1. In the system sketched below, block A slides on a *frictionless surface* and the string connecting the two blocks is *massless*.



- *a.* Sketch the *free-body diagrams for masses A and B* showing all the forces acting on them.
- *b*. Write the *F=ma* relations for masses A and B based on the free-body diagrams of part a.
- c. If $M_B = 2 \ kg$ and the tension in the string $T = 12 \ N$, calculate the acceleration of the objects. $4 \ m/s^2$
- *d.* If $M_B = 2 \ kg$ and the tension in the string $T = 12 \ N$, calculate M_A. $\boxed{3 \ kg}$
- 2. For the system sketched above, $M_B = 4 kg$, $M_A = 8 kg$, $\mu_s = 0.4$, and $\mu_k = 0.2$. $(g = 10 m/s^2)$
 - a. Sketch the *free-body diagrams for masses A and B* showing all the forces acting on them.
 - **b.** Write the *F*=*ma* relations for masses A and B based on the free-body diagrams of part a.
 - c. Calculate the tension in the string and the acceleration of the objects if they begin from rest. $a = 2 m/s^2$; T = 16 N
 - *d*. How would part *c*. be different if $\mu_s = 0.6$, and $\mu_k = 0.4$? $a = 0 m/s^2$; T = 40 N
 - e. Calculate how far M_B travels in 4 s if it begins from rest in part c. d = 16 m
- 3. On the sketch below, the sloping surface is frictionless.



- *a.* Draw free-body diagrams for *masses A and B* showing *all the forces* acting on each.
- *b*. Use the free body diagram to write the *F=ma* relations for both A and B.
- c. If $M_A = 2M_B$, calculate the *angle* θ at which the *acceleration is zero*. $\theta = 30^{\circ}$

- 4. Re. the set of objects sketched below:
 - *a.* Draw free-body diagrams for each of the masses shown (concentrate only on the horizontal forces);
 - **b.** Use the free-body diagrams to write the *F***=ma** relations for each mass;
 - c. From the F=ma relations and Newton's 3^{rd} law, develop expressions for the accelerations of each object;
 - *d.* Develop an expression for the forces of contact with which each object acts on the one adjacent to it.
 - e. Test your expressions for the case F = 70 N, $M_1 = 2$ kg, $M_2 = 4$ kg, and $M_3 = 8$ kg by calculating the acceleration and the forces of contact.

$$\vec{F} = 1 \quad 2 \quad 3$$

- 5. Re. the object sketched below:
 - *a.* Calculate the object's acceleration from rest if the surface is frictionless, M = 12 kg, and

F = 50 N.
$$a = 4.16 \text{ m/s}^2$$

b. Repeat *a.*) for the case $\mu_s = 0.5$ and $\mu_k = 0.2$. $a = 0 \text{ m/s}^2$
c. Repeat *a.*) for the case $\mu_s = 0.35$ and $\mu_k = 0.1$. $a = 3.17 \text{ m/s}^2$

- 6. Re. the object sketched below:
 - *a*. Calculate the object's acceleration from rest if the surface is frictionless, M = 12 kg, and

F = 50 N.
$$a = 4.16 \text{ m/s}^2$$

Repeat *a.*) for the case $\mu_s = 0.5$ and $\mu_k = 0.2$.
 \vec{F} M
 $a = 0 \text{ m/s}^2$

c. Repeat *a.*) for the case $\mu_s = 0.35$ and $\mu_k = 0.1$. $a = 3.17 \text{ m/s}^2$

b.

- 7. Re. the object sketched below:
 - *a.* Sketch the free-body diagram for the object shown · showing all forces including friction;
 - b. Use the free-body diagram to write the F = ma relations (for horizontal & vertical directions);
 - c. Use the F=ma relations to calculate the object's acceleration from rest if the surface is
 - frictionless, $\theta = 30^\circ$, M = 4 kg, and F = 50 N. a = 10.8 m/s²
 - *d*. Repeat *c*.) for the case $\mu_s = 0.5$ and $\mu_k = 0.2$. $a = 7.6 \text{ m/s}^2$
 - e. Repeat c.) for the case $\mu_s = 0.35$ and $\mu_k = 0.1$. $a = 9.33 \text{ m/s}^2$
 - f. Repeat d.) for the case M = 8 kg. $a = 0 \text{ m/s}^2$
 - *d*. Challenge question: for given values of θ , *M*, *and F*, find an expression for the largest value μ_s can have if there will be any acceleration at all.
- 8. Re. the object sketched below:
 - *a.* Sketch the free-body diagram for the object shown showing all forces including friction;
 - **b.** Use the free-body diagram to write the F = ma relations (for horizontal & vertical directions);
 - c. Use the F=ma relations to calculate the object's acceleration from rest if the surface is frictionless, $\theta = 30^\circ$, M = 4 kg, and F = 50 N. a = 10.8 m/s²
 - *d*. Repeat *c*.) for the case $\mu_s = 0.5$ and $\mu_k = 0.2$. $a = 10.1 \text{ m/s}^2$
 - *e*. Repeat *c*.) for the case $\mu_s = 0.35$ and $\mu_k = 0.1$. $a = 10.5 \text{ m/s}^2$



