P2211K

10 / 05 / 2010

Dynamics in 2 dimensions: ***(Chapter 8)***

- Projectile motion when there are forces acting in addition to gravity;
    - o Effect on range (p. 211)
    - o Effect on trajectory (p. 212)
- Uniform circular motion (Sections 8.2 – 8.6, pp. 212-226
    - o Dynamics and forces necessary (pp. 214 – 219)
    - o Circular orbits & gravity (pp. 219 – 221)
    - o "Vertical" circular motion (223-226)
    - o "Centrifugal" force (p. 222)
- Non-uniform circular motion (Section 8.7, pp. 226-227)

- Projectile motion when there are forces acting in addition to gravity;
    - Effect on range (p. 211)
        - Horizontal forces (such as wind, air resistance, etc.) will reduce the range (if they are opposite the direction of motion—headwinds, air resistance, etc.) or they will extend it (tail winds, etc). Describing these is complex and we don't need to go into them in this course.
    - Effect on trajectory (p. 212)
        - Likewise, additional forces will distort a projectile's trajectory from the parabola that results when only gravity acts. A complete description of these effects requires knowledge of the additional forces and the use of mathematical / computational techniques beyond those required here.

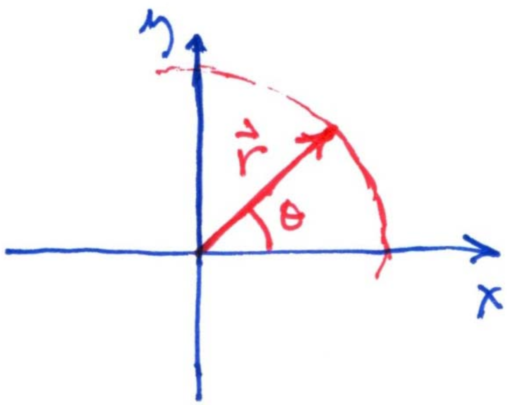
Uniform circular motion (Sections 8.2 – 8.6, pp. 212-226)

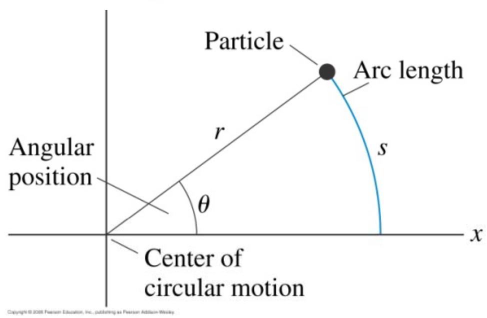
- ***Basic point:*** whenever an object moves in a curved path, at least the direction of its velocity changes. Thus, the object experiences acceleration (because velocity is a vector), and acceleration requires a net force.
- ***Uniform circular motion*** is the special case when the speed remains constant ("uniform") but the path is a circle (of radius r). (Re. class notes from 9/9/2010.)
- ***Characteristics of uniform circular motion, a reminder***:
  - the path is a circle;
  - the magnitude of the velocity remains constant but its direction continuously changes and remains tangent to the path (or perpendicular to the radius);
  - the magnitude of the acceleration remains constant but its direction changes and remains directed towards the center of the path (centripetal).

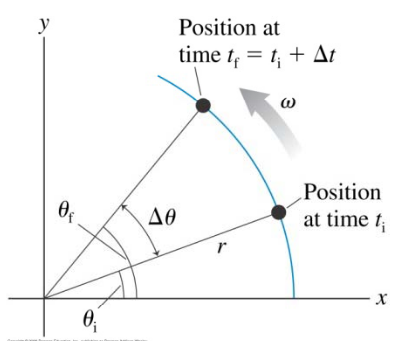
$$a_c = \frac{v_t^2}{r}, \text{ where}$$
$$a_c = \text{centripetal acceleration}$$
$$v_t = \text{tangential speed}$$
$$r = \text{radius of the path}$$

- Thus, for an object of mass ***m*** to travel in a circular path at constant speed, it must be acted upon continuously by the net force of magnitude $ma_c = m(v^2 / r)$ that always is directed towards the path's center.

