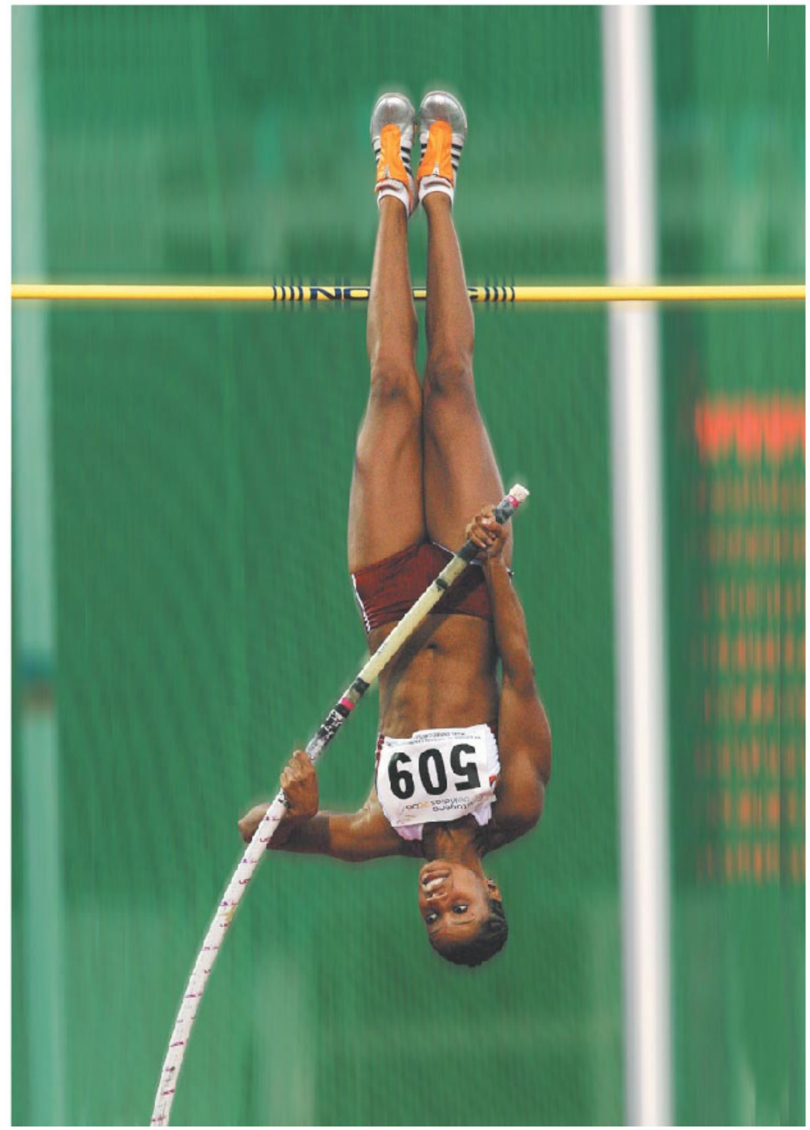
Chapter 10. Energy

This pole vaulter can lift herself nearly 6 m (20 ft) off the ground by transforming the kinetic energy of her run into gravitational potential energy.

Chapter Goal: To introduce the ideas of kinetic and potential energy and to learn a new problem-solving strategy based on conservation of energy.



Chapter 10. Energy

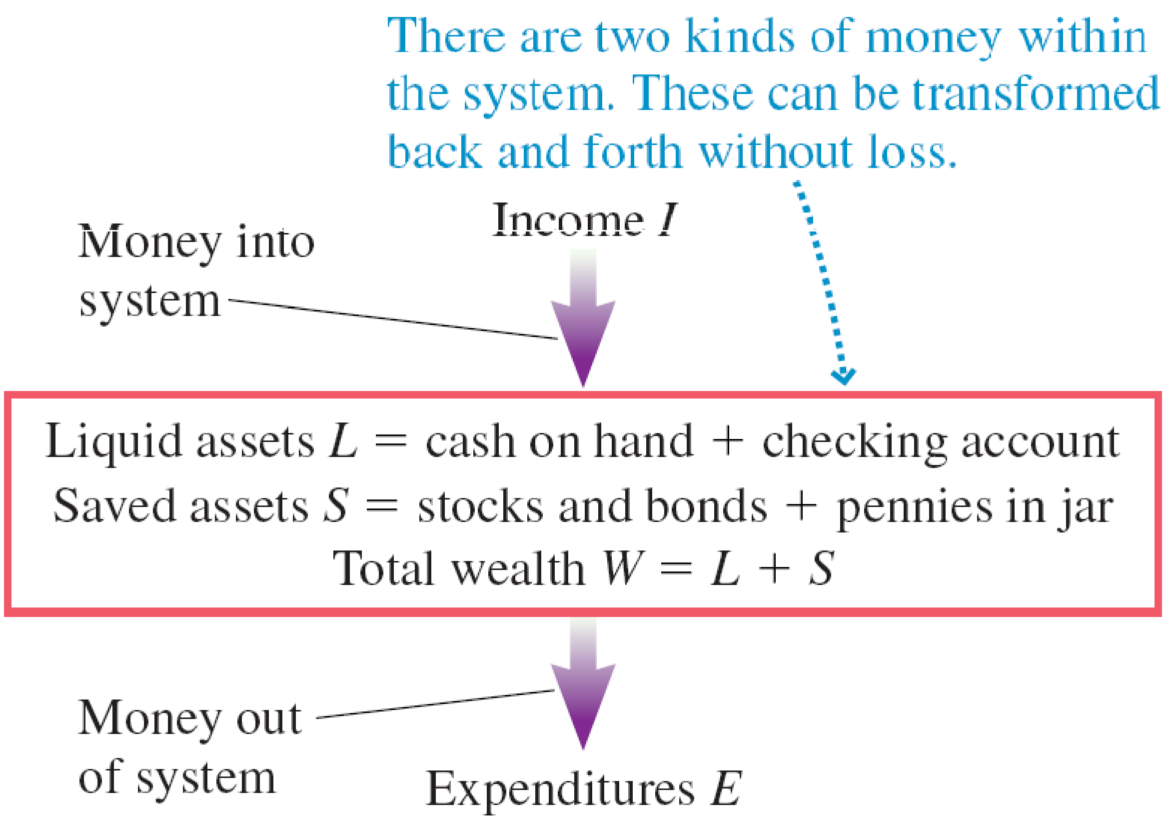
Topics:

- A “Natural Money” Called Energy
- Kinetic Energy and Gravitational Potential Energy
- A Closer Look at Gravitational Potential Energy
- Restoring Forces and Hooke’s Law
- Elastic Potential Energy
- Elastic Collisions
- Energy Diagrams

Money-Energy Analogy

From the *Parable of the Lost Penny*

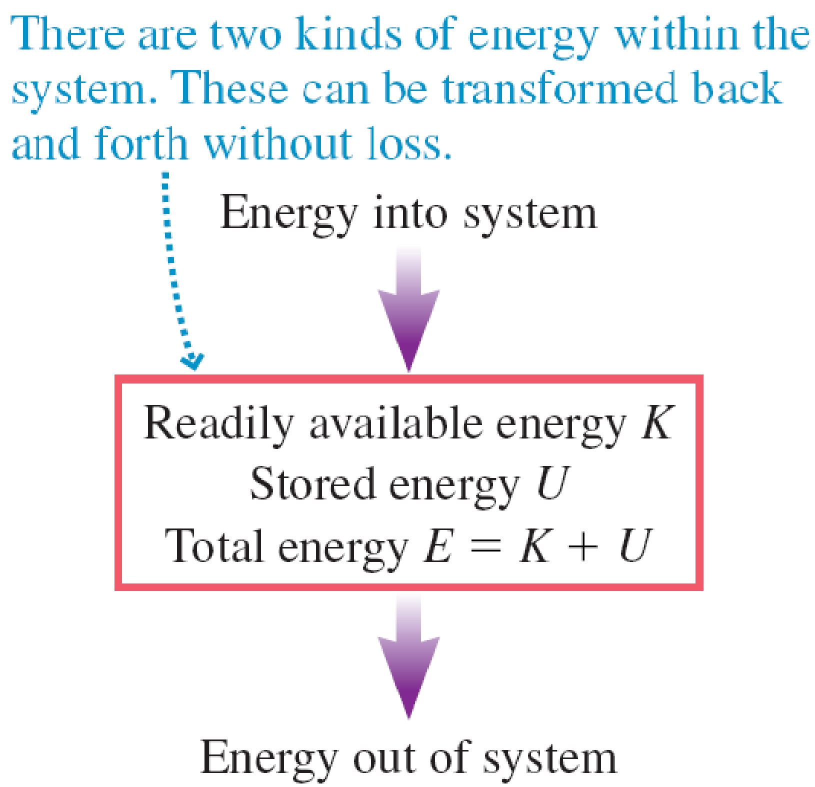
FIGURE 10.1 John's model of the monetary system.



Money—Energy Analogy

From the *law of conservation of energy*

FIGURE 10.2 An initial model of energy. Compare this model to Figure 10.1.



Kinetic and Potential Energy

There are two basic forms of energy. Kinetic energy is an energy of motion

$$K = \frac{1}{2}mv^2 \qquad \text{(kinetic energy)}$$

Gravitational potential energy is an energy of position

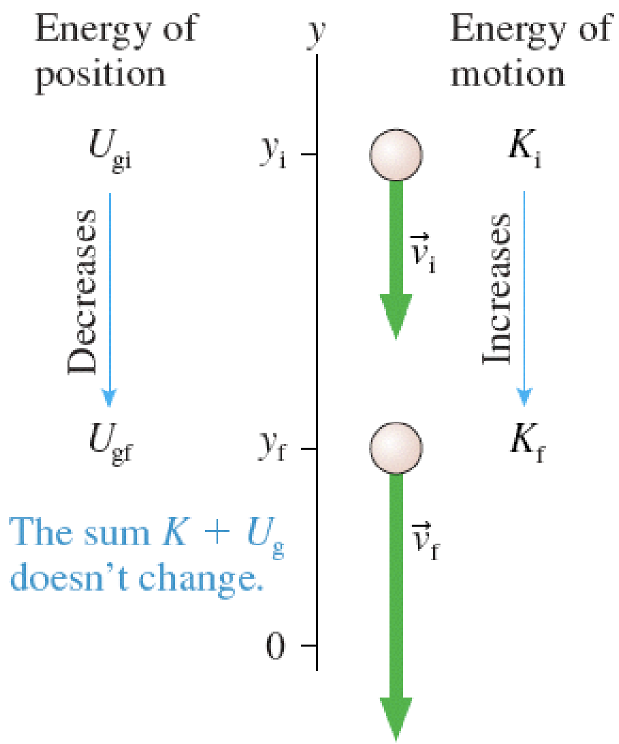
$$U_g = mgy \qquad \text{(gravitational potential energy)}$$

The sum $K + U_g$ is not changed when an object is in freefall. Its initial and final values are equal

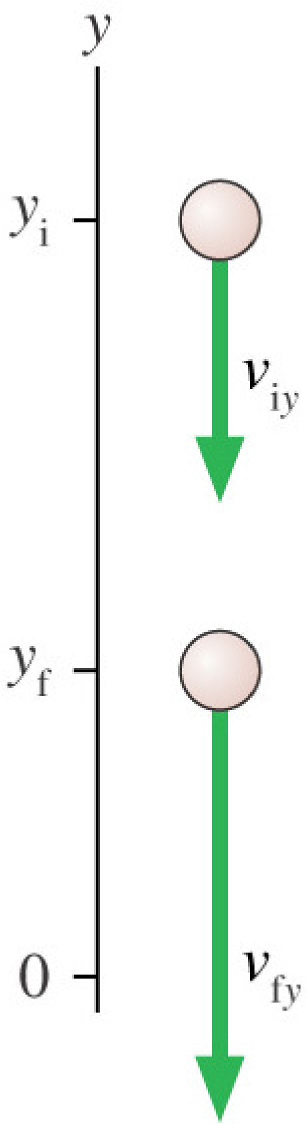
$$K_f + U_{gf} = K_i + U_{gi}$$

Kinetic and Potential Energy

FIGURE 10.4 Kinetic energy and gravitational potential energy.



