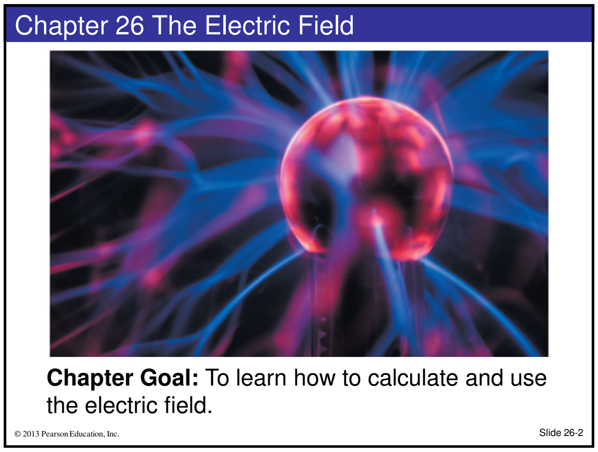
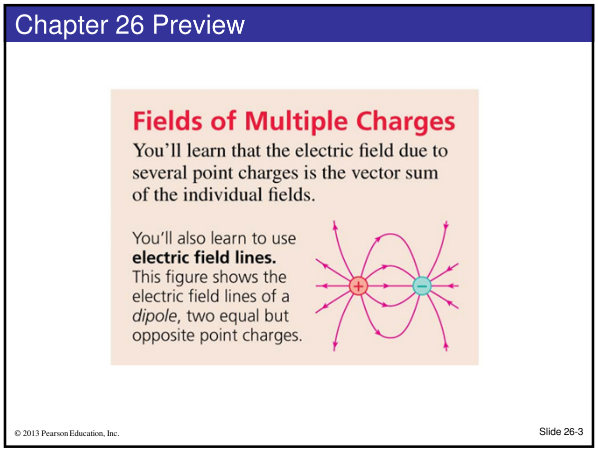


Chapter 26 The Electric Field

Chapter Goal: To learn how to calculate and use the electric field.

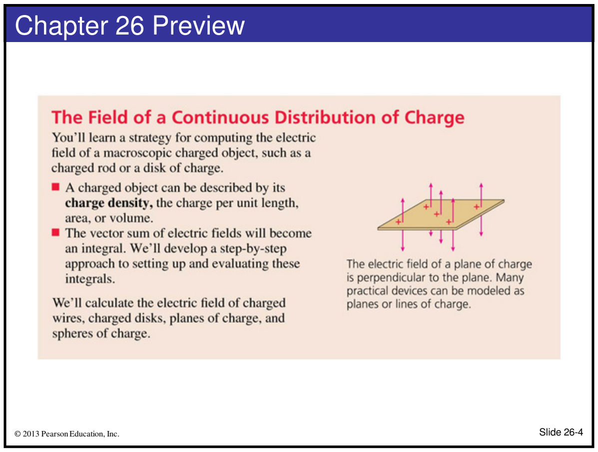


Chapter 26 Preview

Fields of Multiple Charges

You'll learn that the electric field due to several point charges is the vector sum of the individual fields.

You'll also learn to use **electric field lines.** This figure shows the electric field lines of a *dipole,* two equal but opposite point charges.



Chapter 26 Preview

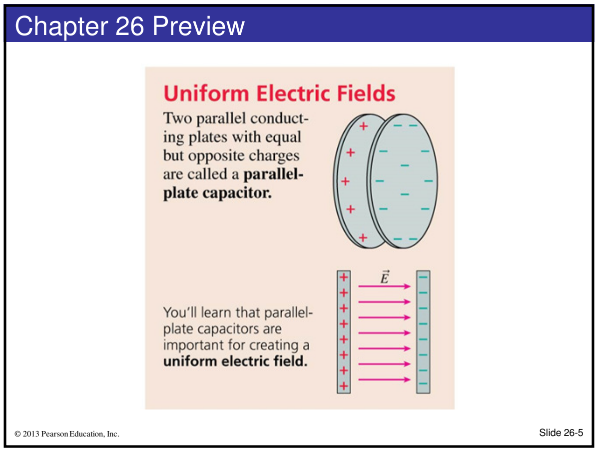
The Field of a Continuous Distribution of Charge

You'll learn a strategy for computing the electric field of a macroscopic charged object, such as a charged rod or a disk of charge.

- A charged object can be described by its **charge density,** the charge per unit length, area, or volume.
- The vector sum of electric fields will become an integral. We'll develop a step-by-step approach to setting up and evaluating these integrals.

We'll calculate the electric field of charged wires, charged disks, planes of charge, and spheres of charge.

The electric field of a plane of charge is perpendicular to the plane. Many practical devices can be modeled as planes or lines of charge.

