P2211K

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Continuing the focus on Newton's 2nd Law: ***(Chapter 6)***

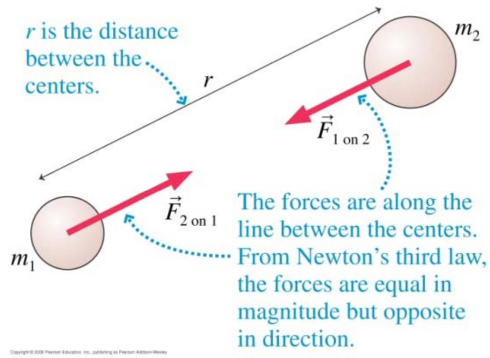
- Models, approximations, & variable forces

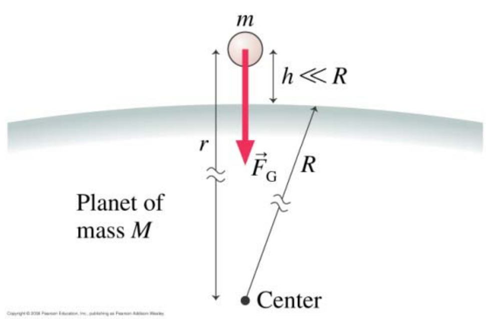
  - Gravity: $F_g = G\dfrac{m_1 m_2}{r^2} = m\left(G\dfrac{m_E}{R_E^2}\right) = mg$

  - Friction: $\left\{\begin{matrix} \Im_s < \mu_s N, \text{static friction} \\ \Im_k = \mu_k N, \text{kinetic friction} \\ \Im_r = \mu_r N, \text{rolling friction} \end{matrix}\right\}$, opposite to $\vec{v}$

  - Air resistance---"drag": $\begin{cases} D_A = kv^2, \text{ Air resistance; opposite to } \vec{v} \\ D_W = kv, \text{ Water resistance; opposite to } \vec{v} \end{cases}$

Gravity:





$$F_g = G\frac{m_1 m_2}{r^2} = m\left(G\frac{m_E}{R_E^2}\right) = mg$$

So,

$$g = \left(G\frac{m_E}{R_E^2}\right)$$

- So how can we justify treating g as a value independent of altitude (h) on the Earth?
- For us, h = 0 is the surface of the earth, or at r = R_E. The question becomes "what is the difference between g @ R_E and g @ R_E + h?" [i.e., $\Delta g = g(R_E+h) - g(R_E)$]:

If $g = \left(G\frac{m_E}{R_E^2}\right)$, then $\left(M_E = 5.98\times10^{24}\,kg \text{ and } R_E = 6.37\times10^{6}\,m\right)$

$$\frac{dg}{dR_E} = -2G\frac{m_E}{R_E^3} = \frac{-2}{R_E}\left(G\frac{m_E}{R_E^2}\right) = \left(\frac{-2}{R_E}\right)g$$

Δg when $\Delta R_E = h$ with $h \ll R_E$:

$$\frac{\Delta g}{\Delta R_E} \cong \frac{dg}{dR_E} \Rightarrow \Delta g \cong \left(\frac{dg}{dR_E}\right)\Delta R_E = \left(\frac{dg}{dR_E}\right)h = -2g\left(\frac{h}{R_E}\right)$$

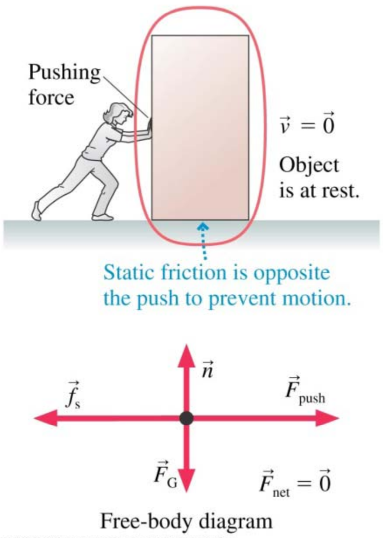
e.g., for $h = 100\ m$, $\Delta g \cong -2g\left(\frac{10^2}{6.37\times10^6}\right) \cong 3\times10^{-4}\,\frac{m}{s^2} \cong 0$

