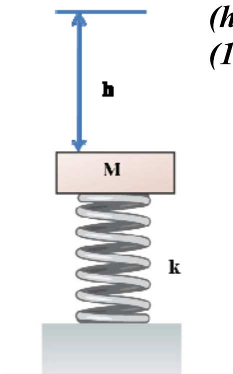
Physics 2211K
Test # 3
November 11, 2010

"I have neither given nor received help on this exam." **VERSION 3, SOLUTION**

In order to evaluate your progress in this course, I must see how you arrive at your answers. THEREFORE, YOU MUST SHOW YOUR WORK IN ORDER TO RECEIVE CREDIT FOR A QUESTION. Always circle your answer and show the units. {*Unless otherwise instructed, use g = 10 m/s² for all questions.*}

1. In the arrangement below, the spring constant is ***k = 1000 N/m*** and the mass ***M = 40 g***. If the spring is compressed ***Δs = 10 cm*** with the mass on it and then released (***from rest***), ***how high (h) above the compressed position will the mass go***? ***(Note: Be careful with the units!!) (10 pts)***



Basic idea :

$$\Delta K + \Delta U = \Delta K + \Delta U_g + \Delta U_{sp} = 0$$

$$@\ start,\ K = 0, U_g = 0, \text{and } U_{sp} = \tfrac{1}{2}k\Delta s^2 \quad \&\ @\ h, K = 0 \Rightarrow$$

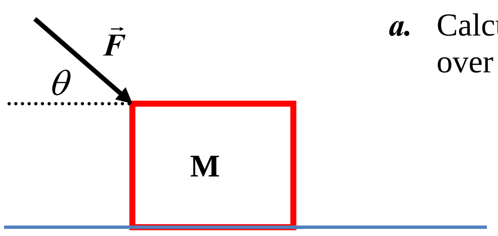
$$@\ h,\ K = 0,\ U_{sp} = 0,\ \text{and } U_g = mgh$$

$$\text{thus, } \Delta K = 0,\ \Delta U_{sp} = -\tfrac{1}{2}k\Delta s^2,\ \text{and } \Delta U_g = mgh$$

$$\Rightarrow \Delta U = mgh - \tfrac{1}{2}k\Delta s^2 = 0$$

$$\therefore\ mgh = \tfrac{1}{2}ks^2\ \&\ \boxed{h = \frac{\frac{1}{2}k\Delta s^2}{mg} = 12.5\ m}$$

2. In the sketch below, the applied force ***F = 100 N*** at ***θ = 30°*** acts on the object of mass ***M = 15 kg*** as it slides on the on the horizontal surface with friction. ***The effect of F and friction is a change in the object's speed from 4.0 m/s to 5.0 m/s over the distance d = 2.5 m.***



a. Calculate the change in kinetic energy and the work done by the force F over this distance; ***(10 pts)***

$$\boxed{\begin{aligned}&\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = 67.5\ J\\ &W_F = Fd\cos\theta = 216.5\ J\end{aligned}}$$

b. Using the results of ***part a.***, calculate the work done by friction and the frictional force. ***(10 pts)***

Basic idea :

$$\Delta K = W_T = W_F + W_f \Rightarrow W_f = \Delta K - W_F$$

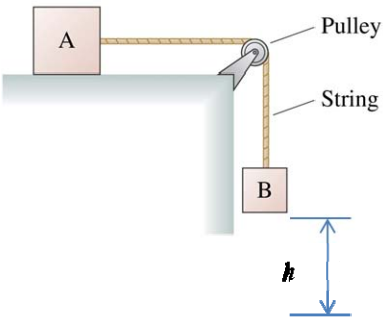
$$W_f = 36\ J - 155.9\ J = -149.0\ J$$

$$f = \frac{W_f}{d} = 59.6\ N$$

$$\text{vert. forces} = 0 = n - Mg - F\sin\theta \Rightarrow n = 200\ N$$

$$\mu_k = \frac{f}{n} = 0.298$$

3. In the system below, blocks ***A and B*** are connected by a ***massless string*** passing over a ***massless pulley***, and the horizontal surface is frictionless. ***Mass B = 6 kg and is observed to travel at 4.0 m/s after being released from rest and falling h = 2.0 m.***



a. What is the kinetic energy of block A? (10 pts)

