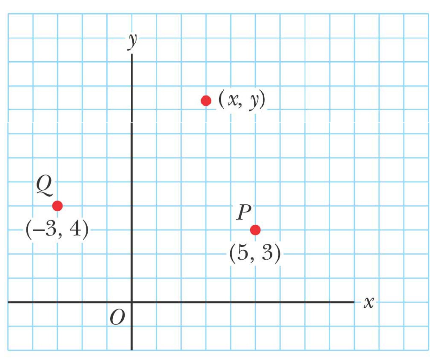
Chapter 3: Vectors and Coordinate Systems

Coordinate Systems

- Used to describe the position of a point in space
- Coordinate system consists of
  - a fixed reference point called the origin
  - specific axes with scales and labels
  - instructions on how to label a point relative to the origin and the axes

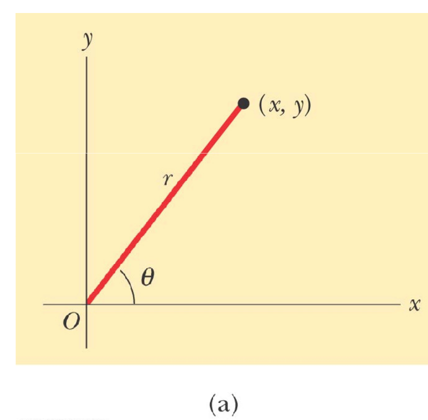
Cartesian Coordinate System

- Also called rectangular coordinate system
- x- and y- axes intersect at the origin
- Points are labeled (x,y)



Polar Coordinate System

- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ, ccw from reference line
- Points are labeled (r,θ)



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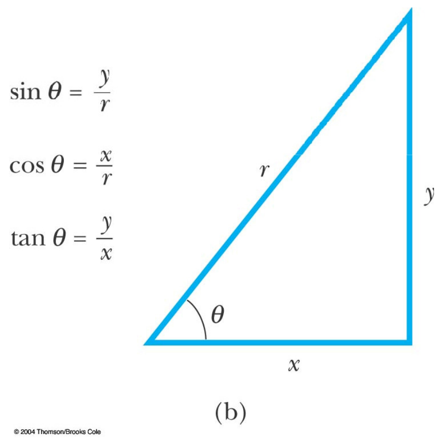
Polar to Cartesian Coordinates

- Based on forming a right triangle from r and θ
- $x = r\cos\theta$
- $y = r\sin\theta$



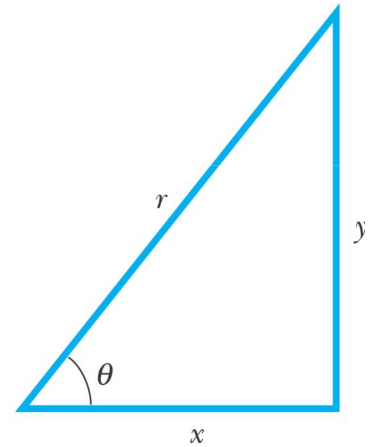
Cartesian to Polar Coordinates

- r is the hypotenuse and θ an angle

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

θ must be ccw from positive x axis for these equations to be valid



Vectors and Scalars

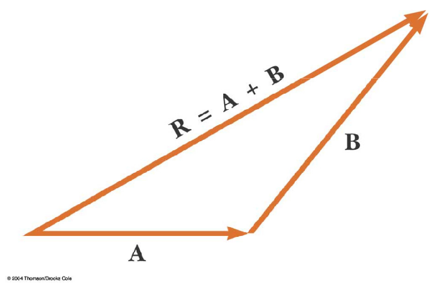
- A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.
- A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

Vector Notation

- When handwritten, use an arrow: $\vec{A}$
- When printed, will be in bold print: **A**
- When dealing with just the magnitude of a vector in print, an italic letter will be used: *A* or |**A**|
- The magnitude of the vector has physical units
- The magnitude of a vector is always a positive number

Adding Vectors Graphically

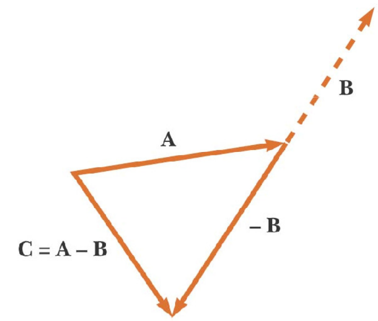
- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of **A** to the end of the last vector
- Measure the length of **R** and its angle
  - Use the scale factor to convert length to actual magnitude



Subtracting Vectors

- Special case of vector addition
- If **A** – **B**, then use **A**+(**-B**)
- Continue with standard vector addition procedure

Vector Subtraction



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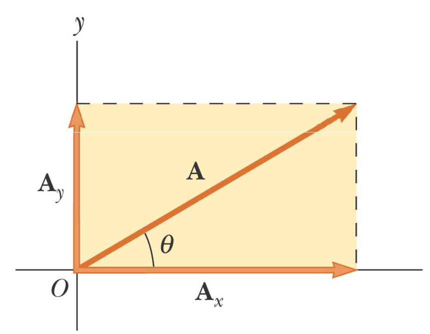
Multiplying or Dividing a Vector by a Scalar

- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes



(a)

Vector Component Terminology

- $\mathbf{A_x}$ and $\mathbf{A_y}$ are the ***component vectors*** of **A**
  - They are vectors and follow all the rules for vectors
- A_x and A_y are scalars, and will be referred to as the ***components*** of **A**

Components of a Vector

- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$

Components of a Vector

- The previous equations are valid ***only if θ is measured with respect to the x-axis***
- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

  - May still have to find θ with respect to the positive x-axis

Unit Vectors

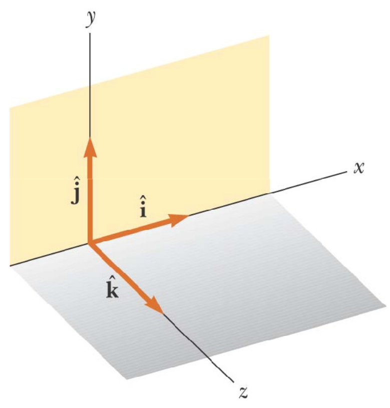
- A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance

Unit Vectors, cont.