Uniform circular motion and rotational motion:







- A particularly important type of 2-dimensional motion is that of an object in a circular path. This is the starting point of discussions involving orbital motion, rotating objects, etc.
- Because the path is a circle it is convenient to describe its instantaneous position as $\vec{r} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$ as shown in the sketch to the left.
- It is convenient to introduce the measure of θ in radians as defined by θ = arc / radius = s / r. (For a full circle, the arc length is the circumference = 2πr and the angle is 2π radians.)
- Also, because the is moving in its path, the value of θ constantly changes. To describe this, it is convenient to introduce the angular speed in radians / sec, which is symbolized by ω.

- In its circular path, the object's speed is v = ds/dt = d(rθ)/dt = r(dθ/dt = rω. Thus, there is a direct connection between the angular speed and the object's speed in its path.
- It is important to note that the object is accelerated even though its speed is constant—its direction is changing and therefore its velocity is not constant.
- Because r is constant (the path is a circle), and for the case of constant ω and v (uniform circular motion, the following relations connect the object's position, velocity, and acceleration:

$$\vec{r} = r(\cos\omega t\ \hat{i} + \sin\omega t\ \hat{j})$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega r(-\sin\omega t\ \hat{i} + \cos\omega t\ \hat{j})$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega^2 r(-\cos\omega t\ \hat{i} - \sin\omega t\ \hat{j})$$

- Although it is not obvious without additional vector techniques (such as the dot product), the velocity and position vectors are perpendicular to each other. This means that the velocity is tangent to the circular path because any vector perpendicular to the radius is a tangent (from basic geometry).
- It is obvious, however, that the acceleration is directed oppositely to the position vector; thus $\vec{a}$ points towards the center of the circular path. In effect, the circular path is the result of a constantly changing velocity due to this ***centripetal*** acceleration. Note also that the acceleration in this case is not constant—its direction is constantly changing.

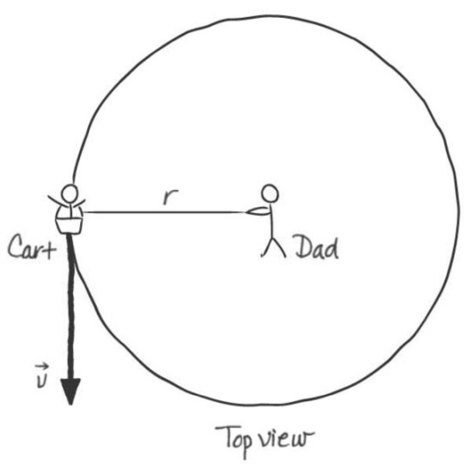
Typical examples:

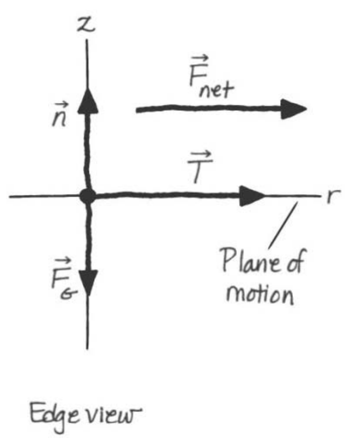
- A mass on the end of a string
- A car on a curve
- Orbital motion in gravity
- Water in a bucket
- etc.

Example: A dad swings his 20 kg child in a 5 kg cart at the end of a 2.0 m rope in a circle at the (tangential) speed 2.5 m/s. (The cart rolls on the ground without friction.)

- Calculate the tension in the rope necessary to accomplish this.

