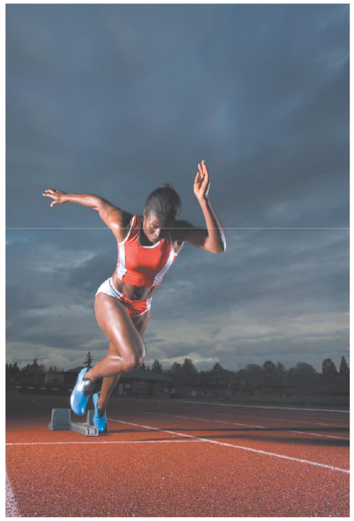
Chapter 2. Kinematics in One Dimension

In this chapter we study kinematics of motion in one dimension—motion along a straight line. Runners, drag racers, and skiers are just a few examples of motion in one dimension.

Chapter Goal: To learn how to solve problems about motion in a straight line.

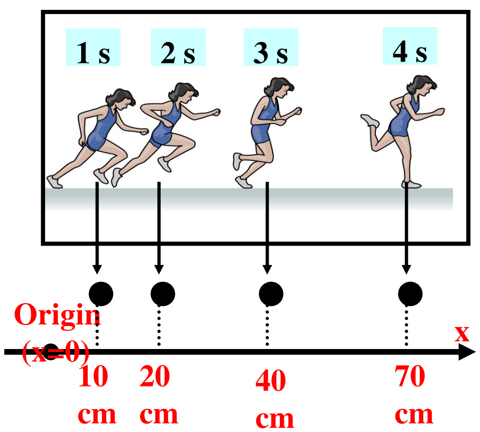


Chapter 2. Kinematics in One Dimension

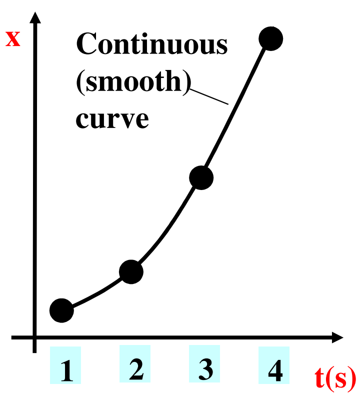
Topics:

- Uniform Motion
- Instantaneous Velocity
- Finding Position from Velocity
- Motion with Constant Acceleration
- Free Fall
- Motion on an Inclined Plane
- Instantaneous Acceleration

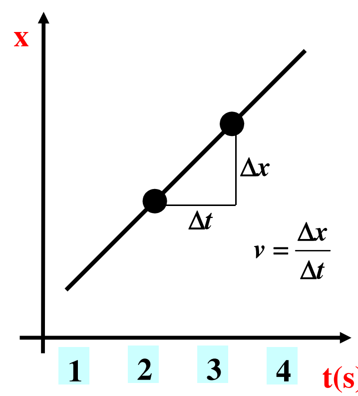
Motion along a straight line



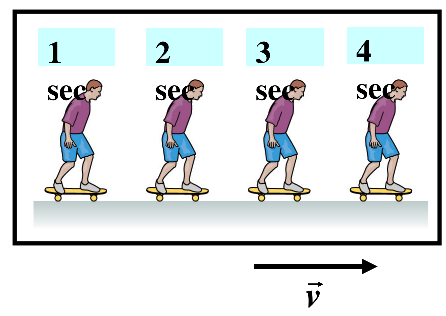
Can be illustrated by position-versus-time graph:



Position graph



$$v = \frac{\Delta x}{\Delta t}$$



$\vec{v}$

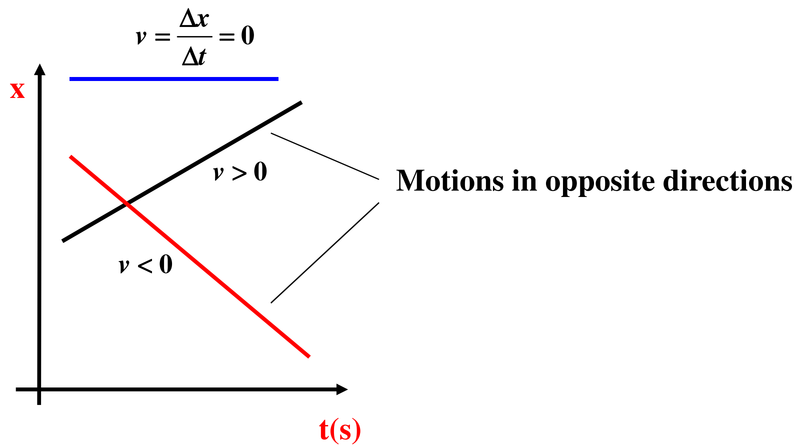
Straight line – uniform motion – the same velocity

