P2211K

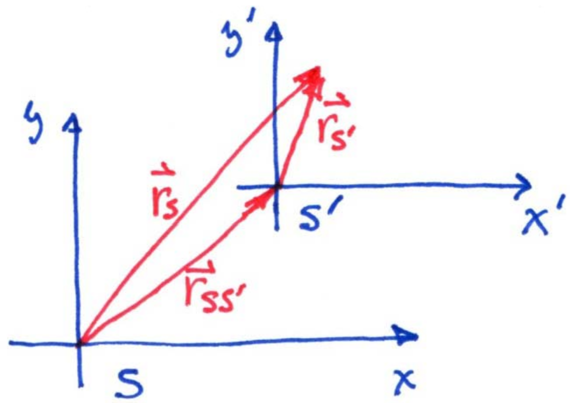
9 / 9 / 2010

(Excerpt from updated syllabus on-line at my website.)

POLICY ON CALCULATORS AND CELL PHONES FOR EXAMS: ***Programmable calculators, graphics calculators, hand held computers and any type of cell phone are not allowed for tests and exams. For these, you need to have a simple "scientific" calculator that has roots and trig functions. (These are available at Walmart, Office Depot, etc., for $10 - $15. An example is the TI-30 series.)***

POLICY ON CELL PHONES IN CLASS: ***Anyone texting or talking on a cell phone during class will be asked to leave.***

Relative position and relative motion:



Because coordinate systems (frames of reference) are not absolute and can be chosen to "suit the problem," it sometimes is useful to "transform" an object's description (position and / or velocity) from one system to another.

The sketch to the left shows the basis for this. (in the sketch, S represents one system and S' represents the other.) The sketch shows the relation between the objects position as expressed in S and S':

$$\vec{r}_S = \vec{r}_{S'} + \vec{r}_{SS'}$$

Similarly, the velocity can be expressed in the two systems by:

$$\vec{v}_S = \frac{d\vec{r}_S}{dt} = \frac{d\vec{r}_{S'}}{dt} + \frac{d\vec{r}_{SS'}}{dt} = \vec{v}_{S'} + \vec{v}_{SS'}$$

The term $\vec{v}_{SS'}$ expresses the velocity of S' relative to S.

Important note: If the relative velocity is not constant (i.e., if one is accelerating), life becomes difficult. We will consider only those cases of constant relative velocity. (Constant relative velocity = "inertial"; other cases are "non-inertial.")

Examples:

1. Coordinate systems S and S' are related according to $\vec{r}_{SS'} = 3\hat{i} + 7\hat{j}$. An object's position is described in S' as $\vec{r}_{S'} = 6\hat{i} - 9\hat{j}$. Describe its position relative to S.

$$\vec{r}_S = \vec{r}_{S'} + \vec{r}_{SS'} = \left(6\hat{i} - 9\hat{j}\right) + \left(3\hat{i} + 7\hat{j}\right) = 9\hat{i} - 2\hat{j}$$

2. $\vec{r}_{SS'}$ is the vector from the origin of S to that of S', thus it describes the origin of S' in system S. How is it related to the vector describing the origin of system S in system S'? $\vec{r}_{S'} = \vec{r}_S - \vec{r}_{SS'} = \vec{r}_S + \vec{r}_{S'S} \Rightarrow \vec{r}_{S'S} = -\vec{r}_{SS'}$

3. Coordinate systems S and S' have the relative velocity $\vec{v}_{SS'} = \left(\hat{i} + 5\hat{j}\right) m/s$. An object has velocity in S' described as $\vec{v}_{S'} = \left(4\hat{i} - 8\hat{j}\right) m/s$. Describe its velocity relative to S.

