Chapter 29 Lecture

physics

FOR SCIENTISTS AND ENGINEERS

a strategic approach

THIRD EDITION

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Chapter 29 Potential and Field

Chapter Goal: To understand how the electric potential is connected to the electric field.

Chapter 29 Preview

Field and Potential

The electric potential and the electric field are intimately connected. They are two different perspectives of how static charges alter the space around them.

You’ll learn to:
- Use the electric potential to find the electric field.
- Use the electric field to find the electric potential.

The mathematical connection is analogous to that between force and potential energy.
Chapter 29 Preview

The electric field and the electric potential can be related to each other geometrically.

You’ll also learn that:
- The electric field is always perpendicular to equipotential surfaces.
- The electric field points “downhill” in the direction of decreasing potential.
- The electric field is stronger where equipotential lines are closer together.

Chapter 29 Preview

Sources of Potential

A potential difference—a voltage—is created by separating positive and negative charges.

We’ll develop a chapter model of a battery in which chemical reactions separate charge to create a potential difference.

You’ll learn that any nonelectrical means of separating charge—e.g., batteries, phototools, and generators—does work and develops what we’ll call as emf.

Chapter 29 Preview

Conductors

You’ll learn several important characteristics of conductors in electrostatic equilibrium, with stationary charges.

- Any excess charge is on the surface.
- The interior electric field is zero.
- The interior electric field is perpendicular to the surface.
- The entire conductor is an equipotential.
Chapter 29 Preview

Capacitors
Capacitors are circuit elements that store charge and energy. They are used in devices ranging from high-speed computers to heart defibrillators.

The flash on your camera uses energy stored in a capacitor. The capacitor can discharge in a few microseconds, much faster than a battery can provide energy.

Chapter 29 Preview

You'll learn to:
- Work with combinations of capacitors called in series and in parallel.
- Calculate the energy stored in a capacitor's electric field.
- Understand capacitors with dielectrics.

Dielectric:
An insulator between the capacitor plates is called a dielectric. It changes the capacitor properties in many useful ways.

Chapter 29 Reading Quiz
Reading Question 29.1

What quantity is represented by the symbol $\mathcal{E}$?

A. Electronic potential.
B. Excitation potential.
C. emf.
D. Electric stopping power.
E. Exosphericity.

C. emf.

Reading Question 29.2

What is the SI unit of capacitance?

A. Capaciton.
B. Faraday.
C. Hertz.
D. Henry.
E. Exciton.
Reading Question 29.2

What is the SI unit of capacitance?

A. Capaciton.
B. Faraday. 🔴
C. Hertz.
D. Henry.
E. Exciton.

Reading Question 29.3

The electric field

A. Is always perpendicular to an equipotential surface.
B. Is always tangent to an equipotential surface.
C. Always bisects an equipotential surface.
D. Makes an angle to an equipotential surface that depends on the amount of charge.
This chapter investigated

A. Parallel capacitors.
B. Perpendicular capacitors.
C. Series capacitors.
D. Both A and B.
E. Both A and C.

Reading Question 29.4

This chapter investigated

A. Parallel capacitors.
B. Perpendicular capacitors.
C. Series capacitors.
D. Both A and B.
E. Both A and C.

Chapter 29 Content, Examples, and QuickCheck Questions
The figure shows the four key ideas of force, field, potential energy, and potential.

- We know, from Chapters 10 and 11, that force and potential energy are closely related.
- The focus of this chapter is to establish a similar relationship between the electric field and the electric potential.

Finding the Potential from the Electric Field

The potential difference between two points in space is:

\[ \Delta V = V_f - V_i = -\int_{s_i}^{s_f} E \cdot ds \]

where \( s \) is the position along a line from point \( i \) to point \( f \).

