P2211K

9 / 2 / 2010

About vectors and scalars:

- Scalars are mathematical objects having only magnitude (an ordinary number).
- Vectors are mathematical objects having ***magnitude*** AND ***direction.*** (The magnitude of a vector is a scalar.)
- When used to represent physical quantities, both scalars and vectors must have the appropriate units. (6 m/s, or 6 m/s north, for example.)

Properties of vectors:

- The magnitude of a vector is independent of the coordinate system, but the direction of a vector can be described only with respect to a specified coordinate system. (The starting point of a vector is not important.)
- Vectors may be added and subtracted;
- Vectors may be multiplied and divided by scalars;
- Later on, we will encounter two versions of vector-vector multiplication (the dot product and the cross product);
- Vector-vector division is not defined.

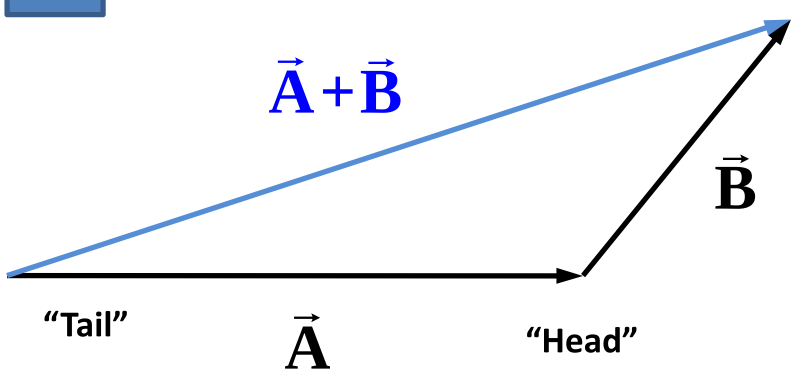
About vectors and scalars:

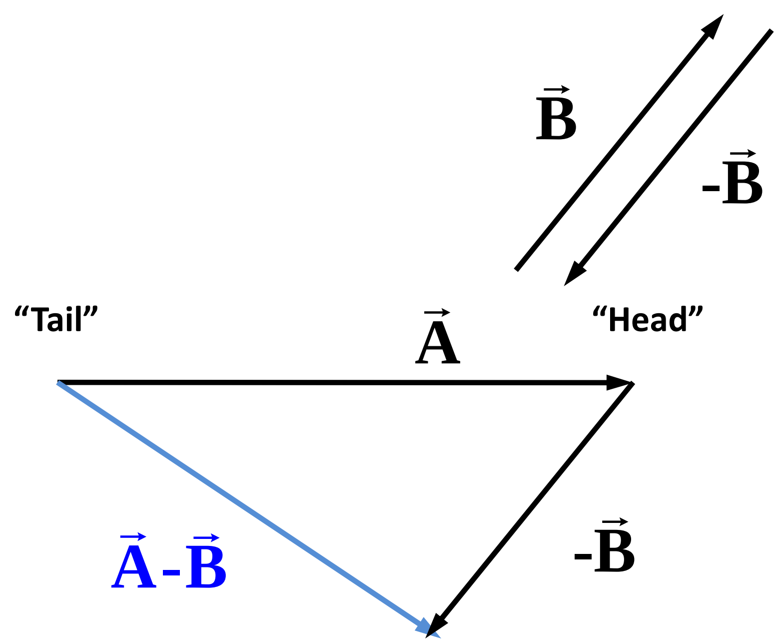
- Geometrically, vectors are added "tail-to-head." The ***resultant*** is the vector from the tail of the first to the head of the last.
- The negative of a vector has the same magnitude but the opposite direction;
- Vector subtraction is accomplished by adding the negative of a vector:

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$$

- Vector addition & subtraction are commutative and associative:

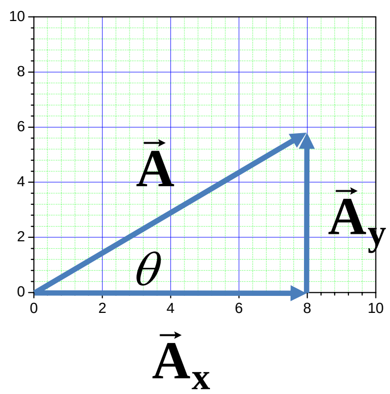
$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$ and $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = \vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}} = (\vec{\mathbf{A}} + \vec{\mathbf{C}}) + \vec{\mathbf{B}}$ etc.





Coordinate systems and vector components:

- Geometric vector addition and subtraction are inconvenient. Use of coordinate systems (or “frames of reference”) and the concept of vector “components” are far more convenient.
- The component vector concept is to think of any vector as the resultant of two perpendicular “components.” These all form a right triangle with all its geometric and trigonometric convenience.



