Physics 2211K
Test # 1
September 16, 2010

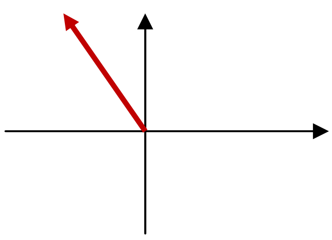
"I have neither given nor received help on this exam." *Name*________________________

In order to evaluate your progress in this course, I must see how you arrive at your answers. <u>Therefore, you must show your work in order to receive credit for a question</u>. Always circle your answer and show the units.

1. Vector $\vec{A} = 4\hat{i} - 3\hat{j}$ and vector $\vec{B} = 3\hat{i} + 2\hat{j}$. Calculate vector $\vec{C}$ such that $2\vec{A} + 3\vec{B} - \vec{C} = 0$. (Express $\vec{C}$ in the $\hat{i}, \hat{j}$ format.)

$$2\vec{A} + 3\vec{B} - \vec{C} = 0 \Rightarrow \vec{C} = 2\vec{A} + 3\vec{B}$$
$$\vec{C} = 2\vec{A} + 3\vec{B} = 2\left(4\hat{i} - 3\hat{j}\right) + 3\left(3\hat{i} + 2\hat{j}\right)$$
$$\vec{C} = \left(17\hat{i} - 0\hat{j}\right) = 17\hat{i}$$

2. Calculate the magnitude and direction of the vector $\vec{V} = -6\hat{i} + 8\hat{j}$. (Express the direction carefully.)



$$V^2 = V_x^2 + V_y^2 = (-6)^2 + (8)^2 = 100$$
$$V = 10 \;@\; \theta = tan^{-1}\left(\frac{-6}{8}\right) = -53.1° \text{ or } +127°$$
$$\left.\begin{matrix} V_x < 0 \\ V_y > 0 \end{matrix}\right\} 2^{nd} \text{ quadrant}; \quad \therefore \theta = +127°\text{, as shown}$$

3. An object traveling initially at ***30 m/s*** is stopped by the uniform acceleration ***6 m/s²***. How long does it take to stop?

$$v_f = v_0 + at \Rightarrow 0 = 30m/s - (6m/s^2)t \Rightarrow t = \frac{30m/s}{6m/s^2} = 5s$$

4. An object traveling initially at ***30 m/s*** is stopped by the uniform acceleration ***6 m/s²***. How far does it travel while stopping?

$$v_f^2 = v_0^2 + 2ad \Rightarrow 0 = \left(30m/s\right)^2 - 2(6m/s^2)d \Rightarrow d = \frac{\left(30m/s\right)^2}{12m/s^2} = 75\ m$$

5. A car traveling at ***15 m/s*** (approx. 35 mi/hr) is on a (circular) curve with radius ***100 m***. What (centripetal) acceleration is necessary for it to maintain this path?

$$a_c = \frac{v^2}{r} = \frac{(15m/s)^2}{100m} = 2.25\ m/s^2$$

6. An object travels in one dimension with the equation of motion $x(t) = (5t - 3t^3)m$. Calculate its velocity at $t = 4s$.

$$\text{if } x(t) = (5t - 3t^3)m, \text{ then } v(t) = \frac{dx}{dt} = (5 - 9t^2)\ m/s$$
$$\text{at } t = 4s, v(t) = \left[5 - 9(4)^2\right] m/s = -139\ m/s$$

7. An object travels in one dimension with the equation of motion $x(t) = (5t - 3t^3)m$. Calculate ***when*** it is at rest.

$$\text{From \# 6}, v(t) = \frac{dx}{dt} = (5 - 9t^2)\ m/s$$
$$\text{at rest means } v = 0, \text{ so}$$
$$v(t) = 0 = \left[5 - 9t^2\right] m/s, \text{ and}$$
$$t = \sqrt{5/9}\ s = 0.75\ s$$