Velocity is the slope

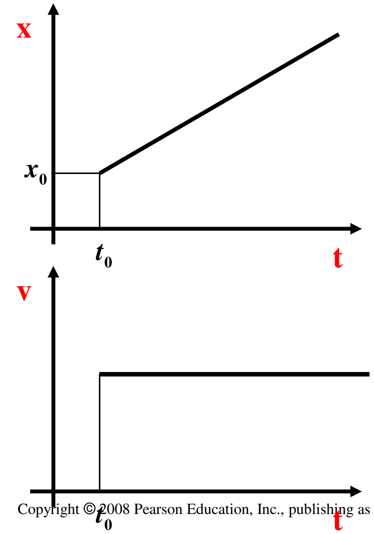
Velocity is the same – zero acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = 0$$

Position graph: uniform motion



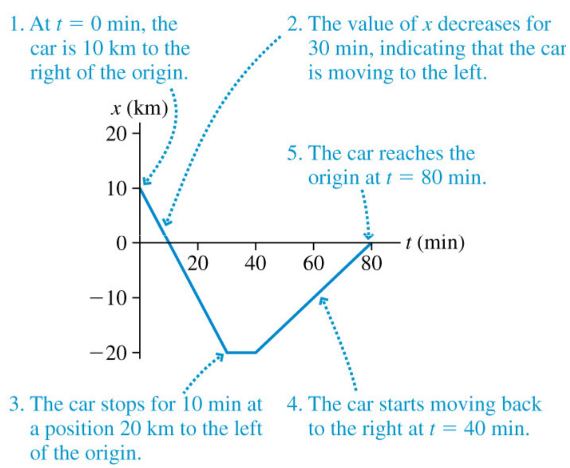
Uniform motion: motion with constant velocity

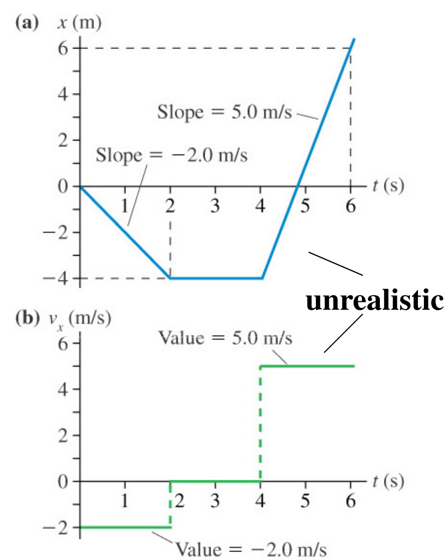


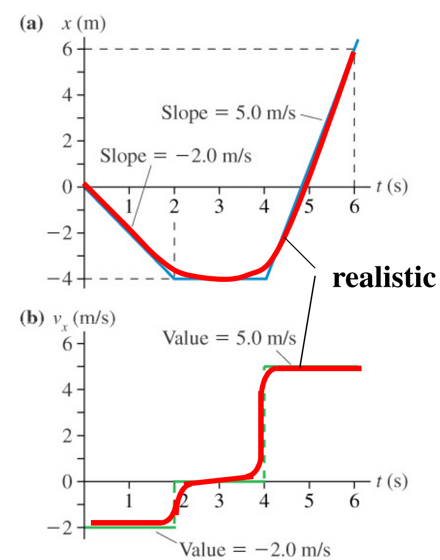
$$v = \frac{\Delta x}{\Delta t} = \frac{x_t - x_0}{t - t_0}$$

$$x_t = x_0 + v(t - t_0)$$

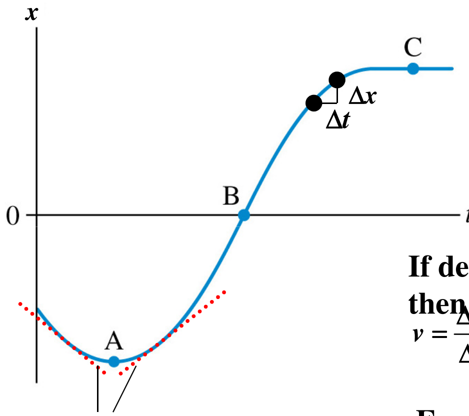
Position graph







Instantaneous velocity



Very small Δt

Instantaneous velocity: $v = \frac{\Delta x}{\Delta t}$

If dependence $x(t)$ **is given, then** $v = \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt}$ **- derivative**

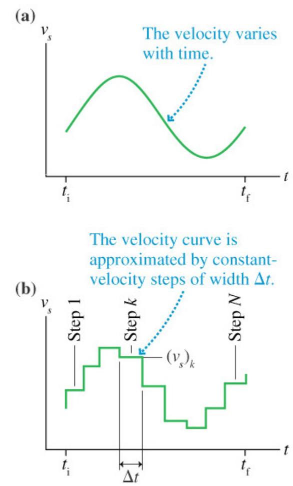
Example:

