Theoretical analysis of phase-matched second-harmonic generation and optical parametric oscillation in birefringent semiconductor waveguides

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We analyze the phase-matching conditions for second-harmonic generation (SHG) and optical parametric oscillation (OPO) in birefringent nonlinear semiconductor waveguides and apply these results to the model system of ZnGeP₂ on a GaP substrate. The analyses and numerical results show that phase matching can be achieved for OPO and SHG for reasonable guide thicknesses throughout much of the infrared, indicating significant potential applications for nonlinear birefringent waveguides. For the fundamental mode of a relatively thick guide the region of phase matching and the phase-matching angles are similar to those in bulk material. However, the waveguide has the added flexibility that phase-matched coupling can occur between the various modes of the guide. For example, the phase-matching region for SHG can be considerably extended by coupling the pump into the guide in the fundamental, m = 0, mode and phase matching to the m = 2 mode of the second harmonic. Significantly, the results indicate, among other things, that ZnGeP₂ waveguides with harmonic output in the m = 2 mode can be used for efficient SHG from input radiation in the 9.6–10.6- μ m region where bulk efficiencies in this wavelength range are too small to be useful. © 2001 Optical Society of America

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1. Introduction

Chalcopyrite semiconductors are promising for optical frequency conversion applications in solid-state laser systems, such as optical parametric oscillators and frequency-doubling devices.^{1,2} ZnGeP₂ is an excellent nonlinear optical material that exhibits good optical transparency across the 0.7–12- μ m wavelength region and is therefore well suited for producing tunable laser output in the near infrared.^{3–8} The feasibility of constructing ZnGeP₂–GaP heterostructures with appropriate refractive-index profiles for waveguiding was reported several years back.⁹

In this paper we examine the phase-matching conditions for second-harmonic generation (SHG) and optical parametric oscillation (OPO) in birefringent nonlinear waveguides and apply that analysis to the

specific case of ZnGeP₂–GaP waveguides. Although the numerical results that we obtain are specific for a given waveguide structure, the theoretical development should be applicable for other nonlinear waveguide configurations as well. To achieve phase-matched conditions, we consider that the optical axis of the ZnGeP₂ lies in the plane of the guide. In that case the angle between the direction of propagation of the wave in the guide and the optical axis of the guide material can be adjusted to achieve the phase-matching condition. As shown below, we found that this can be accomplished for both SHG and OPO in ZnGeP₂ guides on GaP. Moreover the waveguide geometry affords an additional opportunity for phase matching not available in bulk material. It is possible to couple radiation between different modes of the guide. For example, in SHG one can, in principle, couple the long-wavelength radiation into the fundamental, m = 0, mode of the guide and adjust the propagation direction to obtain phase matching to the m = 2 mode of the guide at the second harmonic. We found that this can greatly extend the wavelength region over which phasematched SHG can be achieved in a planar waveguide of ZnGeP₂.

In Section 2 we outline the general theory for the various modes. In Section 3 we give the results and

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discuss the theory applied to the model system of $ZnGeP_2$ on a GaP substrate. In Section 4 we present the conclusions.

2. Theory

If the optical axis of the uniaxial, nonlinear guide material lies in the plane of the guide, optical propagation in the guide will be governed by two indices of refraction, one of which is dependent on the direction of propagation. If the wave is polarized perpendicular to the guide, the TM mode, the index of the guide will be related to the ordinary index of the guide material. This index is independent of the direction of propagation and is referred to as $n_{\rm TM}$. If the wave is polarized in the plane of the guide, the TE mode, the index will depend on the direction of propagation with respect to the optical axis of the guide material. This index is referred to as $n_{\rm TE}$. When standard equations¹⁰ for step-index waveguides are used, $n_{\rm TM}$ and $n_{\rm TE}$ are given by

$$\begin{aligned} kt(n_o^2 - n_{\rm TM}^2)^{1/2} &= (m+1)\pi \\ &- \tan^{-1} \bigg[\frac{n_s^2 (n_o^2 - n_{\rm TM}^2)^{1/2}}{n_o^2 (n_{\rm TM}^2 - n_s^2)^{1/2}} \bigg] \\ &- \tan^{-1} \bigg[\frac{n_c^2 (n_o^2 - n_{\rm TM}^2)^{1/2}}{n_o^2 (n_{\rm TM}^2 - n_c^2)^{1/2}} \bigg], \end{aligned}$$
(1)

$$kt(n^{2} - n_{\rm TE}^{2})^{1/2} = (m+1)\pi - \tan^{-1} \left[\frac{(n^{2} - n_{\rm TE}^{2})^{1/2}}{(n_{\rm TE}^{2} - n_{s}^{2})^{1/2}} \right] - \tan^{-1} \left[\frac{(n^{2} - n_{\rm TE}^{2})^{1/2}}{(n_{\rm TE}^{2} - n_{c}^{2})^{1/2}} \right], \qquad (2)$$

where

$$\left(\frac{1}{n}\right)^2 = \left(\frac{\cos\theta}{n_o}\right)^2 + \left(\frac{\sin\theta}{n_e}\right)^2,\tag{3}$$

 $n_{\rm o}$ and $n_{\rm e}$ are the ordinary and the extraordinary wave indices of the bulk guide material, $n_{\rm s}$ is the index of the substrate material, n_c is the index of the cladding, t is the guide thickness, $k = 2\pi/\lambda$ is the free-space wave vector of the radiation, and m is the mode number for the wave in the guide. In addition, θ is the angle between the propagation direction and the optical axis of the guide material.