- We can find the potential difference between two points if we know the electric field.
- Thus a graphical interpretation of the equation above is:
  \[ V_f = V_i - (\text{area under the } E,\text{-versus-} s \text{ curve between } s_i \text{ and } s_f) \]

Example 29.1 Finding the Potential

The figure below is a graph of \( E_x \), the \( x \)-component of the electric field, versus position along the \( x \)-axis. Find and graph \( V(x) \). Assume \( V = 0 \) V at \( x = 0 \) m.

MODEL: The potential difference is the negative of the area under the curve.

Vectors \( E_x \) is positive throughout this region of space, meaning that \( E_x \) points in the positive \( x \)-direction.
Example 29.1 Finding the Potential

This is a graph of the \( x \)-component of the electric field along the \( x \)-axis. The potential is zero at the origin. What is the potential at \( x = 1 \text{ m} \)?

A. 2000 V.
B. 1000 V.
C. 0 V.
D. -1000 V.
E. -2000 V.

QuickCheck 29.1

This is a graph of the \( x \)-component of the electric field along the \( x \)-axis. The potential is zero at the origin. What is the potential at \( x = 1 \text{ m} \)?

A. 2000 V.
B. 1000 V.
C. 0 V.
D. -1000 V.
E. -2000 V.

\( \Delta V = -\text{area under curve} \)
Tactics: Finding the Potential From the Electric Field

Finding the Potential of a Point Charge

\[ V(r) = V(\infty) + \frac{q}{4\pi\varepsilon_0} \int_r^\infty \frac{ds}{s^2} \]

\[ V_{\text{point charge}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \]

Example 29.2 The Potential of a Parallel-Plate Capacitor

The potential of a parallel-plate capacitor

In Chapter 26, the electric field inside a capacitor was found to be

\[ \vec{E} = \frac{Q}{\varepsilon_0 A} \text{ from positive to negative} \]

Find the electric potential inside the capacitor. Let \( V = 0 \) V at the negative plate.

Model: The electric field inside a capacitor is a uniform field.
Example 29.2 The Potential of a Parallel-Plate Capacitor

**Example 29.2** The potential of a parallel-plate capacitor

**VISUALIZE** The figure below shows the capacitor and establishes a point \( P \) where we want to find the potential. We’ve chosen an \( s \)-axis measured from the negative plate, which is the zero point of the potential.

\[ V = qV \]

\[ V = qV \]

\[ \int F \, dx = -\frac{Q}{\varepsilon_0 A} \]

\[ \int F \, dx = -\frac{Q}{\varepsilon_0 A} \]

\[ E \]

\[ E \]

\[ V = Es \]

\[ V = Es \]

**Assess.** \( V = Es \) is the capacitor potential we deduced in Chapter 28 by working directly with the potential energy. The potential increases linearly from \( V = 0 \) at the negative plate to \( V = Ed \) at the positive plate. Here we found the potential by explicitly recognizing the connection between the potential and the field.

Sources of Electric Potential

- A separation of charge creates an electric potential difference.
- Shuffling your feet on the carpet transfers electrons from the carpet to you, creating a potential difference between you and other objects in the room.
- This potential difference can cause sparks.
The most common source of electric potential is a battery.
The figure shows the charge escalator model of a battery.
Lifting positive charges to a positive terminal requires work to be done, and the chemical reactions within the battery provide the energy to do this work.

\[ \Delta V = q \Delta \mu \]

A battery constructed to have an emf of 1.5 V creates a 1.5 V potential difference between its positive and negative terminals.
QuickCheck 29.2

The charge escalator in a battery does $4.8 \times 10^{-19}$ J of work for each positive ion that it moves from the negative to the positive terminal. What is the battery’s emf?

