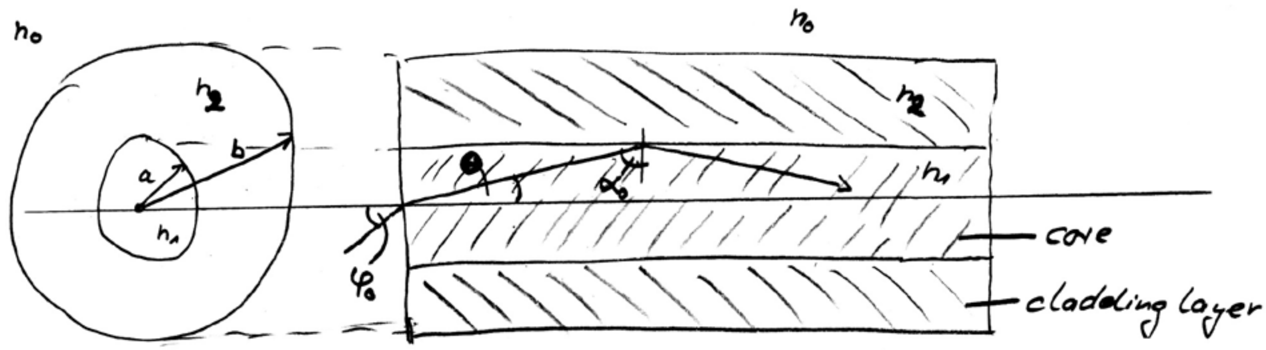
Optical Fibers



In order to get total internal reflection at the interface core – cladding we require $n_2 < n_1$

↳ critical angle: $\sin\alpha_0 = n_2/n_1$

In order to launch light from outside ($n_0 = 1$) into the core of the fiber, the launch angle can be determined according to Snell's law:

$$\frac{\sin\varphi_0}{\sin\theta} = \frac{n_1}{n_0}$$

↳ $\sin\varphi_0 = n_1 \cdot \sin\theta = n_1 \cdot \sqrt{1-\sin^2\theta}$

use $\theta = 90 - \alpha_0$ and $\sin\alpha_0 = n_2/n_1$

to get $\sin\varphi_0 = \sqrt{n_1^2 - n_2^2}$

The numerical aperture NA of a typical fiber: (core ≈ 10 μm, Cladding ϕ = 125 μm)

$$NA = n_1 \cdot \sqrt{2\Delta} \approx 1.46 \cdot \sqrt{2 \cdot 0.003} \approx 0.113$$

⇒

$\rightsquigarrow$ with an acceptance angle φ_0 of

$$\sin\varphi_0 = NA \approx 0.113 \quad \rightsquigarrow \quad \varphi_0 \approx \underline{\underline{6.5^\circ}}$$

Modes of Propagation of lights in fiber:

Again: Maxwell equations

$$\left.\begin{aligned} \nabla\cdot\vec{E} &= \rho/\varepsilon_0 \\ \nabla\cdot B &= 0 \\ \nabla\times\vec{E} &= -\partial B/\partial t \\ \nabla\times\vec{B} &= \frac{1}{c^2}\frac{\partial E}{\partial t} + \mu_0 J \end{aligned}\right\} \quad \begin{aligned} \vec{E} &= -\nabla V - \frac{\partial\vec{A}}{\partial t} \\ \vec{B} &= \nabla\times\vec{A} \end{aligned}$$

$\hookrightarrow$ Wave Equation for dielectric medium

$$\Delta A - \frac{1}{v_p}\frac{\partial^2}{\partial t^2}A = 0$$

Optical fiber represented in cylindrical polar coordinates:

$$x = r\cdot\cos\phi, \quad y = r\cdot\sin\phi, \quad z = z \ (\text{fiber axis})$$

$$\hookrightarrow \quad \Delta\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\frac{\partial\vec{A}}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2\vec{A}}{\partial\phi^2} + \frac{\partial^2\vec{A}}{\partial z^2}$$

Solution: $\quad A_n = (A_r, A_\phi, A_z) \longrightarrow \begin{matrix}(E_r, E_\phi, E_z) \\ (H_r, H_\phi, H_z)\end{matrix}$

time dependency

$$A_n = A_n(r,\phi)\,\exp[i(\omega t - \beta\cdot z)]$$

$$\hookrightarrow \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + (k^2 - \beta^2)\right]\cdot A_z = 0 \qquad \Rightarrow$$

