P2211K

9 / 28 / 2010

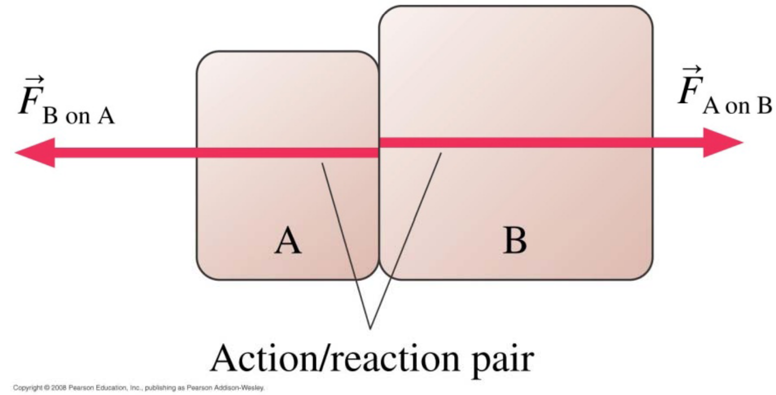
Newton's 3rd Law: ***(Chapter 7)***

Newton's Third Law

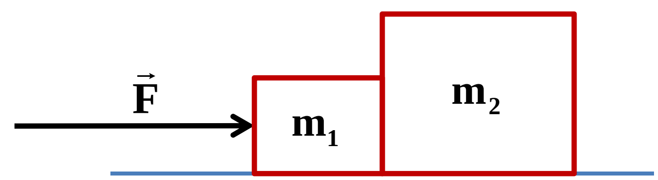
Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:

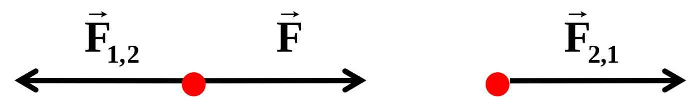
- Act on two *different* objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$



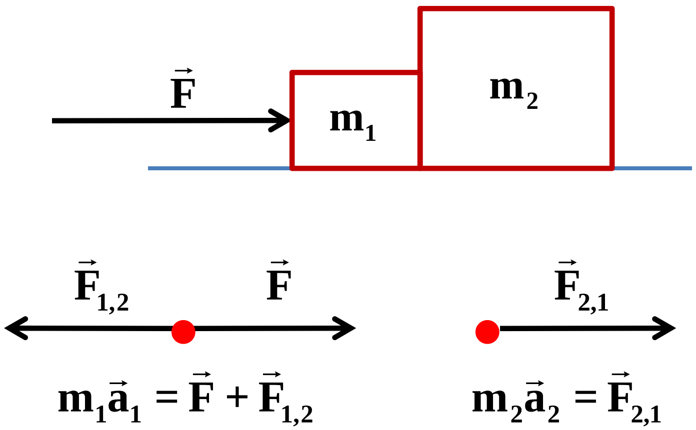
$$\textit{Eq.7.1, (p.185)}\begin{cases} m_1\vec{a}_1 = \vec{F}_{net,1} = \sum\vec{F}_{on\,1} \Rightarrow \vec{a}_1 = \dfrac{1}{m_1}\sum\vec{F}_{on\,1} \\ m_2\vec{a}_2 = \vec{F}_{net,2} = \sum\vec{F}_{on\,2} \Rightarrow \vec{a}_2 = \dfrac{1}{m_2}\sum\vec{F}_{on\,2} \end{cases}$$





$$m_1\vec{a}_1 = \vec{F} + \vec{F}_{1,2} \qquad m_2\vec{a}_2 = \vec{F}_{2,1}$$

Practical constraints: $\boldsymbol{a_1 = a_2 = a}$ — because the objects accelerate together and stay in contact. (***See the discussion on p.191 ff. regarding this issue.***)