Free-Fall motion



$$v_{fy}^2 - v_{iy}^2 = 2g(y_i - y_f)$$

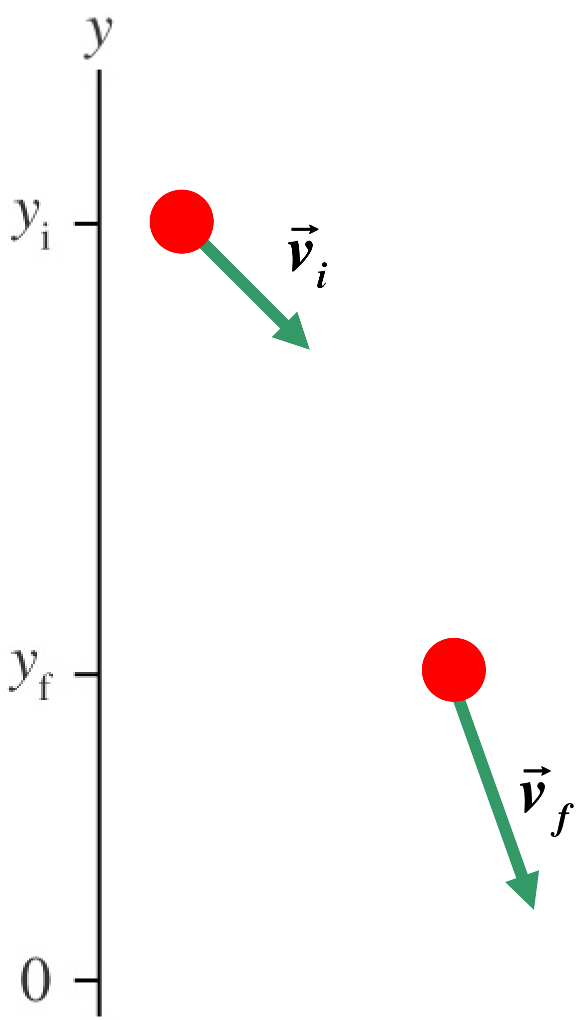
$$\frac{1}{2}v_{fy}^2 + gy_f = \frac{1}{2}v_{iy}^2 + gy_i$$

This is the conservation law for free fall motion: the quantity

$$\frac{1}{2}v_y^2 + gy$$

has the same value before and after the motion.

Free-Fall Motion



$$v_{fy}^2 - v_{iy}^2 = 2g(y_i - y_f)$$

$$v_{fx} = v_{ix}$$

Then

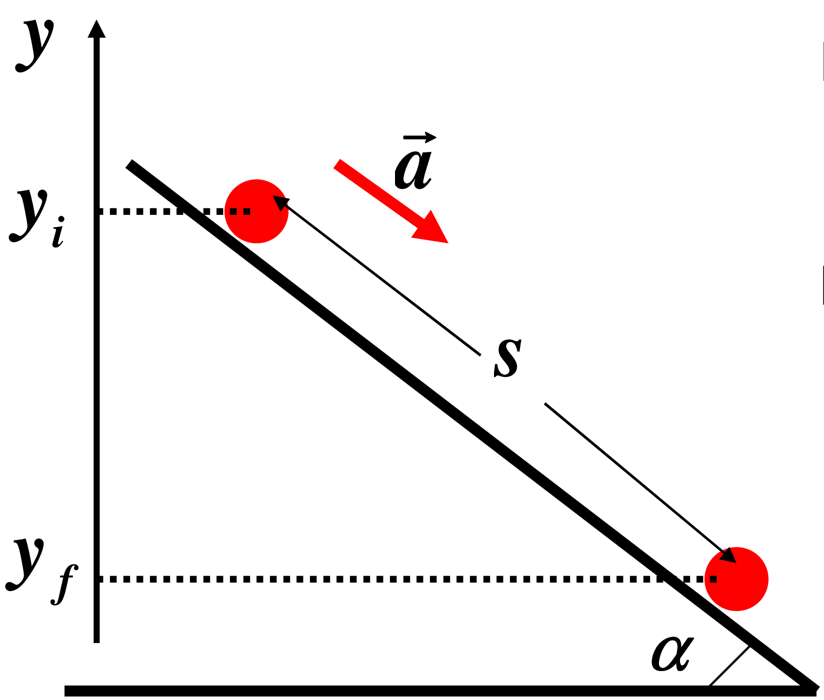
$$\frac{1}{2}v_{fx}^2 + \frac{1}{2}v_{fy}^2 + gy_f = \frac{1}{2}v_{ix}^2 + \frac{1}{2}v_{iy}^2 + gy_i$$

$$\frac{1}{2}v_f^2 + gy_f = \frac{1}{2}v_i^2 + gy_i$$

Conservation law: the quantity

$$\frac{1}{2}v^2 + gy$$

has the same value before and after the motion.



Frictionless surface: acceleration

$$a = g\sin\alpha$$

Motion with constant acceleration:

$$v_f^2 - v_i^2 = 2as$$

$$s = \frac{y_i - y_f}{\sin\alpha}$$

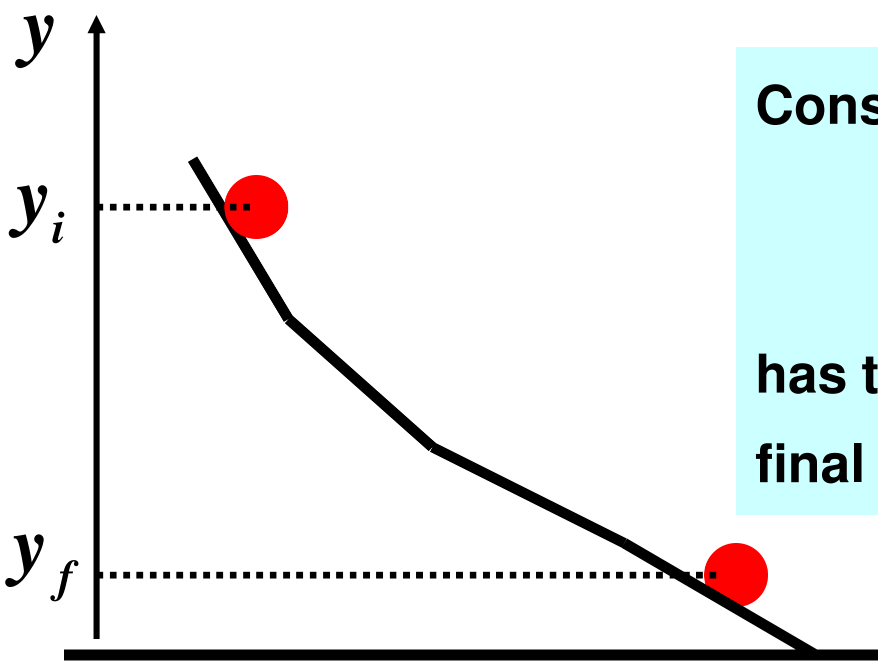
$$v_f^2 - v_i^2 = 2g\sin\alpha\frac{y_i - y_f}{\sin\alpha} = 2gy_i - 2gy$$

Conservation law: the quantity

$$\frac{1}{2}v^2 + gy$$

has the same value before and after the motion.

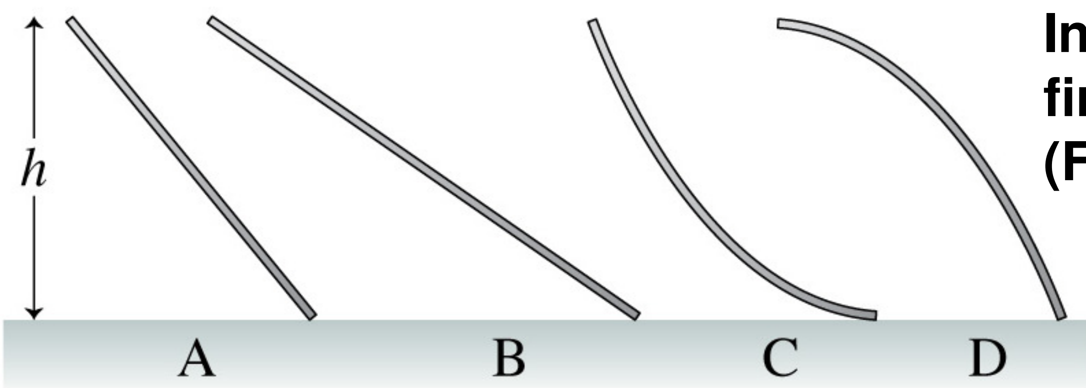
$$\frac{1}{2}v_f^2 + gy_f = \frac{1}{2}v_i^2 + gy_i$$



Conservation law: the quantity

$$\frac{1}{2}v^2 + gy$$

has the same value at the initial and the final points



In all cases the velocity at the final point is the same (FRICTIONLESS MOTION)

Conservation law: $\frac{1}{2}v^2 + gy$ or $\frac{1}{2}mv^2 + mgy$

$K = \frac{1}{2}mv^2$ **Kinetic Energy – energy of motion**

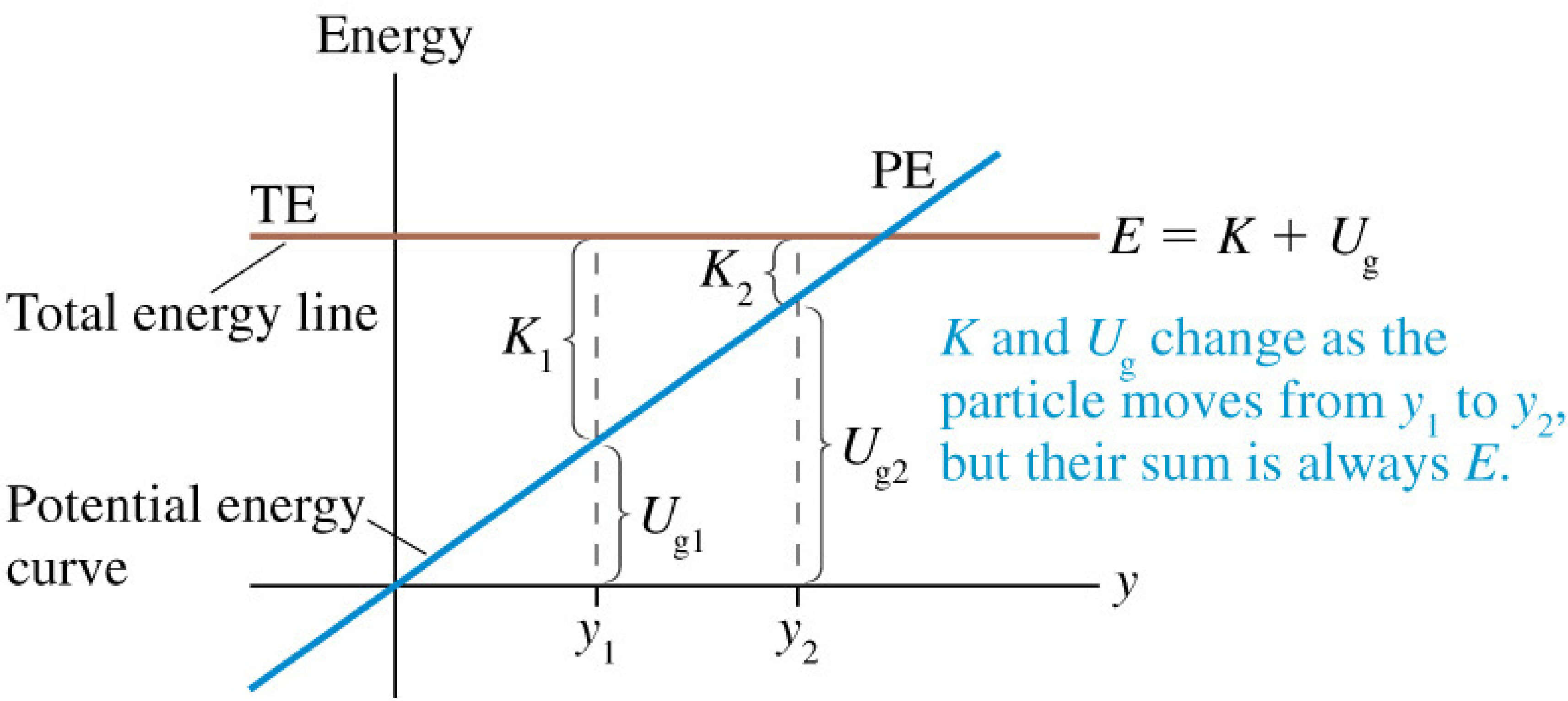
$U_g = mgy$ **Gravitational Potential Energy – energy of position**

