Chapter 13. Newton's Theory of Gravity

The beautiful rings of Saturn consist of countless centimeter-sized ice crystals, all orbiting the planet under the influence of gravity.

Chapter Goal: To use Newton's theory of gravity to understand the motion of satellites and planets.

Chapter 13. Newton's Theory of Gravity

Topics:

- A Little History
- Isaac Newton
- Newton's Law of Gravity
- Little *g* and Big *G*
- Gravitational Potential Energy
- Satellite Orbits and Energies

Chapter 13. Reading Quizzes

Who discovered the basic laws of planetary orbits?

A. Newton
B. Kepler
C. Faraday
D. Einstein
E. Copernicus

What is geometric shape of a planetary or satellite orbit?

A. Circle
B. Hyperbola
C. Sphere
D. Parabola
E. Ellipse

The gravitational force between two objects of masses m_1 and m_2 that are separated by distance r is

A. proportional to r.

B. proportional to $1/r$.

C. proportional to $1/r^2$.

D. $(m_1 + m_2)g$.

E. $(m_1 + m_2)G$.

The value of *g* at the height of the space shuttle's orbit is

A. 9.8 m/s^2.

B. slightly less than 9.8 m/s^2.

C. much less than 9.8 m/s^2.

D. exactly zero.

Chapter 13. Basic Content and Examples

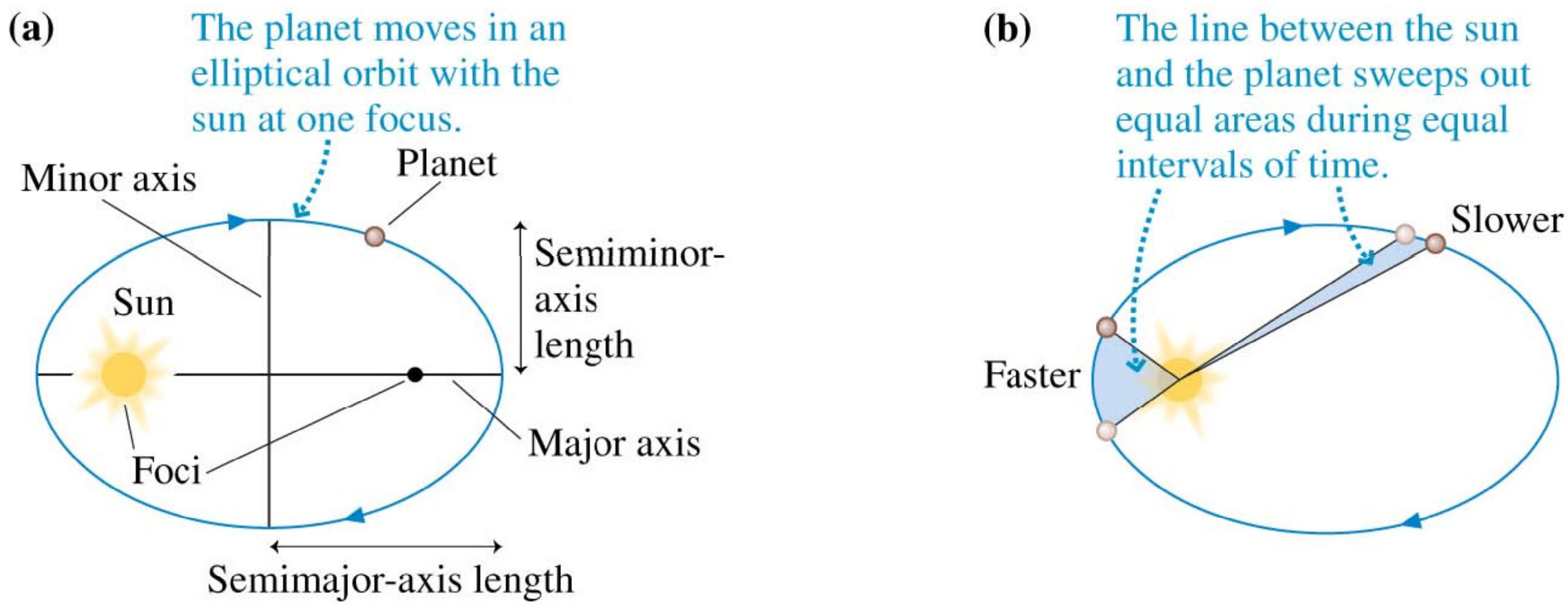
A Little History

Kepler's laws, as we call them today, state that

1. Planets move in elliptical orbits, with the sun at one focus of the ellipse.
2. A line drawn between the sun and a planet sweeps out equal areas during equal intervals of time.
3. The square of a planet's orbital period is proportional to the cube of the semimajor-axis length.

