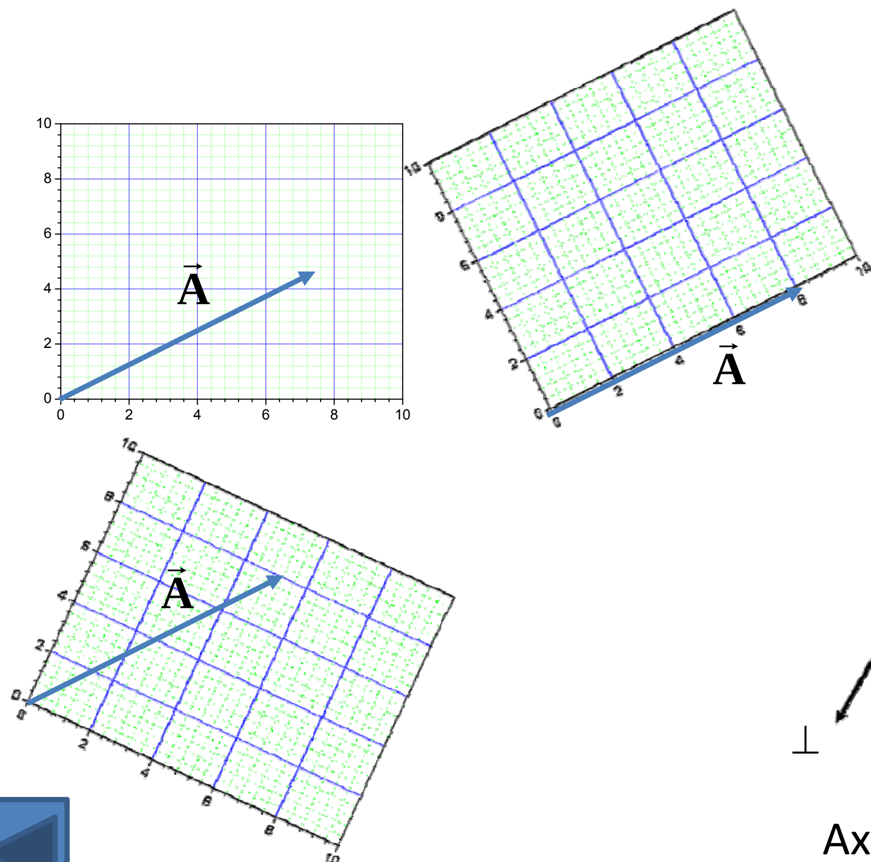
P2211K

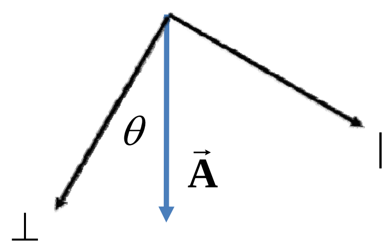
9 / 7 / 2010

More about coordinate systems:

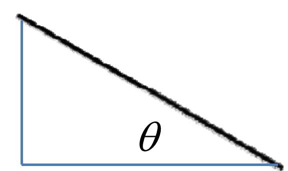
- Vectors may be expressed in a wide variety of Coordinate systems (an infinite variety, actually). The magnitude is the same in all, but the description of the direction is tied directly to the system in use.



- For example, the diagrams show the vector $\vec{\mathbf{A}}$ expressed in three different coordinate systems.
- This actually means that the coordinate system can be chosen to "fit" the situation at hand, an inclined plane for example.

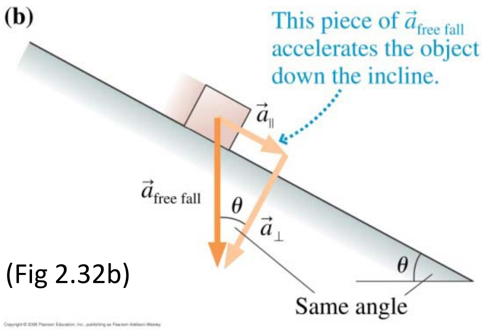


Axes Inclined || and $\perp$ to the plane



Plane Inclined @ θ

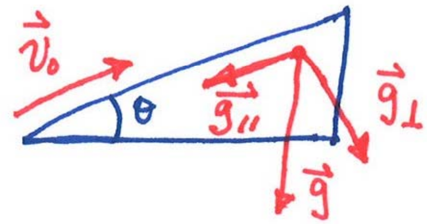
Prob. 2.20: A car traveling at 25 m/s runs out of gas while traveling up a 15° slope. How far up the hill will it coast before starting to roll back down?