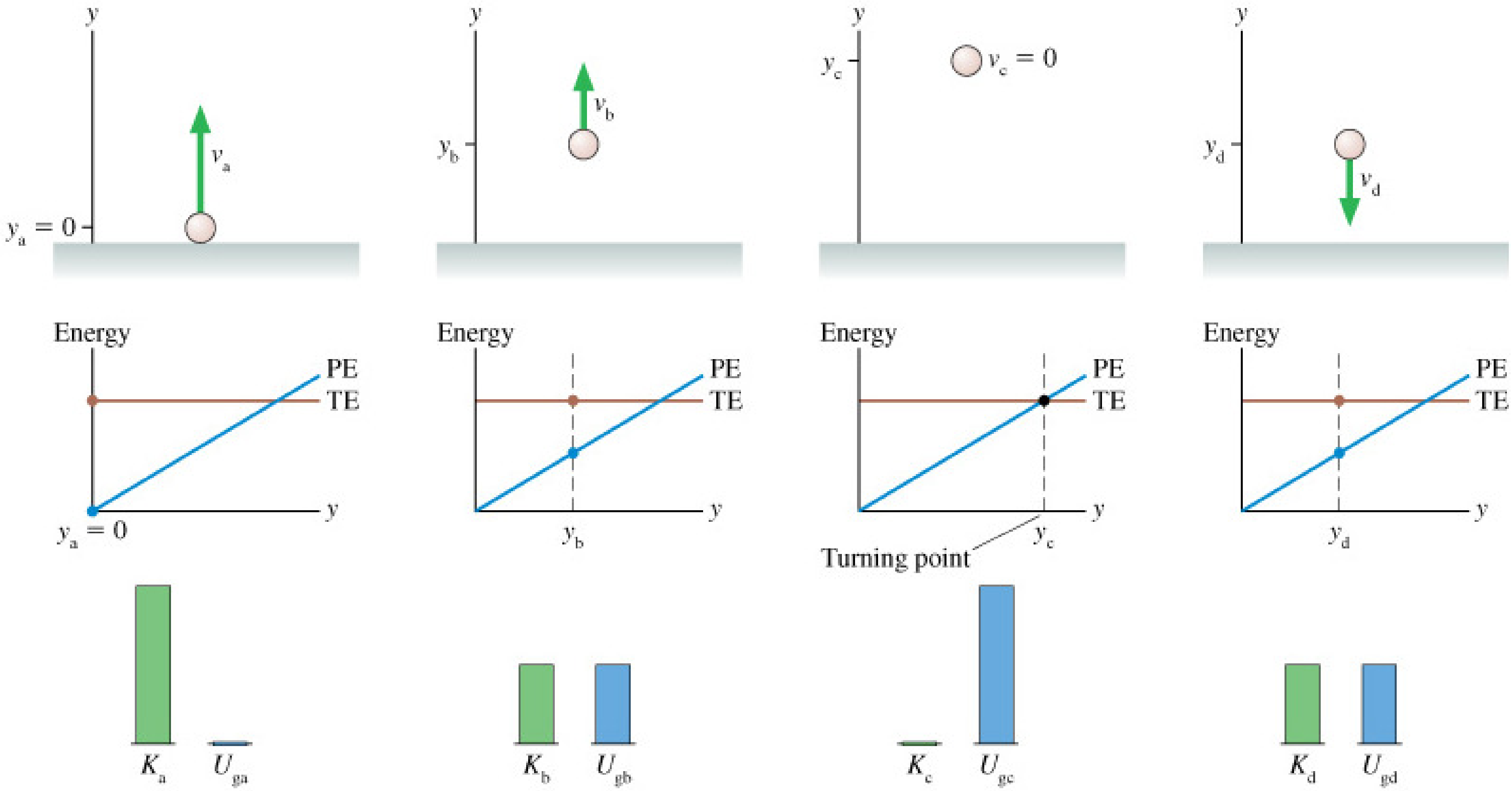
$E_{mech} = K + U_g$ **Mechanical Energy**

Conservation law of mechanical energy (without friction):

$$E_{mech} = K + U = \text{constant}$$

The units of energy is Joule: $J = kg\frac{m^2}{s^2}$


Energy
TE
PE
$E = K + U_g$
Total energy line
K_2
K_1
K and U_g change as the particle moves from y_1 to y_2, but their sum is always E.
U_{g2}
Potential energy curve
U_{g1}
y
y_1
y_2



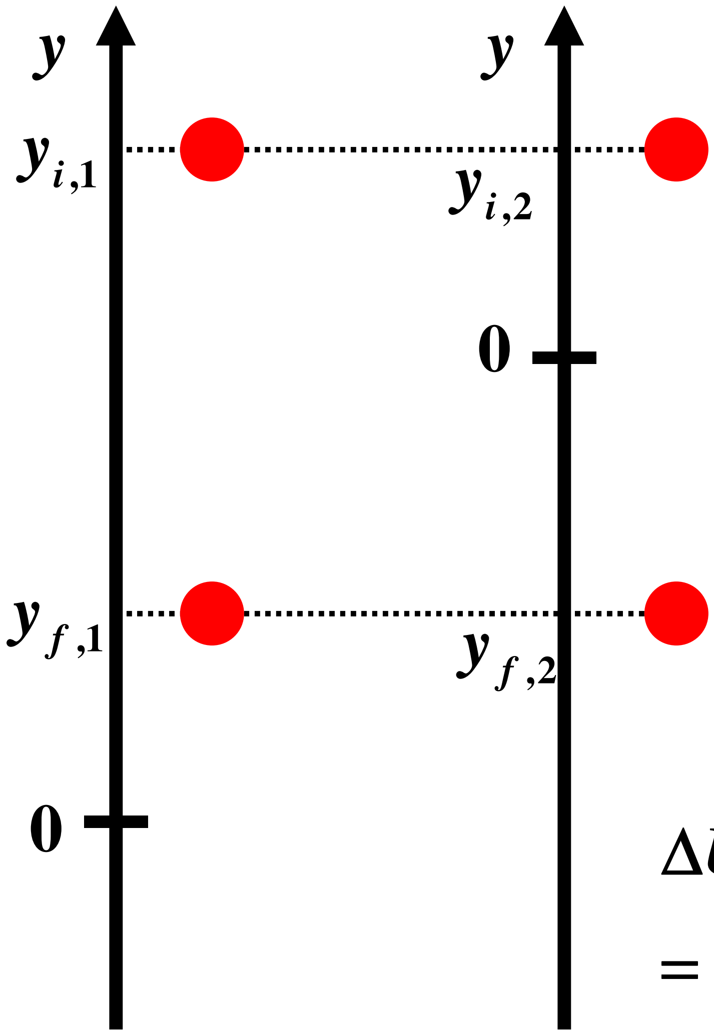
The particle is projected upward. Energy is entirely kinetic.

The particle has gained potential energy, lost kinetic energy.

The energy is entirely potential at the turning point.

The particle gains kinetic energy and loses potential energy as it falls.

Zero of potential energy



$$K_i + U_{g,i,1} = K_{f,1} + U_{g,f,1}$$

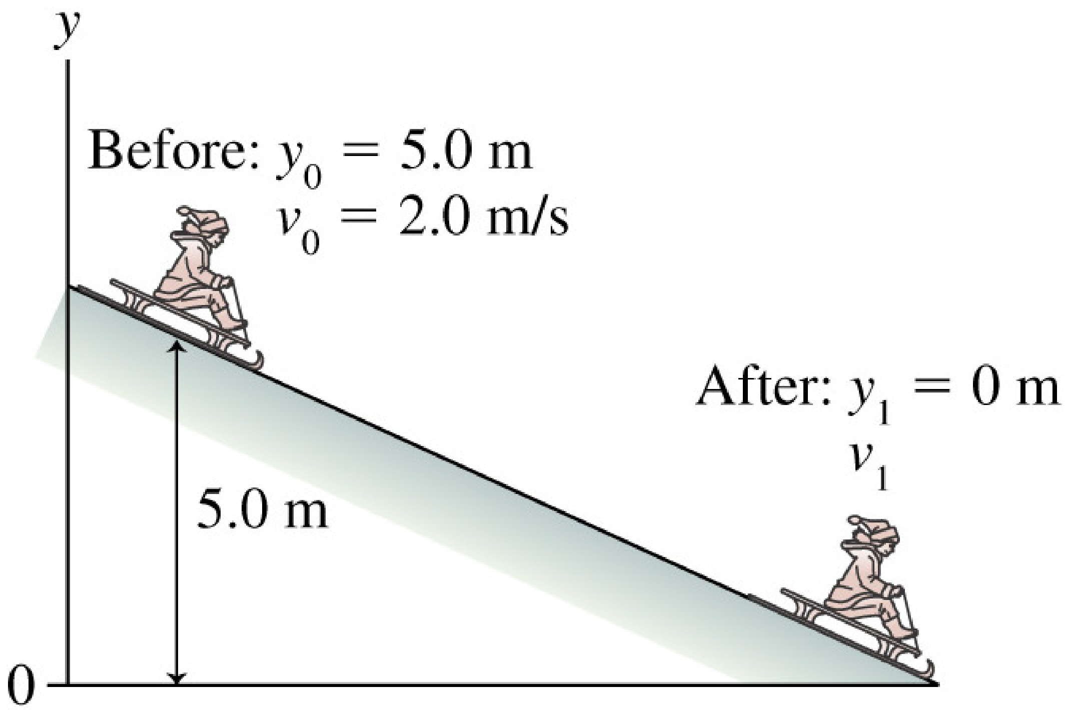
$$K_i + U_{g,i,2} = K_{f,2} + U_{g,f,2}$$

$$K_{f,1} = K_i + (U_{g,i,1} - U_{g,f,1})$$

$$K_{f,2} = K_i + (U_{g,i,2} - U_{g,f,2})$$

$$\Delta U_{g,1} = (U_{g,i,2} - U_{g,f,2}) = mg(y_{i,2} - y_{f,2}) =$$
$$= mg(y_{i,1} - y_{f,1}) = (U_{g,i,1} - U_{g,f,1}) = \Delta U_{g,2}$$