$$\vec{F}$$
 M

- 9. Re. the object sketched below:
 - a. Sketch the free-body diagram for the object shown showing all forces including friction;
 - **b.** Use the free-body diagram to write the F = ma relations for M_1 and M_2 (for horizontal & vertical directions);



c. For the case $M_1 = 12 \ kg$, $M_2 = 4 \ kg$, and $F = 50 \ N$ calculate the acceleration of each object from rest if there is no friction either between M_1 and the horizontal surface or M_1 and M_2 . $\boxed{a_1 = 4.2 \ m/s^2; \ a_2 = 0 \ m/s^2}$

d. Repeat part *c*.) for the case of friction between
$$M_1$$
 and the horizontal surface ($\mu_s = 0.25$ and $\mu_k = 0.1$) and between M_1 and M_2 ($\mu_s = 0.5$ and $\mu_k = 0.2$). $\boxed{a_1 = a_2 = 2.13 \text{ m/s}^2}$

e. Repeat part *d*.) for the case
$$F = 200 N$$
. $a_1 = 8 m/s^2$; $a_2 = 2 m/s^2$

10. Re. the object sketched below:



- a. Sketch the free-body diagram for each mass showing all forces including friction;
- b. Use the free-body diagrams to write the F = ma relations for each mass (for horizontal & vertical directions);
- *c*. For the case of *no friction*, F = 500 N, $M_1 = 12 \text{ kg}$, $M_2 = 8 \text{ kg}$, and $M_3 = 10 \text{ kg}$, calculate the acceleration and the tension in each string (T_1 and T_2);

$$a = 16.7 m/s^2$$
; $T_1 = 300 N$; $T_2 = 166.7 N$

d. Repeat *c*.) for the case of friction for all objects characterized by $\mu_s = 0.5$ and $\mu_k = 0.2$;

$$a = 14.7 \text{ m/s}^2$$
; $T_1 = 300 \text{ N}$; $T_2 = 166.7 \text{ N}$

e. Repeat *c*.) for the case of friction for each object characterized by ($\mu_s = 0.5$ and $\mu_k = 0.2$) for M₁, ($\mu_s = 0.6$ and $\mu_k = 0.1$) for M₂, and ($\mu_s = 0.35$ and $\mu_k = 0.1$) for M₃.

 $a = 15.3 \text{ m/s}^2$; $T_1 = 292.8 \text{ N}$; $T_2 = 162.7 \text{ N}$

- 11. An object with *mass = 4 kg* is whirled in a vertical circular path using a string *1.2 m long*.
 - a. Sketch free-body diagrams for the object at the bottom of its path and at the top of the path;
 - **b.** Write the *F*=*ma* relations for the object at the top and the bottom of its path;
 - *c*. Use the *F*=*ma* relations to calculate the tension in the string when the object travels at $v_t = 5 \text{ m/s}$ at the top and bottom of its path. $T_{top} = 43.2 \text{ N}; T_{bottom} = 123.2 \text{ N}$
 - *d*. Use the *F=ma* relations to find the minimum speed the object can have at the top of its path for it to remain circular. $v_{min, top} = 3.46 \text{ m/s}$
- 12. Mass $M_1 = 4 kg$ travels at $v_1 = 20 m/s$ in the +x direction and collides with $M_2 = 2kg$ at rest. After the collision, they stick together. *Calculate the velocity of the combined masses after the collision.* $v_f = 13.3 m/s, +x direction$
- 13. Mass $M_1 = 4 kg$ travels at $v_1 = 20 m/s$ in the +x direction and collides with $M_2 = 2kg$ traveling at $v_2 = 30 m/s$ in the +y direction. After the collision, they stick together. *Calculate the*

velocity of the combined masses after the collision. $\vec{v}_f = (13.3 \,\hat{i} + 10 \,\hat{j}) \, m/s$

14. Mass $M_1 = 4 kg$ travels at $v_1 = 20 m/s$ in the +x direction and collides with $M_2 = 2kg$ traveling at $v_2 = 15 m/s$ in the -x direction. After the collision, M_1 has velocity components $v_{1fx} = +15$ m/s and $v_{1fy} = -5 m/s$. Calculate the velocity of M_2 after the collision.

 $\vec{v}_{2f} = \left(20\,\hat{i} + 25\,\hat{j}\right)m/s$