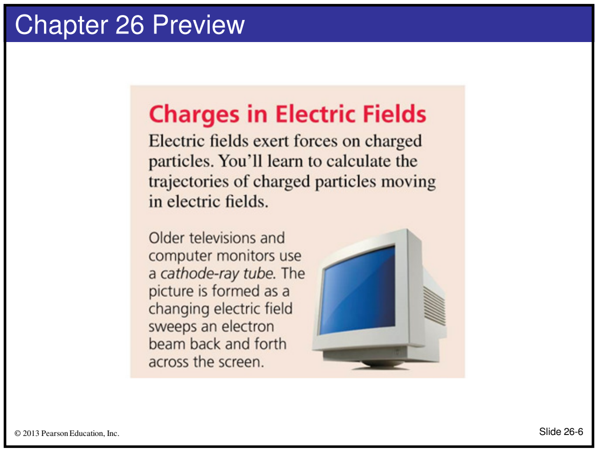


Chapter 26 Preview

Uniform Electric Fields

Two parallel conducting plates with equal but opposite charges are called a **parallel-plate capacitor.**

You'll learn that parallel-plate capacitors are important for creating a **uniform electric field.**

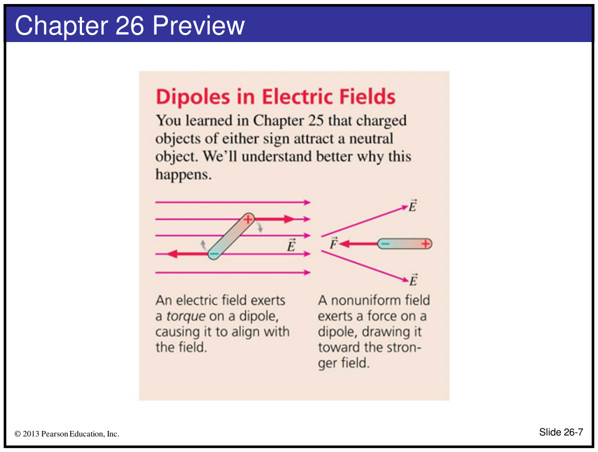


Chapter 26 Preview

Charges in Electric Fields

Electric fields exert forces on charged particles. You'll learn to calculate the trajectories of charged particles moving in electric fields.

Older televisions and computer monitors use a *cathode-ray tube*. The picture is formed as a changing electric field sweeps an electron beam back and forth across the screen.



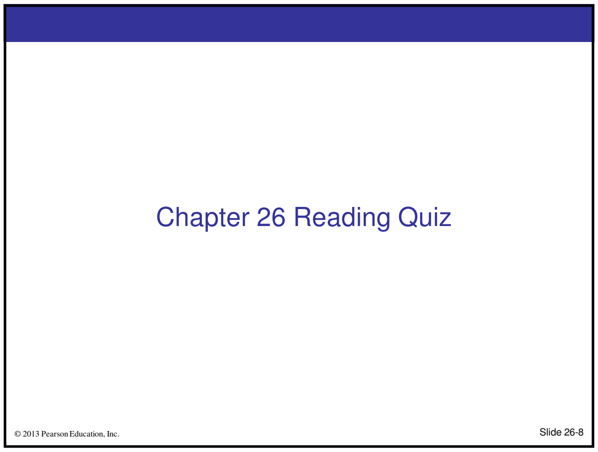
Chapter 26 Preview

Dipoles in Electric Fields

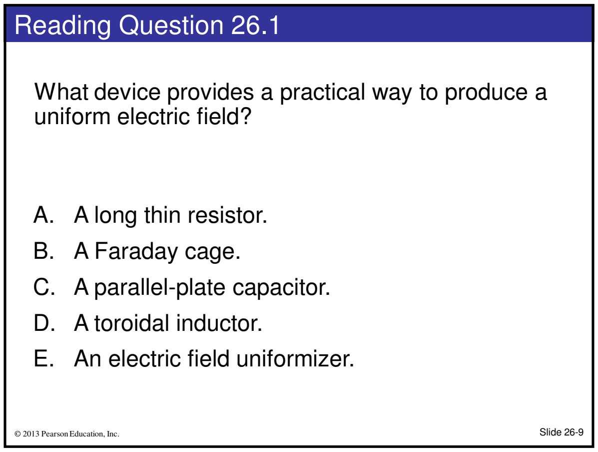
You learned in Chapter 25 that charged objects of either sign attract a neutral object. We'll understand better why this happens.

An electric field exerts a *torque* on a dipole, causing it to align with the field.

A nonuniform field exerts a force on a dipole, drawing it toward the stronger field.



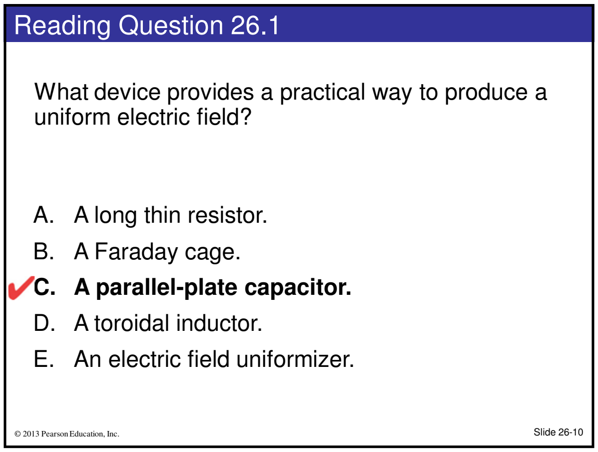
Chapter 26 Reading Quiz



Reading Question 26.1

What device provides a practical way to produce a uniform electric field?