(Fig 2.32b)

$$v_f^2 = v_0^2 + 2ad \Rightarrow 0 = (25m/s)^2 + 2(-g_{\parallel})d$$

$$\therefore d = \frac{(25m/s)^2}{2(-g_{\parallel})} = \frac{(25m/s)^2}{2(-g\sin 15°)} = 123.2m$$

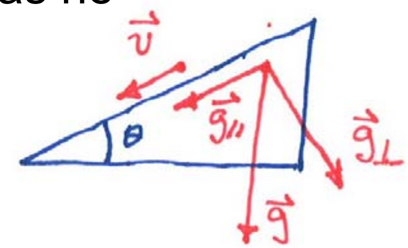


Prob. 2.55: Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of 30°. If Santa slides 9.0m before reaching the edge, what is his speed as he leaves the roof?

$$v_f^2 = v_0^2 + 2ad \Rightarrow v_f^2 = 0 + 2(g_{\parallel})d$$

$$\therefore v_f^2 = 2(9.8m/s^2)\sin 30°(9.0m) = 88.2m^2/s^2$$

$$\& \ v_f = 9.39m/s$$



Motion on an inclined plane actually is a situation needing a force-based analysis. We'll treat it more fully later with the discussions in Ch.'s 5 & 6.

Motion in 2 (or 3) dimensions (Ch. 4)

Basic Idea: Position, velocity, and acceleration, etc., are vector quantities an have components in more than one dimension in general.

Basic relations from 1 dimension transferred to 2 dimensions:

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_{\boldsymbol{0}} + \vec{\boldsymbol{a}}t \qquad \text{(for constant acceleration!!!)}$$

$$\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_{\boldsymbol{0}} + \vec{\boldsymbol{v}}_{\boldsymbol{0}}t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

These can be separated into two-dimensional components (x & y, for example) as follows:

$$\textbf{(eq. 1)} \quad \vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_{\boldsymbol{0}} + \vec{\boldsymbol{a}}t \Rightarrow \begin{cases} v_x = v_{x0} + a_x t \\ v_y = v_{y0} + a_y t \end{cases}$$

$$\textbf{(eq. 2)} \quad \vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_{\boldsymbol{0}} + \vec{\boldsymbol{v}}_{\boldsymbol{0}}t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2 \Rightarrow \begin{cases} r_x = r_{x0} + v_{x0}t + \tfrac{1}{2}a_x t^2 \\ r_y = r_{y0} + v_{y0}t + \tfrac{1}{2}a_y t^2 \end{cases}$$

The relation $v_f^2 = v_0^2 + 2ad$ is tricky and does not transfer directly to vectors because of the speed-squared terms. However, the following are true:

$$\begin{cases} v_{fx}^2 = v_{0x}^2 + 2a_x d_x \\ v_{fy}^2 = v_{0y}^2 + 2a_y d_y \end{cases}$$

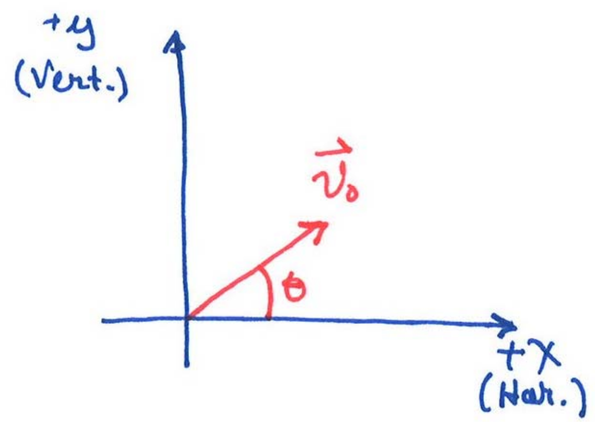
(In these relations, *a* is negative if the object is slowing down, and *a* is positive if it is speeding up. That is, if $v_f > v_0$, $a > 0$; if $v_f < v_0$, $a < 0$.)

$$\textbf{(eq. 1)} \quad \vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_{\boldsymbol{0}} + \vec{\boldsymbol{a}}t \Rightarrow \begin{cases} v_x = v_{x0} + a_x t \\ v_y = v_{y0} + a_y t \end{cases}$$

$$\textbf{(eq. 2)} \quad \vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_{\boldsymbol{0}} + \vec{\boldsymbol{v}}_{\boldsymbol{0}}t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2 \Rightarrow \begin{cases} r_x = r_{x0} + v_{x0}t + \tfrac{1}{2}a_x t^2 \\ r_y = r_{y0} + v_{y0}t + \tfrac{1}{2}a_y t^2 \end{cases}$$

The individual component relations in eq. 1 and eq.2 occur ***simultaneously***, so *t* is the link connecting the *x* and *y* components of the motion. (Technically, these are "*parametric equations*" where *t* is the parameter.) This makes it possible to eliminate *t* to obtain an equation for the path in *x, y* coordinates. We'll see this later.