- The symbols

  $\hat{\mathrm{i}}, \hat{\mathrm{j}}, \text{and } \hat{\mathrm{k}}$

  represent unit vectors
- They form a set of mutually perpendicular vectors



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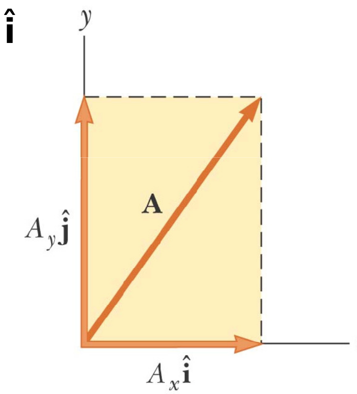
Unit Vectors in Vector Notation

- The complete vector can be expressed as

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$



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Adding Vectors Using Unit Vectors

- Using $\mathbf{R} = \mathbf{A} + \mathbf{B}$
- Then

$$\mathbf{R} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\mathbf{R} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\mathbf{R} = R_x + R_y$$

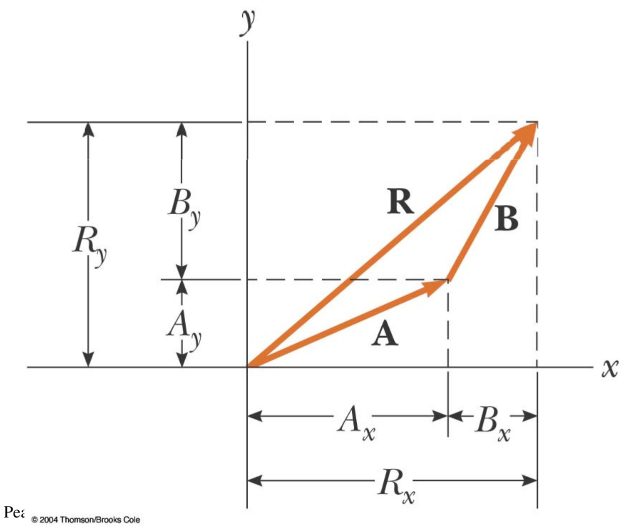
- and so $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Trig Function Warning

- The component equations ($A_x = A \cos \theta$ and $A_y = A \sin \theta$) apply only when the angle is measured with respect to the *x*-axis (preferably ccw from the positive *x*-axis).
- The resultant angle ($\tan \theta = A_y / A_x$) gives the angle with respect to the *x*-axis.

Adding Vectors with Unit Vectors



Adding Vectors Using Unit Vectors – Three Directions

- Using **R** = **A** + **B**

$$\mathbf{R} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\mathbf{R} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

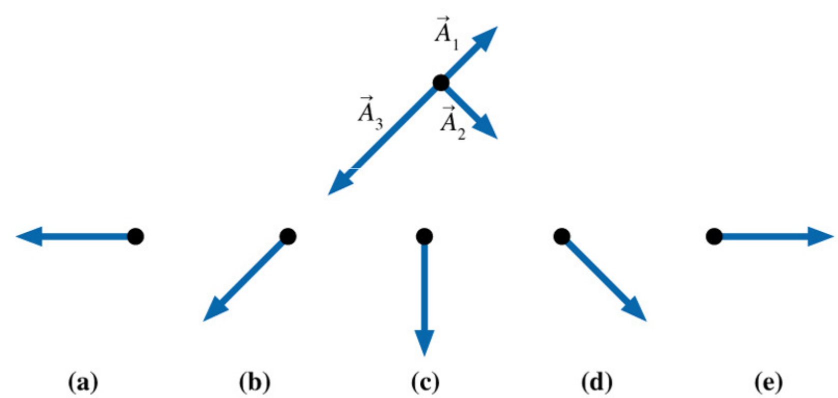
$$\mathbf{R} = R_x + R_y + R_z$$

- $R_x = A_x + B_x$, $R_y = A_y + B_y$ and $R_z = A_z + B_z$

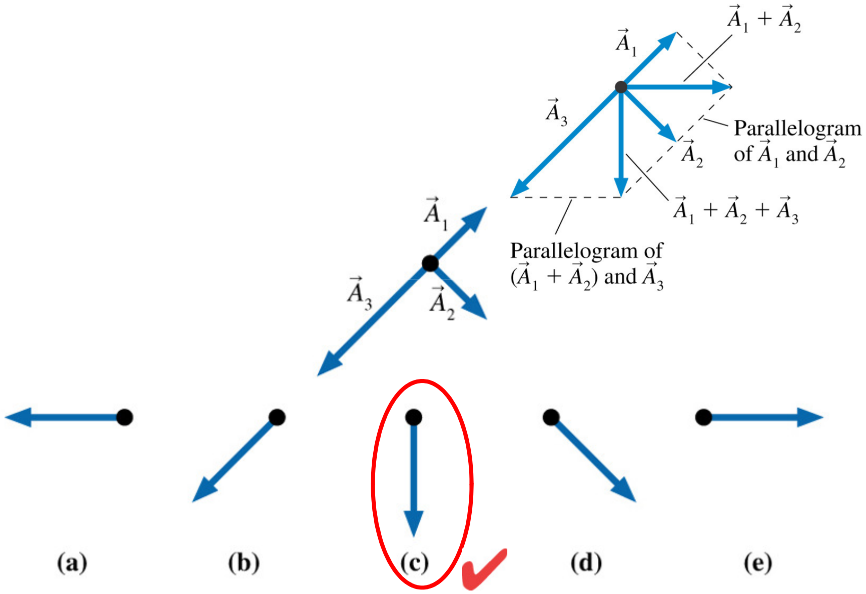
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta_x = \tan^{-1}\frac{R_x}{R} \text{ etc.}$$

Chapter 3. Questions

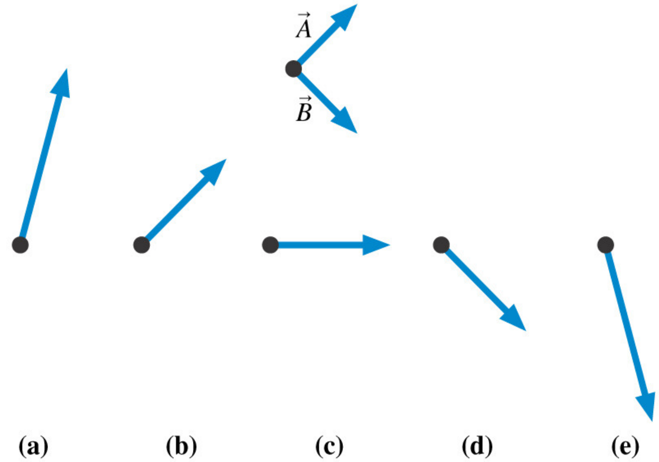
Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



Which figure shows $2\vec{A} - \vec{B}$?