- A. A long thin resistor.
- B. A Faraday cage.
- C. A parallel-plate capacitor.
- D. A toroidal inductor.
- E. An electric field uniformizer.



Reading Question 26.1

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- ✓**C. A parallel-plate capacitor.**
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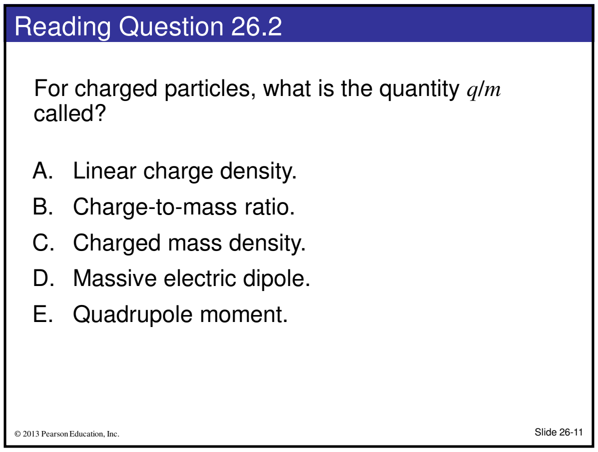
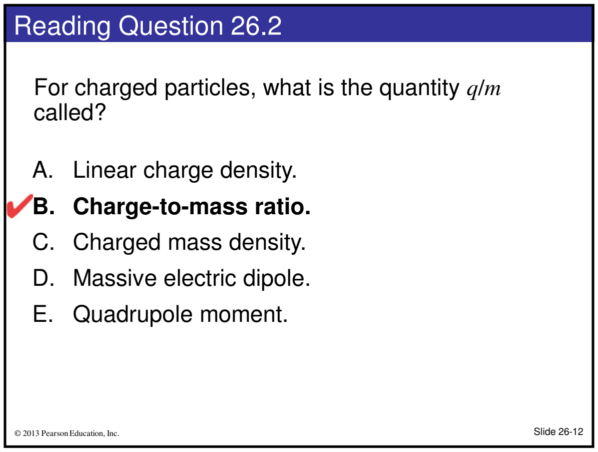


Reading Question 26.2

For charged particles, what is the quantity q/m called?

A. Linear charge density.
B. Charge-to-mass ratio.
C. Charged mass density.
D. Massive electric dipole.
E. Quadrupole moment.



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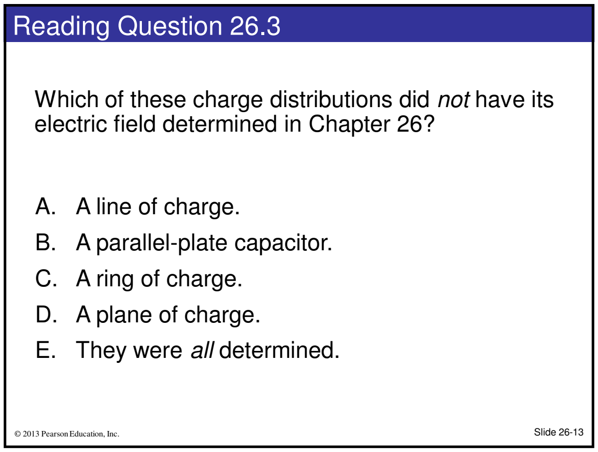
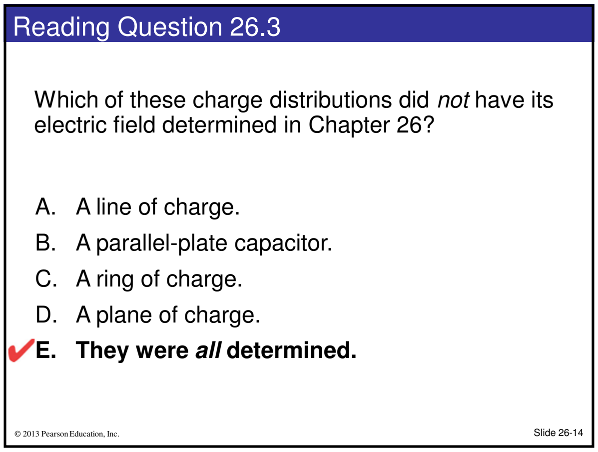


Reading Question 26.3

Which of these charge distributions did *not* have its electric field determined in Chapter 26?

A. A line of charge.
B. A parallel-plate capacitor.
C. A ring of charge.
D. A plane of charge.
E. They were *all* determined.