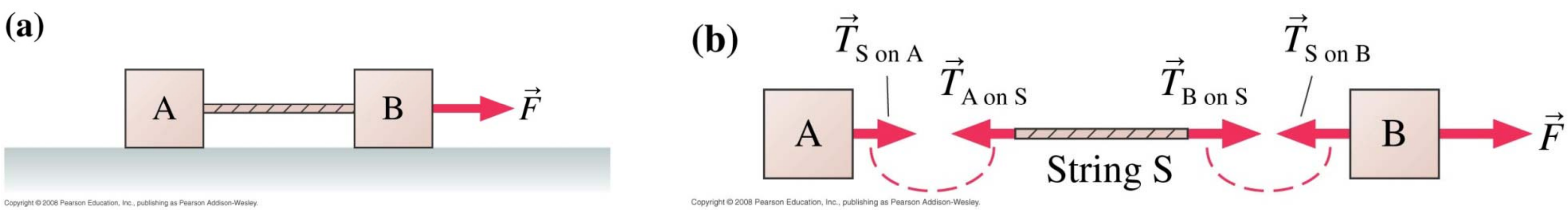


- For example, if F = 20 N, m_1 = 4 kg, and m_2 = 6 kg, then
  - $F_{1,2} = F_{2,1}$ (Newton's 3rd law—magnitudes only described here)
  - (6 kg) a = $F_{2,1} = F_{1,2}$ ($F_{2,1}$ is the only force on m_2)
  - (4 kg) a = (20 N) - $F_{1,2}$ = (20 N) - (6 kg) a, thus
  - A = (20 N) / (4 kg + 6 kg) = 2 m/s^2

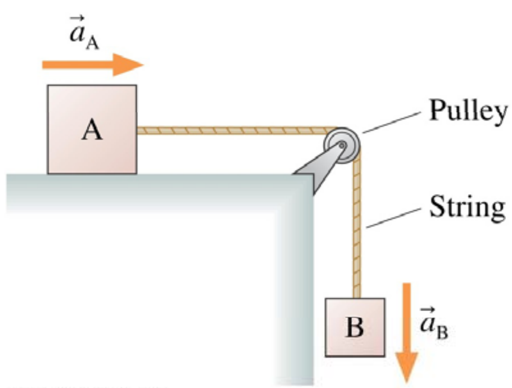
When we fall towards the earth, the earth falls towards us! However, we don't notice the earth's acceleration because its mass is so much greater than ours. By Newton's 3rd law, $M_E\, a_E = m_p\, a_p$, but $\boldsymbol{a_E = a_p\, (m_p / M_E)}$ (***See the discussion on p. 190.)***

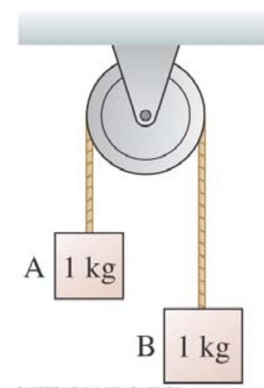
Strings, tension, and pulleys:

- Strings (ropes, chains, cables) are used to ***transmit*** force from one object to another. The force being transmitted creates (and equals) the ***tension*** in the string. (***e.g. Fig 7.16 on p. 196, shown below***) Also, the string may be ***massless*** (meaning that its mass is negligible compared to others), or it may have mass. ***(See the discussion on pp. 196-197.)***

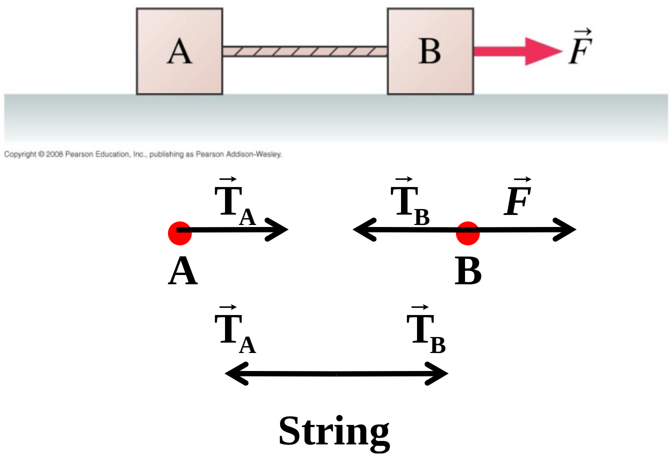


- Pulleys are used to ***change the direction*** of forces being transmitted. ***(See pp. 197-198 for a full discussion & force analysis.)***





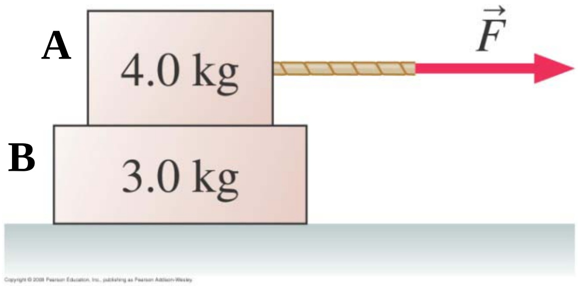
Examples:



$F = 20$ N, $m_A = 6$ kg, and $m_B = 4$ kg.