A. 9 V.
B. 4.8 V.
C. 3 V.
D. $4.8 \times 10^{-19}$ V.
E. I have no idea.

\[ \varepsilon = \frac{W}{q} \] and \( q = e = 1.6 \times 10^{-19} \) C for an ion

Batteries in Series

- The total potential difference of batteries in series is simply the sum of their individual terminal voltages:
  \[ \Delta V_{\text{series}} = \Delta V_1 + \Delta V_2 + \cdots \]
- Flashlight batteries are placed in series to create twice the potential difference of one battery.
- For this flashlight:
  \[ \Delta V_{\text{series}} = \Delta V_1 + \Delta V_2 \]
  \[ = 1.5 \text{ V} + 1.5 \text{ V} \]
  \[ = 3.0 \text{ V} \]
Finding the Electric Field from the Potential

The figure shows two points $i$ and $f$ separated by a small distance $\Delta s$.

The potential difference between the points is:

$$\Delta V = \frac{\Delta V_{\text{work}}}{q} = \frac{W}{q} = -E_s \Delta s$$

The electric field in the $s$-direction is $E_s = -\Delta V / \Delta s$. In the limit $\Delta s \to 0$:

$$E_s = -\frac{dV}{ds}$$

Finding the Electric Field from the Potential

Quick Example

- Suppose we knew the potential of a point charge to be $V = q/4\pi\epsilon_0$ but didn’t remember the electric field.
- Symmetry requires that the field point straight outward from the charge, with only a radial component $E_r$.
- If we choose the $s$-axis to be in the radial direction, parallel to $E_r$, we find:

$$E_r = -\frac{dV}{dr} = -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

- This is, indeed, the well-known electric field of a point charge!

Example 29.3 The Electric Field of a Ring of Charge

\[ V_{\text{ring}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{z^2 + R^2}} \]

Find the on-axis electric field of a ring of charge.
Example 29.3 The Electric Field of a Ring of Charge

**Example 29.3** The electric field of a ring of charge

**SOLVE** Symmetry requires the electric field along the axis to point straight outward from the ring with only a \( z \)-component \( E_z \). The electric field at position \( z \) is

\[
E_z = \frac{dV}{dz} = \frac{d}{dz} \left( \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{z^2 + R^2}} \right)
\]

\[
= \frac{1}{4\pi\varepsilon_0} \frac{-Q}{(z^2 + R^2)^{3/2}}
\]

**ASSESS** This result is in perfect agreement with the electric field we found in Chapter 26, but this calculation was easier because we didn’t have to deal with angles.

Example 29.4 Finding \( E \) From the Slope of \( V \)

**Example 29.4** Finding \( E \) from the slope of \( V \)

This figure below is a graph of the electric potential in a region of space where \( E \) is parallel to the \( x \)-axis. Draw a graph of \( E \) versus \( x \).

**MODEL** The electric field is the negative of the slope of the potential graph.

![Graph of electric potential vs. x](image)

Example 29.4 Finding \( E \) From the Slope of \( V \)

**Example 29.4** Finding \( E \) from the slope of \( V \)

**NOTE** There are two regions of different slope:

- \( x < x_1 \) \( E_x = -20 \text{ V/m} \) \( x_1 = 0 \text{ cm} \) \( E_x = 0 \text{ V/m} \)
- \( x > x_2 \) \( E_x = -50 \text{ V/m} \) \( x_2 = 0 \text{ cm} \) \( E_x = 0 \text{ V/m} \)

The results are shown below.

![Graph of electric field vs. x](image)
Example 29.4 Finding $E$ From the Slope of $V$

At which point is the electric field stronger?

A. At $x_A$
B. At $x_B$
C. The field is the same strength at both.
D. There’s not enough information to tell.
An electron is released from rest at $x = 2$ m in the potential shown. What does the electron do right after being released?

A. Stay at $x = 2$ m.
B. Move to the right ($+x$) at steady speed.
C. Move to the right with increasing speed.
D. Move to the left ($-x$) at steady speed.
E. Move to the left with increasing speed.

Slope of $V$ negative => $E_x$ is positive (field to the right).
Electron is negative => force to the left.
Force to the left => acceleration to the left.