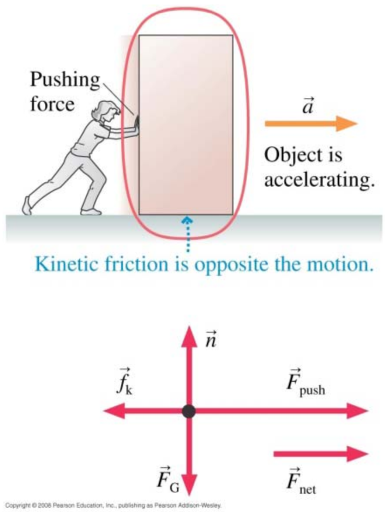
Friction: $\begin{Bmatrix} \Im_s < \mu_s N, \text{static friction} \\ \Im_k = \mu_k N, \text{kinetic friction} \\ \Im_r = \mu_r N, \text{rolling friction} \end{Bmatrix}$, opposite to $\vec{v}$

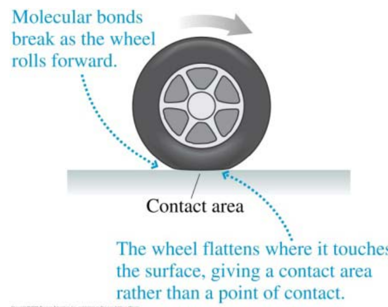
$$\begin{aligned} &\text{Static:} \quad \vec{f}_s \le (\mu_s n, \text{ direction as necessary to prevent motion}) \\ &\text{Kinetic:} \ \vec{f}_k = (\mu_k n, \text{ direction opposite the motion}) \\ &\text{Rolling:} \ \vec{f}_r = (\mu_r n, \text{ direction opposite the motion}) \end{aligned} \quad (6.15)$$

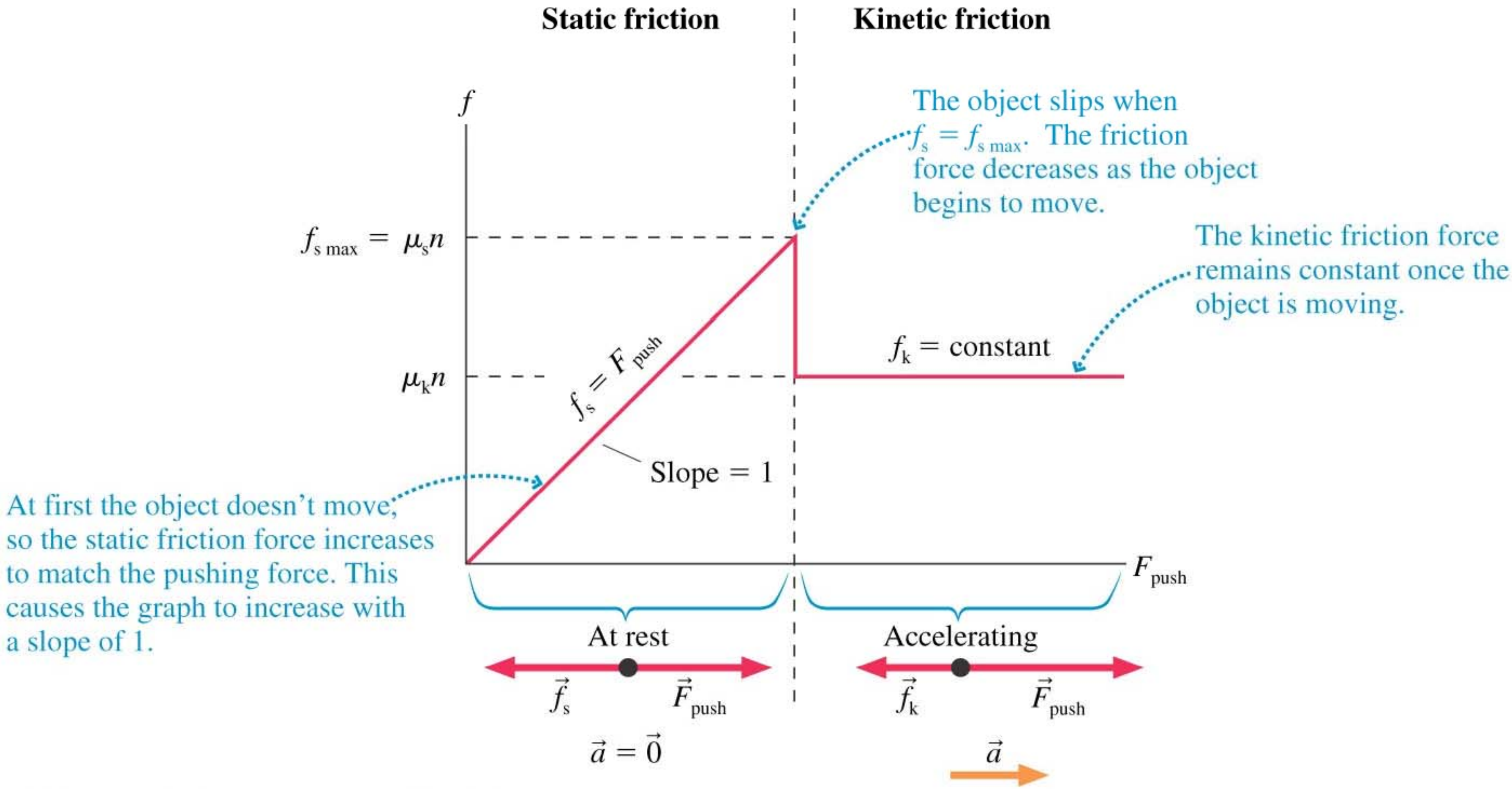
TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	