A Little History

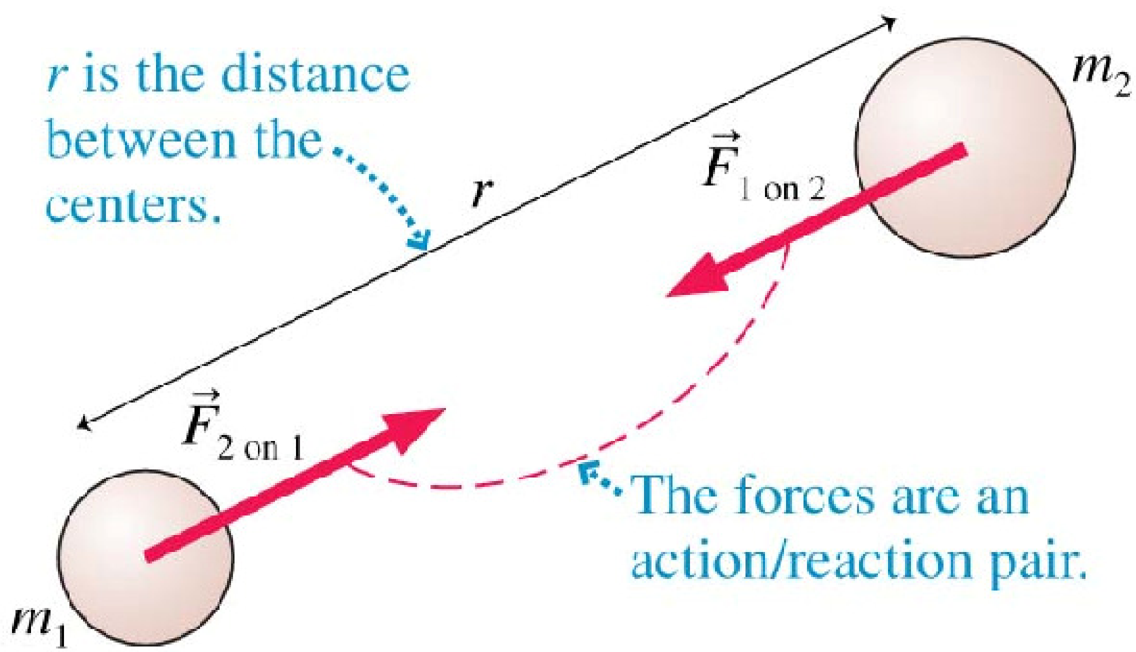
FIGURE 13.2 The elliptical orbit of a planet about the sun.



Newton's Law of Gravity

Newton proposed that *every* object in the universe attracts *every other* object.

FIGURE 13.5 The gravitational forces on masses m_1 and m_2.



Newton's Law of Gravity

Newton's law of gravity If two objects with masses m_1 and m_2 are a distance r apart, the objects exert attractive forces on each other of magnitude

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{Gm_1m_2}{r^2} \qquad (13.2)$$

The forces are directed along the straight line joining the two objects.

The constant G, called the **gravitational constant**, is a proportionality constant necessary to relate the masses, measured in kilograms, to the force, measured in newtons. In the SI system of units, G has the value 6.67×10^{-11} N m^2/kg^2.

Little g and Big G

Suppose an object of mass m is on the surface of a planet of mass M and radius R. The local gravitational force may be written as

$$F_G = mg_{\text{surface}}$$

where we have used a local constant acceleration:

$$g_{\text{surface}} = \frac{GM}{R^2}$$

On earth near sea level it can be shown that $g_{\text{surface}} = 9.80 \text{ m/s}^2$.

Gravitational Potential Energy

When two isolated masses m_1 and m_2 interact over large distances, they have a gravitational potential energy of

$$U_g = -\frac{Gm_1m_2}{r}$$

where we have chosen the zero point of potential energy at $r = \infty$, where the masses will have no tendency, or potential, to move together.