Basic idea :

$$\Delta K + \Delta U = \Delta K_A + \Delta K_B + \Delta U_g = 0$$

$$@\ start,\ K_{Ai} = K_{Bi} = 0$$

$$@\ end,\ K_{Af} = \tfrac{1}{2} M_A v^2 \ \ \&\ \ K_{Bf} = \tfrac{1}{2} M_B v^2$$

$$@\ start,\ U_g = M_B gh \quad (choose\ y_i\ as\ h,\ which\ makes\ y_f = 0)$$

$$@\ end,\ U_g = 0 \ \ (choose\ y_f\ as\ 0)$$

$$\Rightarrow (K_{Af} - 0) + (K_{Bf} - 0) + (0 - M_B gh) = 0$$

$$\therefore \boxed{K_{Af} = M_B gh - K_{Bf} = 120\ J - 48\ J = 72\ J}$$

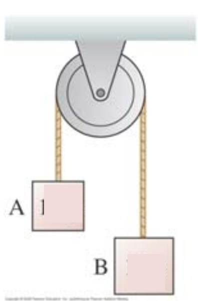
b. What is the mass of block A? (10 pts)

Basic idea :

$$K_{Af} = \tfrac{1}{2} M_A v^2 \Rightarrow \boxed{M_A = \frac{2K_{Af}}{v^2} = \frac{144\ J}{16\ m^2/s^2} = 9.0\ kg}$$

4. In the pulley arrangement below, ***mass A = 6 kg, mass B = 10 kg,*** and they are connected by a massless string. The pulley has radius ***R = 6 cm and unknown rotational inertia***. After starting from rest, ***B falls 2.0 m*** and the masses are observed to travel ***at $v_f = 2.0$ m/s at that moment***.

   a. ***Calculate the rotational kinetic energy of the pulley at this moment; (10 pts)***



Basic idea :

$$\Delta K + \Delta U = \Delta K_A + \Delta K_B + \Delta K_{Pulley} + \Delta U_{gA} + \Delta U_{gB} = 0$$

$$\Delta K_{Pulley} = -(\Delta K_A + \Delta K_B + \Delta U_{gA} + \Delta U_{gB})$$

All start from rest so that

$$K_{Af} = \Delta K_A = \tfrac{1}{2} M_A v_f^2\ ,\ K_{Bf} = \Delta K_B = \tfrac{1}{2} M_B v_f^2,\ \&\ K_{Pulley,f} = \Delta K_{Pulley}$$

$$\Delta U_{gA} = M_A gh \ \ (because\ it\ rises)$$

$$\Delta U_{gB} = -M_B gh \quad (because\ it\ falls)$$

$$K_{Pulley} = \Delta K_{Pulley} = gh(M_B - M_A) - \tfrac{1}{2}(M_B + M_A) v_f^2$$

$$\therefore \boxed{K_{Pulley} = 80\ J - 32\ J = 48\ J}$$

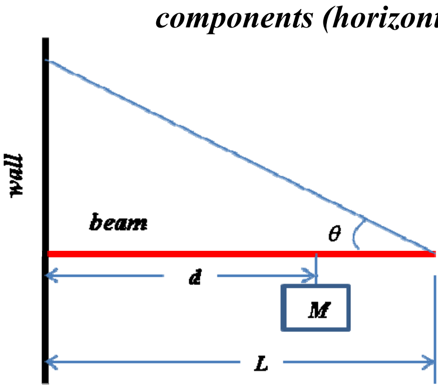
   b. ***Calculate the rotational inertia of the pulley. (10 pts)***

Basic idea :

$$K_{Pulley,f} = \Delta K_{Pulley} = \tfrac{1}{2} I \omega_f^2 = \tfrac{1}{2} I \left(\frac{v_f}{R_{pulley}} \right)^2$$

$$\therefore \boxed{I = 2K_{Pulley,f} \left(\frac{R_{pulley}}{v_f} \right)^2 = 124\ J \left(\frac{6 \times 10^{-2}\ m^2}{2.0\ m^2/s^2} \right)^2 = 8.6 \times 10^{-2}\ kg\ m^2}$$

5. In the system below, ***M = 250 kg, L = 5.0 m, d = 3.0 m, θ = 30°, the uniform beam has mass 150 kg, and the system is in static equilibrium. Calculate the tension in the cable and the components (horizontal and vertical) of the force supporting the beam at the wall. (10 pts)***