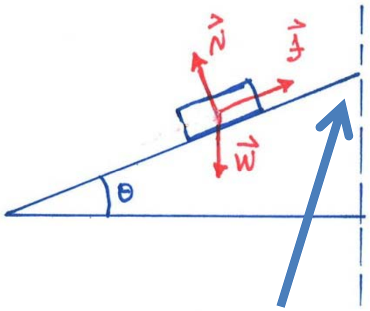


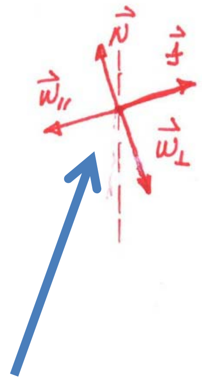


Static friction
Kinetic friction
f
$f_{s\,max} = \mu_s n$
$\mu_k n$
$f_s = F_{push}$
Slope = 1
The object slips when $f_s = f_{s\,max}$. The friction force decreases as the object begins to move.
The kinetic friction force remains constant once the object is moving.
f_k = constant
At first the object doesn't move, so the static friction force increases to match the pushing force. This causes the graph to increase with a slope of 1.
F_{push}
At rest
Accelerating
$\vec{f}_s$
$\vec{F}_{push}$
$\vec{f}_k$
$\vec{F}_{push}$
$\vec{a} = \vec{0}$
$\vec{a}$

Problem 6.20: A 4000 kg truck is parked on a 15° slope. How much is the friction force on the truck?

- Note that the angle indicated by the arrow is 90-θ so that $W_{||}$ = Wcos(90- θ). However, cos(90- θ) = sin θ, and Wsinθ is the usual way of expressing $W_{||}$.



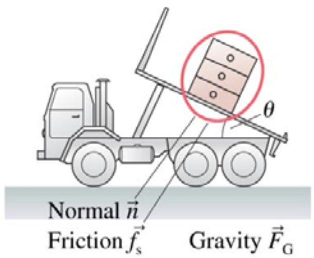


$$|| \ : 0 = \vec{W}_{||} + \vec{\mathfrak{I}} \Rightarrow 0 = mg\cos(90-\theta) - \mathfrak{I}$$

$$\mathfrak{I} = mg\cos(90-\theta) = mg\sin\theta = (4000kg)(9.8\ m/s^2)\sin 15°$$

$$\mathfrak{I} = 10146N$$

Assignment: Read & work through Example 6.7, pp. 166-167

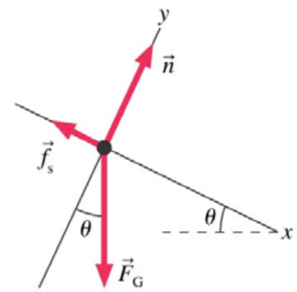


Known

$\mu_s = 0.80 \quad m = 50\ \text{kg}$
$\mu_k = 0.60$

Find

f_s where $\theta = 20°$
θ where cabinet slips



a. What is the acceleration of the object (file cabinet) when θ=20°?

b. What is its acceleration when θ=45°?

c. At what angle will it just begin to slide?