Note that this equation gives the potential energy of masses m_1 and m_2 when their *centers* are separated by a distance r.

EXAMPLE 13.2 Escape speed

QUESTION:

EXAMPLE 13.2 Escape speed

A 1000 kg rocket is fired straight away from the surface of the earth. What speed does the rocket need to "escape" from the gravitational pull of the earth and never return? Assume a nonrotating earth.

EXAMPLE 13.2 Escape speed

MODEL In a simple universe, consisting of only the earth and the rocket, an insufficient launch speed will cause the rocket eventually to fall back to earth. Once the rocket finally slows to a halt, gravity will ever so slowly pull it back. The only way the rocket can escape is to never stop ($v = 0$) and thus never have a turning point! That is, the rocket must continue moving away from the earth forever.

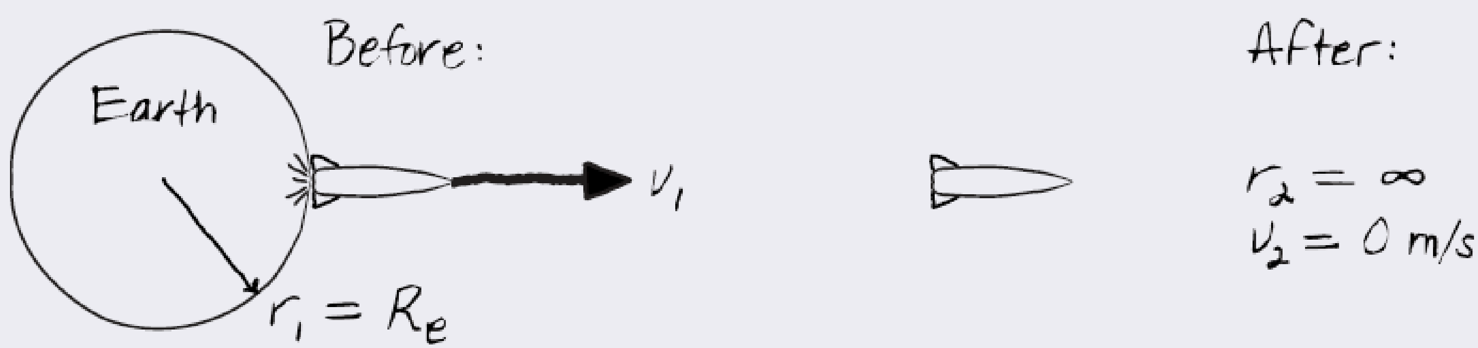
EXAMPLE 13.2 Escape speed

The *minimum* launch speed for escape, which is called the **escape speed,** will cause the rocket to stop ($v = 0$) only as it reaches $r = \infty$. Now ∞, of course, is not a "place," so a statement like this means that we want the rocket's speed to approach $v = 0$ asymptotically as $r \rightarrow \infty$.

EXAMPLE 13.2 Escape speed

VISUALIZE **FIGURE 13.14** is a before-and-after pictorial representation.

FIGURE 13.14 Pictorial representation of a rocket launched with sufficient speed to escape the earth's gravity.



EXAMPLE 13.2 Escape speed

SOLVE Energy conservation $K_2 + U_2 = K_1 + U_1$ is

$$0 + 0 = \frac{1}{2}mv_1^2 - \frac{GM_em}{R_e}$$

where we used the fact that both the kinetic and potential energy are zero at $r = \infty$. Thus the escape speed is

$$v_{escape} = v_1 = \sqrt{\frac{2GM_e}{R_e}} = 11{,}200 \text{ m/s} \approx 25{,}000 \text{ mph}$$