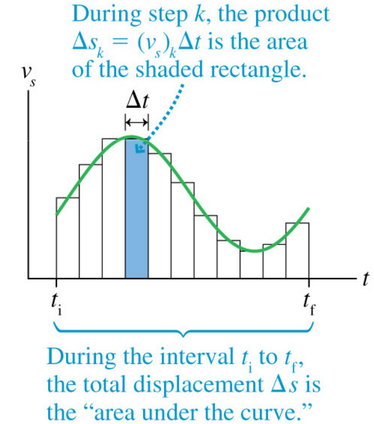
$x(t) = 5t^4 \qquad v = \frac{dx(t)}{dt} = 20t^3$

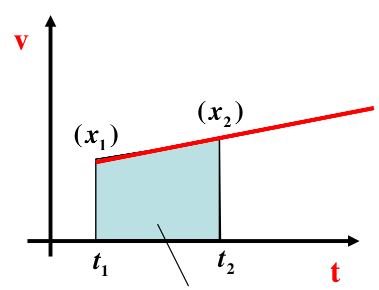
$x(t) = 5\sin(t) \qquad v = \frac{dx(t)}{dt} = 5\cos(t)$

Finding position from velocity

$$v = \frac{dx(t)}{dt} \qquad x(t) = \int v(t)dt$$



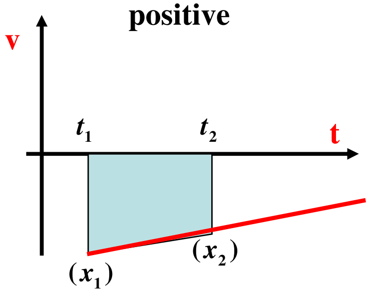




$$x_{2-1} = x_2 - x_1 = \int_{t_1}^{t_2} v(t)dt$$

The displacement is the area, displacement is positive

$$x_{2-1} = x_2 - x_1 = \int_{t_1}^{t_2} v(t)dt > 0 \qquad v(t) > 0$$

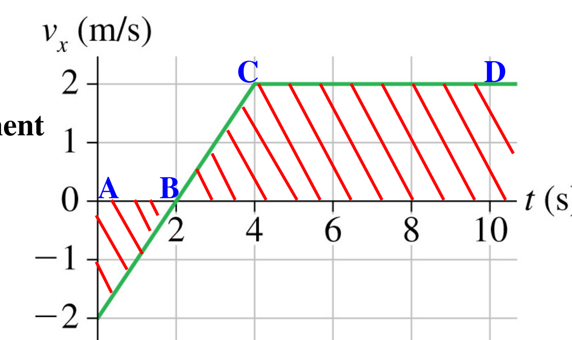


Displacement is negative

$$x_{2-1} = x_2 - x_1 = \int_{t_1}^{t_2} v(t)dt < 0 \qquad v(t) < 0$$

Example:

Find the net displacement



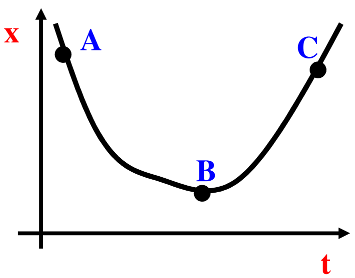
$$x_{A-B} = x_B - x_A = \int_{t_A}^{t_B} v(t)dt = -\frac{1}{2} 2 \cdot 2 = -2m$$

area

$$x_{B-C} = x_C - x_B = \int_{t_B}^{t_C} v(t)dt = \frac{1}{2} 2 \cdot 2 = 2m$$

$$x_{C-D} = x_D - x_C = \int_{t_C}^{t_D} v(t)dt = 2 \cdot (10-4) = 12m$$

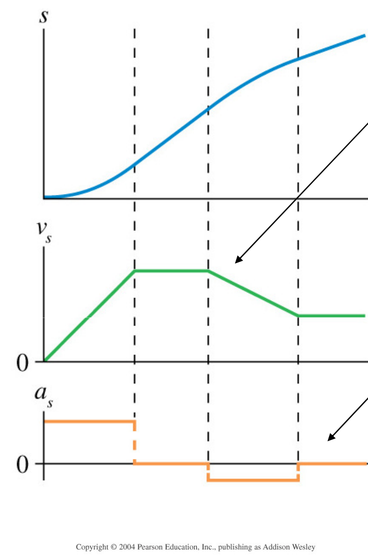
$$x_{A-D} = x_D - x_A = x_D - x_C + x_C - x_B + x_B - x_A = x_{C-D} + x_{B-C} + x_{A-B} = -2 + 2 + 12 = 12m$$