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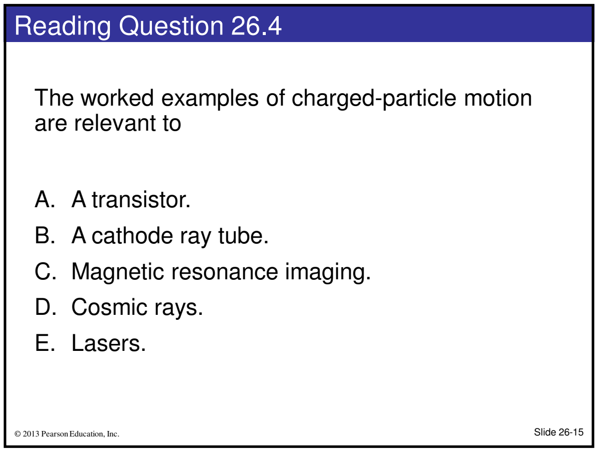
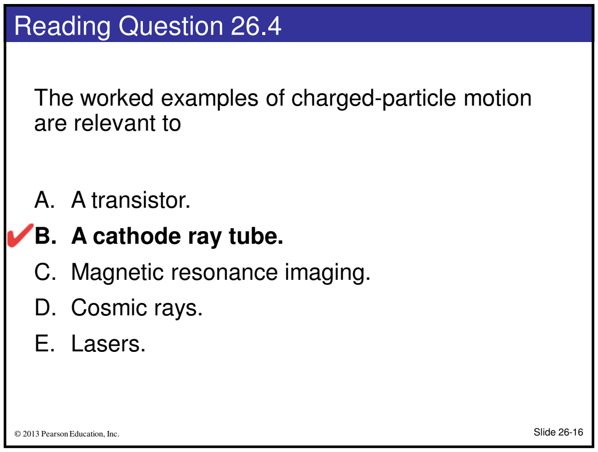
✓**E. They were *all* determined.**



Reading Question 26.4

The worked examples of charged-particle motion are relevant to

A. A transistor.
B. A cathode ray tube.
C. Magnetic resonance imaging.
D. Cosmic rays.
E. Lasers.

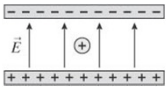


Reading Question 26.4

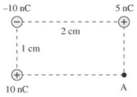
The worked examples of charged-particle motion are relevant to

A. A transistor.
✓**B. A cathode ray tube.**
C. Magnetic resonance imaging.
D. Cosmic rays.
E. Lasers.

1. What are the magnitude and direction of the force experienced by a proton of charge e in an electric field E as in the following diagram?



2. (a) Determine the magnitude and the direction of the electric field at point A.

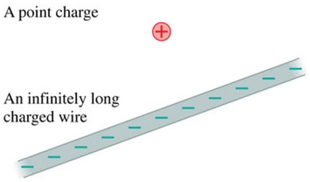


(b) What is the force on a +1nC charge when it is placed at A?

Chapter 26 Content, Examples, and QuickCheck Questions

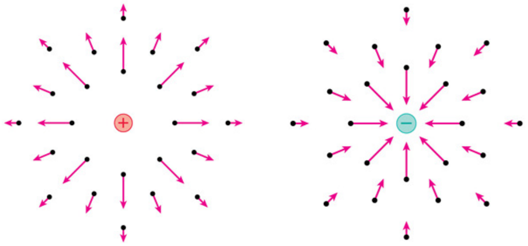
Electric Field Models

- Most of this chapter will be concerned with the *sources* of the electric field.
- We can understand the essential physics on the basis of simplified *models* of the sources of electric field.
- The drawings show models of a positive point charge and an infinitely long negative wire.
- We also will consider an infinitely wide charged plane and a charged sphere.



Electric Field of a Point Charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)}$$



The Electric Field

- The electric field was defined as:

  $\vec{E} = \vec{F}_{\text{on } q} / q$

  where $\vec{F}_{\text{on } q}$ is the electric force on test charge q.
- The SI units of electric field are therefore Newtons per Coulomb (N/C).

TABLE 26.1 Typical electric field strengths

Field location	Field strength (N/C)
Inside a current-carrying wire	10^{-3}–10^{-1}
Near the earth's surface	10^{2}–10^{4}
Near objects charged by rubbing	10^{3}–10^{6}
Electric breakdown in air, causing a spark	3×10^{6}
Inside an atom	10^{11}

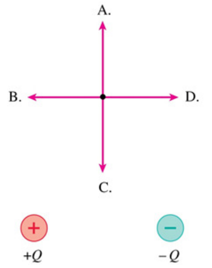
The Electric Field of Multiple Point Charges

- Suppose the source of an electric field is a group of point charges q_1, q_2, …
- The net electric field $\vec{E}_{\text{net}}$ at each point in space is a superposition of the electric fields due to each individual charge:

$$(E_{\text{net}})_x = (E_1)_x + (E_2)_x + \cdots = \sum (E_i)_x$$
$$(E_{\text{net}})_y = (E_1)_y + (E_2)_y + \cdots = \sum (E_i)_y$$
$$(E_{\text{net}})_z = (E_1)_z + (E_2)_z + \cdots = \sum (E_i)_z$$

QuickCheck 26.1

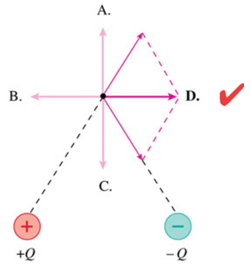
What is the direction of the electric field at the dot?