EXAMPLE 13.2 Escape speed

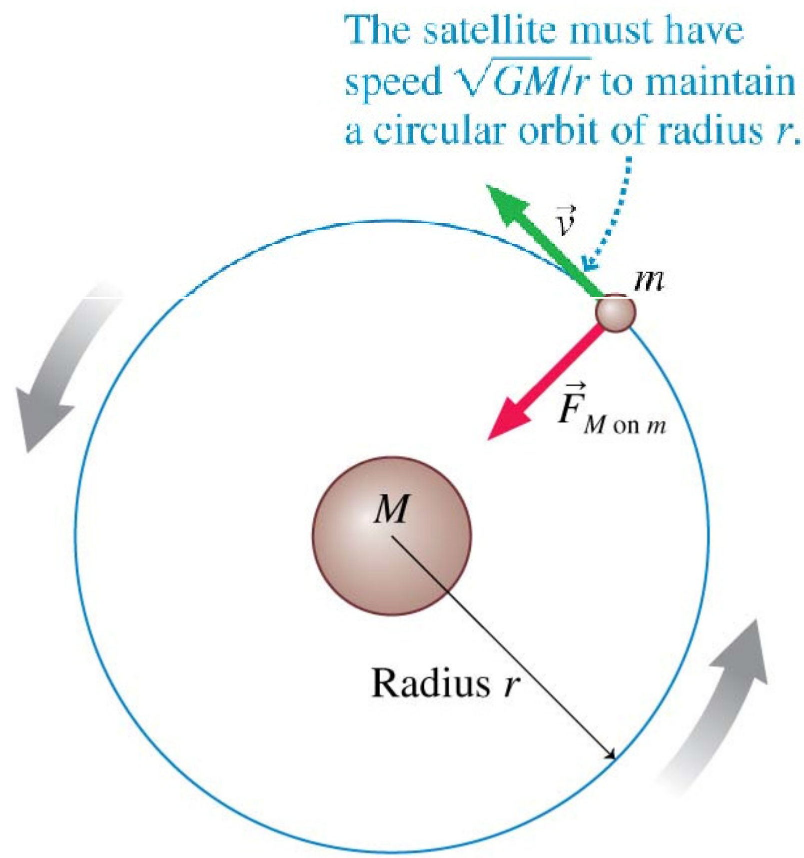
ASSESS The problem was mathematically easy; the difficulty was deciding how to interpret it. That is why—as you have now seen many times—the "physics" of a problem consists of thinking, interpreting, and modeling. We will see variations on this problem in the future, with both gravity and electricity, so you might want to review the *reasoning* involved. Notice that the answer does *not* depend on the rocket's mass, so this is the escape speed for any object.

Satellite Orbits

The mathematics of ellipses is rather difficult, so we will restrict most of our analysis to the limiting case in which an ellipse becomes a circle. Most planetary orbits differ only very slightly from being circular. If a satellite has a circular orbit, its speed is

$$v = \sqrt{\frac{GM}{r}}$$

FIGURE 13.17 The orbital motion of a satellite due to the force of gravity.



EXAMPLE 13.4 The speed of the space shuttle

QUESTION:

EXAMPLE 13.4 The speed of the space shuttle

The space shuttle in a 300-km-high orbit ($\approx$ 180 mi) wants to capture a smaller satellite for repairs. What are the speeds of the shuttle and the satellite in this orbit?

EXAMPLE 13.4 The speed of the space shuttle

SOLVE Despite their different masses, the shuttle, the satellite, and the astronaut working in space to make the repairs all travel side by side with the same speed. They are simply in free fall together. Using $r = R_e + h$ with $h = 300\text{ km} = 3.00 \times 10^5\text{ m}$, we find the speed

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\text{ Nm}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{6.67 \times 10^6\text{ m}}}$$

$$= 7730\text{ m/s} \approx 17{,}000\text{ mph}$$

EXAMPLE 13.4 The speed of the space shuttle

ASSESS The answer depends on the mass of the earth but *not* on the mass of the satellite.

Orbital Energetics

We know that for a satellite in a circular orbit, its speed is related to the size of its orbit by $v^2 = GM/r$. The satellite's kinetic energy is thus

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

But $-GMm/r$ is the potential energy, U_g, so

$$K = -\frac{1}{2}U_g$$

If K and U do not have this relationship, then the trajectory will be elliptical rather than circular. So, the mechanical energy of a satellite in a circular orbit is always:

$$E_{mech} = K + U_g = \frac{1}{2}U_g$$

EXAMPLE 13.6 Raising a satellite

QUESTION:

EXAMPLE 13.6 Raising a satellite

How much work must be done to boost a 1000 kg communications satellite from a low earth orbit with $h = 300$ km, where it is released by the space shuttle, to a geosynchronous orbit?