$v_A < v_B < v_C$

Velocity is the slope $v = \dfrac{ds}{dt}$

$v_A < 0$ $\quad$ $v_C > 0$



Velocity is the slope $v = \dfrac{ds}{dt}$

Acceleration is the slope

$$a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Motion with constant acceleration

Motion with constant velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_t - x_0}{t - t_0} \qquad x_t = x_0 + v(t - t_0)$$

Motion with constant acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_t - v_0}{t - t_0} \qquad v_t = v_0 + a(t - t_0)$$

$x_t = ?$

$$v_t = \frac{dx_t}{dt}$$

$$x_t = x_0 + \int_{t_0}^{t} v_t dt = x_0 + \int_{t_0}^{t} (v_0 + a(t - t_0))dt = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

Motion with constant velocity or constant acceleration

Motion with constant velocity: $x_t = x_0 + v(t - t_0)$

Motion with constant acceleration:

$$v_t = v_0 + a(t - t_0)$$

$$x_t = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

Useful relation:

$$(t - t_0) = \frac{v_t - v_0}{a} \qquad x_t = x_0 + v_0\frac{v_t - v_0}{a} + \frac{1}{2}\frac{(v_t - v_0)^2}{a}$$

then

$$v_t^2 - v_0^2 = 2a(x_t - x_0)$$

Motion with constant velocity or constant acceleration

$t_0 = 0$

Motion with constant velocity: $x_t = x_0 + v\,t$

Motion with constant acceleration:

$$v_t = v_0 + a\,t$$

$$x_t = x_0 + v_0\,t + \frac{1}{2}a\,t^2$$

$$v_t^2 - v_0^2 = 2a(x_t - x_0)$$

Free fall motion - motion with constant acceleration

$\vec{g} = \text{constant}$

y

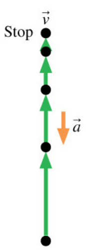
$$a = -g = -9.8\frac{m}{s^2}$$

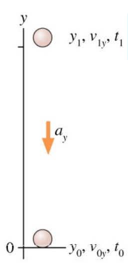
$$v_t = v_0 - g\,t$$

$$v_t^2 - v_0^2 = -2g(y_t - y_0)$$

$$y_t = y_0 + v_0\,t - \frac{1}{2}g\,t^2$$

Free fall motion - motion with constant acceleration





$v_{final} = 0$

$v_0 = 2\,m/s$

$y_0 = 0$

$$g = 9.8\frac{m}{s^2}$$

$$v_t = v_0 - g\,t$$

$$v_t^2 - v_0^2 = -2g(y_t - y_0)$$

$$y_t = y_0 + v_0\,t - \frac{1}{2}g\,t^2$$

$$v_t = 2 - 9.8t$$

$$v_t^2 - 2^2 = -2 \times 9.8 \times y_t$$

$$y_t = 2t - 4.9t^2$$

At final point:

$v_{final} = 0$

$$0 = 2 - 9.8t_f$$

$$-2^2 = -2 \times 9.8 \times y_f$$

$$y_f = 2t_f - 4.9t_f^2$$

EXAMPLE 2.14 Friday night football

QUESTION:

EXAMPLE 2.14 Friday night football

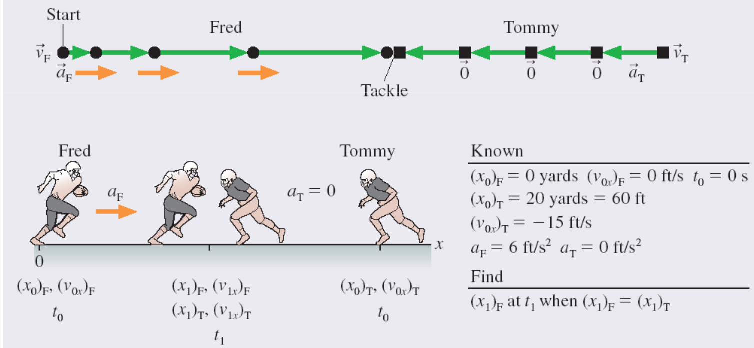
Fred catches the football while standing directly on the goal line. He immediately starts running forward with an acceleration of 6 ft/s^2. At the moment the catch is made, Tommy is 20 yards away and heading directly toward Fred with a steady speed of 15 ft/s. If neither deviates from a straight-ahead path, where will Tommy tackle Fred?

EXAMPLE 2.14 Friday night football

MODEL Represent Fred and Tommy as particles.

EXAMPLE 2.14 Friday night football

VISUALIZE The pictorial representation is shown again in **FIGURE 2.27**. With two moving objects we need the additional subscripts F and T to distinguish Fred's symbols and Tommy's symbols.