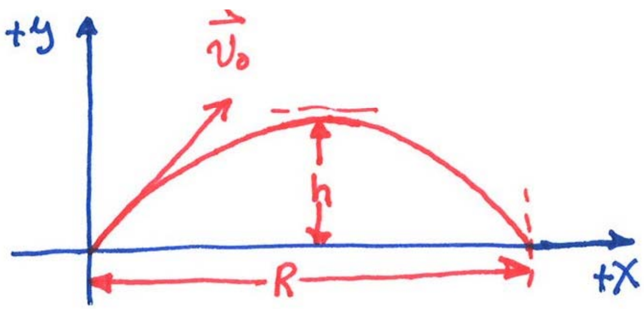
Applications and Examples



Describe the the motion of an object projected with initial velocity at the angle θ = 30° to the horizontal with initial velocity 30 m/s as shown in the sketch. (Think golf ball, for example.) Questions:

a. How long is it in the air?
b. How high does it go--***h***?
c. How far does it go—***R***?

(The motion remains in the *xy* plane because there is no component either of initial velocity or acceleration perpendicular to the plane.)



The object begins at y = 0 and returns there as determined by the y-component of v_0 and the influence of gravity.

a. How long is it in the air (the time-of-flight)?

$$r_y = 0 = 0 + (v_0 \sin\theta)t + \tfrac{1}{2}(-g)t^2 = (30\text{m/s})(\sin 30°)t \text{ - } (4.90\text{ m/s}^2)t^2$$

$$t = 0 \text{ and } t = \frac{(30\text{m/s})(\sin 30°)}{4.90\text{ m/s}^2} = 3.06\text{ s}$$

b. How high does it go--***h***?

Again, this is governed by v_{0y} and g in combination with the fact that the y-component of the velocity is 0 at the peak of the path. (It is the turning point of the vertical motion component.)

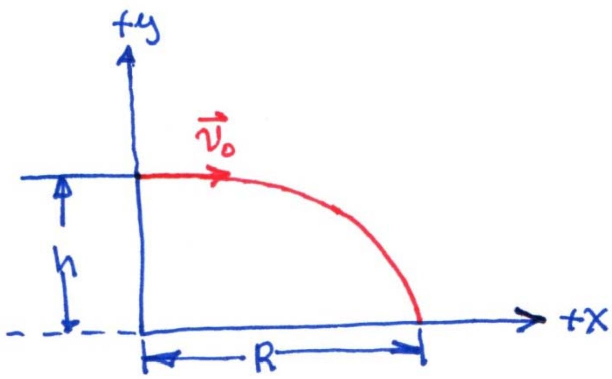
$$v_{fy}^2 = v_{0y}^2 + 2a_y d_y \Rightarrow 0 = \left[(30m/s)(\sin 30°)\right]^2 + 2(-9.8m/s^2)h$$

$$h = \frac{\left[(30m/s)(\sin 30°)\right]^2}{2(9.8m/s^2)} = 11.5\ m$$

c. How far does it go--***R***?

This is governed by the time-of-flight and v_{0x} since there is no acceleration component along x.

$$R = v_{0x}t = (30m/s)(\cos 30°)(3.06s) = 79.5\ m$$



Example 2: (re book's example 4.4 & problem 4.11)

An object is projected horizontally off a cliff at 30m/s.

a. If the cliff is 10 m high (h), how far does it travel before hitting the ground (what is R)?

b. If it hits the ground 50 m from the base of the cliff (R), how high is the cliff (what is h)?

Analysis: for this question, the time-of-flight is controlled by g and h (because there is no component of the initial velocity in the vertical direction), and R is controlled by v_0 and the time-of-flight.

a. $y_f = 0 = h + v_{0y}t + \frac{1}{2}(-g)t^2 = 10\text{m} + 0 - (4.90\ \text{m/s}^2)t^2$

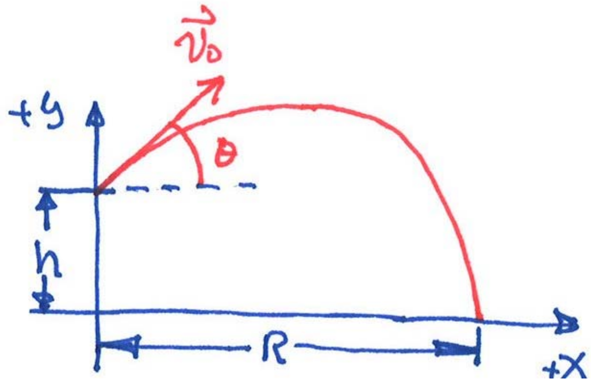
$t = \sqrt{\dfrac{10\ \text{m}}{4.90\ \text{m/s}^2}} = 1.43\ \text{s}$

