

Polarization

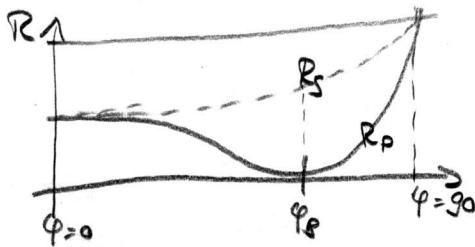
Production of polarized light

1) Polarization by selective Absorption (Dichromism)

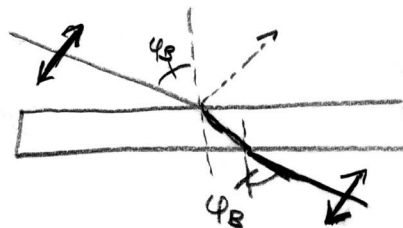
A dichroic polarizer selectively absorbs light with \vec{E} -vibrations along an unique direction — characteristic of the dichroic material

2) Polarization by reflection from dielectric surfaces:

Remember: a) Brewster angle φ_B : p-polarized light is completely transmitted



$$\varphi_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$



Brewster window
 R_p - fully transmitted
 R_s partially rejected
 \hookrightarrow multipaths for complete rejection

3) Polarization by scattering

Scattering: Rayleigh scattering

$$P = \frac{e^2 \omega^4 r_0^2}{12 \pi \epsilon_0 c^3}$$

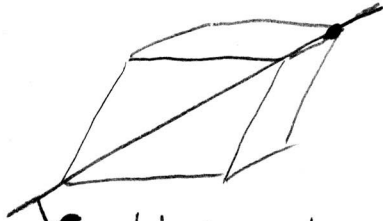
\hookrightarrow weak and imperfect, since statistically distributed.

However, controlled scattering of light from active media, exemplified by stimulated Raman, Rayleigh and Brillouin scattering, the scattered light is modified by the resonant frequencies in the medium.

Polarization

4.) Polarization with two refractive indices (Birefringence)

Refractive index varies as function of crystal-orientation



Crystal symmetry axis = optic axis (OA)

For light entering parallel to (OA) $n_{\perp} = n_{\parallel}$

For light entering perpendicular or under an angle to optical axis
the phase velocity for E_{\parallel} and E_{\perp} is different

$$v_{\perp} = \frac{c}{n_{\perp}} \quad \text{and} \quad v_{\parallel} = \frac{c}{n_{\parallel}}$$

Example: Calcite $n_{\perp} = 1.658$, $n_{\parallel} = 1.486$ ($\lambda = 589.3 \text{ nm}$)

If medium has one optical axis: \leadsto uniaxial birefringent
 [one unique direction through the crystal for which the atoms are arranged symmetrically]

Further: $n_{\parallel} - n_{\perp} > 0$: uniaxial positive

$n_{\parallel} - n_{\perp} < 0$ " negative

Other crystalline systems (triclinic, monoclinic, orthorhombic ...)
possess two optical symmetry axes

\leadsto biaxial crystals

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4. Birefringent - continue

Phase difference = difference in optical paths for the two components. If thickness of the crystal is d ,

$$\Delta = |n_{\perp} - n_{\parallel}| \cdot d$$

and
$$\Delta\varphi = 2\pi \cdot \left(\frac{\Delta}{\lambda_0}\right) = \frac{2\pi}{\lambda_0} \cdot |n_{\perp} - n_{\parallel}| \cdot d, \quad \lambda_0 = \text{vacuum wavelength of light}$$

If thickness of plate is such as to make

$$\Delta\varphi = \frac{\pi}{2} \quad \leadsto \text{QWP (quarter-wave plate)}$$

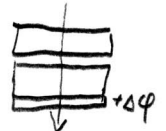
$$\Delta\varphi = \pi \quad \leadsto \text{HWP (half-wave plate)}$$

These are called zero-order (first-order) plates, which are extremely thin.

\leadsto More practical are thicker QWP of higher order,

$$\text{giving } \Delta\varphi = (2\pi) \cdot m + \pi/2 \quad (m = 1, 2, \dots)$$

A thicker composite of two plates may also be joined, in which one plate compensates the retardance of all but the desired $\Delta\varphi$ of the other.



continuously adjustable retardation plate:

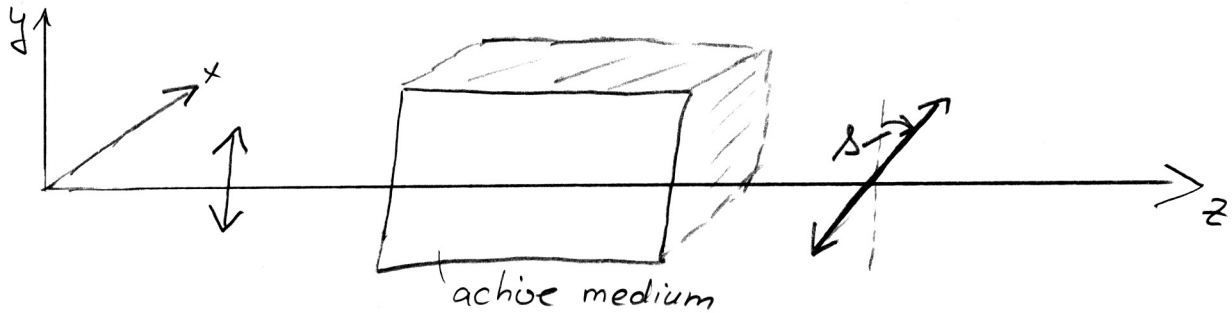
\leadsto Soleil-Babinet compensator



Polarization

5.) Optical Activity

Certain materials possess a property called optical activity.