EXAMPLE 2.14 Friday night football

SOLVE We want to find *where* Fred and Tommy have the same position. The pictorial representation designates time t_1 as *when* they meet. The axes have been chosen so that Fred starts at $(x_0)_F = 0$ ft and moves to the right while Tommy starts at $(x_0)_T = 60$ ft and runs to the left with a *negative* velocity. The second equation of Table 2.2 allows us to find their positions at time t_1. These are:

$$(x_1)_F = (x_0)_F + (v_{0x})_F(t_1 - t_0) + \tfrac{1}{2}(a_x)_F(t_1 - t_0)^2$$
$$= \tfrac{1}{2}(a_x)_F t_1^2$$
$$(x_1)_T = (x_0)_T + (v_{0x})_T(t_1 - t_0) + \tfrac{1}{2}(a_x)_T(t_1 - t_0)^2$$
$$= (x_0)_T + (v_{0x})_T t_1$$

EXAMPLE 2.14 Friday night football

Notice that Tommy's position equation contains the term $(v_{0x})_T t_1$, not $-(v_{0x})_T t_1$. The fact that he is moving to the left has already been considered in assigning a *negative value* to $(v_{0x})_T$, hence we don't want to add any additional negative signs in the equation. If we now set $(x_1)_F$ and $(x_1)_T$ equal to each other, indicating the point of the tackle, we can solve for t_1:

$$\tfrac{1}{2}(a_x)_F t_1^2 = (x_0)_T + (v_{0x})_T t_1$$
$$\tfrac{1}{2}(a_x)_F t_1^2 - (v_{0x})_T t_1 - (x_0)_T = 0$$
$$3t_1^2 + 15t_1 - 60 = 0$$

EXAMPLE 2.14 Friday night football

The solutions of this quadratic equation for t_1 are $t_1 = (-7.62\text{ s}, +2.62\text{ s})$. The negative time is not meaningful in this problem, so the time of the tackle is $t_1 = 2.62$ s. We've kept an extra significant digit in the solution to minimize round-off error in the next step. Using this value to compute $(x_1)_F$ gives

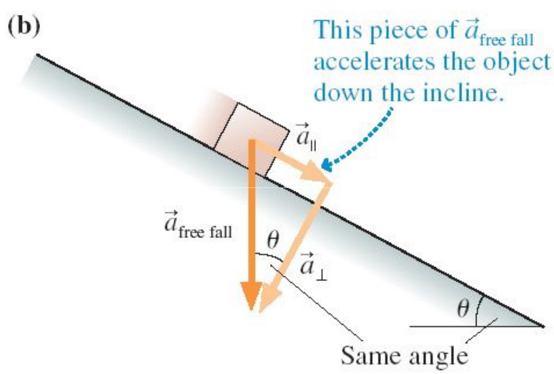
$$(x_1)_F = \frac{1}{2}(a_x)_F t_1^2 = 20.6 \text{ feet} = 6.9 \text{ yards}$$

Tommy makes the tackle at just about the 7-yard line!

EXAMPLE 2.14 Friday night football

ASSESS The answer had to be between 0 yards and 20 yards. Because Tommy was already running, whereas Fred started from rest, it is reasonable that Fred will cover less than half the 20-yard separation before meeting Tommy. Thus 6.9 yards is a reasonable answer.

Motion on an Inclined Plane



$$a_s = \pm g \sin\theta \qquad (2.25)$$

EXAMPLE 2.17 Skiing down an incline

QUESTION:

EXAMPLE 2.17 Skiing down an incline

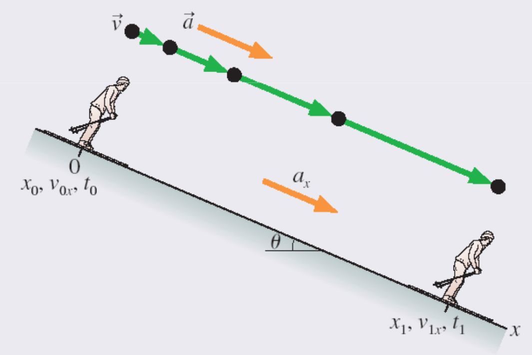
A skier's speed at the bottom of a 100-m-long, frictionless, snow-covered slope is 20 m/s. What is the angle of the slope?

EXAMPLE 2.17 Skiing down an incline

MODEL Represent the skier as a particle. Assume that air resistance is negligible. Assume that the slope is a straight line.

EXAMPLE 2.17 Skiing down an incline

VISUALIZE **FIGURE 2.33** on the next page shows the pictorial representation of the skier. Notice that we've chosen the x-axis to be parallel to the motion. Straight-line motion is almost always easier to analyze if the motion is parallel to a coordinate axis.