E. None of these.

QuickCheck 26.1

What is the direction of the electric field at the dot?



E. None of these.

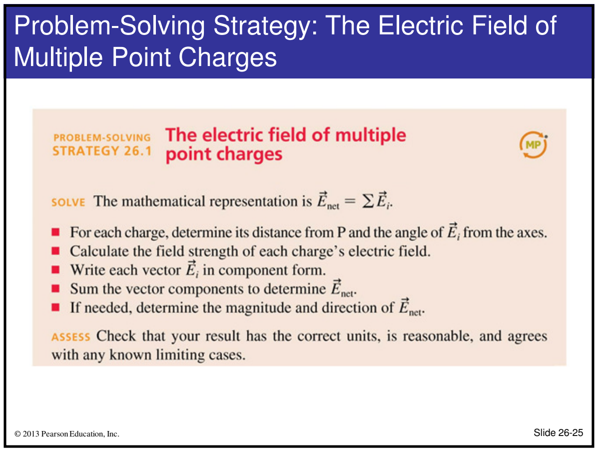
Problem-Solving Strategy: The Electric Field of Multiple Point Charges

PROBLEM-SOLVING STRATEGY 26.1 **The electric field of multiple point charges**

MODEL Model charged objects as point charges.

VISUALIZE For the pictorial representation:

- Establish a coordinate system and show the locations of the charges.
- Identify the point P at which you want to calculate the electric field.
- Draw the electric field of each charge at P.
- Use symmetry to determine if any components of $\vec{E}_{\text{net}}$ are zero.



Problem-Solving Strategy: The Electric Field of Multiple Point Charges

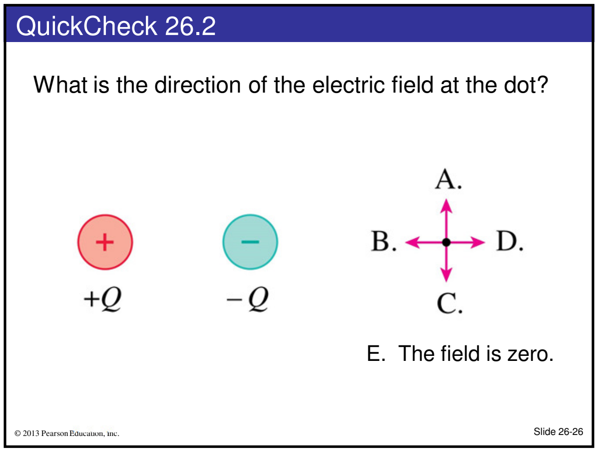
PROBLEM-SOLVING STRATEGY 26.1 **The electric field of multiple point charges**

MP

SOLVE The mathematical representation is $\vec{E}_{\text{net}} = \Sigma \vec{E}_i$.

- For each charge, determine its distance from P and the angle of $\vec{E}_i$ from the axes.
- Calculate the field strength of each charge's electric field.
- Write each vector $\vec{E}_i$ in component form.
- Sum the vector components to determine $\vec{E}_{\text{net}}$.
- If needed, determine the magnitude and direction of $\vec{E}_{\text{net}}$.

ASSESS Check that your result has the correct units, is reasonable, and agrees with any known limiting cases.



QuickCheck 26.2

What is the direction of the electric field at the dot?

$+Q$

$-Q$

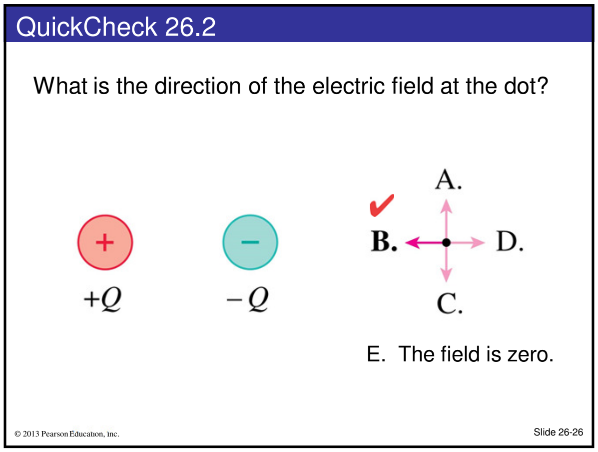
A.

B.

C.

D.

E. The field is zero.



QuickCheck 26.2

What is the direction of the electric field at the dot?