In three dimensions, we can find the electric field from the electric potential as:

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

1. $\vec{E}$ is everywhere perpendicular to the equipotential surfaces.
2. $\vec{E}$ points “downhill” in the direction of decreasing $V$.
3. The field strength is inversely proportional to the spacing, as between the equipotential surfaces.

Direction of decreasing potential
QuickCheck 29.5

Which set of equipotential surfaces matches this electric field?

A. 0 V, 30 V, 0 V, 30 V, 0 V, 30 V
B. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V
C. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V
D. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V
E. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V

QuickCheck 29.5

Which set of equipotential surfaces matches this electric field?

A. 0 V, 30 V, 0 V, 30 V, 0 V, 30 V
B. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V
C. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V
D. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V
E. 30 V, 0 V, 30 V, 0 V, 30 V, 0 V

Stronger field on these equipotentials

QuickCheck 29.6

The electric field at the dot is

A. 100 V/m.
B. -100 V/m.
C. 200 V/m.
D. 300 V/m.
E. -300 V/m.
QuickCheck 29.6

The electric field at the dot is

A. $10\,\text{V/m}$.
B. $-10\,\text{V/m}$.
C. $20\,\text{V/m}$.
D. $30\,\text{V/m}$.
E. $-30\,\text{V/m}$.

The electric field is $20\,\text{V over 2 m}$, pointing toward lower potential.

Kirchhoff’s Loop Law

- For any path that starts and ends at the same point:
  \[ \Delta V_{\text{loop}} = \sum \Delta V_i = 0 \]
- The sum of all the potential differences encountered while moving around a loop or closed path is zero.
- This statement is known as Kirchhoff’s loop law.

QuickCheck 29.7

A particle follows the trajectory shown from initial position $i$ to final position $f$. The potential difference $\Delta V$ is

A. $100\,\text{V}$.
B. $50\,\text{V}$.
C. $0\,\text{V}$.
D. $-50\,\text{V}$.
E. $-100\,\text{V}$.
A particle follows the trajectory shown from initial position \( i \) to final position \( f \). The potential difference \( \Delta V \) is

A. 100 V.
B. 50 V.
C. 0 V.
D. \(-50 \text{ V}\).
E. \(-100 \text{ V}\).

\[ \Delta V = V_{\text{final}} - V_{\text{initial}} \], independent of the path.

---

When a conductor is in equilibrium:

- All excess charge sits on the surface.
- The surface is an equipotential.
- The electric field inside is zero.
- The external electric field is perpendicular to the surface.
- The electric field is strongest at sharp corners of the conductor’s surface.

A corona discharge, with crackling noises and glimmers of light, occurs at pointed metal tips where the electric field can be very strong.
A Conductor in Electrostatic Equilibrium

- The figure shows a negatively charged metal sphere near a flat metal plate.
- Since a conductor surface must be an equipotential, the equipotential surfaces close to each electrode roughly match the shape of the electrode.

QuickCheck 29.8

Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2?

A. 6 V.
B. 3 V.
C. 0 V.
D. Undefined.
E. Not enough information to tell.

Every point on this conductor is at the same potential as the positive terminal of the battery.

A. 6 V.
B. 3 V. ✔
C. 0 V.
D. Undefined.
E. Not enough information to tell.
Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?

A. Same potential.
B. Same electric field.
C. Same charge.
D. Both A and B.
E. Both A and C.

QuickCheck 29.9

Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?

✓ A. Same potential.
B. Same electric field.
C. Same charge.
D. Both A and B.
E. Both A and C.

Capacitance and Capacitors

- The figure shows a capacitor just after it has been connected to a battery.
- Current will flow in this manner for a nanosecond or so until the capacitor is fully charged.

The charge oscillator moves charge from one plate to the other. \( \Delta V \) increases as the charge separation increases.
The figure shows a fully charged capacitor.
Now the system is in electrostatic equilibrium.
Capacitance always refers to the charge per voltage on a fully charged capacitor.