$$\boldsymbol{T = m(v^2/r) = (25\ kg)\ (2.5\ m/s)^2 / 2.0\ m = 78.13\ N}$$



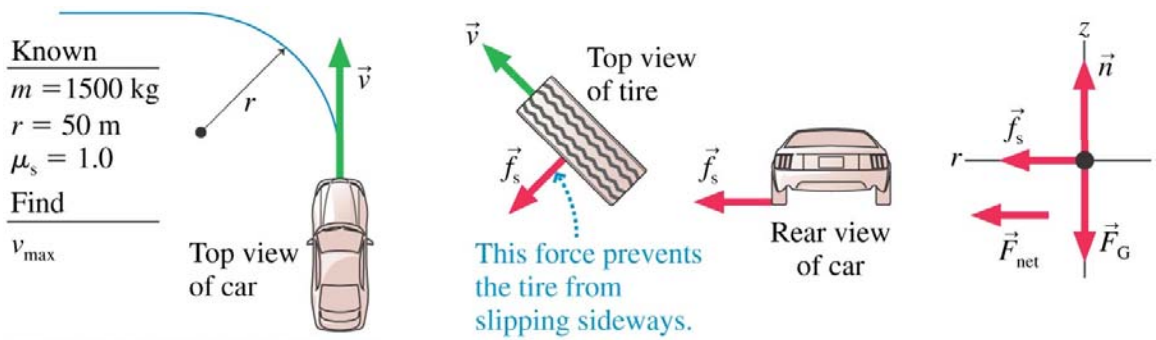


Example ***(re. example 8.4, pp. 216-217)***: A 1500 kg car travels on a curve of 50 m radius on a level (or unbanked) road. The coefficient of static friction is 1.0 between the tires and road. (g = 10 m/s^2)

- What is the maximum speed with which the car can take the curve?

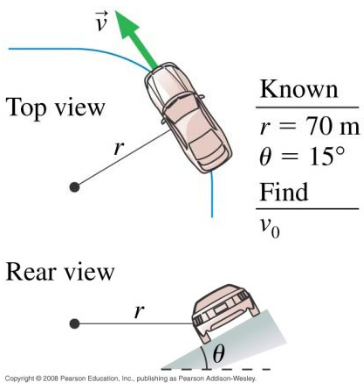
$f_{max} = \mu_s n = 15000\ N = m(v^2 / r) = (1500\ kg)(v^2 / 50m)$

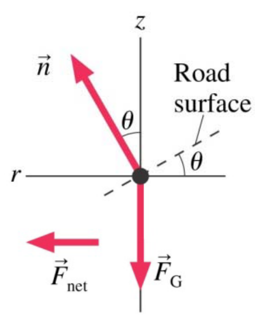
$v_{max} = 22.4\ m/s$



Example ***(re. example 8.5, pp. 217-218)***: A 1500 kg car travels on a curve of 70 m radius on a road banked at 15°. (g = 10 m/s^2)

- What is the speed with which the car can take the curve?
- Basic ideas: Need a component of the force of contact to provide the centripetal force to keep the car in the circular path. However, there is only one value of the speed that will work: too slow & the car will “slide” down the bank; too fast, and it will drift up the bank.





Objectives :

$$\text{Vertical : } F_{net} = 0$$

$$\text{Horizontal : } F_{net} = ma_c = m\left(\frac{v^2}{r}\right)$$

Thus,

$$\text{Vertical : } F_{net} = 0 = n(\cos\theta) - F_G \Rightarrow n = \frac{mg}{\cos\theta}$$

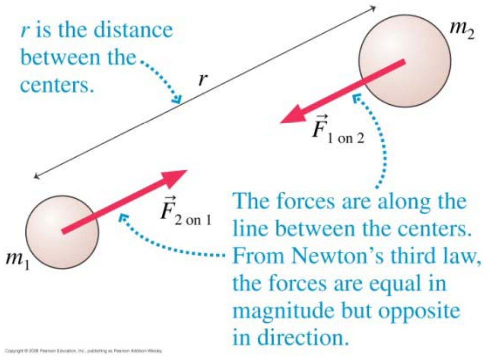
$$\text{Horizontal : } F_{net} = ma_c = m\left(\frac{v^2}{r}\right) = n(\sin\theta)$$

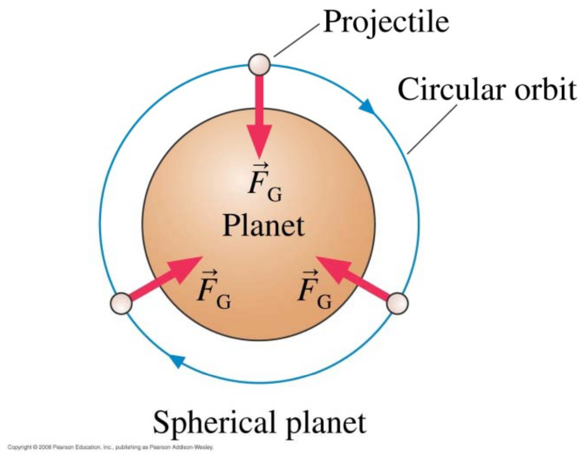
$$m\left(\frac{v^2}{r}\right) = \left(\frac{mg}{\cos\theta}\right)(\sin\theta) \Rightarrow v^2 = gr(\tan\theta)$$

$$v = 14 \text{ m/s}$$

Circular orbits & gravity (pp. 219 – 221)

