Explosions and collisions obey some surprisingly simple laws that make problem solving easier when comparing the situation before and after an interaction.

**Chapter Goal:** To introduce the ideas of impulse and momentum and to learn a new problem-solving strategy based on conservation laws.
Chapter 9. Impulse and Momentum

Topics:

• Momentum and Impulse
• Solving Impulse and Momentum Problems
• Conservation of Momentum
• Inelastic Collisions
• Explosions
• Momentum in Two Dimensions
Momentum

After the collision

What is the velocity of ball A after the collision? ball B?
What is conserved during the collision?

**MOMENTUM**

\[ \vec{p} = m \vec{v} \]

The total momentum is the sum of momentum of ball A and momentum of ball B.
Momentum

The total momentum of the system is conserved during the collision:

\[ m_A v_{A,i} = m_A v + m_B u \]

- Momentum is a vector. It has the same direction as corresponding velocity.
- General expression for the momentum conservation: the total momentum before the collision is equal to the total momentum after the collision
Momentum

- General expression for the momentum conservation: the total momentum before the collision is equal to the total momentum after the collision.

\[
\mathbf{p}_A + \mathbf{p}_B = \mathbf{p}_A' + \mathbf{p}_B'
\]

Usually this equation is written in terms of components.
Example:

After the collision the balls are moving together (have the same velocity). What is their velocity?

Momentum before the collision: \( p_i = m_A v_{A,i} = 10 \frac{kg \cdot m}{s} \)

Momentum after the collision: \( p_f = (m_A + m_B)v = 5v \)

Conservation of momentum: \( p_i = p_f \)

\[ 10 = 5v \quad v = 2 m / s \]
Why do we have conservation of total momentum?

Newton’s second law:

\[ \vec{F}_{\text{net}} = m\vec{a} \]

Acceleration:

\[ \vec{a} = \frac{d\vec{v}}{dt} \]

Then

\[ \vec{F}_{\text{net}} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} \]

After integration

\[ \Delta \vec{p} = \int_{t_1}^{t_2} \vec{F}_{\text{net}} dt \]

The area under \( \vec{F}_{\text{net}}(t) \) curve.

It is called IMPULSE, J.

\[ \vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}} dt \]

The impulse of the force is equal to the change of the momentum of the object.

\[ \Delta \vec{p} = \vec{J} \]
\[ p_i = m v_{ix} \]
\[ p_f = m v_{fx} < 0 \]

\[ J_x = p_f - p_i < 0 \]
The forces during the collision are an action/reaction pair.

Newton’s third law:

\[ F_{x,1on2} = -F_{x,2on1} \]

Then

\[ \int_{t_1}^{t_2} F_{x,1on2} dt = -\int_{t_1}^{t_2} F_{x,2on1} dt \]

\[ m_2v_{fx,2} - m_2v_{ix,2} = -\left( m_1v_{fx,1} - m_1v_{ix,1} \right) \]
The forces during the collision are an action/reaction pair.

The law of conservation of momentum

\[ m_2 v_{fx,2} - m_2 v_{ix,2} = -\left( m_1 v_{fx,1} - m_1 v_{ix,1} \right) \]

\[ m_1 v_{ix,1} + m_2 v_{ix,2} = m_1 v_{fx,1} + m_2 v_{fx,2} \]

\[ p_{ix,1} + p_{ix,2} = p_{fx,1} + p_{fx,2} \]

\[ p_{ix,total} = p_{fx,total} \]
Momentum

The law of conservation of momentum:

The total momentum of an isolated system (no external forces) does not change.

Interactions within system do not change the system’s total momentum

\[ \vec{p} = m \vec{v} \]

\[ m_A \vec{v}_{A,i} + m_B \vec{v}_{B,i} = m_A \vec{v}_{A,f} + m_B \vec{v}_{B,f} \]
The ball is dropped onto a hard floor:

- The ball is not an isolated system (interaction with the floor)
- No conservation of momentum for the ball
- Initial momentum is $\vec{p}_i = m\vec{v}_i$
- Final momentum (after collision) is $\vec{p}_f = m\vec{v}_f$
- The ball + the floor is an isolated system
- The total momentum (ball + floor) is conserved
Example: Find $v_{2x}$

Before:
- $x_0 = 0 \text{ m}$
- $(v_{0x})_B = 0 \text{ m/s}$
- $a_x = 1.0 \text{ m/s}^2$
- $m_B = 75 \text{ kg}$
- $(v_{1x})_B = 8.0 \text{ m}$
- $(v_{1x})_C = 0 \text{ m/s}$
- $m_C = 25 \text{ kg}$

After:
- $(v_{2x})_B = (v_{2x})_C = v_{2x}$

Motion with constant acceleration:

$$ (v_{1x,B})^2 = 2a_x x_1 = 16 $$

$$ v_{1x,B} = 4 \text{ m/s} $$

Isolated system

Momentum before the “collision”:

$$ p_{i,total} = m_B v_{1x,B} + m_C v_{1x,C} = m_B v_{1x,B} = 75 \cdot 4 = 300 \frac{\text{kg} \cdot \text{m}}{\text{s}} $$

Momentum after the “collision”:

$$ p_{f,total} = m_B v_{2x,B} + m_C v_{2x,C} = (m_B + m_C)v_{2x} = 100v_{2x} $$

Conservation of momentum:

$$ p_{f,total} = p_{i,total} $$

$$ 100v_{2x} = 300 $$

$$ v_{2x} = 3 \text{ m/s} $$
Perfectly inelastic collision:

A collision in which the two objects stick together and move with a common final velocity.