- The ratio of the charge \( Q \) to the potential difference \( \Delta V_C \) is called the **capacitance** \( C \):

\[
C = \frac{Q}{\Delta V_C} = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}
\]

- Capacitance is a purely geometric property of two electrodes because it depends only on their surface area and spacing.
- The SI unit of capacitance is the **farad**:

\[
1 \text{ farad} = 1 \text{ F} = 1 \text{ C/V}
\]

- The charge on the capacitor plates is directly proportional to the potential difference between the plates:

\[
Q = C \Delta V_C \quad \text{(charge on a capacitor)}
\]

What is the capacitance of these two electrodes?

A. 8 nF.
B. 4 nF.
C. 2 nF.
D. 1 nF.
E. Some other value.
QuickCheck 29.10

What is the capacitance of these two electrodes?

A. 8 nF.
B. 4 nF.
C. 2 nF.
D. 1 nF.
E. Some other value.

Capacitance and Capacitors

Capacitors are important elements in electric circuits. They come in a variety of sizes and shapes.

The keys on most computer keyboards are capacitor switches. Pressing the key pushes two capacitor plates closer together, increasing their capacitance.

Example 29.6 Charging a Capacitor

Example 29.6 Charging a capacitor

The spacing between the plates of a 1.0 µF capacitor is 0.050 mm.

a. What is the surface area of the plates?
b. How much charge is on the plates if this capacitor is attached to a 1.5 V battery?

Model Assume the battery is ideal and the capacitor is a parallel-plate capacitor.

Solve

a. From the definition of capacitance,

\[ A = \frac{Q}{\Delta V} = \frac{0.050 \text{ mm}}{1.5 \text{ V}} = 5.65 \text{ m}^2 \]

b. The charge is \[ Q = C \Delta V = 1.5 \times 10^{-6} \text{ C} = 1.5 \mu \text{C}. \]

Assess The surface area needed to construct a 1.0 µF capacitor (a fairly typical value) is enormous. We’ll see in Section 29.7 how the area can be reduced by inserting an insulator between the capacitor plates.
Forming a Capacitor

- The figure shows two arbitrary electrodes charged to $\pm Q$.
- It might appear that the capacitance depends on the amount of charge, but the potential difference is proportional to $Q$.
- Consequently, the capacitance depends only on the geometry of the electrodes.

Combinations of Capacitors

- In practice, two or more capacitors are sometimes joined together.
- The circuit diagrams below illustrate two basic combinations: parallel capacitors and series capacitors.

Capacitors Combined in Parallel

- Consider two capacitors $C_1$ and $C_2$ connected in parallel.
- The total charge drawn from the battery is $Q = Q_1 + Q_2$.
- In figure (b) we have replaced the capacitors with a single "equivalent" capacitor:

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

$$C_{eq} = C_1 + C_2$$
If capacitors $C_1$, $C_2$, $C_3$, ... are in parallel, their equivalent capacitance is:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$  
(parallel capacitors)

QuickCheck 29.11

The equivalent capacitance is

A. 9 $\mu$F.
B. 6 $\mu$F.
C. 3 $\mu$F.
D. 2 $\mu$F.
E. 1 $\mu$F.

QuickCheck 29.11

The equivalent capacitance is

✓ A. 9 $\mu$F.  Parallel => add
B. 6 $\mu$F.
C. 3 $\mu$F.
D. 2 $\mu$F.
E. 1 $\mu$F.
Consider two capacitors $C_1$ and $C_2$ connected in series.

The total potential difference across both capacitors is $\Delta V_C = \Delta V_1 + \Delta V_2$.

The inverse of the equivalent capacitance is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

If capacitors $C_1$, $C_2$, $C_3$, ... are in series, their equivalent capacitance is:

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

QuickCheck 29.12

The equivalent capacitance is $3 \mu F$.