What is the tension in the rope?

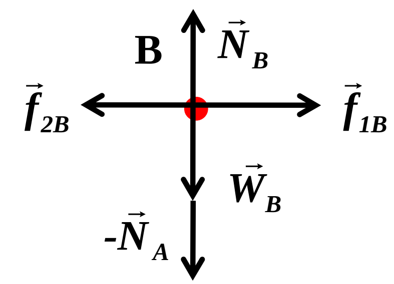
- $a_A = a_B = a$ (both objects move together)
- $T_A = T_B = T$ (massless string & Newton's 3rd law)
- $m_A a = T$ (only T acts on A)
- $m_B a = F - T = F - T = F - m_A a$
- Thus, ***a = (20 N) / (4 kg + 6 kg) = 2 m/s²*** $\left(a = (20\ \text{N}) / (4\ \text{kg} + 6\ \text{kg}) = 2\ \text{m/s}^2\right)$
- ***$T = m_A a = 12$ N***

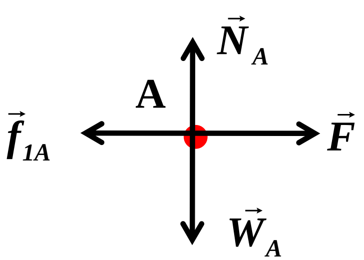


Problem 7.33: The coefficient of static friction is 0.60 between the two blocks in figure. The coefficient of kinetic friction between the lower block and the floor is 0.20. Force $\vec{F}$ causes both blocks to cross a distance of 5.0 m, starting from rest.

- What is the least amount of time in which this motion can be completed without the top block sliding on the lower block?

Analysis: need to know the max acceleration for both that will allow them to slide together. Friction between blocks A & B is the force keeping Block A from sliding along B. So what is the max value for that frictional force? (It will be independent of the applied force!)





$$f_{1A} = f_{1B} = f_1 \ \textit{(Newton's 3}^{rd}\textit{ Law)}$$

$$N_A = m_A g \quad (N_A \textit{ is due only to } W_A)$$

$$f_1 = \mu_s N_A = \mu_s (m_A g) \quad \textit{(static f is maximum for } f_1\textit{)}$$

$$0 = N_B - W_B - N_A = N_B - m_B g - m_A g \Rightarrow N_B = (m_B g + m_A g)$$

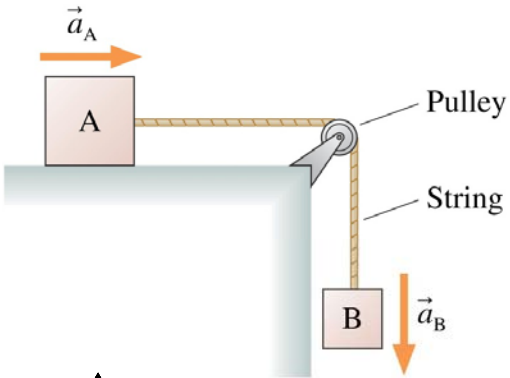
$$f_{2B} = \mu_k N_B = \mu_k (m_A g + m_B g) = \mu_k g (m_A + m_B)$$

$$m_B a_B = f_{1s} - f_{2B} = \mu_s (m_A g) - \mu_k g (m_A + m_B)$$

$$a_B = g\left[(\mu_s - \mu_k)\frac{m_A}{m_B} - \mu_k\right] \quad \textit{(maximum possible for } a_B\textit{)}$$

$$a_{B,max} = 3.34\ m/s^2 \quad \Rightarrow \quad t_{min} = 1.73\ s$$

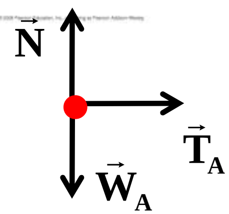
Example: Block A has mass 6 kg and block B has mass 2 kg.



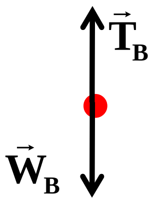
a. If the horizontal surface on which A slides is frictionless, what is the acceleration of block A?

b. What is the tension in the string?

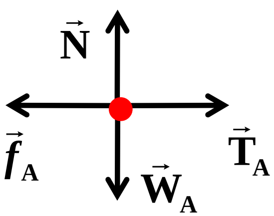
c. Repeat a. and b. for the case with the surface having coefficients of friction μ_s = 0.3 and μ_k = 0.2.