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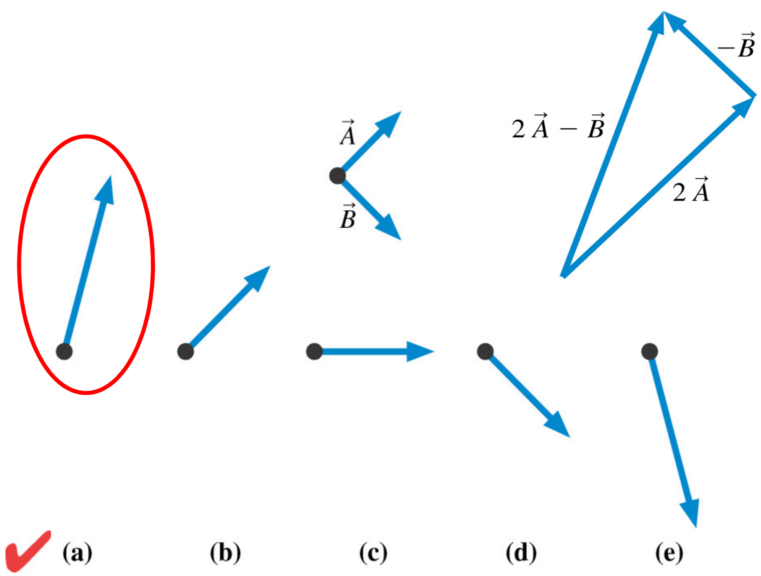
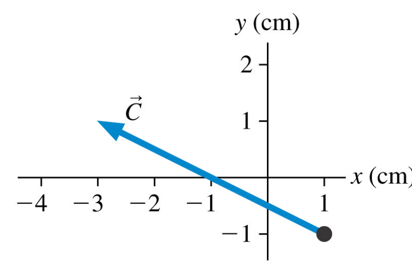
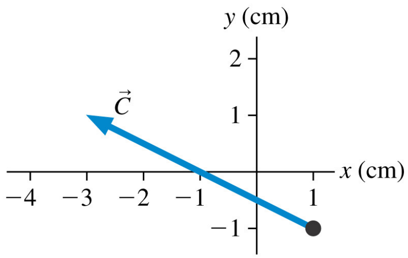


What are the x- and y-components C_x and C_y of vector $\vec{C}$?



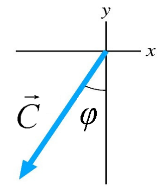
A. $Cx = 1$ cm, $Cy = -1$ cm
B. $C_x = -3$ cm, $C_y = 1$ cm
C. $Cx = -2$ cm, $Cy = 1$ cm
D. $C_x = -4$ cm, $C_y = 2$ cm
E. $C_x = -3$ cm, $C_y = -1$ cm

What are the x- and y-components C_x and C_y of vector $\vec{C}$?



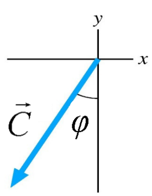
A. Cx = 1 cm, Cy = –1 cm
B. C_x = –3 cm, C_y = 1 cm
C. Cx = –2 cm, Cy = 1 cm
✔ **D. C_x = –4 cm, C_y = 2 cm**
E. C_x = –3 cm, C_y = –1 cm

Angle φ that specifies the direction of $\vec{C}$ is given by



A. $\tan^{-1}(C_y/C_x)$
B. $\tan^{-1}(C_x/|C_y|)$
C. $\tan^{-1}(C_y/|C_x|)$
D. $\tan^{-1}(C_x/C_y)$
E. $\tan^{-1}(|C_x|/|C_y|)$

Angle φ that specifies the direction of $\vec{C}$ is given by



A. $\tan^{-1}(C_y/C_x)$
B. $\tan^{-1}(C_x/|C_y|)$
C. $\tan^{-1}(C_y/|C_x|)$
D. $\tan^{-1}(C_x/C_y)$
✔ **E. $\tan^{-1}(|C_x|/|C_y|)$**

Back to the concepts of motion: Chapter 1

Chapter 1. Concepts of Motion

The universe we live in is one of change and motion. Although we all have intuition about motion, based on our experiences, some of the important aspects of motion turn out to be rather subtle.

Chapter Goal: To introduce the fundamental concepts of motion.