A. $9 \mu F$.
B. $6 \mu F$.
C. $3 \mu F$.
D. $2 \mu F$.
E. $1 \mu F$. 
QuickCheck 29.12

The equivalent capacitance is \[ \frac{3 \, \mu \text{F}}{6 \, \mu \text{F}} \]

A. 9 \( \mu \text{F} \).
B. 6 \( \mu \text{F} \).
C. 3 \( \mu \text{F} \).
D. 2 \( \mu \text{F} \). \( \text{Series} \Rightarrow \text{inverse of sum of inverses} \)
E. 1 \( \mu \text{F} \).

The Energy Stored in a Capacitor

- The figure shows a capacitor being charged.
- As a small charge \( dq \) is lifted to a higher potential, the potential energy of the capacitor increases by:
  \[ dU = dq \Delta V = \frac{q \cdot dq}{C} \]
- The total energy transferred from the battery to the capacitor is:
  \[ U_c = \int_0^{Q} q \cdot dq = \frac{Q^2}{2C} \]

The Energy Stored in a Capacitor

- Capacitors are important elements in electric circuits because of their ability to store energy.
- The charge on the two plates is \( \pm q \) and this charge separation establishes a potential difference \( \Delta V = q/C \) between the two electrodes.
- In terms of the capacitor’s potential difference, the potential energy stored in a capacitor is:
  \[ U_c = \frac{Q^2}{2C} = \frac{1}{2} C(\Delta V_c)^2 \]
A capacitor can be charged slowly but then can release the energy very quickly.

An important medical application of capacitors is the defibrillator.

A heart attack or a serious injury can cause the heart to enter a state known as fibrillation in which the heart muscles twitch randomly and cannot pump blood.

A strong electric shock through the chest completely stops the heart, giving the cells that control the heart’s rhythm a chance to restore the proper heartbeat.

QuickCheck 29.13

A capacitor charged to 1.5 V stores 2.0 mJ of energy. If the capacitor is charged to 3.0 V, it will store

A. 1.0 mJ.
B. 2.0 mJ.
C. 4.0 mJ.
D. 6.0 mJ.
E. 8.0 mJ.

U_C = \frac{1}{2} C V^2

\boxed{E. 8.0 \text{ mJ.}}
Example 29.8 Storing Energy in a Capacitor

**Example 29.8 Storing energy in a capacitor**

How much energy is stored in a 2.0 μF capacitor that has been charged to 5000 V? What is the average power dissipation if this capacitor is discharged in 10 μs?

**Solve:** The energy stored in the charged capacitor is

\[
U_C = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2}(2.0 \times 10^{-6} \text{ F})(5000 \text{ V})^2 = 25 \text{ J}
\]

If this energy is released in 10 μs, the average power dissipation is

\[
P = \frac{\Delta E}{\Delta t} = \frac{25 \text{ J}}{1.0 \times 10^{-3} \text{ s}} = 2.5 \times 10^6 \text{ W} = 2.5 \text{ MW}
\]

**Assess:** The stored energy is equivalent to raising a 1 kg mass 2.5 m. This is a rather large amount of energy, which you can see by imagining the damage a 1 kg mass could do after falling 2.5 m. When this energy is released very quickly, which is possible in an electric circuit, it provides an enormous amount of power.

---

The Energy in the Electric Field

The energy density of an electric field, such as the one inside a capacitor, is:

\[
\begin{align*}
\varepsilon_E &= \frac{\text{energy stored}}{\text{volume in which it is stored}} = \frac{U_C}{Ad} = \frac{e_0}{2} E^2
\end{align*}
\]

The energy density has units J/m³.
The figure shows a parallel-plate capacitor with the plates separated by a vacuum. When the capacitor is fully charged to voltage \( \Delta V_0 \), the charge on the plates will be \( \pm Q_0 \), where \( Q_0 = C_0 \Delta V_0 \). In this section the subscript 0 refers to a vacuum-filled capacitor.