Block A :



Block B :



Block A : (*for part c.*)

- $a_A = a_B = a$ (both blocks move together)
- $T_A = T_B = T$ (massless string & Newton's 3rd law)
- $m_A a = T$ for Block A because N and W_A add to zero
- $m_B a = W_B - T = W_B - m_A a$

a. ***$a_A = a = W_B / (m_A + m_B) = 2.5\ m/s^2$***

b. ***$T = m_A a = 15\ N$***

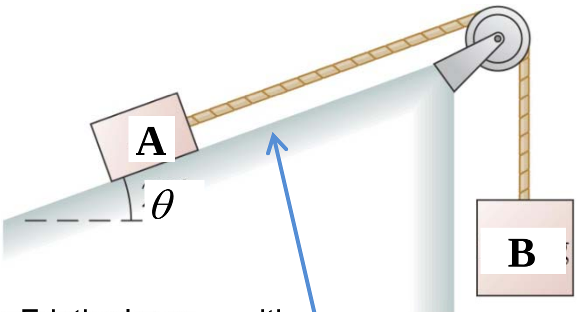
c. ***For part c., we need to add the frictional force to the diagram for Block A and modify its equation:***

- $m_A a = T - f_A = T - \mu N = T - \mu W_A$
- $T = m_A a + \mu W_A$ (from the relation just above)
- $m_B a = W_B - T = W_B - (m_A a + \mu W_A)$
- ***$a_A = a = (W_B - \mu W_A)/(m_A + m_B)$ = 0 unless $W_B > \mu_s W_A$***
- ***For this case, $a_A = 1.0\ m/s^2$***
- ***$T = m_A a + \mu_k W_A = 18\ N$***

Example: Block A has mass 6 kg, block B has mass 4 kg, the inclined surface is frictionless, and θ= 30°. (***g = 10 m/s***^{2})

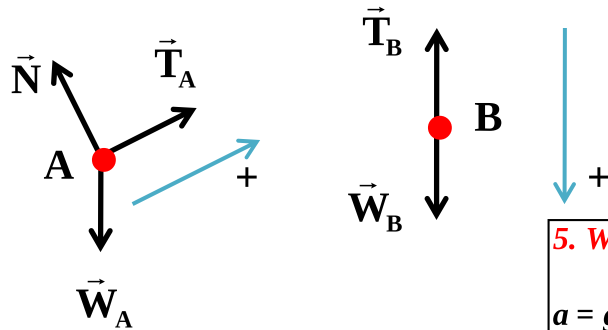
a. What is the acceleration of B?

b. What is the tension in the string?



Frictionless, or with coefficients μ_s and μ_k.

Analysis:



1. Separate $\vec{W}_A$ into components $\parallel$ and $\perp$ to the incline:

$$\vec{W}_A = \vec{W}_{A\parallel} + \vec{W}_{A\perp}$$

$$W_{A\parallel} = W_A \sin\theta \quad \& \quad W_{A\perp} = W_A \cos\theta$$

2. For A:

$$\perp \text{ incline: } m_A a_{A\perp} = 0 = N - W_{A\perp} = N - (m_A g)\cos\theta$$

$$\therefore\ N = (m_A g)\cos\theta \ \text{(needed for friction)}$$

$$\parallel \text{ incline: } m_A a_{A\parallel} = T_A - W_{A\parallel} = T_A - (m_A g)\sin\theta$$

3. For B:

$$m_B a_B = W_B - T_B = m_B g - T_B$$

$$T_B = m_B(g - a_B)$$

4. For A and B:

$$a_{A\parallel} = a_B = a \ \text{(objects move together)}$$

$$T_A = T_B = T \ \text{(massless string)}$$

$$T = T_B = m_B(g - a)$$

$$m_A a = T - (m_A g)\sin\theta = m_B(g - a) - (m_A g)\sin\theta$$

$\therefore$ *by algebraic rearrangement:*

$$a = g\left(\frac{m_B - m_A \sin\theta}{m_A + m_B}\right) \quad (= 1.0\ m/s^2 \text{ for the given \#'s})$$

$$T = g\left(\frac{m_A m_B}{m_A + m_B}\right)(1 + \sin\theta) \quad (= 36\ N \text{ for the given \#'s})$$

5. With friction:

$$a = g\left[\frac{m_B - m_A(\sin\theta + \mu\cos\theta)}{m_A + m_B}\right]$$

$$T = g\left(\frac{m_A m_B}{m_A + m_B}\right)(1 + \sin\theta + \mu\cos\theta)$$

Try it!!

Assignment:

Begin reading and working on Chapter 8.