Before:

\[ m_1 = 200 \text{ g}, \quad (v_{ix})_1 = 3.0 \text{ m/s}, \quad (v_{ix})_2 \quad \text{and} \quad m_2 = 400 \text{ g} \]

\[ p_{i,\text{total}} = m_1 v_{ix,1} + m_2 v_{ix,2} \]

After:

\[ v_{fx} = -0.50 \text{ m/s} \]

\[ p_{f,\text{total}} = (m_1 + m_2)v_{fx} \]

Find: \((v_{ix})_2\)

\[ v_{ix,2} = \left( \frac{m_1}{m_2} + 1 \right)v_{fx} - \frac{m_1}{m_2}v_{ix,1} = -\frac{3}{2} \cdot 0.5 - \frac{3}{2} = -2.25 \text{ m/s} \]
Chapter 9. Summary Slides
General Principles

**Law of Conservation of Momentum**

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_t = \vec{P}_i$$

**Newton’s Second Law**

In terms of momentum, Newton’s second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$
General Principles

Solving Momentum Conservation Problems

**MODEL**  Choose an isolated system or a system that is isolated during at least part of the problem.

**VISUALIZE**  Draw a pictorial representation of the system before and after the interaction.

**SOLVE**  Write the law of conservation of momentum in terms of vector components:

\[
(p_{tx})_1 + (p_{tx})_2 + \cdots = (p_{tx})_1 + (p_{tx})_2 + \cdots
\]

\[
(p_{ty})_1 + (p_{ty})_2 + \cdots = (p_{ty})_1 + (p_{ty})_2 + \cdots
\]

**ASSESS**  Is the result reasonable?
Important Concepts

Momentum \( \vec{p} = m\vec{v} \)

Impulse \( J_x = \int_{t_i}^{t_f} F_x(t) \, dt = \) area under force curve

Impulse and momentum are related by the impulse-momentum theorem

\[ \Delta p_x = J_x \]

This is an alternative statement of Newton’s second law.
Important Concepts

**System**  A group of interacting particles.

**Isolated system**  A system on which there are no external forces or the net external force is zero.

**Before-and-after pictorial representation**

- Define the system.
- Use two drawings to show the system before and after the interaction.
- List known information and identify what you are trying to find.
**Applications**

**Collisions** Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.

**Explosions** Two or more particles move away from each other.

**Two dimensions** No new ideas, but both the $x$- and $y$-components of $P$ must be conserved, giving two simultaneous equations.
Applications

**Momentum bar charts** display the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$ in graphical form.
The cart’s change of momentum is

A. 30 kg m/s.
B. 10 kg m/s.
C. –10 kg m/s.
D. –20 kg m/s.
E. –30 kg m/s.
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D. –20 kg m/s.
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A 10 g rubber ball and a 10 g clay ball are thrown at a wall with equal speeds. The rubber ball bounces, the clay ball sticks. Which ball exerts a larger impulse on the wall?

A. They exert equal impulses because they have equal momenta.
B. The clay ball exerts a larger impulse because it sticks.
C. Neither exerts an impulse on the wall because the wall doesn’t move.
D. The rubber ball exerts a larger impulse because it bounces.
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☑️ D. The rubber ball exerts a larger impulse because it bounces.
Objects A and C are made of different materials, with different “springiness,” but they have the same mass and are initially at rest. When ball B collides with object A, the ball ends up at rest. When ball B is thrown with the same speed and collides with object C, the ball rebounds to the left. Compare the velocities of A and C after the collisions. Is \( v_A \) greater than, equal to, or less than \( v_C \)?

**Before**

- B
- \( A \)
- \( m_A = m_C \)

**After**

- \( v = 0 \)
- \( v_A \)
- \( v_C \)

**A.** \( v_A > v_C \)

**B.** \( v_A < v_C \)

**C.** \( v_A = v_C \)
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A. $v_A > v_C$
B. $v_A < v_C$
C. $v_A = v_C$

B. $v_A < v_C$
The two particles are both moving to the right. Particle 1 catches up with particle 2 and collides with it. The particles stick together and continue on with velocity \( v_f \). Which of these statements is true?

A. \( v_f = v_2 \).
B. \( v_f \) is less than \( v_2 \).
C. \( v_f \) is greater than \( v_2 \), but less than \( v_1 \).
D. \( v_f = v_1 \).
E. \( v_f \) is greater than \( v_1 \).
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D. $v_f = v_1$.
E. $v_f$ is greater than $v_1$.

$\checkmark$ C. $v_f$ is greater than $v_2$, but less than $v_1$. 
An explosion in a rigid pipe shoots out three pieces. A 6 g piece comes out the right end. A 4 g piece comes out the left end with twice the speed of the 6 g piece. From which end does the third piece emerge?

A. Right end
B. Left end
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