- This opens the general topic of central forces, which are forces always directed towards a point central to the motion of any object under their influence. By their basic nature, central forces can provide an acceleration always directed towards the center of a circular path. (Gravitational and electrostatic forces are prime examples of central forces. In general, central forces lead to trajectories described by "conic sections," which include circles, ellipses, parabolas, and hyperbolas. Hyperbolas occur if and only if the central forces are repulsive; e.g., for charged particles of like sign.)





$$F_G = G\frac{m_1 m_2}{r^2} = m_{satellite}\left(G\frac{m_E}{r_{orbit}^2}\right) = m_{satellite}\left(\frac{v_{satellite}^2}{r_{orbit}}\right)$$

$$v_{satellite} = \sqrt{\left(G\frac{m_E}{r_{orbit}}\right)} = \omega_{orbit} r_{orbit}$$

$$\omega_{orbit} = \frac{1}{r_{orbit}}\sqrt{Gm_E}$$

Example of a circular orbit: Communications satellites sometimes are launched so that they remain "parked" over a specific location on the surface of the earth. These also are called "geosynchronous" orbits.) (R_E = 6.37 x 10^6 m)

a. What is the radius of a geosynchronous orbit?

b. In a geosynchronous orbit, how high is the satellite above the surface of the earth?

$$F_G = G\frac{m_1 m_2}{r^2} = m_{satellite}\left(G\frac{m_E}{r_{orbit}^2}\right) = m_{satellite}\left(G\frac{m_E}{R_E^2}\right)\frac{R_E^2}{r_{orbit}^2} = m_{satellite}\left(\frac{v_{satellite}^2}{r_{orbit}}\right)$$

$$\left(G\frac{m_E}{R_E^2}\right)\frac{R_E^2}{r_{orbit}^2} = g\left(\frac{R_E^2}{r_{orbit}^2}\right) = \left(\frac{v_{satellite}^2}{r_{orbit}}\right), \textbf{\textit{because}}\ g = \left(G\frac{m_E}{R_E^2}\right) = 9.8\ m/s^2$$

$$also, v_{satellite} = \frac{2\pi r_{orbit}}{T}, where\ T = orbital\ \ period$$

$$so, g\left(\frac{R_E^2}{r_{orbit}^2}\right) = \left(\frac{v_{satellite}^2}{r_{orbit}}\right) = \left[\frac{\left(2\pi r_{orbit}/T\right)^2}{r_{orbit}}\right] \Rightarrow r_{orbit}^3 = g\left(\frac{R_E T}{2\pi}\right)^2$$

In this relation, we know everything but r_{orbit}*, so*

$$\boldsymbol{r_{orbit} = \sqrt[3]{g\left(\frac{R_E T}{2\pi}\right)^2} = 4.22\times 10^7\ m}$$

Analysis:

- To remain above the same point on the surface, the satellite must have an orbital period of exactly one day (24 h x 3600 s/h = 86,400 s).
- Also, gravity at its orbital radius must exactly provide the centripetal force necessary for the circular orbit.

$$\boldsymbol{h = r_{orbit} - R_E = 3.58\times 10^7\ m\ (\cong 22{,}000\ miles)}$$

Also, at this altitude, the gravitational acceleration is

$$\boldsymbol{g(r_{orbit}) = 0.221\ m/s^2 \quad (much\ different\ from\ 9.8\ m/s^2\ !!)}$$

Thus, the satellite is acted on by gravity and actually is "falling" towards the earth! It is just that the tangential speed causes it to move sideways as it falls in exactly the right proportion to maintain the circular orbit at constant r.

