum that is the same.)
- In fact, ***if the only forces acting in a multi-object system are those between members of the system, then Newton's 3rd law leads to the conclusion that the total momentum of the system remains constant.***
- This is the ***"Law of Conservation of Momentum."*** ***(***Don't forget, however, that the momenta of the individual members will change as a result of any internal interactions!!!***)***

Prob. 9-21. Bob has a mass of 75 kg and can throw a 500 g rock with a speed of 30 m/s. He throws a rock while standing on frictionless ice.

- Find Bob's recoil speed, using momentum conservation.

$$\textit{Conservation principle}:\ \vec{p}_{before} = \vec{p}_{after}$$

$$\vec{p}_{Bob,before} + \vec{p}_{Rock,before} = 0 = \vec{p}_{Bob,after} + \vec{p}_{rock,after}$$

$$\Rightarrow \vec{p}_{Bob,after} = -\vec{p}_{rock,after}$$

$$m_{Bob} v_{Bob,after} = -\, m_{Rock} v_{Rock,after}$$

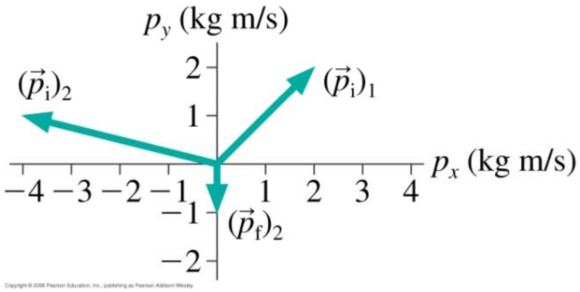
$$(75\ kg) v_{Bob,after} = -\,(0.5\ kg)(30\ m/s)$$

$$v_{Bob,after} = -\,\frac{(0.5\ kg)}{75\ kg}(30\ m/s) = -\,0.2\ m/s$$

(- means that Bob moves opposite to the Rock)

Prob. 9-23. Two particles collide and bounce apart. The figure shows the initial momenta of both and the final momentum of particle 2.

- What is the final momentum of particle 1? (Write your answer in component form.)



Conservation principle

$$\vec{p}_{before} = \vec{p}_{after} \Rightarrow (\vec{p}_1 + \vec{p}_2)_{before} = (\vec{p}_1 + \vec{p}_2)_{after}$$

$$(\vec{p}_1)_{after} = (\vec{p}_1 + \vec{p}_2)_{before} - (\vec{p}_2)_{after}$$

Before :

$p_{1x} = +2$ kg m/s $\qquad p_{1y} = +2$ kg m/s

$p_{2x} = -4$ kg m/s $\qquad p_{2y} = +1$ kg m/s

$p_{before,x} = -2$ kg m/s $\qquad p_{before,y} = +3$ kg m/s

After :

$p_{2x} = 0$ kg m/s $\qquad p_{2y} = -1$ kg m/s

Thus :

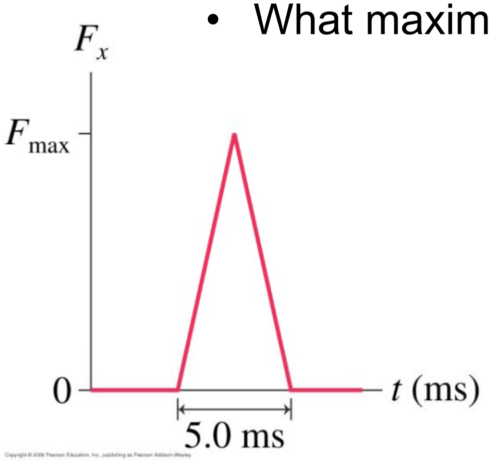
$p_{1x} = -2$ kg m/s $\qquad p_{1y} = +4$ kg m/s

and

$$\vec{p}_{1f} = \left(-2\hat{i} + 4\hat{j}\right) \text{ kg m/s}$$

Prob. 9-29. A 150 g ball is dropped from a height of 2.0 m, bounces on a hard floor, and rebounds to a height of 1.0 m. The figure shows the impulse received from the floor.