EXAMPLE 13.6 Raising a satellite

SOLVE The required work is $W_{ext} = \Delta E_{mech}$, and from Equation 13.31 we see that $\Delta E_{mech} = \frac{1}{2}\Delta U_g$. The initial orbit has radius $r_{shuttle} = R_e + h = 6.67 \times 10^6$ m. We earlier found the radius of a geosynchronous orbit to be 4.22×10^7 m. Thus

$$W_{ext} = \Delta E_{mech} = \frac{1}{2}\Delta U_g = \frac{1}{2}(-GM_e m)\left(\frac{1}{r_{geo}} - \frac{1}{r_{shuttle}}\right) = 2.52 \times 10^{10}\ \mathrm{J}$$

EXAMPLE 13.6 Raising a satellite

ASSESS It takes a lot of energy to boost satellites to high orbits!

Chapter 13. Summary Slides

General Principles

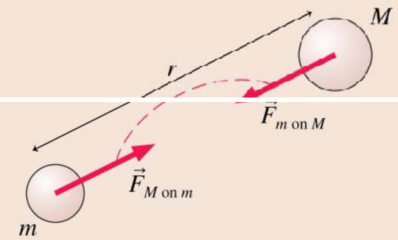
Newton's Theory of Gravity

1. Two objects with masses M and m a distance r apart exert attractive **gravitational forces** on each other of magnitude

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

where the **gravitational constant** is $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$.

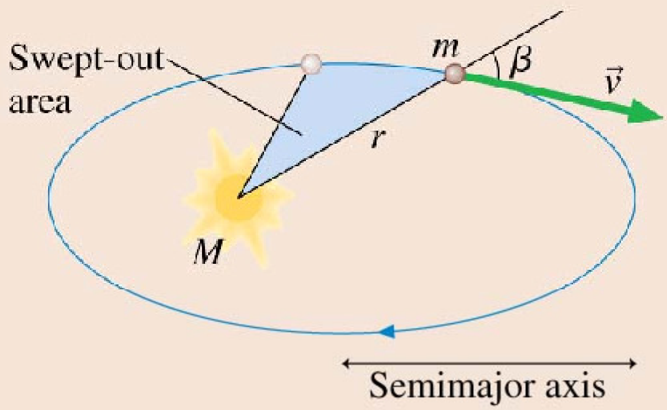
2. Gravitational mass and inertial mass are equivalent.
3. Newton's three laws of motion apply to satellites, planets, and stars.



Important Concepts

Orbital motion of a planet (or satellite) is described by **Kepler's laws:**

1. Orbits are ellipses with the sun (or planet) at one focus.
2. A line between the sun and the planet sweeps out equal areas during equal intervals of time.
3. The square of the planet's period T is proportional to the cube of the orbit's semimajor axis.



Circular orbits are a special case of an ellipse. For a circular orbit around a mass M,

$$v = \sqrt{\frac{GM}{r}} \quad \text{and} \quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Important Concepts

Conservation of angular momentum

The angular momentum $L = mrv \sin\beta$ remains constant throughout the orbit. Kepler's second law is a consequence of this law.

Important Concepts

Orbital energetics

A satellite's mechanical energy $E_{\text{mech}} = K + U_{\text{g}}$ is conserved, where the gravitational potential energy is

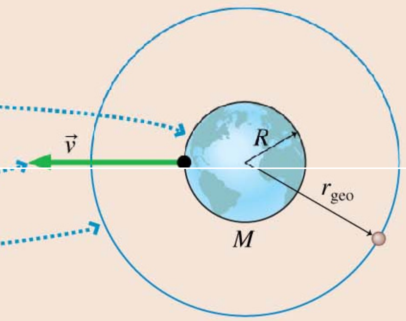
$$U_{\text{g}} = -\frac{GMm}{r}$$

For circular orbits, $K = -\frac{1}{2}U_{\text{g}}$ and $E_{\text{mech}} = \frac{1}{2}U_{\text{g}}$. Negative total energy is characteristic of a **bound system.**

Applications

For a planet of mass M and radius R,

- The free-fall acceleration on the surface is $g_{\text{surface}} = \dfrac{GM}{R^2}$
- The **escape speed** is $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}}$
- The radius of a **geosynchronous orbit** is $r_{\text{geo}} = \left(\dfrac{GM}{4\pi^2}T^2\right)^{1/3}$



Chapter 13. Questions

A satellite orbits the earth with constant speed at a height above the surface equal to the earth's radius. The magnitude of the satellite's acceleration is