Forces

$$x:\ 0 = F_x - T\cos\theta \Rightarrow \boxed{F_x = T\cos\theta = 3897\ N}$$

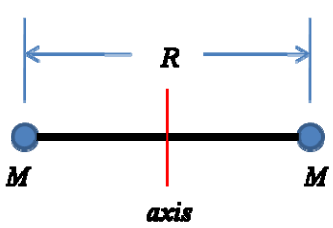
$$y:\ 0 = F_y + T\sin\theta - gM_{beam} - gM \Rightarrow \boxed{F_y = gM_{beam} + gM - T\sin\theta = 1750\ N}$$

Torques *(choose CR @ wall)*

$$\tau_{cw} = \tau_{ccw}$$

$$\left(\frac{L}{2}\right) gM_{beam} + dgM = LT\sin\theta \Rightarrow \boxed{T = \frac{\left(\frac{L}{2}\right) gM_{beam} + dgM}{L\sin\theta} = 4500\ N}$$

6. The dumbbell-shaped object below has ***M = 0.4 kg, R = 1.2 m***, and it rotates about the axis through its mid point at the rate ***50 rpm***. With an internal mechanism, ***R is suddenly changed to 0.6 m. Calculate the new rotational speed. (10 pts)***



Basic idea :

Angular momentum is conserved so that

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$$

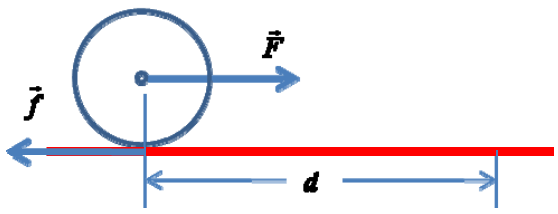
For point masses,

$I = MR^2$***, thus***

$I_i = 2MR_i^2$***, &*** $I_f = 2MR_f^2$***, so that***

$$\therefore \boxed{\omega_f = \left(\frac{R_i^2}{R_f^2}\right)\omega_i = \left(\frac{(0.6m)^2}{(0.3m)^2}\right) 50\ rpm = 200.0\ rpm}$$

7. The solid cylindrical object sketched below ***(M = 15 kg, R = 40 cm, and I=½MR²)*** is acted on, through its center of mass, by the (horizontal) force ***F = 300 N***. It ***rolls without slipping*** on the horizontal surface as indicated. If its ***initial speed*** (center of mass) is ***2.5 m/s, calculate its final speed after the force acts over the distance d = 4.0 m. (10 pts)***



Basic idea :

$$\Delta K = W_T$$

$$\Delta K = \Delta K_{linear} + \Delta K_{rot} = \left(\tfrac{1}{2}Mv_{cm,f}^2 - \tfrac{1}{2}Mv_{cm,i}^2\right) + \left(\tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2\right)$$

But, for rolling without slipping,

$$\omega = \frac{v_{cm}}{R}, \text{ so}$$

$$\Delta K = \left(\tfrac{1}{2}Mv_{cm,f}^2 - \tfrac{1}{2}Mv_{cm,i}^2\right) + \left[\tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_{cm,f}^2}{R^2}\right) - \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_{cm,i}^2}{R^2}\right)\right]$$

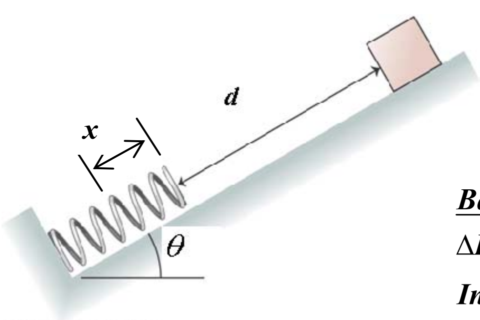
$$\Rightarrow \Delta K = W_T = \tfrac{3}{4}M\left(v_{cm,f}^2 - v_{cm,i}^2\right) \Rightarrow v_{cm,f}^2 = \frac{\tfrac{4}{3}W_T}{M} + v_{cm,i}^2$$

$$W_T = Fd = 1200\ J$$

$$\therefore \boxed{v_{cm,f} = \sqrt{\frac{\tfrac{4}{3}W_T}{M} + v_{cm,i}^2} = 10.6\ m/s}$$

Extra credit. You may earn up to 10 additional points for success with this problem. ***Do not attempt it until you have done your best on the rest of the exam.***

8. An ***M= 25 kg*** box slides (from rest) ***d = 4.5 m*** down the frictionless ramp with ***θ = 30°*** as shown in the figure, and then collides with a spring whose spring constant is ***k = 250 N/m***.