$+Q$

$-Q$

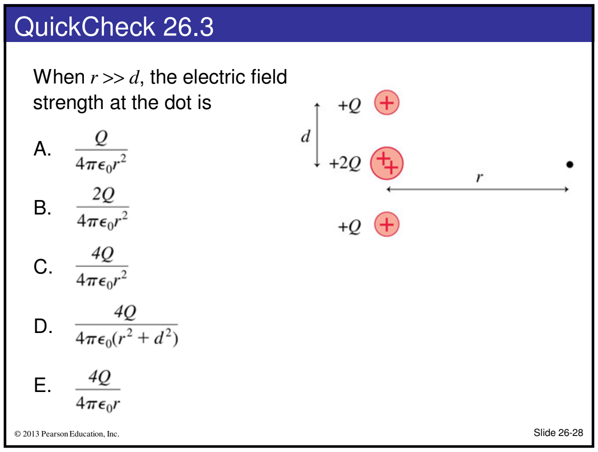
A.

✓ **B.**

C.

D.

E. The field is zero.



QuickCheck 26.3

When $r >> d$, the electric field strength at the dot is

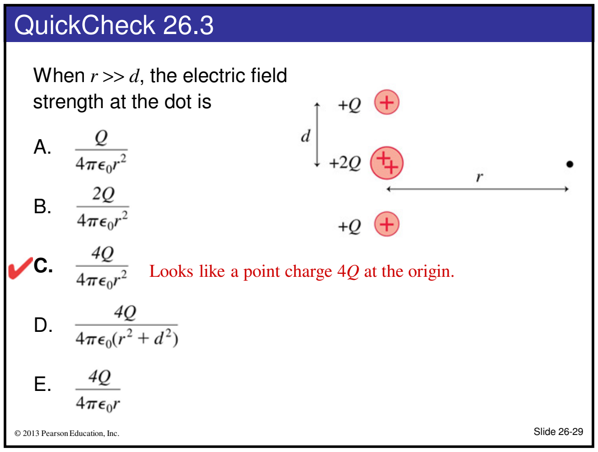
A. $\frac{Q}{4\pi\epsilon_0 r^2}$

B. $\frac{2Q}{4\pi\epsilon_0 r^2}$

C. $\frac{4Q}{4\pi\epsilon_0 r^2}$

D. $\frac{4Q}{4\pi\epsilon_0 (r^2 + d^2)}$

E. $\frac{4Q}{4\pi\epsilon_0 r}$



QuickCheck 26.3

When $r >> d$, the electric field strength at the dot is

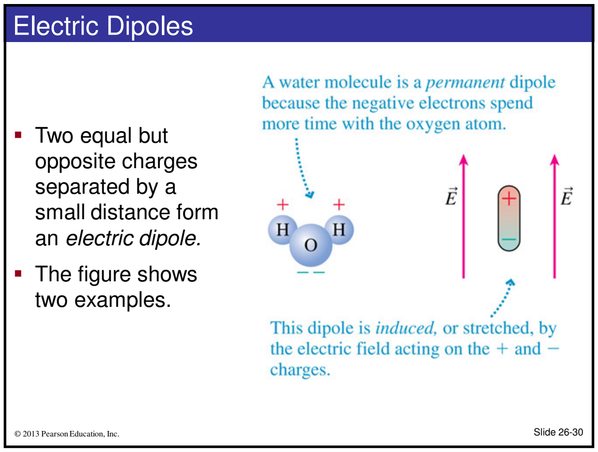
A. $\frac{Q}{4\pi\epsilon_0 r^2}$

B. $\frac{2Q}{4\pi\epsilon_0 r^2}$

✓C. $\frac{4Q}{4\pi\epsilon_0 r^2}$ Looks like a point charge $4Q$ at the origin.

D. $\frac{4Q}{4\pi\epsilon_0 (r^2 + d^2)}$

E. $\frac{4Q}{4\pi\epsilon_0 r}$

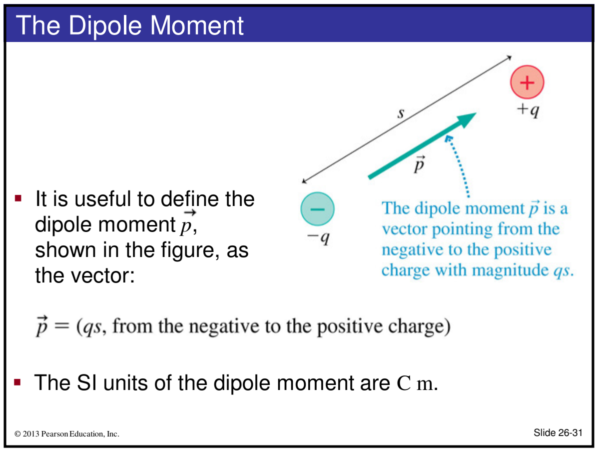


Electric Dipoles

- Two equal but opposite charges separated by a small distance form an *electric dipole.*
- The figure shows two examples.