Solutions:

a. What is the acceleration of the object (file cabinet) when θ=20°?

Will the object slip at all?

From the diagram, $ma_{\parallel} = F_{net\parallel} = f_s - F_{G\parallel}$

$$\left.\begin{aligned} &f_s = \mu_s N \quad (\text{maximum } f_s) \\ &N = mg\cos\theta \\ &F_{G\parallel} = mg\sin\theta \end{aligned}\right\} \Rightarrow ma_{\parallel} = mg(\mu_s\cos\theta - \sin\theta)$$

$$a_{\parallel} = g(\mu_s\cos\theta - \sin\theta) = +4.09\ m/s^2 \quad (\text{for } \theta = 20°)$$

Interpretation: $a_{\parallel} > 0$ *in this case means that Max.* $f_s > F_{G\parallel}$

Thus the object does not slip and $a_{\parallel} = 0$.

b. What is its acceleration when θ=45°?

From part a.), max. $f_s = \mu_s\, mg\cos\theta = 282.8\ N$ *at* $\theta = 45°$

Also, at $\theta = 45°$, $F_{G\parallel} = mg\sin\theta = 353.6\ N$ $(F_{G\parallel} > \max f_s)$

Thus, the object slips and the acceleration is determined by μ_k :

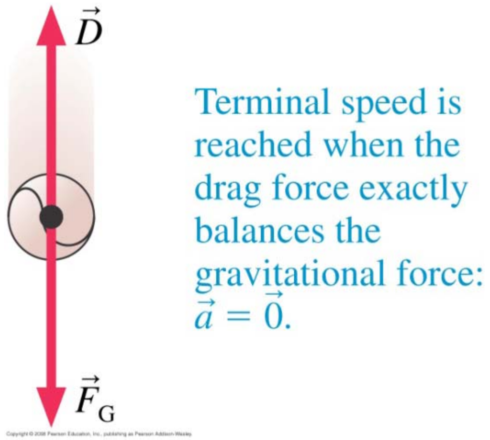
$a_{\parallel} = g(\mu_k\cos\theta - \sin\theta) = -4.25\ m/s^2$ *(" - " means* $a_{\parallel}$ *is down the bed)*

c. At what angle will it just begin to slide?

Also, from a.), the object slips when $F_{G\parallel} > f_s$ *, or*

$mg\sin\theta > \mu_s\, mg\cos\theta \Rightarrow \tan\theta > \mu_s$

Air resistance---"drag": $\begin{cases} D_A = kv^2\text{, Air resistance; opposite to } \vec{v} \\ D_W = kv\text{, Water resistance; opposite to } \vec{v} \end{cases}$



Eq. 6.16 in the book (p. 168):

Drag= ¼Av^2 (an approximate model, but it applies for some typical cases)

$$\vec{F}_{net} = \vec{D} + \vec{W} \quad (\vec{W} = \vec{F}_g)$$

$$ma = mg - kv^2 \text{ (using down as +)}$$

Note that a goes to 0 when the velocity rises enough:

$$ma = 0 = mg - kv_T^2$$

$$\text{Thus, } v_T = \sqrt{mg/k} \text{ is the "terminal" speed}$$

More on air resistance:

- We can see that the characteristic that Drag increases with speed logically leads to a maximum speed for an object acted on by other constant forces (such as gravity for “parachutes, etc., or vehicles with constant propulsion).
- However, how can we describe the velocity (or speed) of the object as it approaches the terminal velocity?

$$ma = mg - kv^2$$

Re-write a as $\dfrac{dv}{dt}$ and divide by m:

$$\frac{dv}{dt} = g - \frac{k}{m}v^2 \Rightarrow \frac{dv}{\left(g - \frac{k}{m}v^2\right)} = dt$$

This can be solved for $v(t)$ by integration as follows:

$$\int_{v_0}^{v(t)} \frac{dv}{\left(g - \frac{k}{m}v^2\right)} = \int_0^t dt$$

(It is important to correlate the integral limits on both sides of the equation. $v(t)$ is the velocity at time t, and v_0 is the initial value at $t = 0$.)

Assignment:

Begin reading and working on Chapter 7.