Displacement - vector

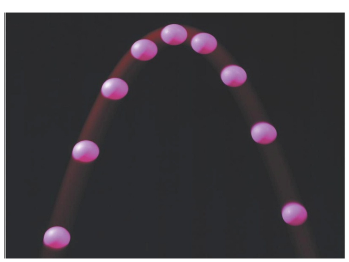
Velocity - vector

Acceleration – vector

Different types of motion

Translational Motion

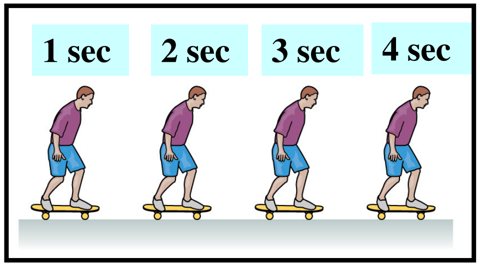
Circular Motion

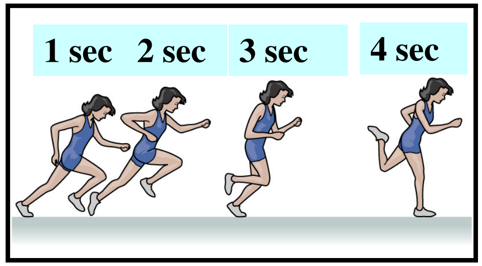


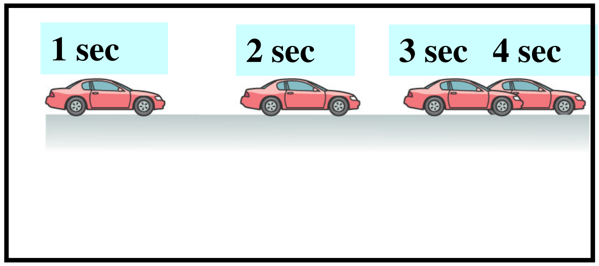
Projectile Motion

Rotational Motion

Different types of motion



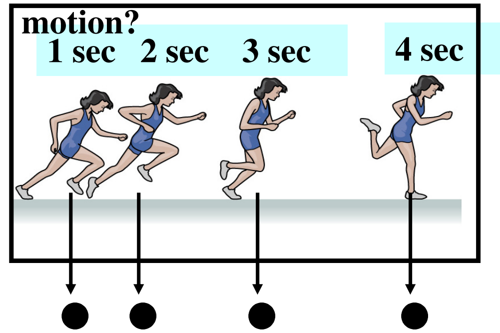




What is the difference between these motions?

How can we characterize these motions?

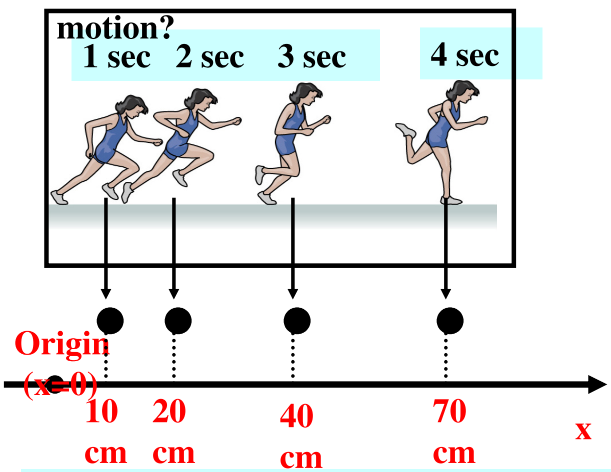
How can we characterize the motion?



The first step: PARTICLE MODEL – MOTION DIAGRAM

We consider object as a single point without size or shape, disregard internal motion of the object.

How can we characterize the motion?



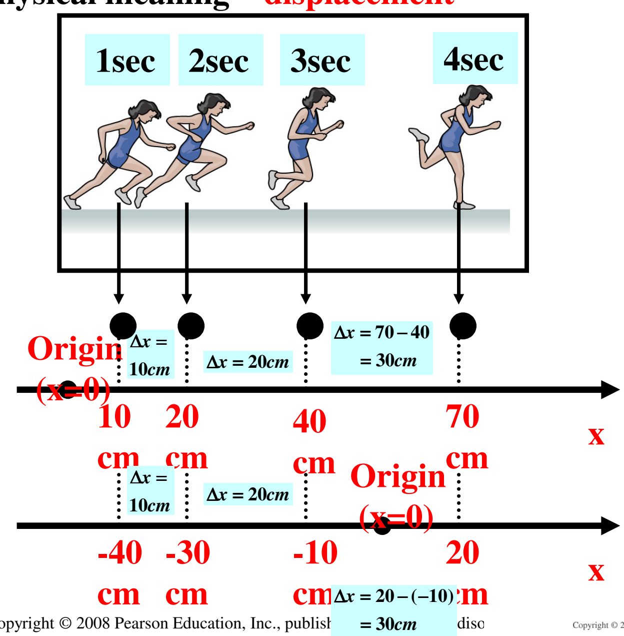
The second step: POSITION OF THE OBJECT (POINT) – COORDIANTE SYSTEM - DISPLACEMENT

We introduce coordinate system: for motion along a line - only *x* (which means that y=0); for a motion in a plane – x and y.

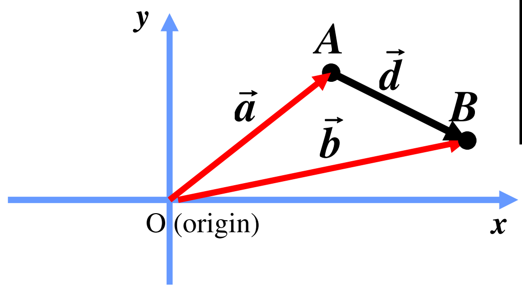
Different origins – different coordinates

$\Delta x = x_2 - x_1$

Physical meaning – displacement -



Displacement



A - initial position of the object
If O is an origin then vector $\vec{a}$ characterizes initial position of the object

B - final position of the object
Vector $\vec{b}$ characterizes the final position of the object

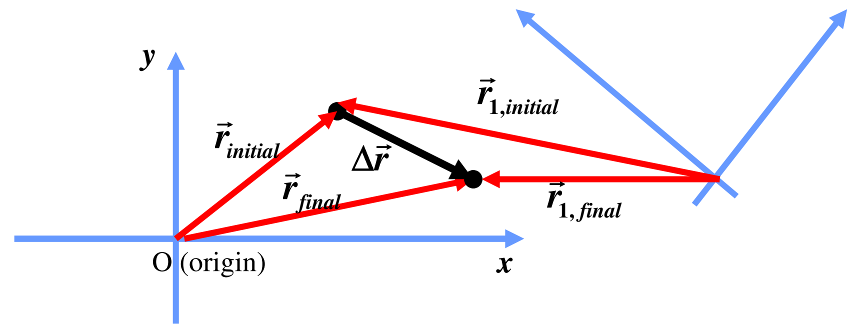
Vector $\vec{d}$ is a displacement (final position minus initial position – does not depend on coordinate system)