“The moon is falling…! by Isaac Newton”

“In the year 1666 he retired again from Cambridge to his mother in Lincolnshire. Whilst he was pensively meandering in a garden it came into his thought that the power of gravity (which brought an apple from a tree to the ground) was not limited to a certain distance from earth, but that this power must extend much further than was usually thought. Why not as high as the Moon said he to himself & if so, that must influence her motion & perhaps retain her in her orbit, whereupon he fell a calculating what would be the effect of that supposition.”

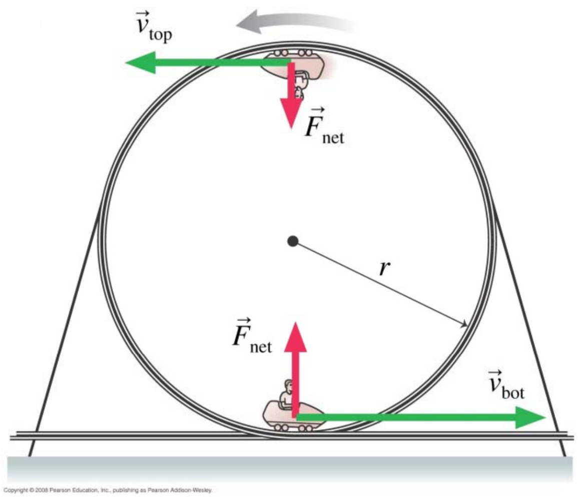
Stukeley, William. "Memoirs of Sir Isaac Newton's Life"

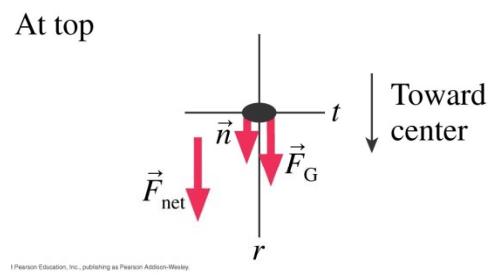
“Vertical” circular motion (223-226)

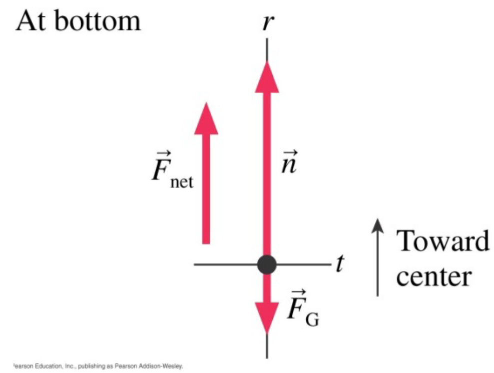
How is it possible to swing a water-filled bucket, with no lid, in a circular path so that it passes overhead with no water spilling?

Basic ideas:

- “Not spilling” means that the water must remain with the bucket as it goes through the path of the motion.
- As the water & bucket pass overhead, the water “falls” due to the gravitational effect. However, if the centripetal acceleration required for circular motion is no less than the gravitational acceleration, the water will stay with the bucket and not spill out of it.
- If the necessary centripetal acceleration equals g, the water is in free fall, but that is the same acceleration as the bucket so they stay together;
- If the necessary acceleration rate is greater than g, the bottom of the bucket “pushes” against the water with sufficient force to cause the water to accelerate with the bucket at the required rate;
- However, if the necessary centripetal acceleration rate is less than g, the water will accelerate at g and will spill out of the bucket!!!







The bucket of water & motion in vertical circles:

In general, n and F_G must combine (in a vector sense) to provide the necessary centripetal force. Because F_G is constant (down and equal to mg), the magnitude and direction of n is different at each point along the path.

To move in a circle,

$$F_{net} = m\frac{v^2}{r}, \text{towards the center of the circle at all times}$$

Thus,

$$F_{net} = F_G + n = m\frac{v^2}{r}$$

Worst case : at the top of the path

$$F_{net} = F_G = m\frac{v_{min}^2}{r} = mg \Rightarrow \frac{v_{min}^2}{r} = g$$

$$v_{min} = \sqrt{gr}$$

Example : r = 1.0 m (an arm's length)
$v_{min} = \sqrt{gr} = 3.2\ m/s$

Assignment:

Begin reading and working on Chapter 9.