Now an insulating material is slipped between the capacitor plates. An insulator in an electric field is called a dielectric. The charge on the capacitor plates does not change \( (Q = Q_0) \). However, the voltage has decreased:

\[ \Delta V < \Delta V_0 \]

The figure shows how an insulating material becomes polarized in an external electric field. The insulator as a whole is still neutral, but the external electric field separates positive and negative charge.
The dielectric constant, like density or specific heat, is a property of a material. Easily polarized materials have larger dielectric constants than materials not easily polarized. Vacuum has $\kappa = 1$ exactly. Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant:

$$C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0} = \kappa Q_0 = \kappa C_0$$
The production of a practical capacitor, as shown, almost always involves the use of a solid or liquid dielectric.

All materials have a maximum electric field they can sustain without breakdown—the production of a spark.

The breakdown electric field of air is about $3 \times 10^6$ V/m.

A material's maximum sustainable electric field is called its dielectric strength.

### Table 29.1 Properties of dielectrics

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant $\varepsilon$</th>
<th>Dielectric strength $E_{\text{break}}$ (V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>Air (1 atm)</td>
<td>1.0006</td>
<td>3</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>60</td>
</tr>
<tr>
<td>Polystyrene plactic</td>
<td>2.6</td>
<td>24</td>
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<tr>
<td>Mylar</td>
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<td>7</td>
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<td>Paper</td>
<td>3.7</td>
<td>16</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>4.7</td>
<td>14</td>
</tr>
<tr>
<td>Pure water (20°C)</td>
<td>80</td>
<td>—</td>
</tr>
<tr>
<td>Trisilic acid</td>
<td>110</td>
<td>6</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>300</td>
<td>8</td>
</tr>
</tbody>
</table>

**Example 29.9 A Water-Filled Capacitor**

**Example 25.9** A water-filled capacitor

A 5.0 nF parallel-plate capacitor is charged to 160 V. It is then disconnected from the battery and immersed in distilled water. What are (a) the capacitance and voltage of the water-filled capacitor and (b) the energy stored in the capacitor before and after its immersion?

**MODEL** Pure distilled water is a good insulator. (The conductivity of tap water is due to dissolved ions.) Thus the immersed capacitor has a dielectric between the electrodes.
Example 29.9 A Water-Filled Capacitor

**EXAMPLE 29.9**

**A water-filled capacitor**

**SOLVE**

a. From Table 29.1, the dielectric constant of water is $\kappa = 80$.

The presence of the dielectric increases the capacitance to

$$C = \kappa C_0 = 80 \times 5.0 \text{nF} = 400 \text{nF}$$

At the same time, the voltage decreases to

$$\Delta V_C = \frac{(\Delta V_C)_b}{\kappa} = \frac{160 \text{ V}}{80} = 2.0 \text{ V}$$


Example 29.9 A Water-Filled Capacitor

**EXAMPLE 29.9**

b. The presence of a dielectric does not alter the derivation leading to Equation 29.26 for the energy stored in a capacitor. Right after being disconnected from the battery, the stored energy was

$$(U_C)_b = \frac{1}{2} C_0 (\Delta V_C)_b^2 = \frac{1}{2} (5.0 \times 10^{-9} \text{ F})(160 \text{ V})^2 = 6.4 \times 10^{-5} \text{ J}$$

After being immersed, the stored energy is

$$U_C = \frac{1}{2} C (\Delta V_C)^2 = \frac{1}{2} (400 \times 10^{-9} \text{ F})(2.0 \text{ V})^2 = 8.0 \times 10^{-5} \text{ J}$$


Example 29.9 A Water-Filled Capacitor

**EXAMPLE 29.9**

**A water-filled capacitor**

**ASSESS**

Water, with its large dielectric constant, has a big effect on the capacitor. But where did the energy go? We learned in Chapter 26 that a dipole is drawn into a region of stronger electric field. The electric field inside the capacitor is much stronger than just outside the capacitor, so the polarized dielectric is actually pulled into the capacitor. The “lost” energy is the work the capacitor’s electric field did pulling in the dielectric.
Example 29.10 Energy Density of a Defibrillator