$$\vec{d} = \vec{b} - \vec{a}$$

Standard notation for displacement is $\Delta\vec{r}$

$$\Delta\vec{r} = \vec{r}_{final} - \vec{r}_{initial}$$

Displacement

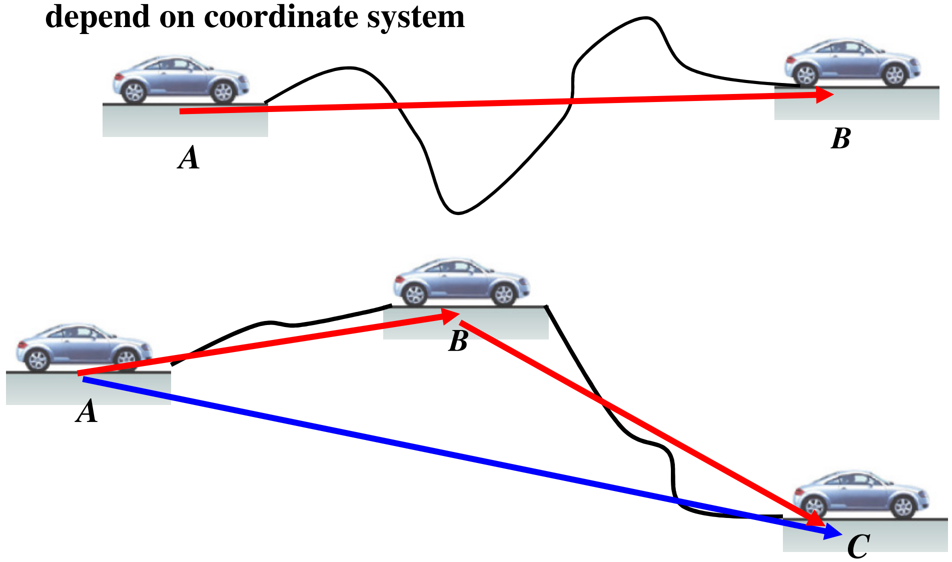


Displacement $\Delta\vec{r}$ does not depend on coordinate system

$$\Delta\vec{r} = \vec{r}_{final} - \vec{r}_{initial} = \vec{r}_{1,final} - \vec{r}_{1,initial}$$

Displacement

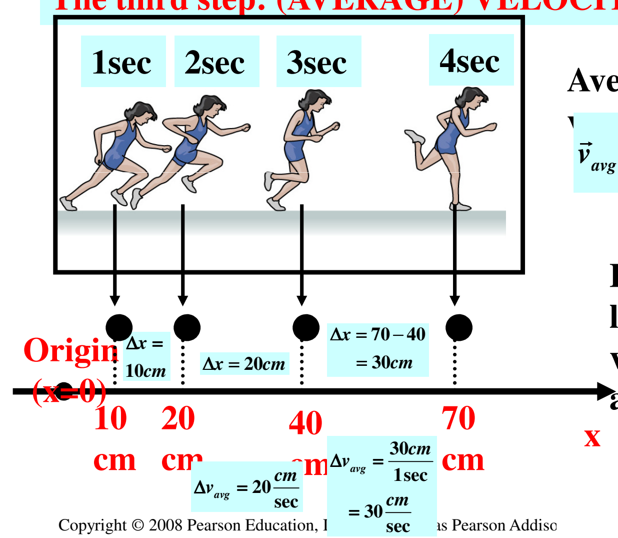
Displacement is a vector, it does not depend on coordinate system



How can we characterize the motion?

The first step: PARTICLE MODEL – MOTION DIAGRAM, The second step: POSITION OF THE OBJECT (POINT) – DISPLACEMENT

The third step: (AVERAGE) VELOCITY



Average velocity is a vector

$$\vec{v}_{avg} = \frac{displacement}{time} = \frac{\Delta\vec{r}}{\Delta t}$$

For a motion along the line – direction of velocity is along the line and the magnitude

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

AVERAGE VELOCITY

$$\vec{v}_{avg} = \frac{displacement}{time} = \frac{\Delta\vec{r}}{\Delta t}$$

The magnitude of velocity (vector) is called speed

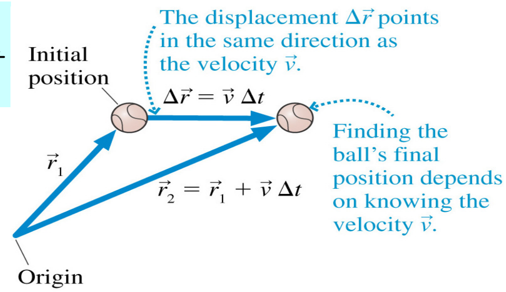
Example: We know initial position of the object (in some coordinate system) $\vec{r}_1$

We know the average velocity $\vec{v}$ of the object during time

Then: What is the final position $\vec{r}_2$ of the object?

$$\vec{v} = \frac{displacement}{time} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t}$$

$$\vec{r}_2 = \vec{r}_1 + \vec{v}\,\Delta t$$



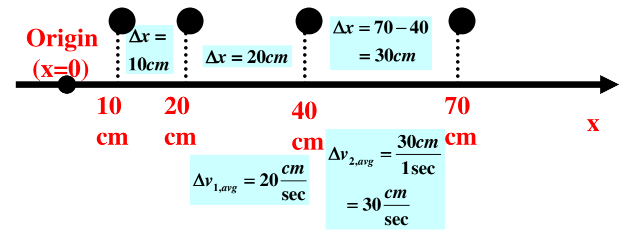
How can we characterize the motion?