Only the change of potential energy has the physical meaning



Find: v_1

$$K_i + U_{g,i} = K_f + U_{g,f}$$

$$K_i = \frac{1}{2}mv_0^2$$

$$U_{g,i} = mgy_0$$

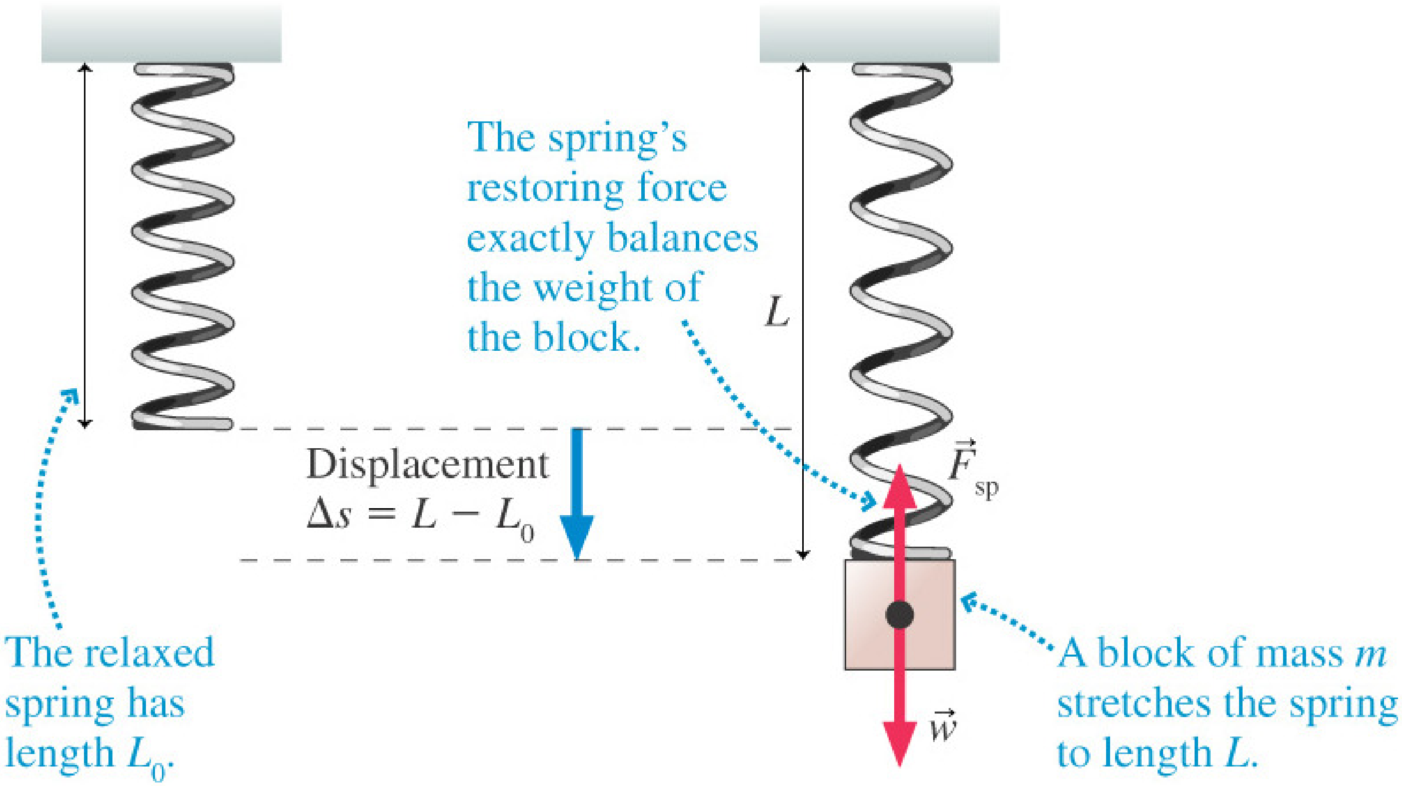
$$U_{g,f} = mgy_1 = 0$$

$$K_f = \frac{1}{2}mv_1^2$$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + mgy_0$$

$$v_1 = \sqrt{v_0^2 + 2gy_0} = \sqrt{4+100} \approx 10.2m/s$$

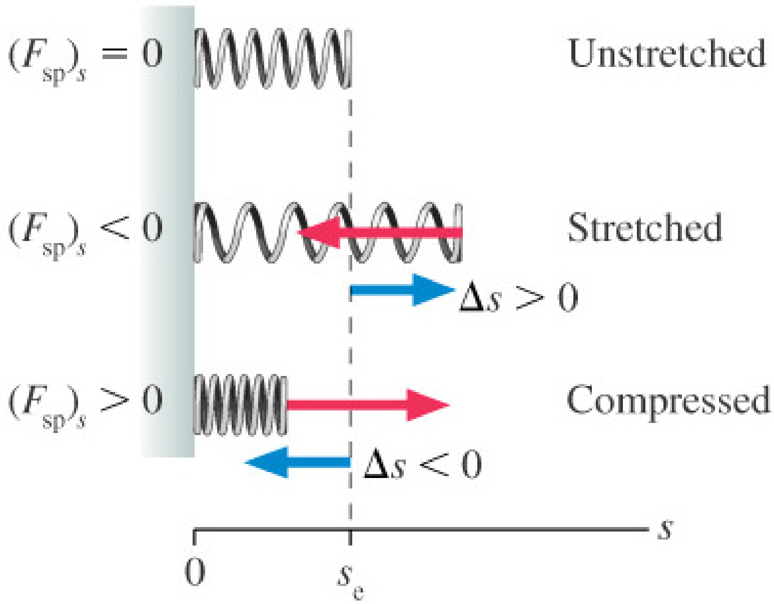
Restoring Force: Hooke's Law

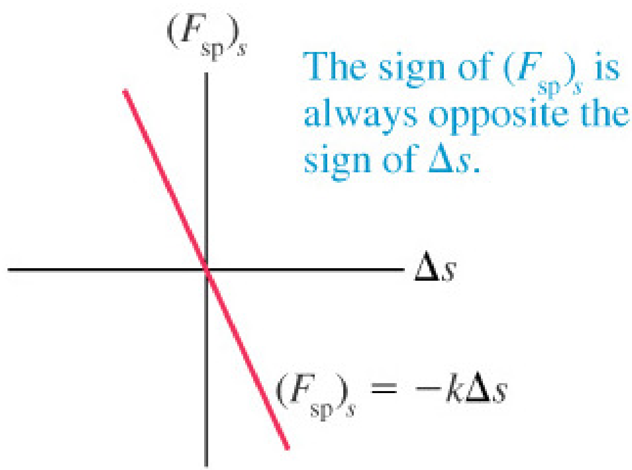


Hooke's Law: $$F_{sp} = -k\Delta s$$

spring constant

Restoring Force: Hooke's Law

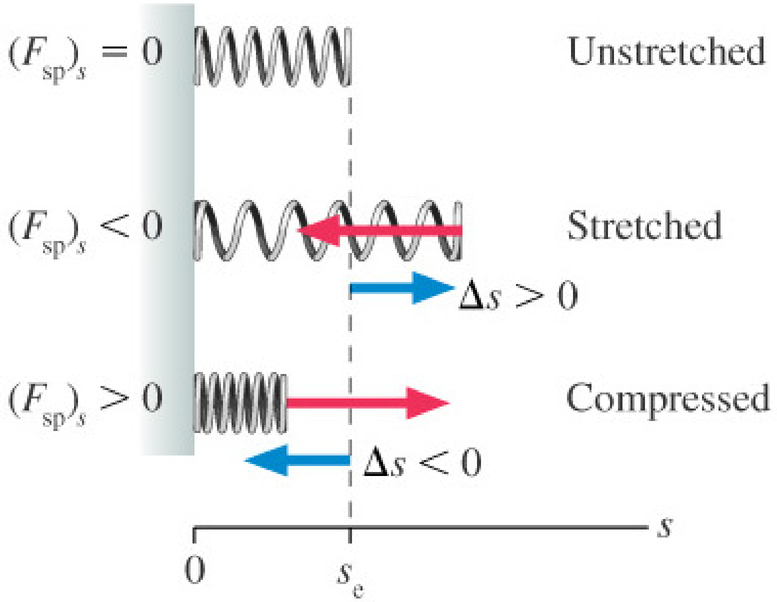




$$F_{sp} = -k\Delta s$$

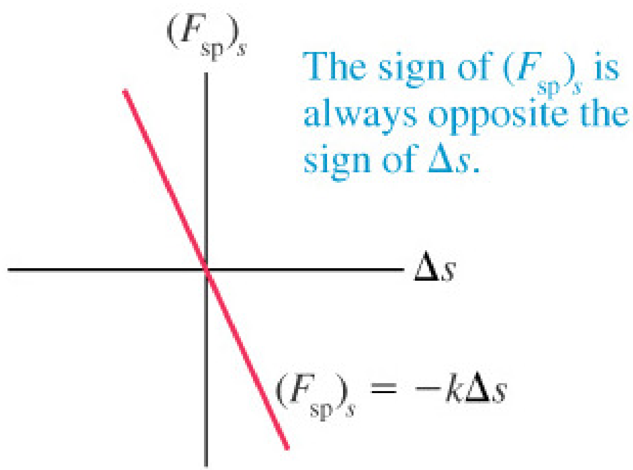
The sign of a restoring force is always opposite to the sign of displacement