The rotation (in deg.) produced by a 1mm plate of optically active material is called its "specific rotation".

$$\left. \begin{array}{l} \text{net angle of rotation} \\ \text{in active liquids} \end{array} \right\} \rho = \rho \cdot \alpha \cdot d \quad \left| \begin{array}{l} \alpha: \text{light path} \\ d: \text{grams of active} \\ \quad \text{solute per cm}^3 \\ \rho: \text{density of liquid} \end{array} \right.$$

6.) Photoelasticity

= birefringent induced by mechanical stress applied to normally isotropic substances.

Mostly: degree of birefringent is proportional to strain induced.

Stokes Parameter [Representation of polarization state by
Stokes vector $\vec{S} = (S_0, S_1, S_2, S_3)$]

Assume two plane waves: $\vec{E}_1 = \vec{E}_1 \cdot E_1 \cdot \exp[i\vec{k} \cdot \vec{r} - i\omega t]$

$$\vec{E}_2 = \vec{E}_2 \cdot E_2 \cdot \exp[i\vec{k} \cdot \vec{r} - i\omega t]$$

with \vec{E}_1 and \vec{E}_2 are the polarization vectors, E_1 and E_2 are the complex amplitudes and $\vec{k} = k \cdot \hat{n}$ the propagation direction.

→ Combined EM-plane wave

$$E_C(\vec{r}, t) = \vec{E}_1 + \vec{E}_2 = (\vec{E}_1 \cdot E_1 + \vec{E}_2 \cdot E_2) \cdot \exp[i\vec{k} \cdot \vec{r} - i\omega t]$$

Linear polarization: E_1 and E_2 have the same phase $\left. \begin{array}{l} E_1 = a_1 e^{i\delta_1} \\ E_2 = a_2 e^{i\delta_2} \end{array} \right\} =$
with magnitude $E = \sqrt{E_1^2 + E_2^2}$
and phase angle $\theta = \tan^{-1}(E_2/E_1)$

→ Stokes vector:

$$S_0 = |\vec{E}_1 \cdot \vec{E}|^2 + |\vec{E}_2 \cdot \vec{E}|^2 = a_1^2 + a_2^2$$

$$S_1 = |\vec{E}_1 \cdot \vec{E}|^2 - |\vec{E}_2 \cdot \vec{E}|^2 = a_1^2 - a_2^2$$

$$S_2 = 2 \cdot \text{Re}\{(\vec{E}_1 \cdot \vec{E})^* (\vec{E}_2 \cdot \vec{E})\} = 2 \cdot a_1 a_2 \cos(\delta_2 - \delta_1)$$

$$S_3 = 2 \cdot \text{Im}\{(\vec{E}_1 \cdot \vec{E})^* (\vec{E}_2 \cdot \vec{E})\} = 2 \cdot a_1 a_2 \sin(\delta_2 - \delta_1)$$

If E_1 and E_2 have different phase, the wave

is elliptically polarized. Simplest case is, if

E_1 and E_2 have the same amplitude → circular polarization ⇒

Circular polarization:

$$E_1 = a_1 e^{i\delta_1}, \quad E_2 = a_2 e^{i\delta_2}, \quad a_1 = a_2 = E_0 : E(\vec{r}, t) = E_0 e^{i(\delta_1 + \delta_2)} (\vec{E}_1 + \vec{E}_2) e^{i(\vec{k}\vec{r} - \omega t)}$$

if $|\delta_1 - \delta_2| = 90^\circ \rightarrow \vec{E}(\vec{r}, t) = E_0 (\vec{E}_1 \pm i\vec{E}_2) \exp[-i(\vec{k}\vec{r} - \omega t)]$

$\vec{E}_1 + i\vec{E}_2$: counter clockwise rotation = left circularly polarized

$\vec{E}_1 - i\vec{E}_2$: clockwise rotation = positive helicity

= right circularly polarized
= negative helicity

\rightarrow orthogonal unit vectors $\vec{E}_\pm = \frac{1}{\sqrt{2}} (\vec{E}_1 \pm i\vec{E}_2)$

with $\vec{E}_\pm^* \cdot \vec{E}_\mp = 0$; $\vec{E}_\pm^* \cdot \vec{E}_\pm = 1$

\rightarrow general wave description

$$\vec{E}(\vec{r}, t) = [E_+ \vec{E}_+ + E_- \vec{E}_-] \cdot e^{i(\vec{k}\vec{r} - \omega t)}$$

E_+, E_- complex amplitudes, $E_+ \neq E_-$: elliptically

$E_+ = E_-$: circularly

Stokes vector with $E_+ = a_+ e^{i\delta_+}$, $E_- = a_- e^{i\delta_-}$

$$S_0 = |E_+^* \cdot E|^2 + |E_-^* \cdot E|^2 = a_+^2 + a_-^2 \quad - \text{(relative intensity)}$$

$$S_1 = 2 \cdot \text{Re}[(E_+^* \cdot E)^* \cdot (E_-^* \cdot E)] = 2a_+ a_- \cos(\delta_- - \delta_+)$$

$$S_2 = 2 \cdot \text{Im}[(E_+^* \cdot E)^* \cdot (E_-^* \cdot E)] = 2a_+ a_- \sin(\delta_- - \delta_+)$$

} degree of helicity

$$S_3 = |E_+^* \cdot E|^2 - |E_-^* \cdot E|^2 = a_+^2 - a_-^2 \quad - \text{(difference in relative intensity)}$$