The first step: PARTICLE MODEL – MOTION DIAGRAM

The second step: POSITION OF THE OBJECT (POINT) – DISPLACEMENT

The third step: (AVERAGE) VELOCITY

The forth step: (AVERAGE) ACCELERATION



$$a_{avg} = \frac{v_{2,avg} - v_{1,avg}}{\Delta t} = \frac{(30-20)\frac{cm}{sec}}{1\,sec} = 10\,\frac{cm}{sec^2}$$

The change in position is characterized by average velocity,

The change in velocity is characterized by average acceleration

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

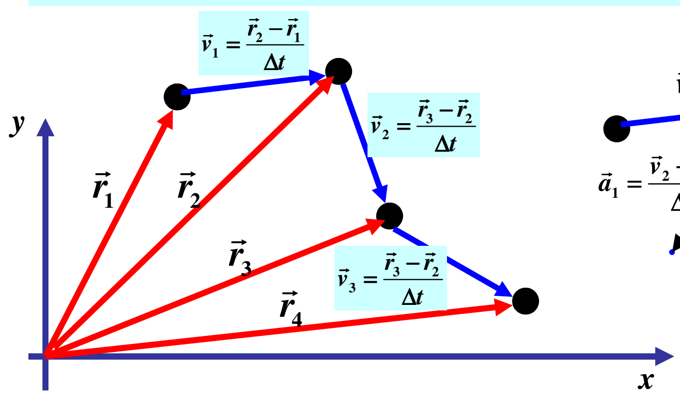
How can we characterize the motion?

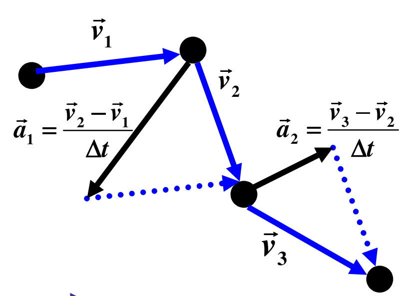
The first step: PARTICLE MODEL – MOTION DIAGRAM

The second step: POSITION OF THE OBJECT (POINT) – DISPLACEMENT

The third step: (AVERAGE) VELOCITY $\quad \vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \quad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

The forth step: (AVERAGE) ACCELERATION



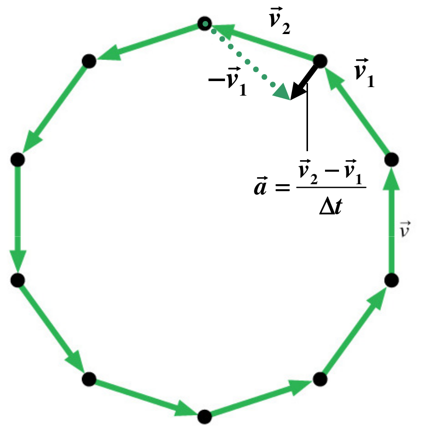


Acceleration

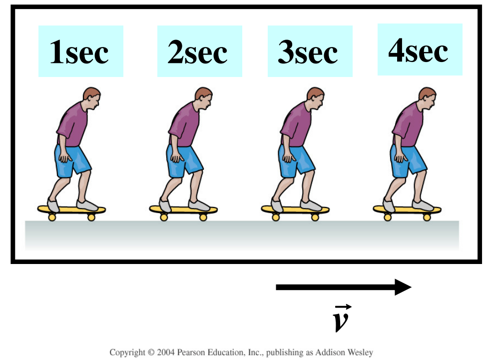
Because velocity is a vector, it can change in two possible ways.

1. The magnitude can change, indicating a change in speed, or
2. The direction can change, indicating that the object has changed direction.

Acceleration is the change of velocity (speed can be the same)

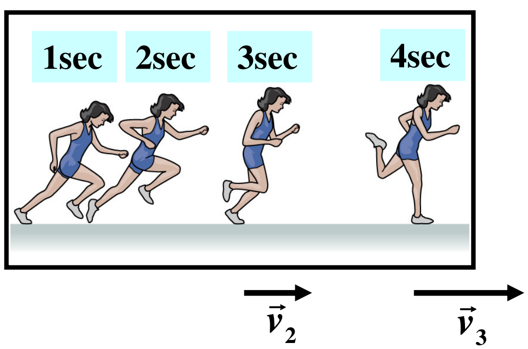


The lengths of the velocity vectors are the same, indicating constant speed, but the direction of each vector is different. This is a changing velocity.



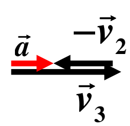
Velocity is the same – zero acceleration

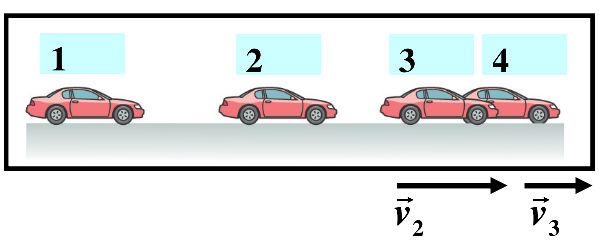
$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = 0$$



Velocity is increasing – acceleration has the same direction as velocity

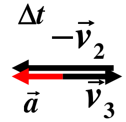
$$\vec{a} = \frac{\vec{v}_3 - \vec{v}_2}{\Delta t}$$



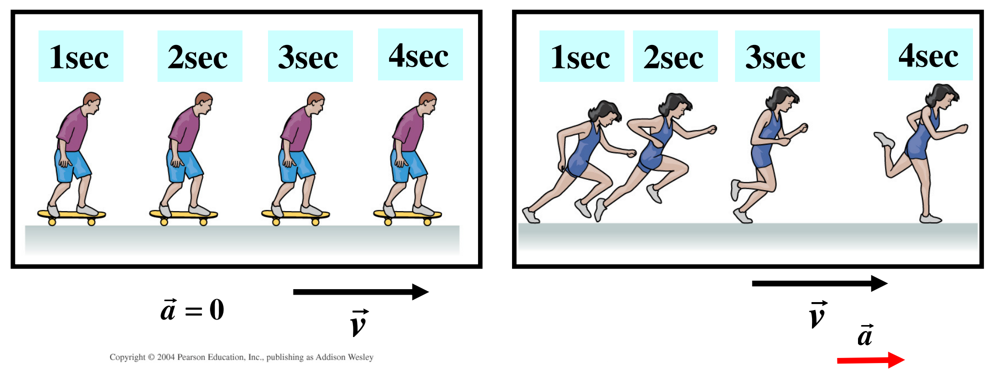


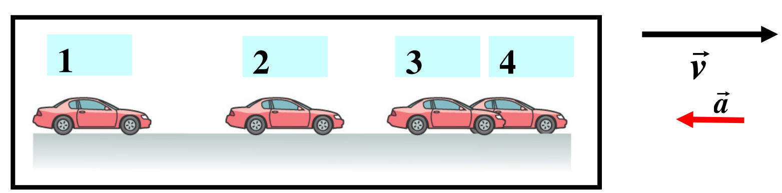
Velocity is decreasing – acceleration has the opposite direction

$$\vec{a} = \frac{\vec{v}_3 - \vec{v}_2}{\Delta t}$$



What is the difference between these motions?