**EXAMPLE 29.10 Energy density of a defibrillator**

A defibrillator unit contains a 150 \( \mu \text{F} \) capacitor that is charged to 2100 V. The capacitor plates are separated by a 0.050-mm-thick insulator with dielectric constant 120.

a. What is the area of the capacitor plates?
b. What are the stored energy and the energy density in the electric field when the capacitor is charged?

**MODEL** Model the defibrillator as a parallel-plate capacitor with a dielectric.

\[ A = \frac{C \varepsilon_0 \varepsilon_r}{k \varepsilon_0} = \frac{(150 \times 10^{-6} \text{ F})(5.0 \times 10^{-3} \text{ m})}{(20)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 7.1 \text{ m}^2 \]

Although the surface area is very large, Figure 29.33 below shows how very large sheets of very thin metal can be rolled up into capacitors that you hold in your hand.

**SOLVE**

b. The energy stored in the capacitor is

\[ U_\varepsilon = \frac{1}{2} C \varepsilon_0 \varepsilon_r (V C)^2 = \frac{1}{2} (150 \times 10^{-6} \text{ F})(2100 \text{ V})^2 = 330 \text{ J} \]

Because the dielectric has increased \( C \) by a factor of \( \varepsilon \), the energy density of Equation 29.28 is increased by a factor of \( \varepsilon \) to \( u_\varepsilon = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 \). The electric field strength in the capacitor is

\[ E = \frac{\Delta V}{d} = \frac{2100 \text{ V}}{5.0 \times 10^{-3} \text{ m}} = 4.2 \times 10^7 \text{ V/m} \]

Consequently, the energy density is

\[ u_\varepsilon = \frac{1}{2} \varepsilon_0 (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4.2 \times 10^7 \text{ V/m})^2 = 9.4 \times 10^3 \text{ J/m}^3 \]
Example 29.10 Energy Density of a Defibrillator

**EXAMPLE 29.10 Energy density of a defibrillator**

**ASSESS** 330 J is a substantial amount of energy—equivalent to that of a 1 kg mass traveling at 25 m/s. And it can be delivered very quickly as the capacitor is discharged through the patient’s chest.

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Chapter 29 Summary Slides

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General Principles

**Connecting \( V \) and \( E \)**

The electric potential and the electric field are two different perspectives of how source charges alter the space around them. \( V \) and \( E \) are related by

\[
\Delta V = V_f - V_i = \int E_x \, dx
\]

where \( x \) is measured from point \( i \) to point \( f \) and \( E_x \) is the component of \( E \) parallel to the line of integration.

Graphically

- \( \Delta V \) = the negative of the area under the \( E_x \) graph and
- \( E_x = \frac{\Delta V}{dx} \) = the negative of the slope of the potential graph.
General Principles

The Geometry of Potential and Field
The electric field:
- Is perpendicular to the equipotential surfaces.
- Points “downhill” in the direction of decreasing V.
- Is inversely proportional to the spacing \( \Delta s \) between the equipotential surfaces.

General Principles

Conservation of Energy
The sum of all potential differences around a closed path is zero.
\[ \Sigma (\Delta V) = 0 \]

Important Concepts

A battery is a source of potential. The charge oscillator in a battery uses chemical reactions to move charges from the negative terminal to the positive terminal.

\[ \Delta V_{\text{cell}} = \mathcal{E} \]
where the end \( \mathcal{E} \) is the work per charge done by the charge oscillator.
Important Concepts

For a conductor in electrostatic equilibrium
- The interior electric field is zero.
- The exterior electric field is perpendicular to the surface.
- The surface is an equipotential.
- The interior is at the same potential as the surface.