A. $g_{\text{on earth}}$.

B. $\frac{1}{4}\,g_{\text{on earth}}$.

C. $\frac{1}{2}\,g_{\text{on earth}}$.

D. $4g_{\text{on earth}}$.

E. $2g_{\text{on earth}}$.

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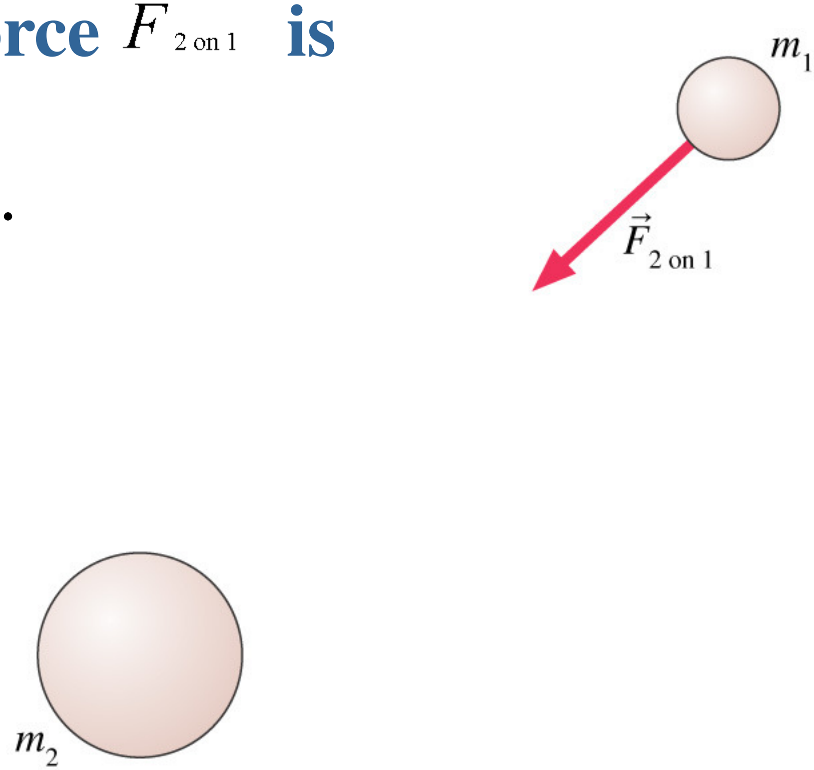
The figure shows a binary star system. The mass of star 2 is twice the mass of star 1. Compared to $\vec{F}_{1\text{ on }2}$, the magnitude of the force $\vec{F}_{2\text{ on }1}$ is



A. one quarter as big.
B. half as big.
C. the same size.
D. twice as big.
E. four times as big.

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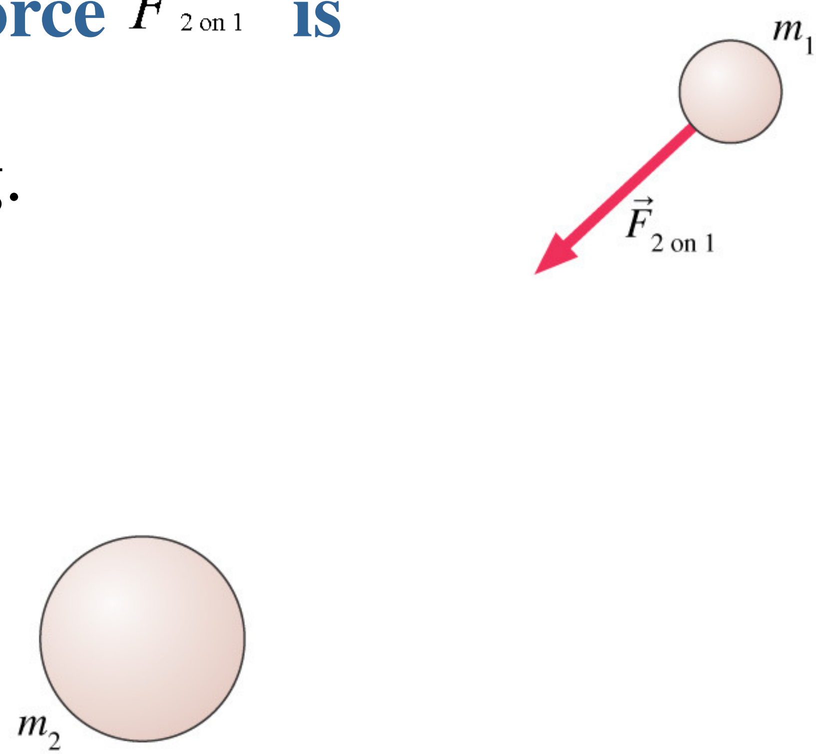