$\text{R} = v_{0x}t = (30\ \text{m/s})(1.43\ \text{s}) = 42.86\ \text{m}$

b. $\text{R} = v_{0x}t = 50\ \text{m} = (30\ \text{m/s})t \Rightarrow t = \dfrac{50\ \text{m}}{30\ \text{m/s}} = 1.67\ \text{s}$

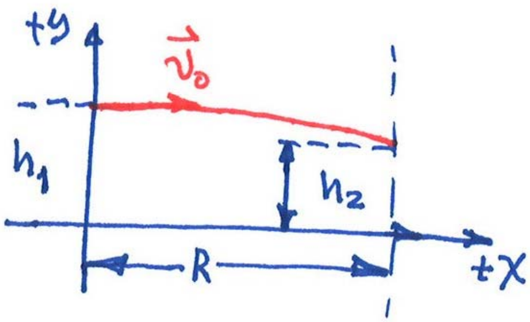
$\text{y}_\text{f} = 0 = \text{h} + \frac{1}{2}(\text{-g})t^2 \Rightarrow \text{h} = \frac{1}{2}(9.8\ \text{m/s}^2)(1.67\ \text{s})^2 = 13.6\ \text{m}$

Other Scenarios:



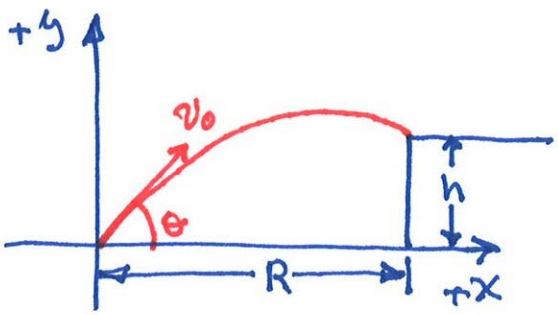
Example questions for scenario #1:

- Given v_0, θ, and h, what is R?
- Given v_0, θ, and R, what is h?
- Given v_0 and h, what θ is needed to reach R?
- Given v_0 and h, what θ maximizes R?



Example questions for scenario #2:

- Given v_0, R, and h_1, what is h_2?
- Given h_1, h_2, and R, what is v_0?
- Given h_1, h_2, and v_0, what is R?



Example questions for scenario #3:

- Given v_0, θ, and R, will the object get to the top of the ledge (@ y = h)?
- Given h, θ, and R, what minimum v_0 is necessary for the object to reach the ledge (y = h @ x = R)?
- Given R, v_0, and h, what θ is necessary for the object to reach the ledge (y = h @ x = R)?

Trajectory (flight path) for projectiles:

If we revisit eq. 2 above for the case $a_x = 0$ and a_y = constant (=-g), we can eliminate t and express the y position as a function of x:

$$x = 0 + v_{0x}t + 0 \quad \Rightarrow \quad x = v_{0x}t \ \Rightarrow t = x/v_{0x}$$

$$y = 0 + v_{0y}t + \tfrac{1}{2}(-g)t^2 \ \Rightarrow \ y = v_{0y}t - \tfrac{1}{2}gt^2$$

$$\left.\begin{array}{l} t = x/v_{0x} \\ y = v_{0y}t - \tfrac{1}{2}gt^2 \end{array}\right\} \Rightarrow y = v_{0y}\left(x/v_{0x}\right) - \tfrac{1}{2}g\left(x/v_{0x}\right)^2$$

$$y = \left(v_{0y}/v_{0x}\right)x - \left(\tfrac{1}{2}g/v_{0x}\right)x^2 = Ax - Bx^2$$

The resulting expression for y(x) is that of a downward-opening parabola with its vertex at the point of maximum height (the turning point). Thus, ***in the absence of air resistance***, the trajectory of a projectile ***near the surface of the earth*** is a parabola.

Speed along the flight path for projectiles:

- At any point in its path, the velocity is the vector sum of its x- and y-components. (Because $a_x = 0$, the x-component is constant, but the y-component changes due to g.) The magnitude of the velocity (speed) is $v^2 = v_x^2 + v_y^2$.
- For example, the "impact speed" can be calculated from v_{0x} and v_y at the time of impact.