Restoring Force: Elastic Potential Energy



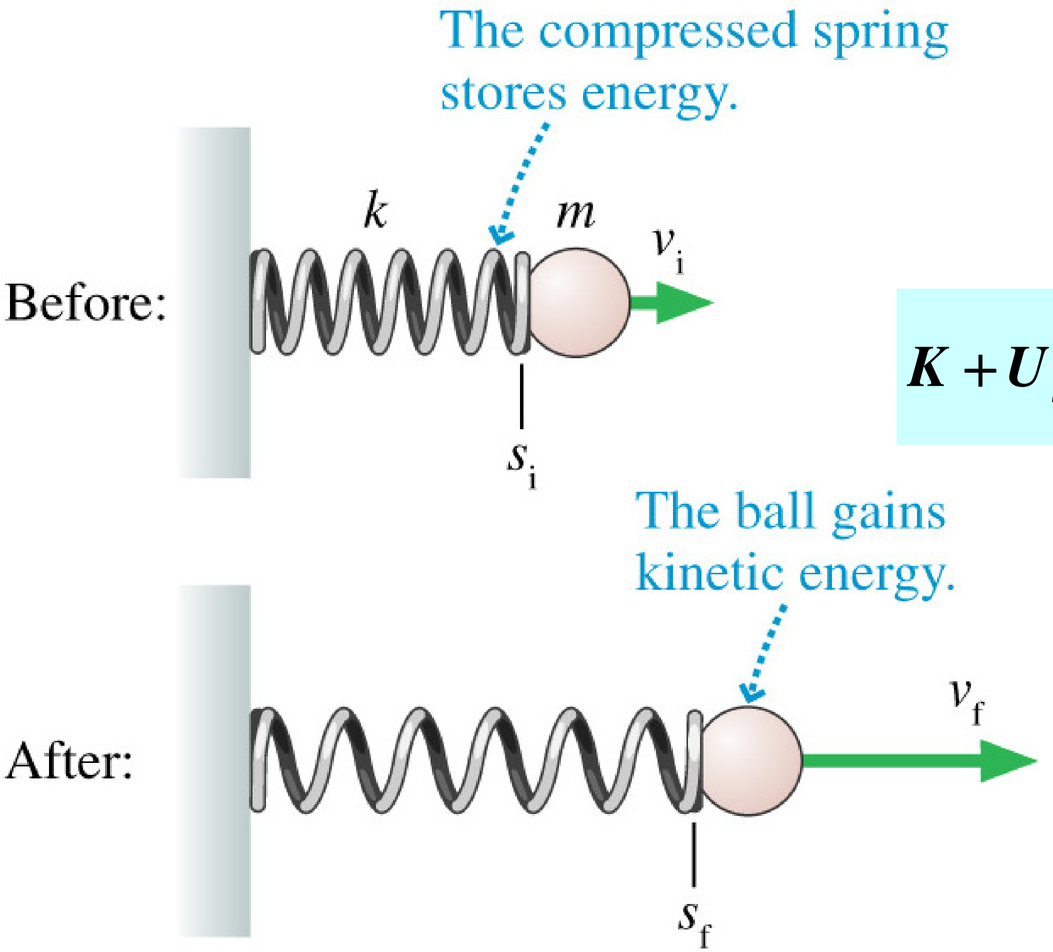
$$F_{sp} = -k\Delta s$$

$$U_s = \frac{1}{2}k(\Delta s)^2$$



The elastic potential energy is always positive

Restoring Force: Elastic Potential Energy

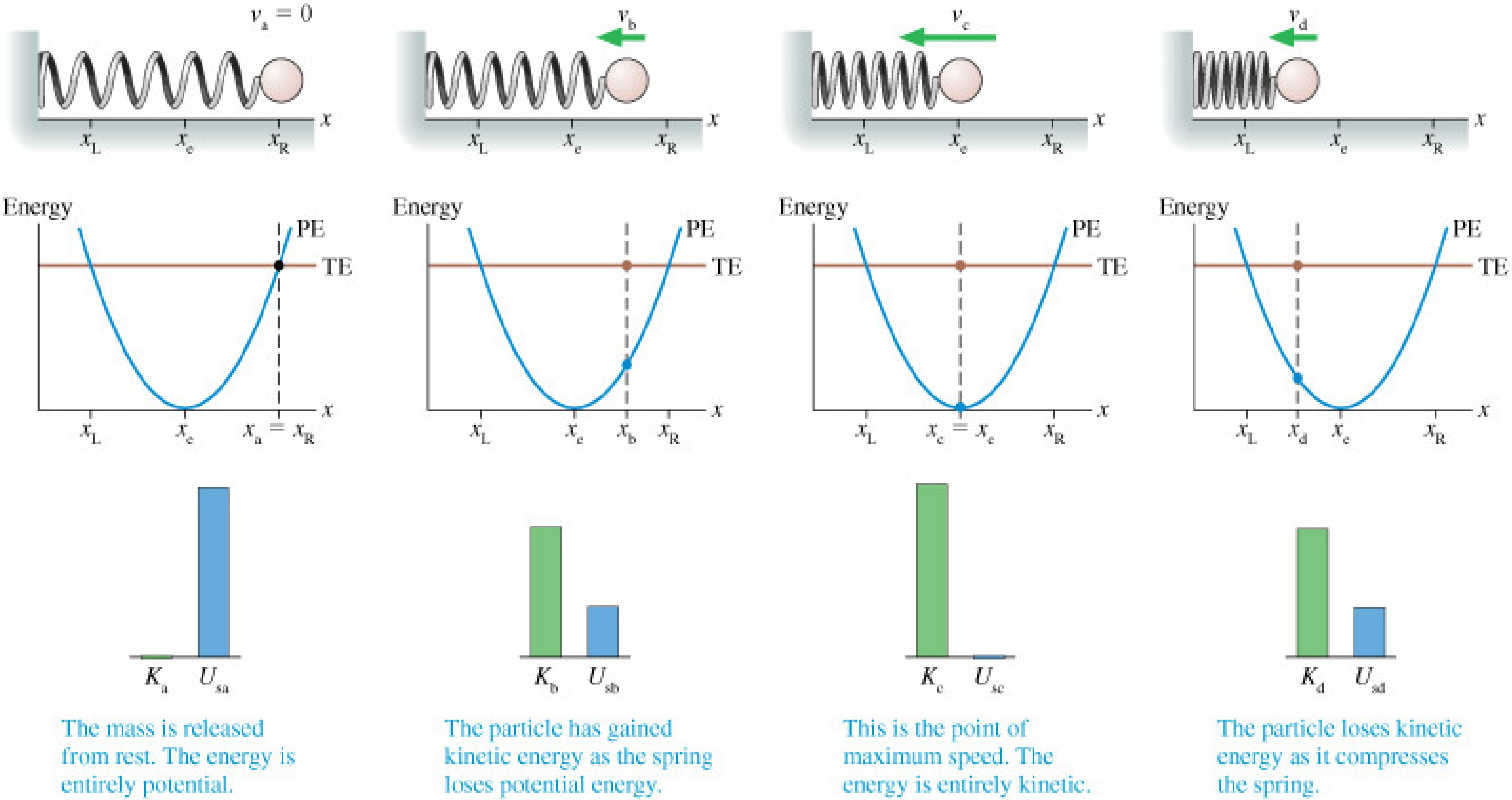


$$U_s = \frac{1}{2}k(\Delta s)^2$$

$$K + U_s = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta s)^2 = \text{constant}$$

Restoring Force: Elastic Potential Energy

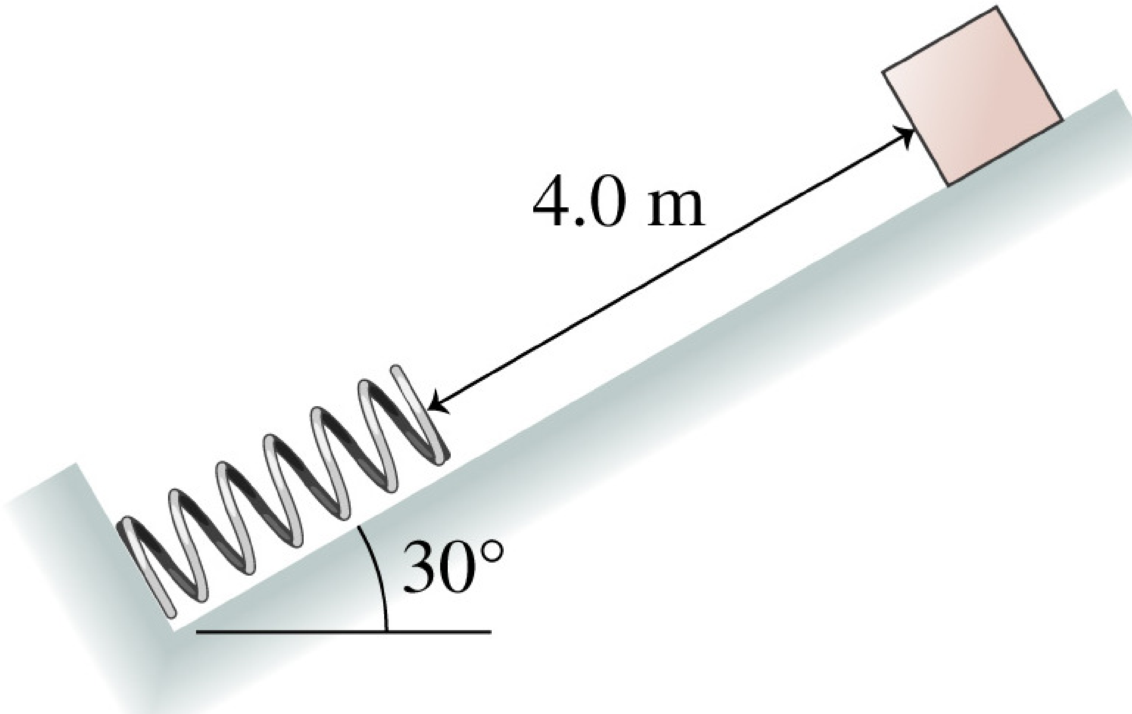
$$U_s = \frac{1}{2}k(\Delta s)^2$$

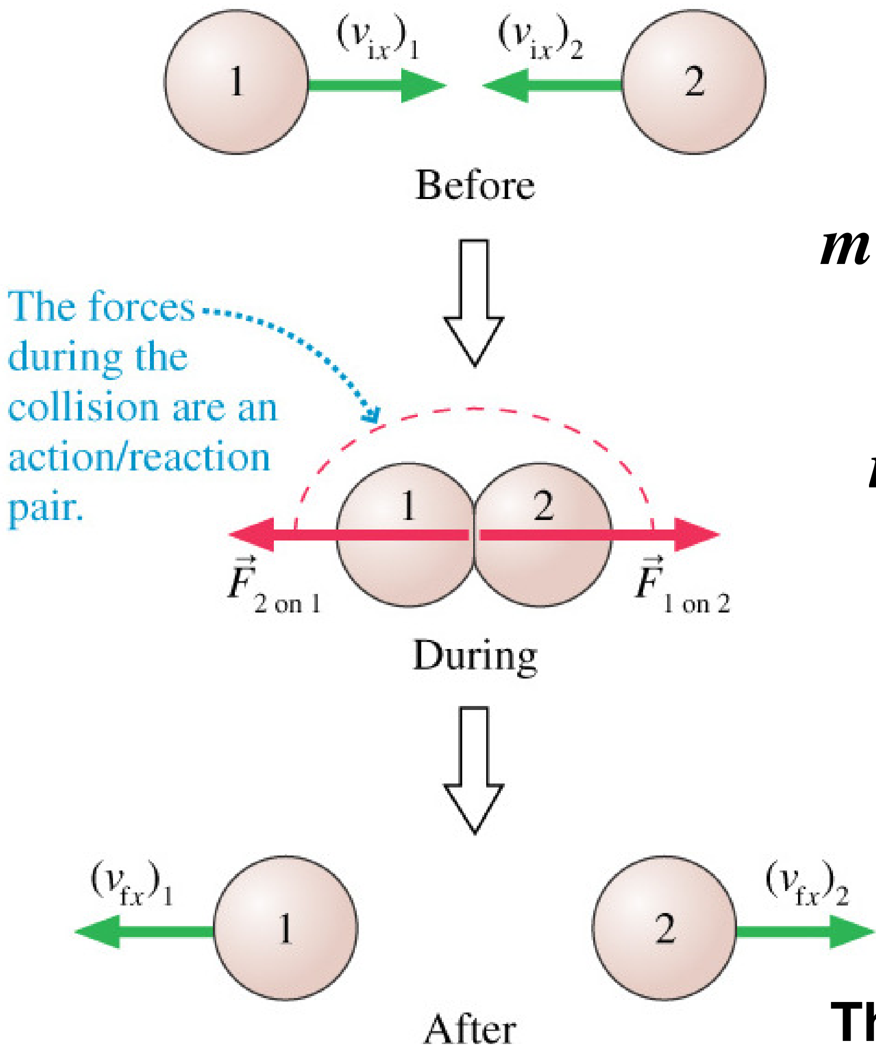


$$K + U_s = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta s)^2 = \text{constant}$$