a. What is the ***maximum compression*** of the spring? **(Use g = 10 m/s²)**

Basic idea :

$$\Delta K + \Delta U = \Delta K + \Delta U_g + \Delta U_{sp} = 0$$

$$\textit{Initially, } K = 0 \ \ \& \ \textit{finally, } K = 0 \ \Rightarrow \Delta K = 0, \textit{ so}$$

$$\Delta K + \Delta U_g + \Delta U_{sp} = 0 \Rightarrow \Delta U_g + \Delta U_{sp} = 0 \Rightarrow \ \Delta U_{sp} = -\Delta U_g$$

$$\textit{Initially, } U_{sp} = 0 \ \ \& \ \ U_g = 0 \ (h = 0 \textit{ is M's starting point})$$

$$\textit{Finally, } U_{sp} = \tfrac{1}{2}kx^2 \ \ \& \ U_g = -Mg(d+x)\sin\theta \ \ (\textit{M has dropped this vertical distance})$$

$$\Delta U_g = -Mg(d+x)\sin\theta - 0$$

$$\Delta U_{sp} = \tfrac{1}{2}kx^2 - 0$$

$$\Delta U_{sp} = -\Delta U_g \Rightarrow \tfrac{1}{2}kx^2 = Mg(d+x)\sin\theta \Rightarrow \left(\tfrac{1}{2}k\right)x^2 - \left(Mg\sin\theta\right)x - Mgd\sin\theta = 0$$

$$\therefore \boxed{x = \frac{Mg\sin\theta \pm \sqrt{\left(Mg\sin\theta\right)^2 + 2kMgd\sin\theta}}{k} = 2.68\ m}$$

b. At what compression of the spring does the box have its ***maximum speed***?

Important note :

(The object continues to pick up speed until the accelerating effect of the spring overcomes that of gravity)

Method # 1 :

Find an expression for v(x) and take its derivative to find the x where it is maximum

$$\Delta K + \Delta U = \Delta K + \Delta U_g + \Delta U_{sp} = 0$$

$$\text{Initially, } K = 0 \ \& \text{ finally, } K = \tfrac{1}{2}Mv^2 \Rightarrow \Delta K = \tfrac{1}{2}Mv^2 \text{, so}$$

$$\Delta K + \Delta U_g + \Delta U_{sp} = 0 \Rightarrow \Delta K = -\left(\Delta U_g + \Delta U_{sp}\right)$$

$$\text{Initially, } U_{sp} = 0 \ \& \ U_g = 0 \text{ (for this, } h = 0 \text{ is M's starting position)}$$

$$\text{Finally, } U_{sp} = \tfrac{1}{2}kx^2 \ \& \ U_g = -Mg(d+x)\sin\theta \text{ (M has fallen this far at any moment after contacting the spring)}$$

$$\Delta U_g = 0 - Mg(d+x)\sin\theta$$

$$\Delta U_{sp} = \tfrac{1}{2}kx^2 - 0$$

$$\Delta K = -\left(\Delta U_g + \Delta U_{sp}\right) \Rightarrow \tfrac{1}{2}Mv^2 = Mg(d+x)\sin\theta - \tfrac{1}{2}kx^2$$

$$v = \sqrt{g(d+x)\sin\theta - \frac{\tfrac{1}{2}kx^2}{M}} = v(x)$$

$$v \text{ is max when } \frac{dv}{dx} = 0 \Rightarrow \text{using chain rule, etc.,} \Rightarrow 0 = g\sin\theta - \frac{kx}{M} \text{, or } x = \frac{Mg\sin\theta}{k}$$

$$\therefore \boxed{x = \frac{Mg\sin\theta}{k} = 0.50 \text{ m}}$$

Part *b.*, method 2: At what compression of the spring does the box have its *maximum speed*?

Important note :

(The object continues to pick up speed until the accelerating effect of the spring overcomes that of gravity)

Method # 2 :

Find the position where $F_g = F_{sp}$, which is where acceleration = 0 and the speed no longer increases.

$$F_g = Mg\sin\theta \text{, (down the incline)}$$

$$F_{sp} = kx \text{, (up the incline)}$$

$$\text{acceleration} = 0 \text{ when } Mg\sin\theta = kx$$

$$\Rightarrow \boxed{x = \frac{Mg\sin\theta}{k} = 0.50 \text{ m}}$$