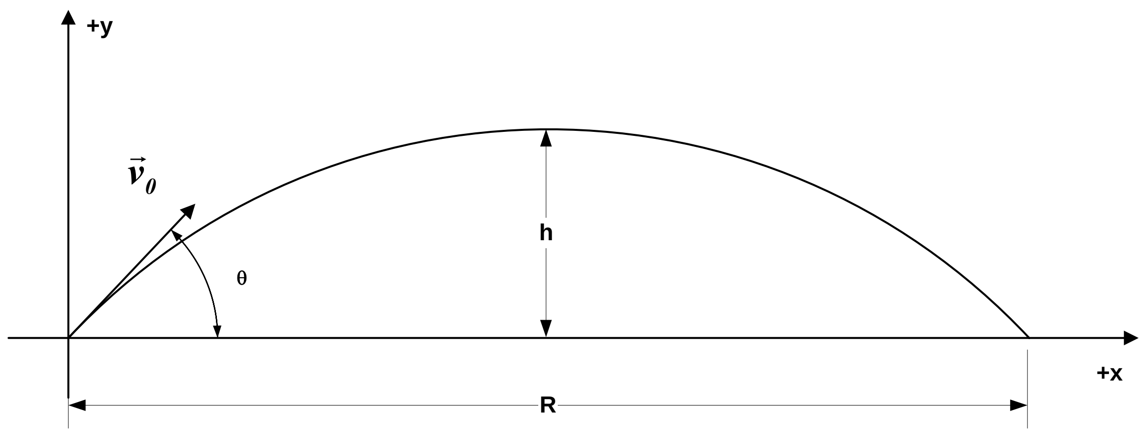
8. An object travels in one dimension with the equation of motion $x(t) = (5t - 3t^3)m$. Calculate ***where*** it is at rest.

$$\text{From \# 7, the object is at rest at } t = 0.75s$$
$$\text{if } x(t) = (5t - 3t^3)m, \text{ then}$$
$$\text{at } t = 0.75s, x(t) = \left[5(0.75) - 3(0.75)^3\right] ms = +2.49\ m$$

9. An object travels in one dimension with the equation of motion $x(t) = (5t - 3t^3)m$. Is its acceleration constant? (Yes or no & why you say so.)

$$\text{If } x(t) = (5t - 3t^3)m, \text{ then}$$
$$v(t) = \frac{dx}{dt} = (5 - 9t^2)\ m/s, \text{ and}$$
$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -18t\ m/s^2$$

NO---a is not constant because it depends on time: $a(t) = -18t\ m/s^2$



10. A golf ball is hit from the tee (on perfectly level ground) with initial velocity $\vec{v}_0 = 45\ m/s$ at the angle to the horizontal $\theta = 30°$. **(Use g= 10 m/s²)**

   a. Calculate the *x*- and *y*-components of the initial velocity.

$$v_{0x} = v_0 \cos\theta = (45\ m/s)\cos 30° = 38.97\ m/s$$
$$v_{0y} = v_0 \sin\theta = (45\ m/s)\sin 30° = 22.50\ m/s$$

   b. Write the equations of motion for ***x(t)*** and ***y(t)*** (using numbers and appropriate units for the velocity and acceleration components);

$$x(t) = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \ \textit{(for constant acceleration)}$$
$$x_0 = 0 \ \textit{(choice of origin)}$$
$$a_x = 0 \ \textit{for projectiles}$$
$$\therefore\ x(t) = 0 + (38.97\ m/s)t + 0 = (38.97\ m/s)t$$

$$y(t) = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \ \textit{(for constant acceleration)}$$
$$y_0 = 0 \ \textit{(choice of origin)}$$
$$a_y = -g = -10\ m/s^2 \ \textit{for projectiles}$$
$$\therefore\ y(t) = (22.50\ m/s)t - \left(5\ m/s^2\right)t^2$$

   c. Calculate how long it takes to reach the ground again after leaving the tee (the time-of-flight). ***The object is at ground level when y = 0:***

$$y(t) = 0 = (22.5\ m/s)t - (5\ m/s^2)t^2 = t\left[(22.5\ m/s) - (5\ m/s^2)t\right]$$
$$\Rightarrow\ t = 0\ \ \&\ t = \frac{22.5\ m/s}{5\ m/s^2} = 4.5\ s.$$
$$t = 0 \text{ is the start, so } t_f = \text{time-of-flight} = 4.5\ s$$

   d. Calculate how far it travels horizontally (**R**) before returning to the ground. ***Horizontal motion is controlled by x(t) & time-of-flight:***

$$R = v_{0x}t_f = (38.97\ m/s)\times(4.5\ s) = 175.4\ m$$

   e. Calculate its maximum height (**h**) in the trajectory. ***Maximum height is governed by motion in the y-direction, and h occurs when $v_y = 0$:***

