Physics 8110 - Electromagnetic Theory II

Assignment #6

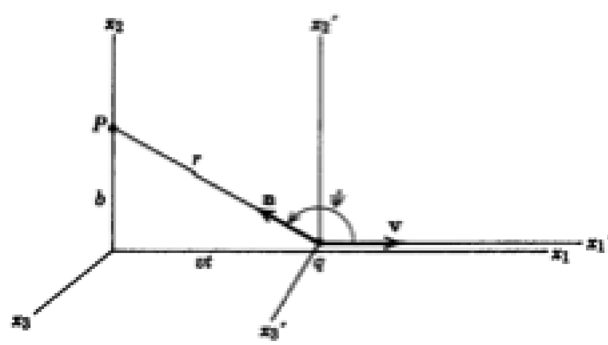
(due to Wednesday, April 09, 2018)

1. Jackson 11.3: Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity $v = \frac{v_1 + v_2}{1 + (v_1 v_2 / c^2)}$. This is an alternative way to derive the parallel-velocity addition law.

2. In a Compton effect, a γ-ray of wavelength λ strikes a free, but initially stationary, electron of mass m. The photon is scattered at an angle θ (measured from the incident direction), and its scattered wavelength is λ'. The electron recoils at an angle ϕ (measured from the incident direction).
   a. Write the relativistic equations for momentum and energy conservation.
   b. Find an expression for the change $\lambda' - \lambda$ in the photon wavelength.
   c. Show that the kinetic energy, $T=E-mc^2$ (E is the total energy of the electron after collision, and mc^2 is the rest energy of the electron before collision), of the recoiled electron is
   $$T = \left(\frac{hc}{\lambda}\right) \frac{\left(\frac{2h}{\lambda mc}\right)\sin^2\theta/2}{1+\left(\frac{2h}{\lambda mc}\right)\sin^2\theta/2}.$$

3. Two events are specified in frame S by (5.0 m, 3.0 m, 0, 10 ns) and (8.0 m, 7.0 m, 0, 15 ns). Observer is in frame S', which moves in the x-direction at a constant speed relative to S,
   a. Find v_x, the velocity of S' relative to S for these two events to be simultaneous in S'.
   b. When do these two events occur in S' then?
   c. What is the distance between these two events in S? in S'?
   d. Is there a reference frame S'' moving in the x-direction relative to S in which these two events have the same x'' coordinate? Explain.
   e. What is the nature of the separation between these two events?

4. Find the fields of a uniformly moving charged particle by the following steps (refer to Fig.)
   a. Write out the vector and scalar potentials in the K' system where the particle is at rest.
   b. Obtain the vector and scalar potentials in the K system using the transformation of potentials.
   c. Find the fields from the potentials in the K system.



5. Jackson 11.13: An infinitely long straight wire of negligible cross-sectional area is at rest and has a uniform linear charge density q_0 in the inertial frame K'. The frame K' (and the wire) move with velocity **v** parallel to the direction of the wire with respect to the laboratory frame K.
   a. Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the laboratory.
   b. What are the charge and current densities associated with the wire inits rest frame? In the laboratory?
   c. From the laboratory charge and current densities, calculate directly the electric and magnetic fields in the laboratory. Compare with the results of part (a).