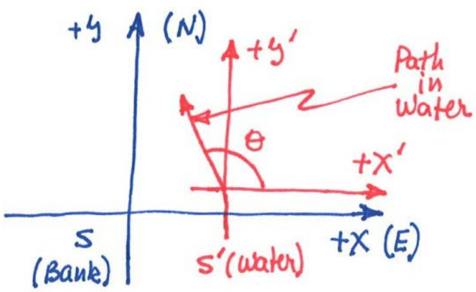
$$\vec{v}_S = \vec{v}_{S'} + \vec{v}_{SS'} = \left[\left(4\hat{i} - 8\hat{j}\right) + \left(\hat{i} + 5\hat{j}\right)\right] m/s = \left(5\hat{i} - 3\hat{j}\right) m/s$$

4. For the systems of #3, an object has velocity $\vec{v}_S = \left(8\hat{i} + 3\hat{j}\right) m/s$. What is its velocity as expressed in system S'?

$$\vec{v}_{S'} = \vec{v}_S - \vec{v}_{SS'} = \left[\left(8\hat{i} + 3\hat{j}\right) - \left(\hat{i} + 5\hat{j}\right)\right] m/s = \left(7\hat{i} - 2\hat{j}\right) m/s$$

Prob. 4.56. A kayaker needs to paddle north across a 100-m-wide harbor. The tide is going out, creating a tidal current that flows to the east at 2.0 m/s. The kayaker can paddle with a speed of 3.0 m/s.

a. In which direction should he paddle in order to travel straight across the harbor?



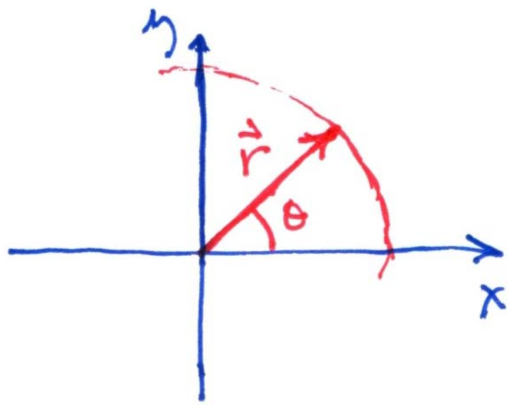
Let North = +y, East = +x, S be a system fixed to the bank, and S' be a system moving with the water. Then $\vec{v}_{SS'} = (2.0m/s)\hat{i}$ and $\vec{v}_{S'} = (3.0m/s)(\cos\theta\hat{i} + \sin\theta\hat{j})$. To go directly across, it is necessary that the x-component of $\vec{v}_S$ is 0. Thus, we need θ such that v_{sx} = 0 = (2.0 + 3.0 cosθ)m/s = 0. This requires cosθ = -2/3, or θ = 131.8°.

(Alternately, it could be approached by seeking θ to make d_{Sx} = 0.)

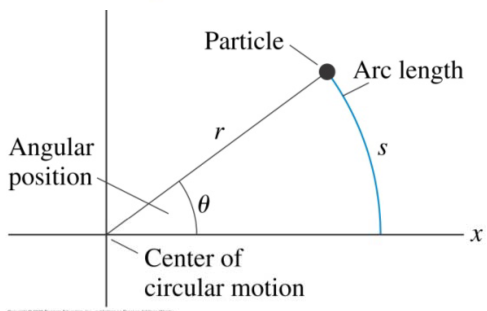
b. How long will it take him to cross?

T = d/v_{Sy} = 100m / (3 sinθ)m/s = 44.7 s

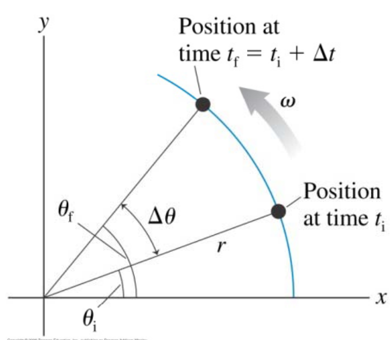
Uniform circular motion and rotational motion:



- A particularly important type of 2-dimensional motion is that of an object in a circular path. This is the starting point of discussions involving orbital motion, rotating objects, etc.
- Because the path is a circle it is convenient to describe its instantaneous position as $\vec{r} = r(\cos\theta\hat{i} = \sin\theta\,\hat{j})$ as shown in the sketch to the left.



- It is convenient to introduce the measure of θ in radians as defined by θ = arc / radius = s / r. (For a full circle, the arc length is the circumference = 2πr and the angle is 2π radians.)



- Also, because the is moving in its path, the value of θ constantly changes. To describe this, it is convenient to introduce the angular speed in radians / sec, which is symbolized by ω.