A planet has 4 times the mass of the earth, but the acceleration due to gravity on the planet's surface is the same as on the earth's surface. The planet's radius is

A. $\frac{1}{4}R_e$.

B. $\frac{1}{2}R_e$.

C. $4R_e$.

D. R_e.

E. $2R_e$.

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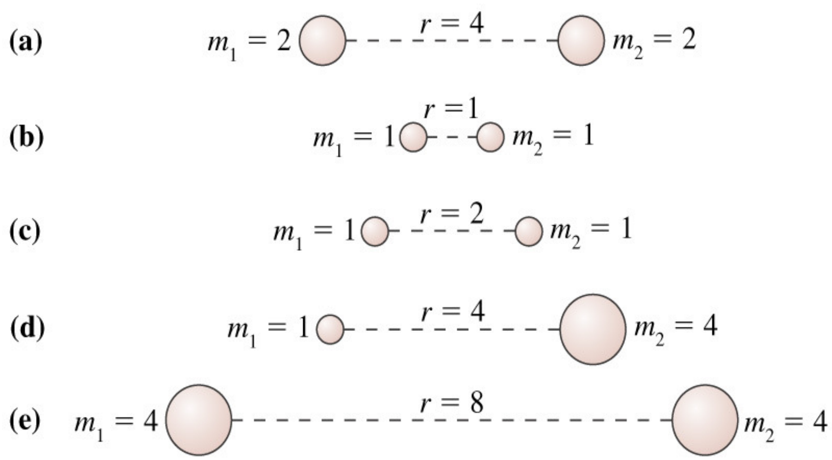
Rank in order, from largest to smallest, the absolute values $|U_g|$ of the gravitational potential energies of these pairs of masses. The numbers give the relative masses and distances.



In absolute value:

A. $U_e > U_d > U_a > U_b = U_c$

B. $U_b > U_c > U_d > U_a > U_e$

C. $U_e > U_a = U_b = U_d > U_c$

D. $U_e > U_a = U_b > U_c > U_d$

E. $U_b > U_c > U_a = U_d > U_e$

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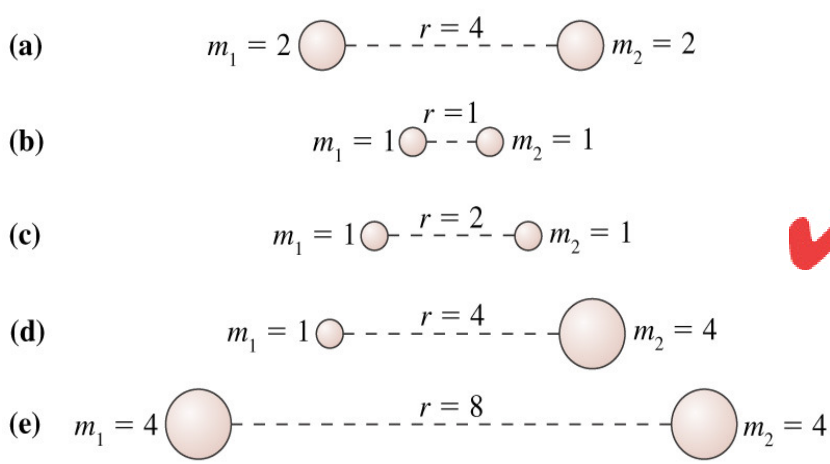

In absolute value:

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B. $U_b > U_c > U_d > U_a > U_e$
C. $U_e > U_a = U_b = U_d > U_c$
D. $U_e > U_a = U_b > U_c > U_d$
E. $U_b > U_c > U_a = U_d > U_e$

Two planets orbit a star. Planet 1 has orbital radius r_1 and planet 2 has $r_2 = 4r_1$. Planet 1 orbits with period T_1. Planet 2 orbits with period

A. $T_2 = T_1$.

B. $T_2 = T_1/2$.

C. $T_2 = 8T_1$.

D. $T_2 = 4T_1$.

E. $T_2 = 2T_1$.

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