SI units

Basic Units:

Time – seconds (s)

Length – meters (m)

Mass – kilogram (

Units of velocity:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} \rightarrow \frac{m}{s}$$

Units of acceleration:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} \rightarrow \frac{m/s}{s} = \frac{m}{s^2}$$

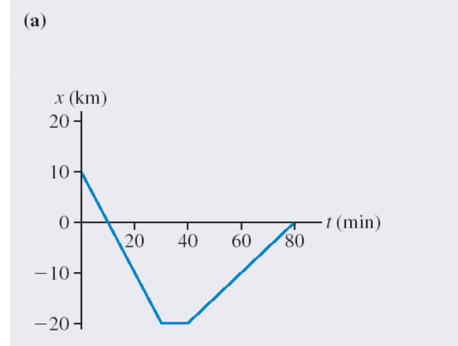
TABLE 1.3 Useful unit conversions

1 in = 2.54 cm
1 mi = 1.609 km
1 mph = 0.447 m/s
1 m = 39.37 in
1 km = 0.621 mi
1 m/s = 2.24 mph

EXAMPLE 1.7 Interpreting a position graph

EXAMPLE 1.7 Interpreting a position graph

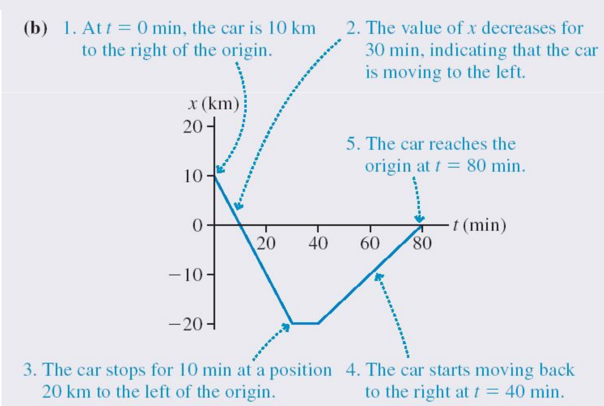
The graph in **FIGURE 1.22a** represents the motion of a car along a straight road. Describe the motion of the car.



EXAMPLE 1.7 Interpreting a position graph

MODEL Represent the car as a particle.

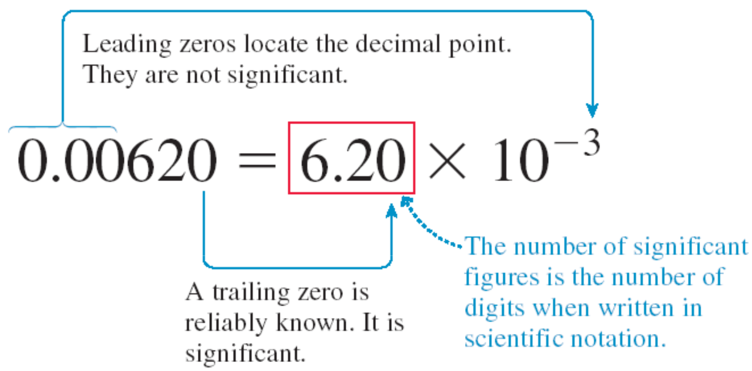
VISUALIZE As **FIGURE 1.22b** shows, the graph represents a car that travels to the left for 30 minutes, stops for 10 minutes, then travels back to the right for 40 minutes.



General Problem-Solving Strategy

ASSESS Is your result believable? Does it have proper units? Does it make sense?

FIGURE 1.25 Determining significant figures.



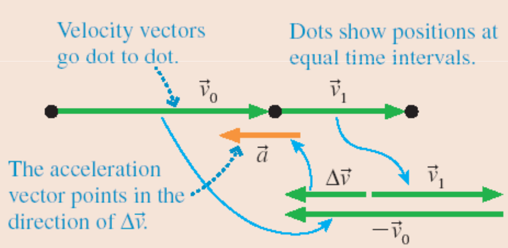
- The number of significant figures ≠ the number of decimal places.
- Changing units shifts the decimal point but does not change the number of significant figures.

Chapter 1. Summary Slides

General Strategy

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



▶ These are the average velocity and the average acceleration vectors.

General Strategy

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

- **Pictorial representation**
- **Graphical representation**

SOLVE Use a **mathematical representation** to find numerical answers.

ASSESS Does the answer have the proper units? Does it make sense?

Important Concepts

The particle model represents a moving object as if all its mass were concentrated at a single point.

Important Concepts

Position locates an object with respect to a chosen coordinate system. Change in position is called displacement.

Velocity is the rate of change of the position vector $\vec{r}$.

Acceleration is the rate of change of the velocity vector $\vec{v}$.

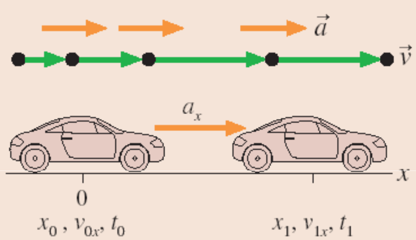
An object has an acceleration if it

- Changes speed and/or
- Changes direction.

Important Concepts

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.



Known

$x_0 = v_{0x} = t_0 = 0$

$a_x = 2.0\ \text{m/s}^2$ $\quad t_1 = 2.0\ \text{s}$

Find

x_1

Applications

For **motion along a line:**

- Speeding up: $\vec{v}$ and $\vec{a}$ point in the same direction, v_x and a_x have the same sign.
- Slowing down: $\vec{v}$ and $\vec{a}$ point in opposite directions, v_x and a_x have opposite signs.
- Constant speed: $\vec{a} = \vec{0}$, $a_x = 0$.

