P2211K Quiz # 1, FA 2010

Version 1: The driver of a car traveling at ***30 m/s*** sees a stoplight change to red. The person's reaction time between seeing the light and pressing the brakes is ***0.40 s***. The brakes allow the car to stop with the maximum constant acceleration rate ***-5.0 m/s²***.

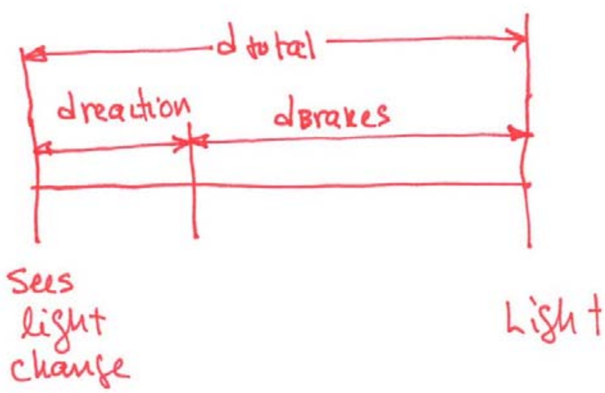
- Calculate the minimum total stopping distance for the car from the time the driver first sees the light.

Version 2: The driver of a car traveling at ***20 m/s*** sees a stoplight change to red when it is ***80.0 m*** from the car. The person's reaction time between seeing the light and pressing the brakes is ***0.50 s***.

- Calculate the minimum (constant) acceleration rate for the car to stop in time after the brakes are first applied.

Version 3: The driver of a car traveling at ***30 m/s*** sees a stoplight change to red when it is ***75.0 m*** from the car. The brakes allow the car to stop with the maximum constant acceleration rate ***-8.0 m/s²***.

- Calculate the maximum reaction time for the driver between seeing the light and pressing the brakes in order to stop in time.



SOLUTIONS:

Basic Analysis: All three versions have in common that the car travels the distance $d_{reaction}$ at its initial speed v_0 before the driver can press the brakes. When the brakes are pressed, the car accelerates at a constant rate to bring it to a stop with $v_{final} = 0$.

Thus, the total distance to come to a complete stop after the driver sees the light is

$$d_{total} = d_{reaction} + d_{brakes}$$

In all cases, the distance the car travels during the reaction time is $d_{reaction} = v_0 t_{reaction}$. The distance traveled with the brakes on is most conveniently obtained from the relation $v_{final}^2 = v_0^2 + 2ad$ following rearrangement to the form $d_{brakes} = \frac{v_{final}^2 - v_0^2}{2a} = -\frac{v_0^2}{2a}$, because $v_{final} = 0$.

Version 1 asks for the stopping distance given the initial speed (*30 m/s*), the reaction time (*0.40 s*), and the braking acceleration (*-5.0 m/s²*). With these values, the result is

$$\boldsymbol{d_{total} = d_{reaction} + d_{brakes} = (30\,m/s)(0.4\,s) - \frac{(30\,m/s)^2}{2(-5m/s^2)} = 12m + 90\,m = 102\,m}$$

Version 2 asks for the minimum acceleration rate to stop given the initial speed, the reaction time, and the distance to the light.   This is the acceleration rate to stop exactly at the light because a rate any slower will not stop before getting there.

In this case, the acceleration rate must stop the car within the braking distance,

$$d_{brakes} = d_{total} - d_{reaction} = 80\, m - (20\, m/s)(0.5\, s) = 70m = -\frac{v_0^2}{2a} = -\frac{(20m/s)^2}{2a}.$$

Thus,

$$70m = -\frac{(20\, m/s)^2}{2a} \rightarrow \boldsymbol{a} = -\frac{\mathbf{200}}{\mathbf{70}}\, \boldsymbol{m/s^2} = \mathbf{-2.86}\, \boldsymbol{m/s^2}$$

Version 3 asks for the maximum reaction time to allow stopping before reaching the light given the initial speed, the distance to the light, and the acceleration rate. This is the time to stop at the light because a reaction any slower will stop beyond it.

In this case the reaction time must be quick enough to allow the brakes to be applied before going beyond the maximum reaction distance

$$d_{reaction} = d_{total} - d_{brakes} = 75m - \left(-\frac{v_0^2}{2a}\right) = 75\, m - \frac{(30\, m/s)^2}{16m/s^2} = 75m - 56.25\, m = 18.75m$$

Thus, the reaction time must be short enough that the car travels no farther than 18.75 m at 30 m/s. This means that it must be no longer than $\frac{\mathbf{18.75}}{\mathbf{30}}\, \boldsymbol{s} = \mathbf{0.63}\, \boldsymbol{s}$