- In its circular path, the object's speed is v = ds/dt = d(rθ)/dt = r(dθ/dt = rω. Thus, there is a direct connection between the angular speed and the object's speed in its path.
- It is important to note that the object is accelerated even though its speed is constant—its direction is changing and therefore its velocity is not constant.
- Because r is constant (the path is a circle), and for the case of constant ω and v (uniform circular motion, the following relations connect the object's position, velocity, and acceleration: $\vec{r} = r(\cos\omega t\ \hat{i} + \sin\omega t\ \hat{j})$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega r(-\sin\omega t\ \hat{i} + \cos\omega t\ \hat{j})$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega^2 r(-\cos\omega t\ \hat{i} - \sin\omega t\ \hat{j})$$

- Although it is not obvious without additional vector techniques (such as the dot product), the velocity and position vectors are perpendicular to each other. This means that the velocity is tangent to the circular path because any vector perpendicular to the radius is a tangent (from basic geometry).
- It is obvious, however, that the acceleration is directed oppositely to the position vector; thus $\vec{a}$ points towards the center of the circular path. In effect, the circular path is the result of a constantly changing velocity due to this ***centripetal*** acceleration. Note also that the acceleration in this case is not constant—its direction is constantly changing.

Summary of circular motion:

- An object traveling in a circular path is accelerated. The evidence is that its direction changes and therefore it velocity is changing.
- If the path is a circle, and the speed is constant, the acceleration is towards the center of the path with magnitude:

$$a = \omega^2 r = \frac{v^2}{r}$$

- {Even if the speed changes, the relation holds for an arc sufficiently short that the speed is virtually constant over it.}
- The velocity is tangent to the path with magnitude:

$$v = \omega r$$

Example 1:

A car travels around a curve at 30 m/s (67.1 mi / hr).

a. If the first part of the curve is circular with radius 150m, what is the car's acceleration?

$$a = \frac{v^2}{r} = \frac{(30m/s)^2}{150m} = 6.0m/s^2$$

b. If the car's maximum acceleration is 9 m/s^2, what minimum curve's radius can it travel?

$$r = \frac{v^2}{a} = \frac{(30m/s)^2}{9.0m/s^2} = 100m$$

Example 2:

Laboratory centrifuges typically spin at a few thousand revolutions per minute (rpm). The objective is to create accelerations several time that of gravity. For example, if such a centrifuge spins at 3000 rpm and has the specimen at the end of an arm 10 cm (0.10 m) long, how many times g (=9.8 m/s^2) is the acceleration of the specimen?

$$a = \frac{v^2}{r} = \omega^2 r = \left(\frac{3x10^3 x 2\pi radians}{60s} \right)^2 (0.10m) = 9870m/s^2$$

$$\frac{9870m/s^2}{9.80m/s^2} = 1007 \quad !!!$$

Rotational Kinematics:

The motion of rotating objects, which we'll explore more fully later, are conveniently describes by angular parameters (measured in radians): angular position (θ), angular speed (ω), and angular acceleration (α). Angular acceleration, the rate at which angular speed changes is:

$$\alpha = \frac{d\omega}{dt}$$

With this suite of parameters, it is possible to develop a set of rotational relations for the condition of constant angular acceleration like those for linear motion under constant acceleration:

TABLE 4.1 Rotational and linear kinematics for constant acceleration

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha \Delta t$	$v_{fs} = v_{is} + a_s \Delta t$
$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$

Example:

A high-speed drill rotating ccw at 2000 rpm comes to a halt in 3.00s.

a. What is the drill's angular acceleration?

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_0}{\Delta t} = \frac{0 - \left(2x10^3 x 2\pi / 60\right) rad/s}{3s} = 69.8\ rad/s^2$$

b. How many revolutions does it make as it stops?

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \theta = \frac{\omega_f^2 - \omega_0^2}{2\alpha} = \frac{0 - \left(2x10^3 x 2\pi / 60\ rad/s\right)^2}{2\left(69.8\ rad/s^2\right)} = 314.159\ rad$$

$$314.159\ rad = 314.159/2\pi\ revolutions = 50\ revolutions$$