Acceleration a_x is positive if $\vec{a}$ points right, negative if $\vec{a}$ points left. The sign of a_x does *not* imply speeding up or slowing down.

Applications

Significant figures are reliably known digits. The number of significant figures for:

- **Multiplication, division, powers** is set by the value with the fewest significant figures.
- **Addition, subtraction** is set by the value with the smallest number of decimal places.

The appropriate number of significant figures in a calculation is determined by the data provided.

Chapter 1. Questions

Which car is going faster, A or B? Assume there are equal intervals of time between the frames of both movies.

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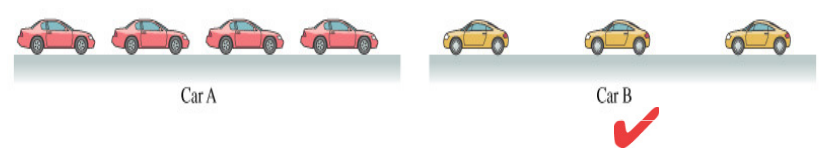
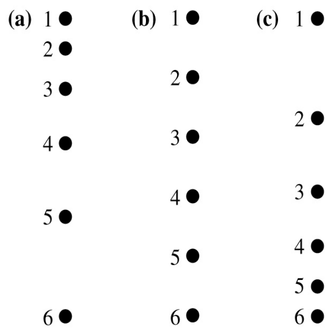


B is going faster.

Three motion diagrams are shown. Which is a dust particle settling to the floor at constant speed, which is a ball dropped from the roof of a building, and which is a descending rocket slowing to make a soft landing on Mars?



A. (a) is ball, (b) is dust, (c) is rocket

B. (a) is ball, (b) is rocket, (c) is dust

C. (a) is rocket, (b) is dust, (c) is ball

D. (a) is rocket, (b) is ball, (c) is dust

E. (a) is dust, (b) is ball, (c) is rocket

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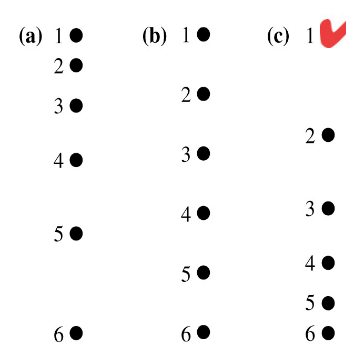

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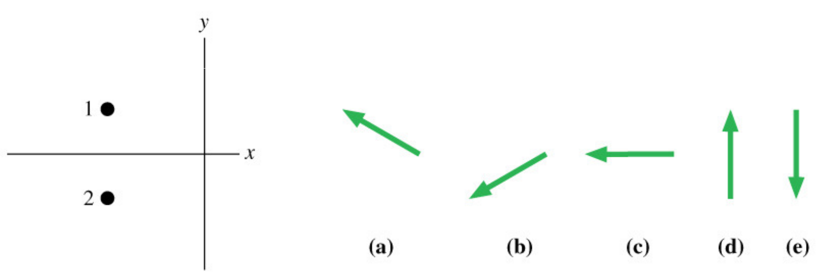
A particle moves from position 1 to position 2 during the interval Δt. Which vector shows the particle's average velocity?



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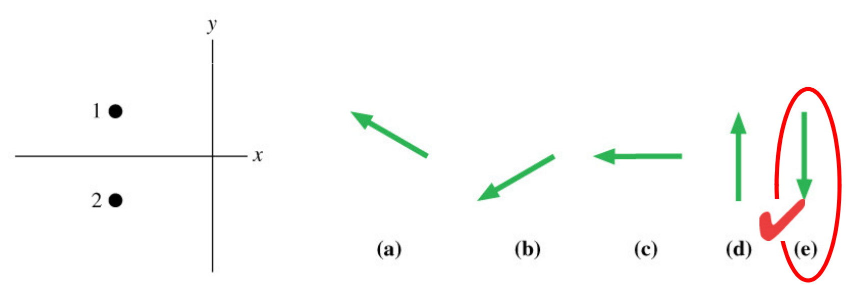
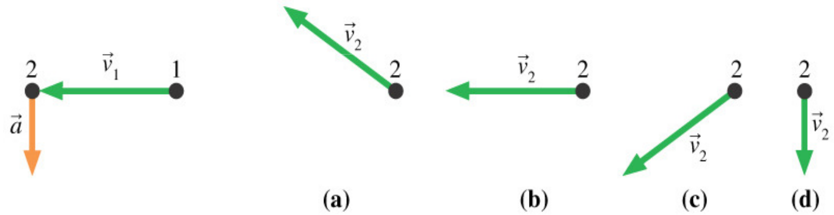


A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the velocity vector $\vec{v}_2$ as the object moves away from point 2?



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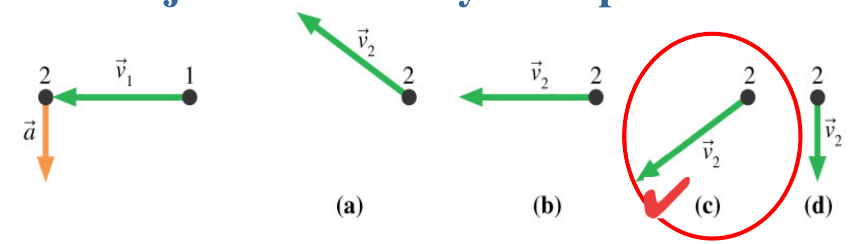

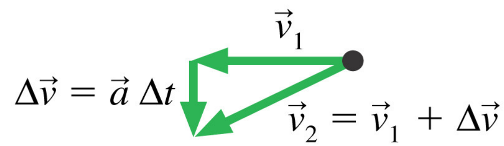

Rank in order, from the most to the least, the number of significant figures in the following numbers. For example, if b has more than c, c has the same number as a, and a has more than d, you could give your answer as b > c = a > d.

a. 8200 b. 0.0052 c. 0.430 d. 4.321×10^{-10}

A. a = b = d > c
B. b = d > c > a
C. d > c > b = a
D. d > c > a > b
E. b > a = c = d

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A. a = b = d > c
B. b = d > c > a
✔ **C. d > c > b = a**
D. d > c > a > b
E. b > a = c = d