Law of Conservation of Mechanical Energy

$$K + U_g + U_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k(\Delta s)^2 = \text{constant}$$





$$m_2 v_{fx,2} - m_2 v_{ix,2} = -\left(m_1 v_{fx,1} - m_1 v_{ix,1}\right)$$

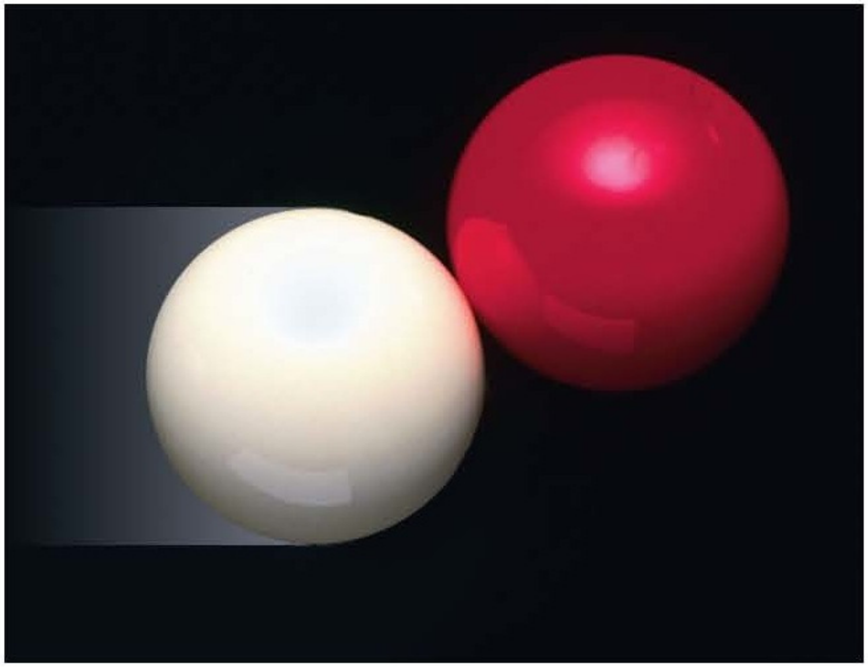
$$m_1 v_{ix,1} + m_2 v_{ix,2} = m_1 v_{fx,1} + m_2 v_{fx,2}$$

$$p_{ix,1} + p_{ix,2} = p_{fx,1} + p_{fx,2}$$

$$p_{ix,total} = p_{fx,total}$$

The law of conservation of momentum

Elastic Collisions

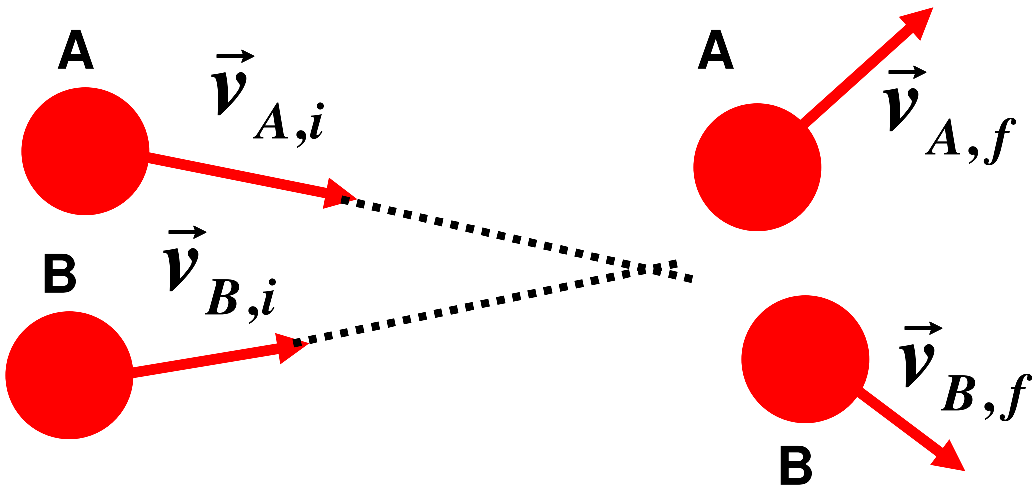


A perfectly elastic collision conserves both momentum and mechanical energy.

Perfectly Elastic Collisions

During the collision the kinetic energy will be transformed into elastic energy and then elastic energy will transformed back into kinetic energy

Perfectly Elastic Collision: Mechanical Energy is Conserved (no internal friction)



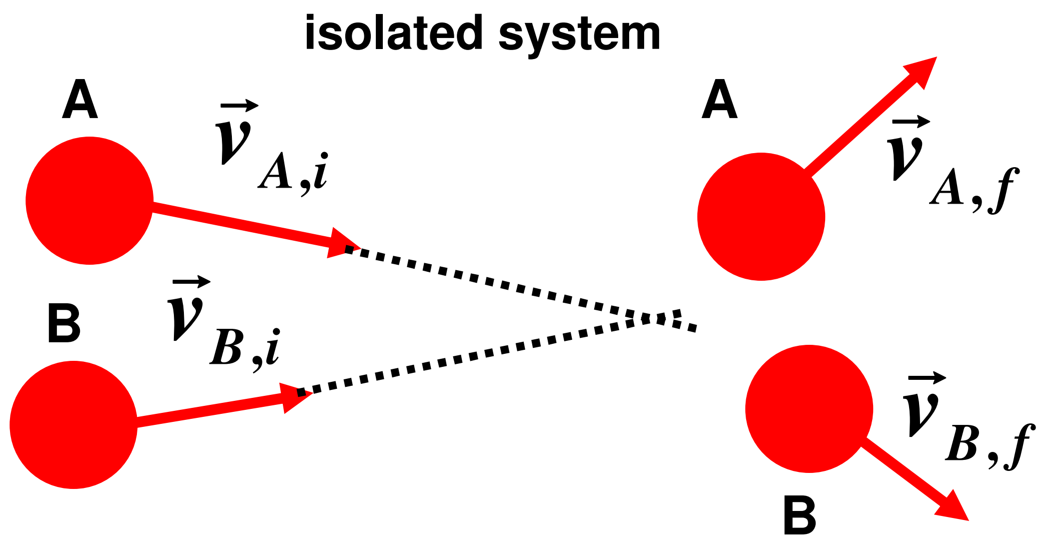
$$\frac{1}{2}m_A v_{A,i}^2 + \frac{1}{2}m_B v_{B,i}^2 =$$
$$= \frac{1}{2}m_A v_{A,f}^2 + \frac{1}{2}m_B v_{B,f}^2$$

Perfectly inelastic collision:

A collision in which the two objects stick together and move with a common final velocity – NO CONSERVATION OF MECHANICAL ENERGY

Perfectly Elastic Collisions

Conservation of Momentum and Conservation of Energy

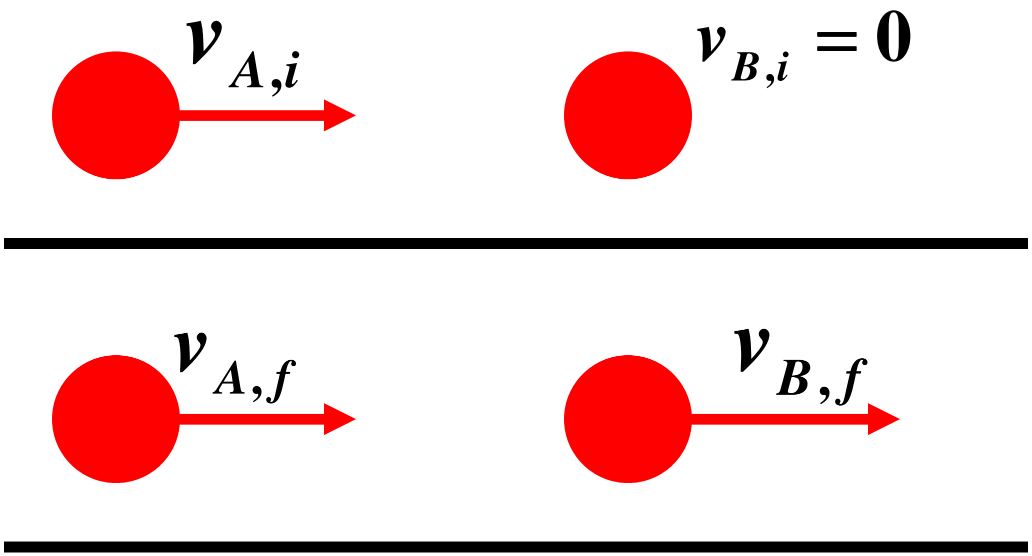


$$m_A \vec{v}_{A,i} + m_B \vec{v}_{B,i} = m_A \vec{v}_{A,f} + m_B \vec{v}_{B,f}$$

$$\frac{1}{2} m_A v_{A,i}^2 + \frac{1}{2} m_B v_{B,i}^2 = \frac{1}{2} m_A v_{A,f}^2 + \frac{1}{2} m_B v_{B,f}^2$$

Now we have enough equations to find the final velocities.

Example:



$$m_A v_{A,i} = m_A v_{A,f} + m_B v_{B,f}$$

$$\frac{1}{2} m_A v_{A,i}^2 = \frac{1}{2} m_A v_{A,f}^2 + \frac{1}{2} m_B v_{B,f}^2$$

$$v_{A,f} = \frac{m_A - m_B}{m_A + m_B} v_{A,i}$$

$$v_{B,f} = \frac{2m_A}{m_A + m_B} v_{A,i}$$

$v_{B,f}$ is always positive, $v_{A,f}$ can be positive (if $m_A > m_B$) or negative (if $m_A < m_B$)

$$v_{A,f} = 0$$
$$v_{B,f} = v_{A,i}$$

if $m_A = m_B$

Chapter 10. Summary Slides

General Principles

Law of Conservation of Mechanical Energy

If there are no friction or other energy-loss processes (to be explored more thoroughly in Chapter 11), then the mechanical energy $E_{mech} = K + U$ of a system is conserved. Thus

$$K_f + U_f = K_i + U_i$$

- K is the sum of the kinetic energies of all particles.
- U is the sum of all potential energies.

General Principles

Solving Energy Conservation Problems

MODEL Choose a system without friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation.

SOLVE Use the law of conservation of energy:

$$K_f + U_f = K_i + U_i$$

ASSESS Is the result reasonable?

Important Concepts

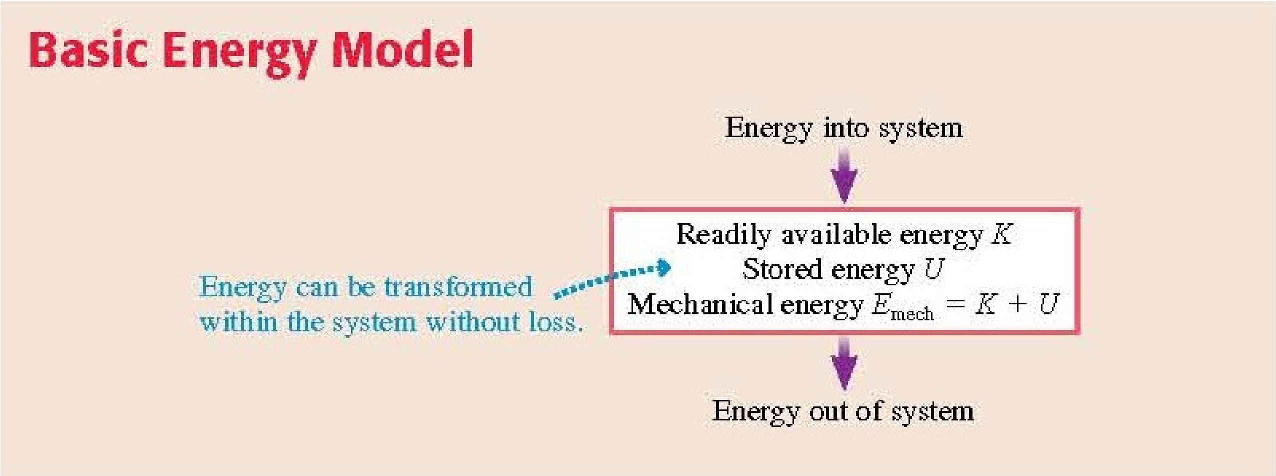
Kinetic energy is an energy of motion:

$$K = \frac{1}{2}mv^2$$

Potential energy is an energy of position

- **Gravitational:** $U_g = mgy$
- **Elastic:** $U_s = \frac{1}{2}k(\Delta s)^2$

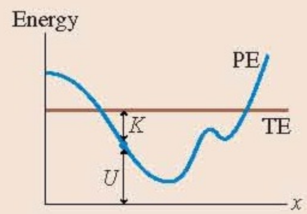
Important Concepts



Important Concepts

Energy diagrams

These diagrams show the potential-energy curve PE and the total mechanical energy line TE.



- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a **turning point.**
- Minima in the PE curve are points of **stable equilibrium.** Maxima are points of **unstable equilibrium.**

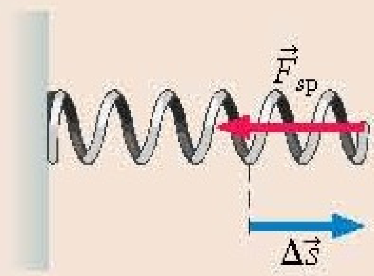
Applications

Hooke's law

The restoring force of an ideal spring is

$$(F_{sp})_s = -k\,\Delta s$$

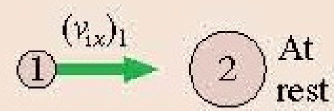
where k is the spring constant and $\Delta s = s - s_e$ is the displacement from equilibrium.



Applications

Perfectly elastic collisions

Both mechanical energy and momentum are conserved.

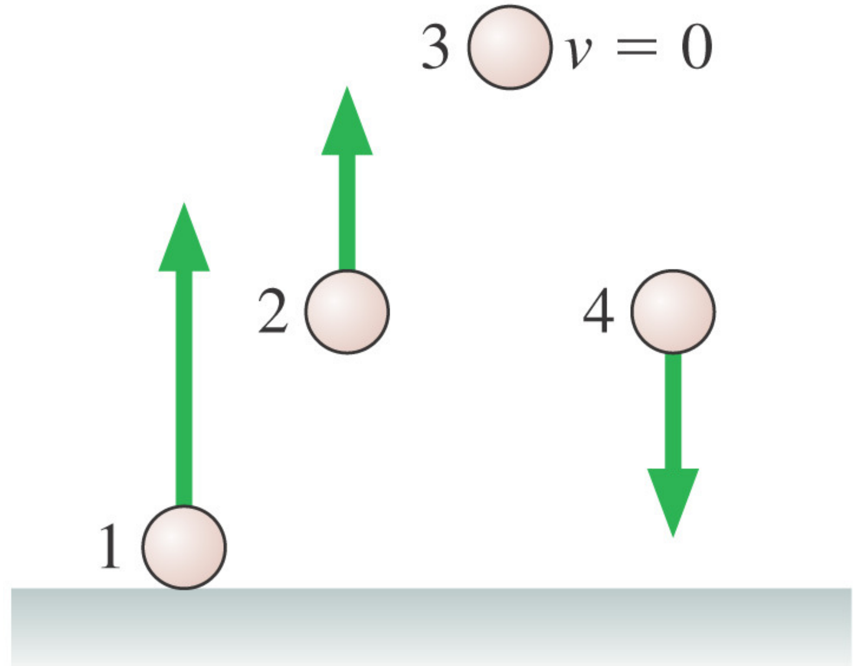


$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

If ball 2 is moving, transform to a reference frame in which ball 2 is at rest.

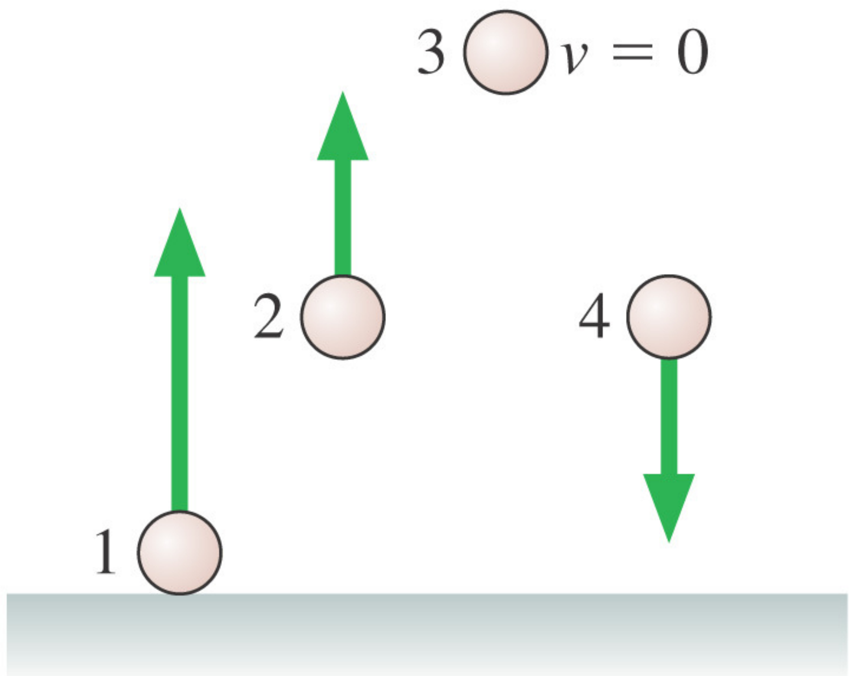
Chapter 10. Clicker Questions

Rank in order, from largest to smallest, the gravitational potential energies of balls 1 to 4.



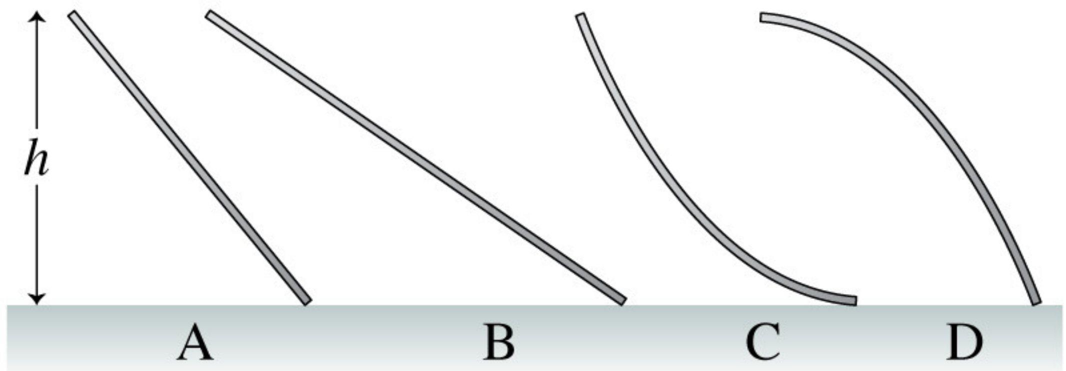
A. $(U_g)_1 > (U_g)_2 = (U_g)_4 > (U_g)_3$
B. $(U_g)_4 > (U_g)_3 > (U_g)_2 > (U_g)_1$
C. $(U_g)_1 > (U_g)_2 > (U_g)_3 > (U_g)_4$
D. $(U_g)_4 = (U_g)_2 > (U_g)_3 > (U_g)_1$
E. $(U_g)_3 > (U_g)_2 = (U_g)_4 > (U_g)_1$

Rank in order, from largest to smallest, the gravitational potential energies of balls 1 to 4.



A. $(U_g)_1 > (U_g)_2 = (U_g)_4 > (U_g)_3$
B. $(U_g)_4 > (U_g)_3 > (U_g)_2 > (U_g)_1$
C. $(U_g)_1 > (U_g)_2 > (U_g)_3 > (U_g)_4$
D. $(U_g)_4 = (U_g)_2 > (U_g)_3 > (U_g)_1$
E. $(U_g)_3 > (U_g)_2 = (U_g)_4 > (U_g)_1$

A small child slides down the four frictionless slides A–D. Each has the same height. Rank in order, from largest to smallest, her speeds v_A to v_D at the bottom.



A. $v_C > v_A = v_B > v_D$

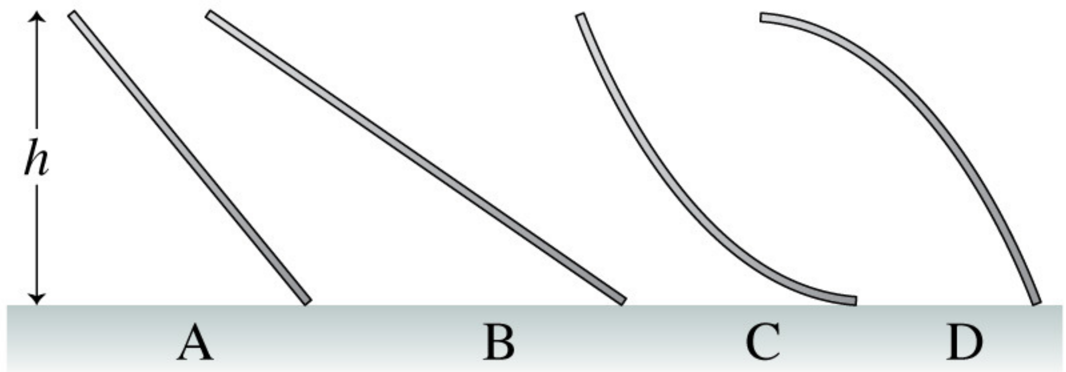
B. $v_C > v_B > v_A > v_D$

C. $v_D > v_A > v_B > v_C$

D. $v_A = v_B = v_C = v_D$

E. $v_D > v_A = v_B > v_C$

A small child slides down the four frictionless slides A–D. Each has the same height. Rank in order, from largest to smallest, her speeds v_A to v_D at the bottom.



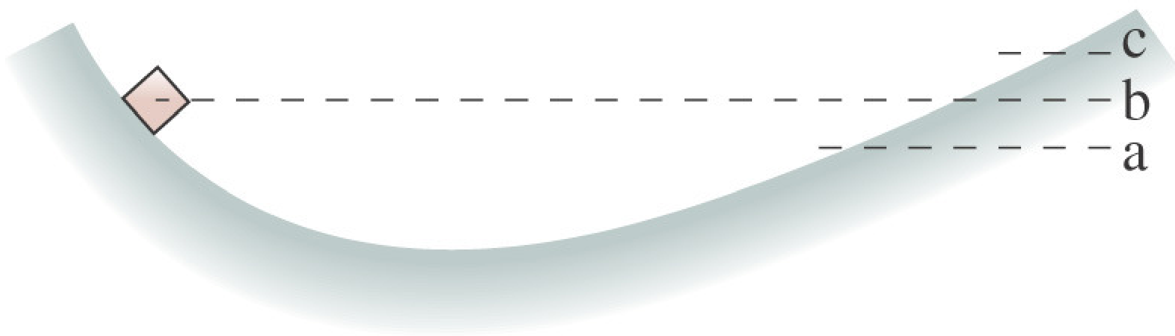
A. $v_C > v_A = v_B > v_D$

B. $v_C > v_B > v_A > v_D$

C. $v_D > v_A > v_B > v_C$

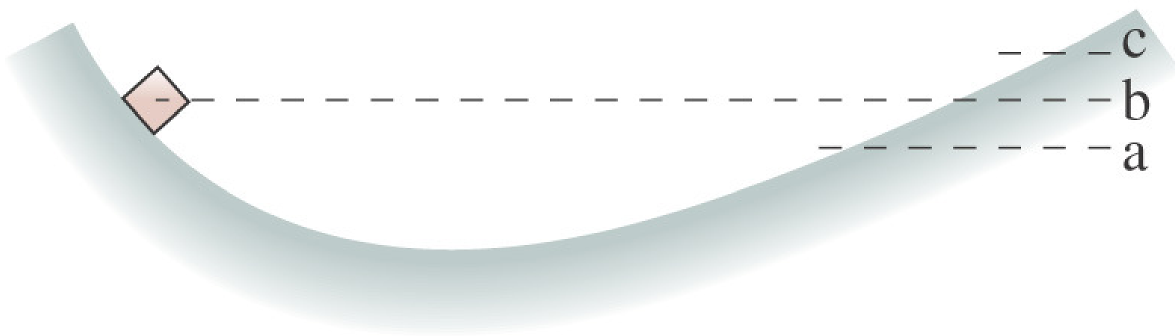
D. $v_A = v_B = v_C = v_D$

E. $v_D > v_A = v_B > v_C$



A box slides along the frictionless surface shown in the figure. It is released from rest at the position shown. Is the highest point the box reaches on the other side at level a, at level b, or level c?