Using Eqs. (1)–(3) and knowledge of n_o and n_e for the guide material, n_s for the substrate, and n_c for the cladding as a function of wavelength and choosing a value of guide thickness and propagation direction, one can calculate the values of $n_{\rm TM}$ and $n_{\rm TE}$ for the guide. These can then be used to obtain phasematching conditions for the different nonlinear processes.

We chose to examine the specific examples of OPO and SHG type I phase matching in a ZnGeP_2 planar waveguide on a GaP substrate and with GaP cladding. The optical (or *z*) axis of the ZnGeP_2 lies in the plane of the guide as does the *x* (or *y*) axis for the optimum type 1 nonlinear coupling. For SHG type I phase matching the guide is pumped in the TE mode



Fig. 1. Calculated phase-matching angles for SHG and OPO in a 16- μ m ZnGeP₂ waveguide with a GaP substrate and cladding layer compared with similar curves for bulk material.

with propagation at an angle Θ with respect to the optical axis of the ZnGeP₂, and phase matching occurs for angle Θ when

$$n_{\rm TE}(2\lambda) = n_{\rm TM}(\lambda). \tag{4}$$

For OPO type I phase matching the guide is pumped in the TM mode with propagation at angle Θ with respect to the optical axis of the ZnGeP₂, and phase matching occurs for angle Θ when

$$\frac{1}{\lambda_P} = \frac{1}{\lambda_S} + \frac{1}{\lambda_I},\tag{5}$$

where λ_p is the wavelength of the pump, λ_S is the wavelength of the output signal, λ_I is the wavelength of the idler wave, and

$$\frac{n_{\rm TM}(\lambda_P)}{\lambda_P} = \frac{n_{\rm TE}(\lambda_{\rm s})}{\lambda_{\rm s}} + \frac{n_{\rm TE}(\lambda_I)}{\lambda_I}.$$
 (6)

3. Results and Discussion

Using fits to the literature values¹¹ for the ordinary and the extraordinary indices of bulk ZnGeP_2 (Ref. 12) and GaP (Ref. 13) and Eqs. (1)–(6), we predict the phase-matching angles shown in Fig. 1 for SHG and OPO for a 16-µm planar ZnGeP₂ guide on GaP operation in the m = 0 mode. The corresponding phase-matching angles for bulk ZnGeP₂ are shown for comparison. These results show phasematching angles that are quite similar to those of bulk ZnGeP₂ (Refs. 3–8) as would be expected for a



Fig. 2. Calculated phase-matching angles for SHG in the $16\mu m$ ZnGeP₂ waveguide of Fig. 1 pumped in the m = 0 mode with output in the m = 2 mode compared with the m = 0 to m = 0 coupling of Fig. 1 and that of the bulk material.

guide this thick. This simply shows that phasedmatched SHG and OPO should be readily obtainable in waveguides of $ZnGeP_2$ that operate in the m = 0 mode. The applicability of the bulk indices for the waveguide will depend on the guide thickness, method of growth, degree of doping, lattice mismatch, and internal strain. Bulk indices should be a good approximation for a wellconstructed 16-µm-thick guide with reasonable lattice match as could be obtained with $ZnGeP_2$ on GaP.

However, we can also examine the case in which coupling into the guide occurs in one mode of the guide, the nonlinear process within the guide couples energy to another mode, and coupling out of the guide occurs from the second mode. Although we have not calculated in detail the strength of coupling, we estimate that nonlinear coupling from, for example, the m = 0 to the m = 2 mode could be as large as 20% of the coupling to the m = 0 mode of the guide for similar phase-matching conditions. For the purpose of illustration, we consider the case of SHG where the input wave is in the m = 0 mode with the second-harmonic output considered in the m = 2 mode. In this case we use the value of $n_{\text{TM}}(\lambda)$ for the m = 2 mode but $n_{\text{TE}}(2\lambda)$ for the m =0 mode. The resulting phase-matching angle is plotted in Fig. 2 along with the SHG curve from Fig. 1 and for the bulk material for comparison. Note that in this case phase matching for SHG can be obtained over a wider wavelength region than for m = 0 to m = 0 or for bulk ZnGeP₂.

4. Conclusions

In conclusion, we have theoretically analyzed nonlinear propagation in birefringent waveguide material for the generation of coherent IR laser light by second-harmonic generation and by optical parametric oscillation. The specific guide material used in the calculation was $ZnGeP_2$ on a GaP substrate. This structure has the added flexibility that phasematched coupling can occur between the various modes of the guide. Moreover the coupling is a function of the guide thickness. These added degrees of freedom will allow for high gain in conditions in which it is difficult or impossible to achieve in bulk material. The results in Fig. 1 indicate that for a 16-µm waveguide the phase-matching angles for SHG and OPO in the m = 0 mode are not significantly changed from those of the bulk material given in Refs. 14-16. However, the phase-matching angles for SHG coupling from the m = 0 to the m = 2mode for a 16-µm guide shown in Fig. 2 are significantly different and much more attractive than those of the bulk material. For example, Fig. 2 indicates that type I SHG from 10.6-µm radiation occurs at a phase-matching angle of ~ 48 deg, which is nearly optimal. In bulk material, SHG from radiation in the 10.6- μ m region is virtually unachievable as the phase-matching angle approaches 90 deg. Our results thus indicate, among other things, that ZnGeP₂ waveguides with harmonic output in the m = 2 mode can be used for efficient SHG from input radiation around 10.6 µm where bulk efficiencies in this wavelength range are too small to be useful. The realization of such a structure would thus pave the way for myriad novel optoelectronic devices.

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