A water molecule is a *permanent* dipole because the negative electrons spend more time with the oxygen atom.

This dipole is *induced,* or stretched, by the electric field acting on the + and − charges.



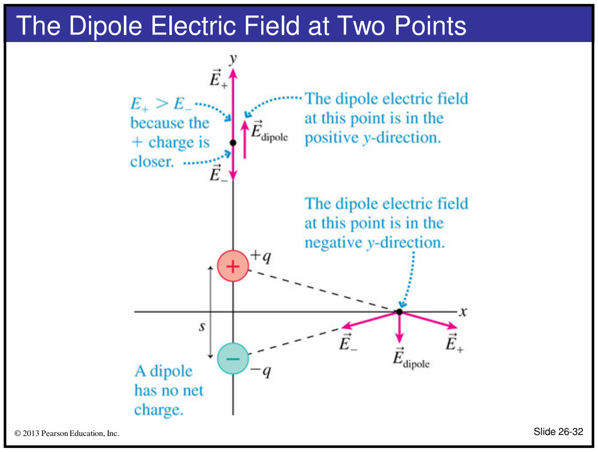
The Dipole Moment

- It is useful to define the dipole moment $\vec{p}$, shown in the figure, as the vector:

$$\vec{p} = (qs,\ \text{from the negative to the positive charge})$$

- The SI units of the dipole moment are C m.

The dipole moment $\vec{p}$ is a vector pointing from the negative to the positive charge with magnitude qs.



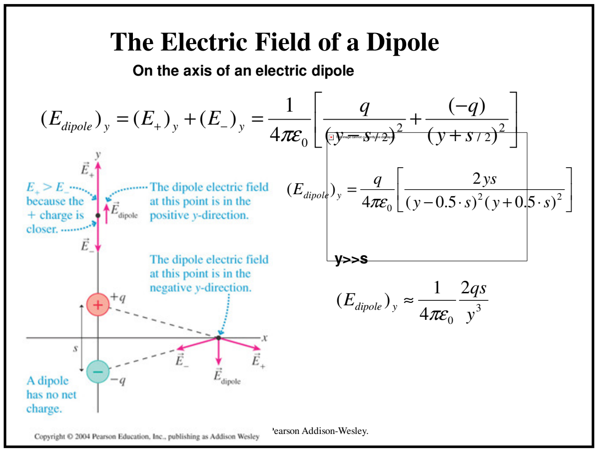
The Dipole Electric Field at Two Points

$E_+ > E_-$ because the + charge is closer.

The dipole electric field at this point is in the positive y-direction.

The dipole electric field at this point is in the negative y-direction.

A dipole has no net charge.



The Electric Field of a Dipole

On the axis of an electric dipole

$$(E_{dipole})_y = (E_+)_y + (E_-)_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(y - s/2)^2} + \frac{(-q)}{(y + s/2)^2}\right]$$

$$(E_{dipole})_y = \frac{q}{4\pi\varepsilon_0}\left[\frac{2ys}{(y - 0.5\cdot s)^2 (y + 0.5\cdot s)^2}\right]$$

y>>s

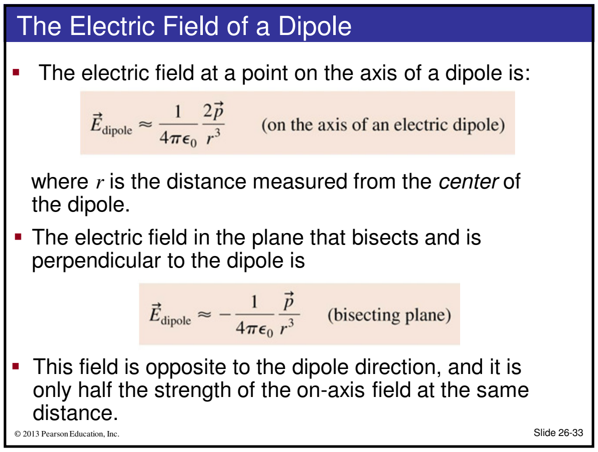
$$(E_{dipole})_y \approx \frac{1}{4\pi\varepsilon_0}\frac{2qs}{y^3}$$

$E_+ > E_-$ because the + charge is closer.

The dipole electric field at this point is in the positive y-direction.

The dipole electric field at this point is in the negative y-direction.

A dipole has no net charge.



The Electric Field of a Dipole

- The electric field at a point on the axis of a dipole is:

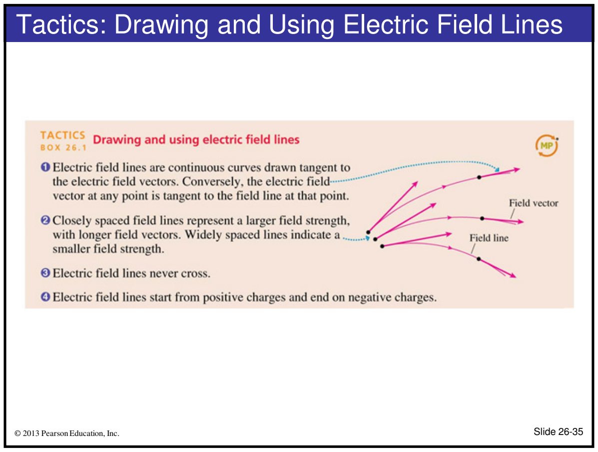
$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad \text{(on the axis of an electric dipole)}$$

where r is the distance measured from the *center* of the dipole.