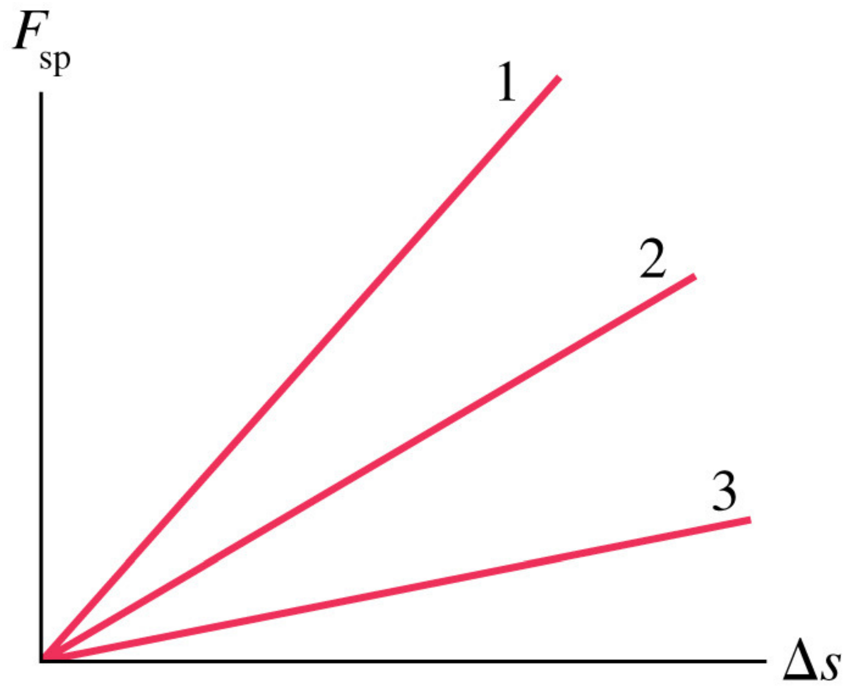
A. At level a
B. At level b
C. At level c



A box slides along the frictionless surface shown in the figure. It is released from rest at the position shown. Is the highest point the box reaches on the other side at level a, at level b, or level c?

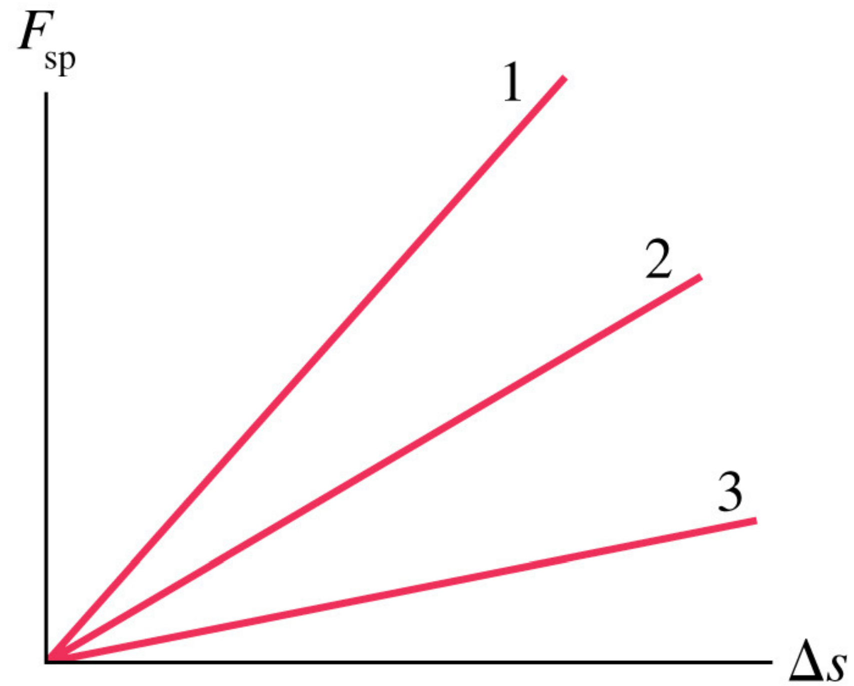
A. At level a
B. At level b
C. At level c

The graph shows force versus displacement for three springs. Rank in order, from largest to smallest, the spring constants k_1, k_2, and k_3.



A. $k_1 > k_3 > k_2$
B. $k_3 > k_2 > k_1$
C. $k_1 = k_3 > k_2$
D. $k_2 > k_1 = k_3$
E. $k_1 > k_2 > k_3$

The graph shows force versus displacement for three springs. Rank in order, from largest to smallest, the spring constants k_1, k_2, and k_3.



A. $k_1 > k_3 > k_2$
B. $k_3 > k_2 > k_1$
C. $k_1 = k_3 > k_2$
D. $k_2 > k_1 = k_3$
E. $k_1 > k_2 > k_3$

A spring-loaded gun shoots a plastic ball with a speed of 4 m/s. If the spring is compressed twice as far, the ball's speed will be

A. 1 m/s.
B. 2 m/s.
C. 4 m/s.
D. 8 m/s.
E. 16 m/s.

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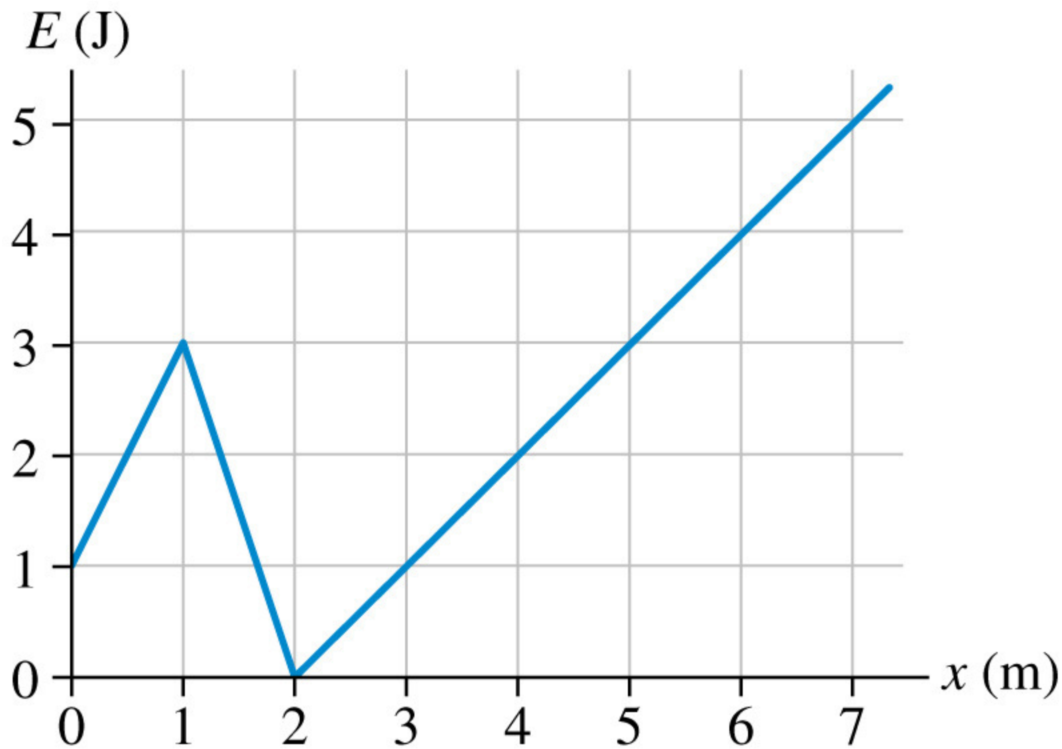
A particle with the potential energy shown in the graph is moving to the right. It has 1 J of kinetic energy at $x = 1$ m. Where is the particle's turning point?



A. $x = 1$ m
B. $x = 2$ m
C. $x = 5$ m
D. $x = 6$ m
E. $x = 7.5$ m

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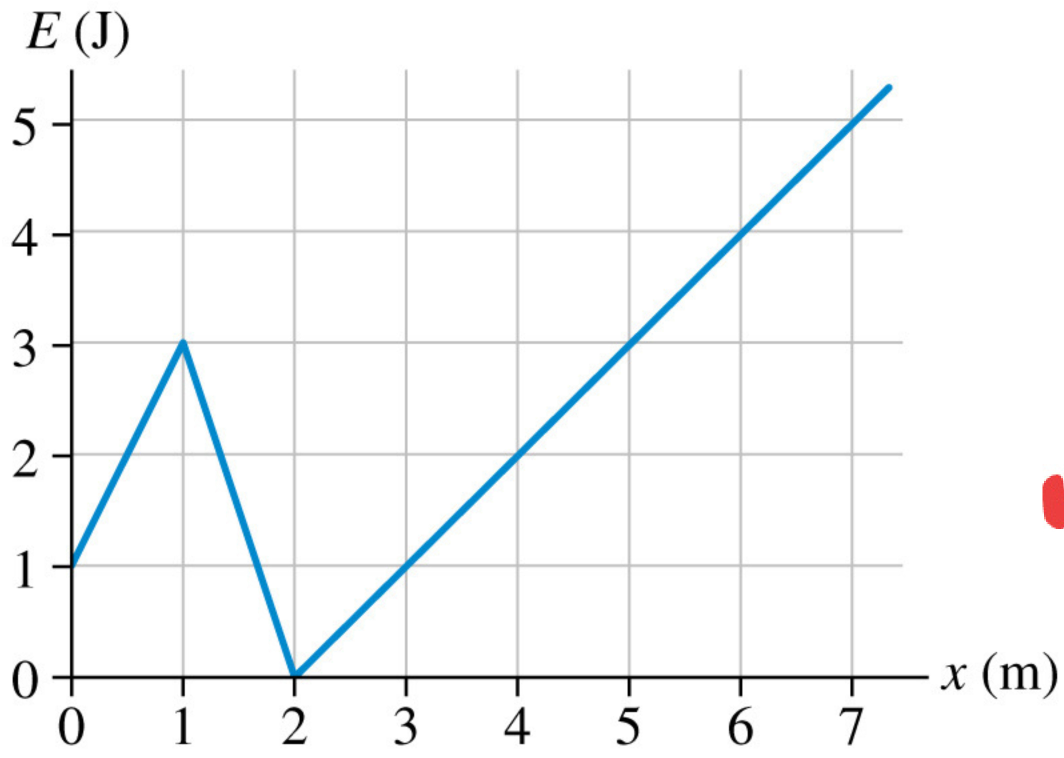

A. $x = 1$ m

B. $x = 2$ m

C. $x = 5$ m

D. $x = 6$ m

E. $x = 7.5$ m