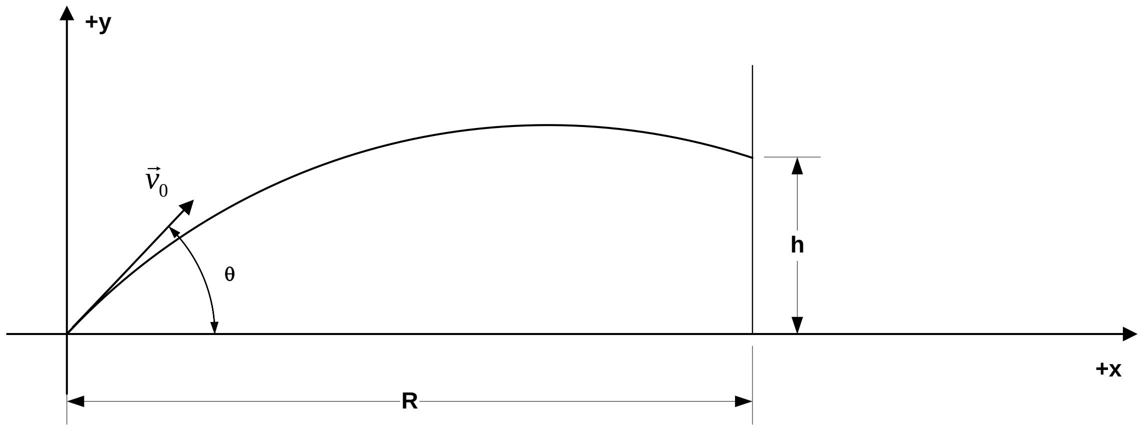
$$\left.\begin{array}{l} v_{fy}^2 = v_{0y}^2 + 2a_y d_y \\ v_{0y} = 22.50\ m/s \ \textit{(initial speed in y direction)} \\ v_{fy} = 0 \ \textit{(is at rest momentarily at max height)} \\ a_y = -10\ m/s^2 \ \textit{(acceleration due to gravity)} \\ d_y = h \ \ \textit{(y distance from start when } v_y = 0\textit{)} \end{array}\right\}\ h = \frac{v_{0y}^2}{-2a_y} = \frac{(22.5\ m/s)^2}{-2(-10\ m/s^2)} = 25.3\ m$$

   f. Calculate how fast it is traveling just before it hits the ground.

$$\textit{Need to calculate } v_{fx} \textit{ and } v_{fy} \textit{ to get } v_f^2 = v_{fx}^2 + v_{fy}^2 :$$
$$v_{fx} = v_{0x} = 38.97\ m/s \ \ \textit{because } a_x = 0$$
$$v_{fy} = v_y \textit{ at } t_f :\ v_{fy} = v_{0y} + a_y t_f = 22.5\ m/s - (5m/s^2)(4.5s) = -22.50\ m/s,$$
$$\textit{(Also, on return to the ground, } d_y = 0 \textit{ and } v_{fy}^2 = v_{0y}^2 + 2a_y d_y = v_{0y}^2\textit{)}$$
$$v_f = \sqrt{(38.97\ m/s)^2 + (-22.50\ m/s)^2} = 45.0\ m/s$$

Extra credit. You may earn up to 10 additional points for success with this problem. ***Do not attempt it until you have done your best on the rest of the exam.***

11. Georgia State's place kicker has just made a successful field goal from the line of scrimmage approximately at the **35 yard line**. In doing so, the football left the tee with $\vec{v}_0$ at the angle $\theta = 45°$ above the horizontal, traveled $R = 65m$ from the tee to the net behind the goalposts, and hit the net $h = 4m$ above the ground. **(Use g= 10 m/s²)**



a. How long was the football in flight (from leaving the tee to hitting the net)? ***Need to check the equations of motion to see how to proceed:***

$$x(t) = v_{0x}t \;\; (a_x = 0) \Rightarrow \; R = x(T) = (v_0 \cos\theta)\, T \;\; (T = time\text{-}of\text{-}flight)$$

$$y(t) = v_{0y}t + \tfrac{1}{2}a_y t^2 \Rightarrow h = y(T) = (v_0 \sin\theta)\, T - \tfrac{1}{2}gT^2$$

We know R, h, g, and θ; we want T and v_0.

Thus. we have a system of two equations and two unknowns.

From the x(T) equation, we have $v_0 = R/(T\cos\theta)$; Substituting that into y(T) gives

$$h = \left(\frac{R}{T\cos\theta}\right)(\sin\theta)\,T - \tfrac{1}{2}gT^2 = R\tan\theta - \tfrac{1}{2}gT^2, \text{and algebraic rearrangement gives}$$

$$T = \sqrt{\frac{2(R\tan\theta - h)}{g}} = 3.49\ s$$

b. How fast was it traveling (v_0) when it left the tee?

From the x(T) relation and T from part a.,

$$v_0 = \frac{R}{T\cos\theta} = 26.32\ m/s$$