EXAMPLE 2.17 Skiing down an incline

SOLVE The motion diagram shows that the acceleration vector points in the positive x-direction. Thus the one-dimensional acceleration is $a_x = +g\sin\theta$. This is constant-acceleration motion. The third kinematic equation from Table 2.2 is

$$v_{1x}{}^2 = v_{0x}{}^2 + 2a_x\Delta x = 2g\sin\theta\Delta x$$

where we used $v_{0x} = 0$ m/s. Solving for $\sin\theta$, we find

$$\sin\theta = \frac{v_{1x}{}^2}{2g\Delta x} = \frac{(20 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(100 \text{ m})} = 0.204$$

Thus

$$\theta = \sin^{-1}(0.204) = 12°$$

EXAMPLE 2.17 Skiing down an incline

ASSESS A 100-m-long slope and a speed of 20 m/s ≈ 40 mph are fairly typical parameters for skiing. A 1° angle or an 80° angle would be unrealistic, but 12° seems plausible.

Chapter 2. Summary Slides

General Principles

Kinematics describes motion in terms of position, velocity, and acceleration.

General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt =$ slope of position graph

Instantaneous acceleration $a_s = dv_s/dt =$ slope of velocity graph

Final position $s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Final velocity $v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

General Principles

Motion with constant acceleration is uniformly accelerated motion. The kinematic equations are:

$$v_{fs} = v_{is} + a_s\Delta t$$

$$s_f = s_i + v_{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$$

Uniform motion is motion with constant velocity and zero acceleration:

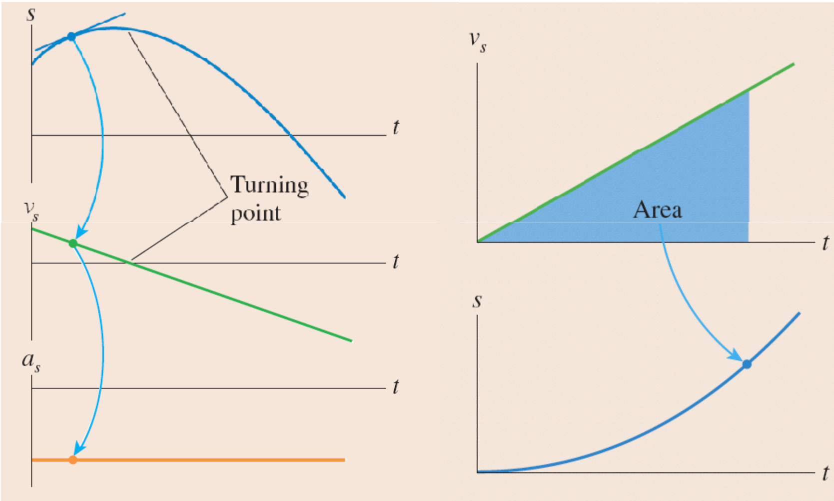
$$s_f = s_i + v_s\Delta t$$

Important Concepts

Position, velocity, and acceleration are related **graphically.**

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.
- Displacement is the area under the velocity curve.

Important Concepts



Applications

The **sign of v_s** indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

The **sign of a_s** indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

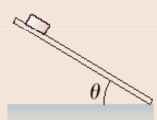
- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

Applications

An object is **speeding up** if and only if v_s and a_s have the same sign.
An object is **slowing down** if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with

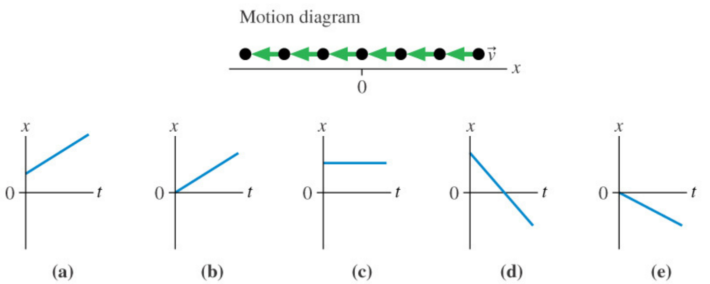
$$a_y = -g = -9.80 \text{ m/s}^2$$



Motion on an inclined plane has $a_s = \pm g \sin\theta$.
The sign depends on the direction of the tilt.

Chapter 2. Questions

Which position-versus-time graph represents the motion shown in the motion diagram?



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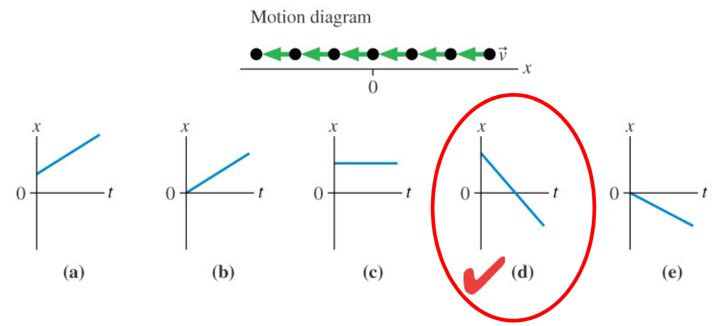
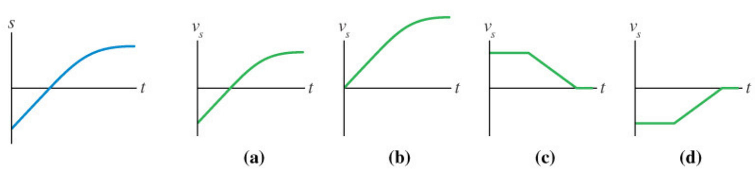


Which velocity-versus-time graph goes with the position-versus-time graph on the left?