- On the sketch, A_x and A_y, are the x- and y-components of A & $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$.
- This also makes the magnitude of A obvious: by the Pythagorean theorem, $|A|^2 = A_x^2 + A_y^2$.
- Also, it is clear that

$$A_x = |A| \cos\theta, \text{ and}$$
$$A_y = |A| \sin\theta$$

Unit vectors:

- It is useful to introduce unit vectors along the coordinate axes for vector algebra. (Unit vectors have magnitude 1 but carry the direction information. The unit vector along x is $\hat{\mathbf{i}}$ and that along y is $\hat{\mathbf{j}}$.)
- With this approach, $\vec{\mathbf{A}} = \mathbf{A}_x \hat{\mathbf{i}} + \mathbf{A}_y \hat{\mathbf{j}}$

Representing vectors:

- Vectors may be represented either by the component format or by their magnitude and direction: $\vec{\mathbf{A}} = \mathbf{A}_x \hat{\mathbf{i}} + \mathbf{A}_y \hat{\mathbf{j}}, \mathbf{or}\ \vec{\mathbf{A}} = |\mathbf{A}|, \theta$
- Interconversion between the two representations is straightforward via trigonometric approaches:

$$\mathbf{A_x} = |\mathbf{A}| \cos\theta \qquad |\mathbf{A}| = \sqrt{(\mathbf{A_x})^2 + (\mathbf{A_y})^2}$$

$$\mathbf{A_y} = |\mathbf{A}| \sin\theta \qquad \theta = \arctan\left(\frac{\mathbf{A_y}}{\mathbf{A_x}}\right)$$

Examples:

$\vec{A} = 5\hat{i} + 2\hat{j}$, and $\vec{B} = -3\hat{i} - 5\hat{j}$.

Calculate (in the $\hat{i}, \hat{j}$ and the magnitude, θ formats)

a. $\vec{C} = \vec{A} + \vec{B}$ ($\vec{C} = 2\hat{i} - 3\hat{j}$; or 3.6 @ -56.3° from +x)

b. $\vec{D} = 2\vec{A} + 3\vec{B}$ ($\vec{D} = \hat{i} - 11\hat{j}$; or 11.04 @ -84.8° from +x)

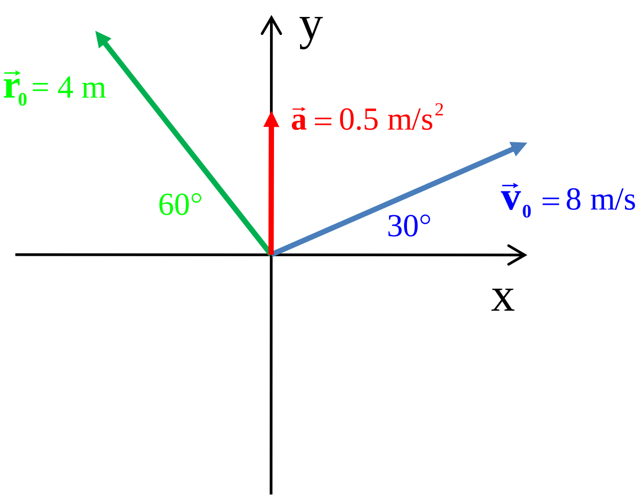
$\vec{A}$ = 8 units @ 30° to +x, and $\vec{B}$ = 10 units @ 120° to +x.

Calculate (in the $\hat{i}, \hat{j}$ and the magnitude, θ formats)

($\vec{A} = 6.93\hat{i} + 4\hat{j}$; $\vec{B} = -5\hat{i} + 8.67\hat{j}$)

a. $\vec{C} = \vec{A} + \vec{B}$ ($\vec{C} = 1.93\hat{i} + 12.67\hat{j}$; or 12.81 @ 81.3° from +x)

b. $\vec{D} = 3\vec{A} - 2\vec{B}$ ($\vec{D} = 30.79\hat{i} - 25.34\hat{j}$; or 39.88 @ -39.5° from +x)



a. Calculate the x- and y components of the initial velocity and initial position vectors;
b. Calculate the sum of the vectors shown;
c. Calculate the velocity after 20 s;.
d. Calculate the position after 20 s.

a. $\vec{\mathbf{v}}_0 = (6.93\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ **m/s**

$\vec{\mathbf{a}} = (0.5\,\hat{\mathbf{j}})$ **m/s^2**

$\vec{\mathbf{r}}_0 = (-2\hat{\mathbf{i}} + 6.93\hat{\mathbf{j}})$ **m**

b. Can't; not all the same type!!!

c. $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = [(6.93\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) + (0.5\hat{\mathbf{j}})(20)]$ **m/s**

$= (6.93\hat{\mathbf{i}} + 14\hat{\mathbf{j}})$ **m/s**

d. $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2 = [(-2\hat{\mathbf{i}} + 6.93\hat{\mathbf{j}}) + (6.93\hat{\mathbf{i}} + 4\hat{\mathbf{j}})(20) + \frac{1}{2}(0.5\hat{\mathbf{j}})(400)]$**m**

$= (136.6\hat{\mathbf{i}} + 186.93\hat{\mathbf{j}})$**m**