- What maximum force does the floor exert on the ball?



- ***Analysis:*** To answer this question, we need to know the change in the ball's momentum due to the impact with the floor. To get the change in momentum, we need to now the ball's speed just before impact and just as it leaves the floor on the rebound. These can be calculated from the height information of the drop and the rebound:

$$\textit{Dropping (up = +):}$$

$$v_f^2 = v_0^2 + 2ad_{drop} \quad (a = g = -10\ m/s^2;\ d_{drop} = -2.0\ m)$$

$$v_f = -\sqrt{40}\ m/s, \text{ down (just before impact)}$$

$$\textit{Rebound (up = +):}$$

$$v_f^2 = v_0^2 + 2ad_{rebound} \quad (a = -g = -10\ m/s^2;\ d_{rebound} = 1.0\ m)$$

$$v_0 = \sqrt{20}\ m/s, \text{ up (just as it leaves the floor)}$$

$$\therefore \Delta p = p_f - p_i = (0.15\ kg)\left[\sqrt{20} - \left(-\sqrt{40}\right)\right] m/s = 1.62\ kg\ m/s$$

Δp gives the impulse, which is related to the force of impact as the area under (or integral of) the F(t) curve shown. The shape of the F(t) curve is triangular so that its area is that of a triangle (a=½hw) having height F_{max} and width Δt :

$$J = \Delta p = F_{av}\Delta t = \tfrac{1}{2} F_{max}\Delta t$$

$$\therefore F_{max} = 2\frac{\Delta p}{\Delta t} = (2)\left(\frac{1.62\ kg\ m/s}{5 \times 10^{-3}\ s}\right) = 6.48 \times 10^{-2}\ N$$

Prob. 9-39. A firecracker in a coconut blows the coconut into three pieces. Two pieces of equal mass fly off south (-y) and west (-x), perpendicular to each other, at 20 m/s. The third piece has twice the mass of the other two.

- What is the velocity of the third piece?

All forces are internal; thus momentum is conserved and

$$\vec{p}_{initial} = 0 = \vec{p}_{final} = m\,(20\text{ m/s})(-\hat{i} - \hat{j}) + 2m\,\vec{v}_{3rd}$$

$$\therefore \vec{v}_{3rd} = \frac{-(20\text{ m/s})(-\hat{i} - \hat{j})}{2} = (10\text{ m/s})(\hat{i} + \hat{j})$$

Prob. 9-41. A 10 g bullet is fired into a 10 kg wood block that is at rest on a wood table. The block, with the bullet embedded, slides 5.0 cm across the table. The coefficient of kinetic friction for wood sliding on wood is 0.20.

- What was the speed of the bullet?

Analysis: This is an example of a "perfectly inelastic" collision. That is, the two objects stick together and move as one following the collision. Nevertheless, momentum is conserved in the interaction so that the total just before impact equals that just after impact.

To answer the question, we need to know the speed of the bullet + block immediately following the collision. The information on how far they slide under the friction conditions allows computing the speed.

Sliding following the collision :

$$v_f^2 = v_0^2 + 2ad$$

$$d = 5\text{ cm} = 0.05\text{ m}$$

$$a = -\mu_k g = -2.0\text{ m/s}^2$$

$$v_0 = \sqrt{0.2}\text{ m/s} = 0.447\text{ m/s}$$

Momentum conservation during the collision :

$$\vec{p}_{initial} = \vec{p}_{bullet} + \vec{p}_{block} = \vec{p}_{final} = \vec{p}_{bullet+block}$$

$$p_{initial} = m_{bullet} v_{bullet} = (0.01\text{ kg}) v_{bullet}$$

$$p_{final} = (m_{bullet+block}) v_{bullet+block} = (10.01\text{ kg})(0.447\text{ m/s})$$

$$v_{bullet} = \frac{(10.01\text{ kg})(0.447\text{ m/s})}{0.01\text{ kg}} = 4.47 \times 10^2\text{ m/s}$$

Assignment:

Begin reading and working on Chapter 10.