$\rightsquigarrow$ separation of variables:

$$A_z(r,\phi) = A_z(r)\cdot \exp(\pm i\,\ell\cdot\phi)\ , \quad \ell = 0,1,2,\dots$$

$$\hookrightarrow \boxed{\frac{\partial^2 A_z(r)}{\partial r^2} + \frac{1}{r}\,\frac{\partial A_z(r)}{\partial r} + \left(k^2 - \beta^2 - \frac{\ell^2}{r^2}\right)\cdot A_z(r) = 0}$$

Bessel Differential Equation

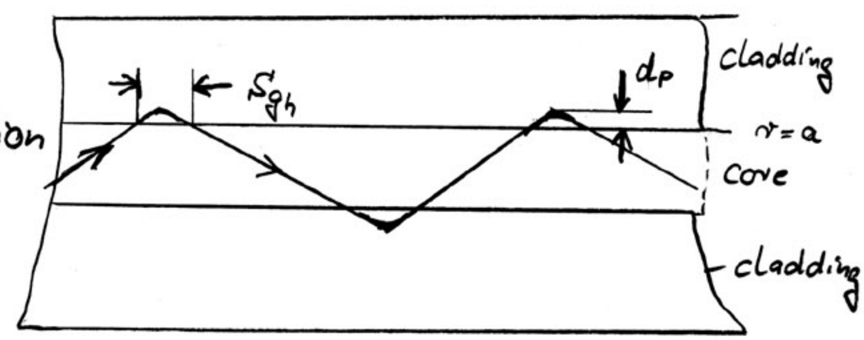
Solutions: $A_z(r) = C_1\, J_\ell(h\cdot r) + C_2\, Y_\ell(h\cdot r)\ , \quad h^2 = k^2 - \beta^2 > 0$

$A_z(r) = C_1\cdot I_\ell(q\cdot r) + C_2\, K_\ell(q\cdot r)\ , \quad -q^2 = k^2 - \beta^2 < 0$

with J_ℓ, Y_ℓ : Bessel functions of first and second kind of order.

I_ℓ, K_ℓ : modified Bessel functions

<u>Boundary Conditions:</u>



Evanescent wave penetration into cladding layer leads to

Goos-Haenchen shift S_{gh} and moves effective reflective boundary by d_p into cladding layer. However, the evanescent wave in the cladding must decay exponentially with increasing r. The functions J_ℓ and I_ℓ do not meet this Boundary condition: $\Rightarrow$ <u>$C_1 = 0$</u> $\rightsquigarrow$

→ <u>In the cladding layer:</u>

$$E_z = C \cdot K_\ell(q \cdot r) \exp\{i(\omega \cdot t + \ell \cdot \phi - \beta \cdot z)\}$$

$$H_z = D \cdot K_\ell(q \cdot r) \exp\{i(\omega \cdot t + \ell \cdot \phi - \beta \cdot z)\}$$

Since guiding requires $\boxed{\beta > n_2 \cdot \omega / c}$

Continuous tangential field components at $r = a$ can not be met with I_ℓ for finite fields at $r = 0 \Rightarrow C_2 = 0$

↳ <u>Inside core layer:</u>

$$E_z = A \cdot J_\ell(q \cdot r) \exp\{i(\omega t + \ell \cdot \phi - \beta \cdot z)\}$$

$$H_z = B \cdot J_\ell(q \cdot r) \exp\{i(\omega \cdot t + \ell \cdot \phi - \beta \cdot z)\}$$

Continuous tangential field components for distinct set of propagation constant β_m, $m = 1, 2, 3 \ldots$

↳ labeling of modes of propagation

$EH_{\ell m}$ or $HE_{\ell m}$

$\boxed{A = C = 0}$ for EH_{0m} modes

$\boxed{B = D = 0}$ " HE_{0m} modes

i.e. only transverse $\vec{E}$ - component for EH_{0m} ⇒ labeled TE_{0M}

" " $\vec{H}$ - components H_ϕ for HE_{0m} ⇒ TM_{0M} modes

For small Δn: → $\beta = n_2 \cdot \omega/c \approx n_1 \cdot \omega/c$ → linearly polarized modes of propagation: <u>$LP_{\ell m}$</u>