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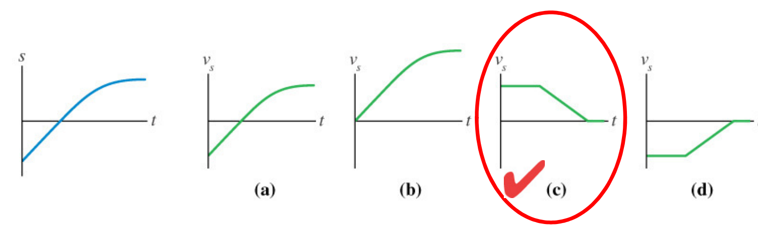
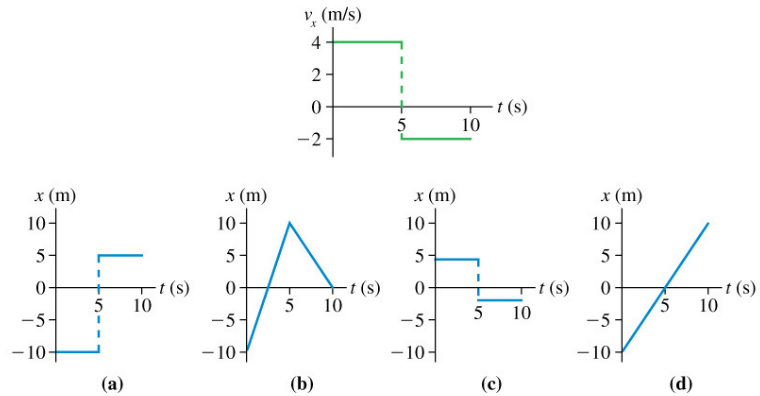


Which position-versus-time graph goes with the velocity-versus-time graph at the top? The particle's position at $t_i = 0$ s is $x_i = -10$ m.



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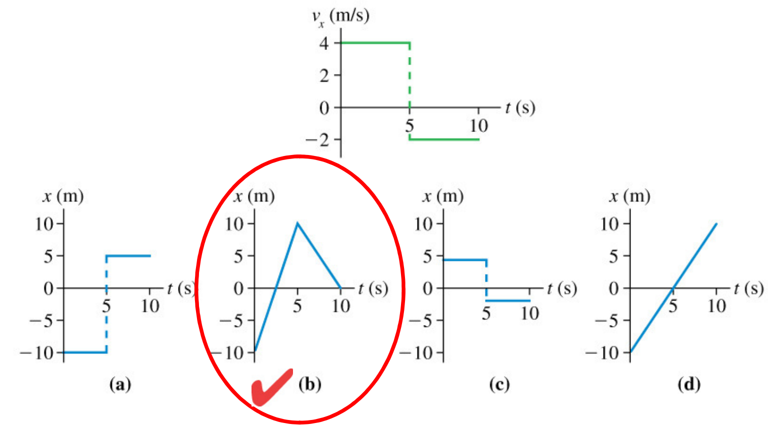
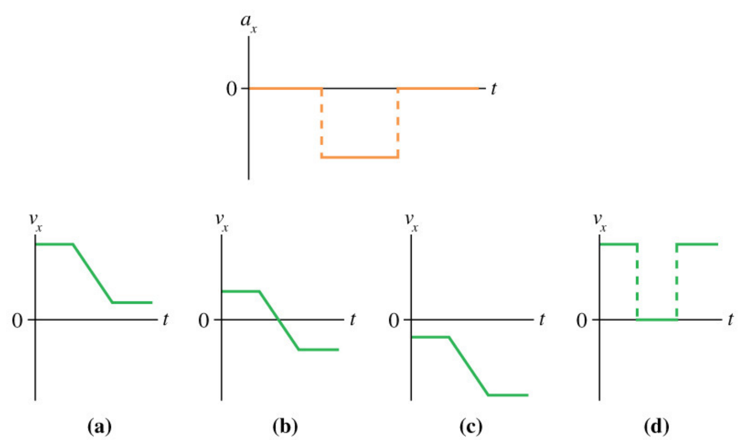


Which velocity-versus-time graph or graphs goes with this acceleration-versus-time graph? The particle is initially moving to the right and eventually to the left.



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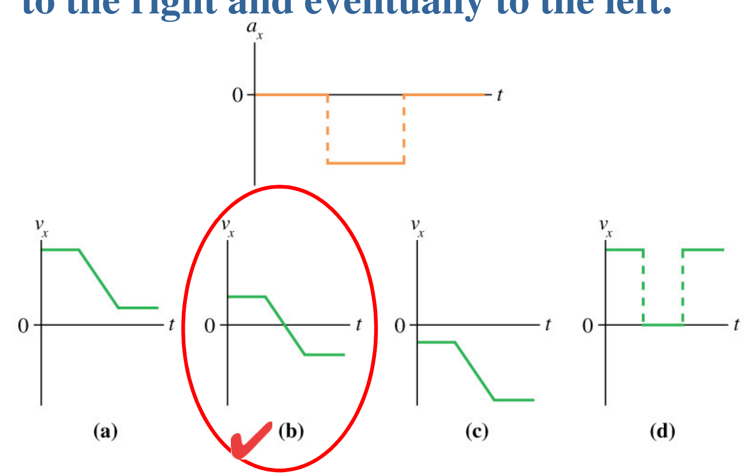
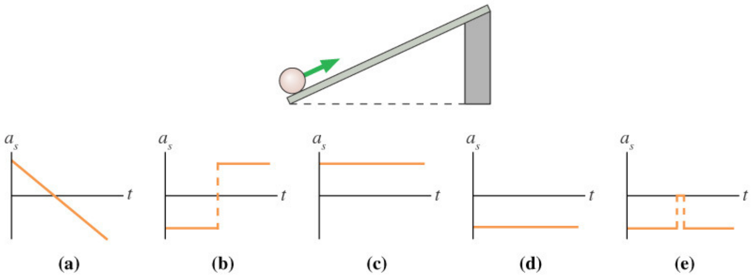


The ball rolls up the ramp, then back down. Which is the correct acceleration graph?



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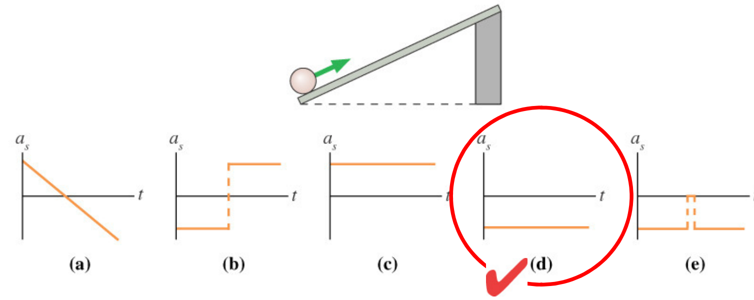
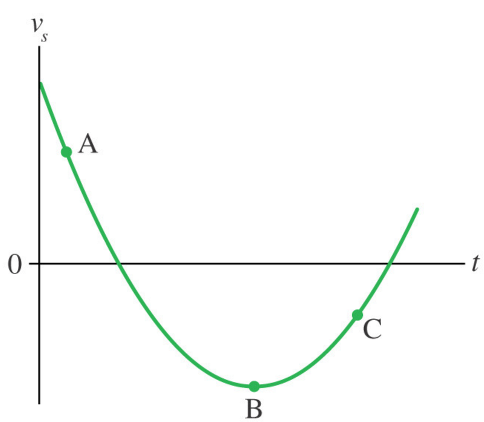
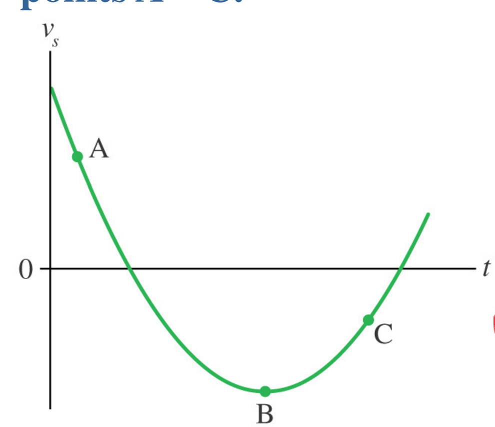


Rank in order, from largest to smallest, the accelerations $a_A - a_C$ at points A – C.



A) $a_A > a_B > a_C$
B) $a_A > a_C > a_B$
C) $a_B > a_A > a_C$
D) $a_C > a_A > a_B$
E) $a_C > a_B > a_A$

Rank in order, from largest to smallest, the accelerations $a_A - a_C$ at points A – C.



A) $a_A > a_B > a_C$
B) $a_A > a_C > a_B$
C) $a_B > a_A > a_C$
D) $a_C > a_A > a_B$
E) $a_C > a_B > a_A$