- The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \quad \text{(bisecting plane)}$$

- This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.



Tactics: Drawing and Using Electric Field Lines

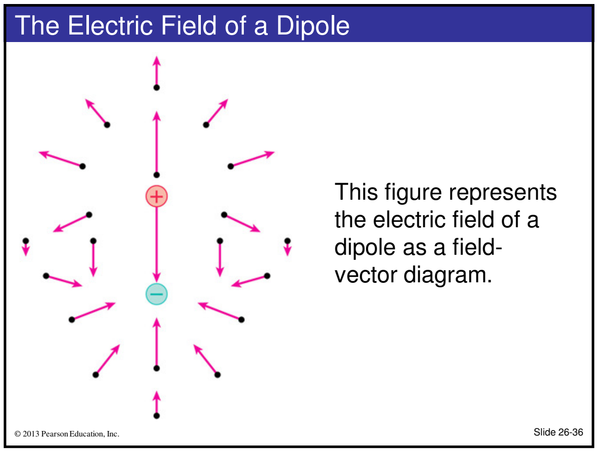
TACTICS BOX 26.1 **Drawing and using electric field lines**

❶ Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.

❷ Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.

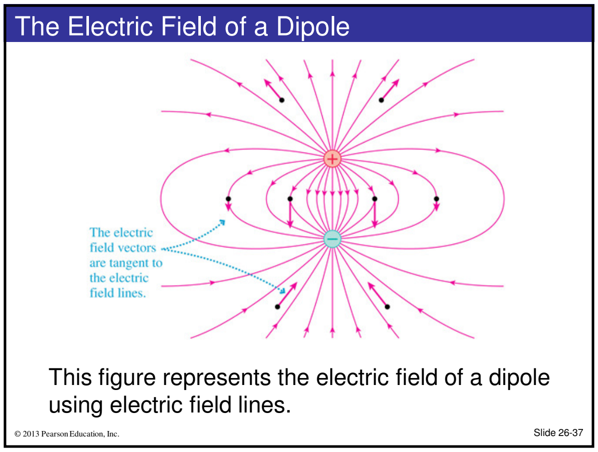
❸ Electric field lines never cross.

❹ Electric field lines start from positive charges and end on negative charges.



The Electric Field of a Dipole

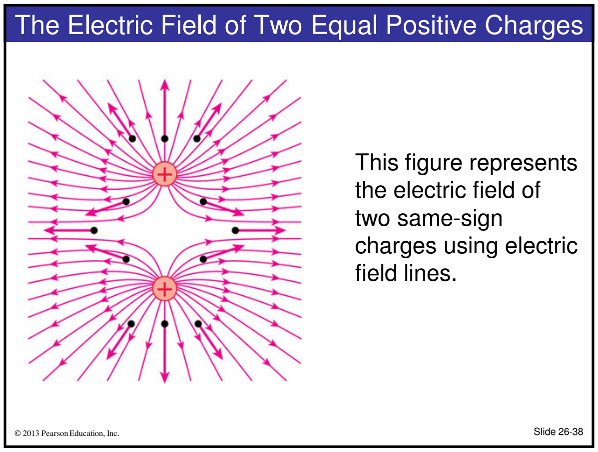
This figure represents the electric field of a dipole as a field-vector diagram.



The Electric Field of a Dipole

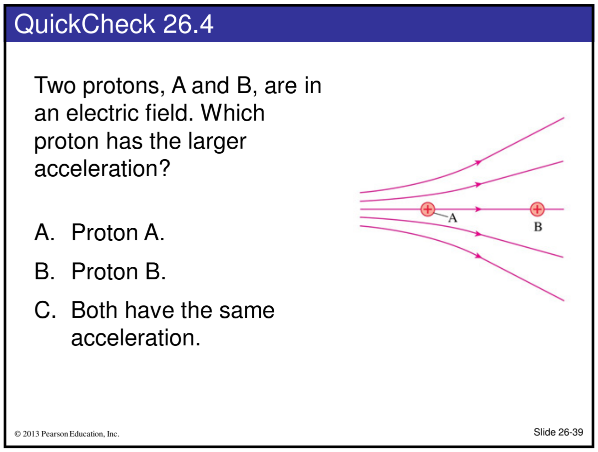
The electric field vectors are tangent to the electric field lines.

This figure represents the electric field of a dipole using electric field lines.



The Electric Field of Two Equal Positive Charges

This figure represents the electric field of two same-sign charges using electric field lines.



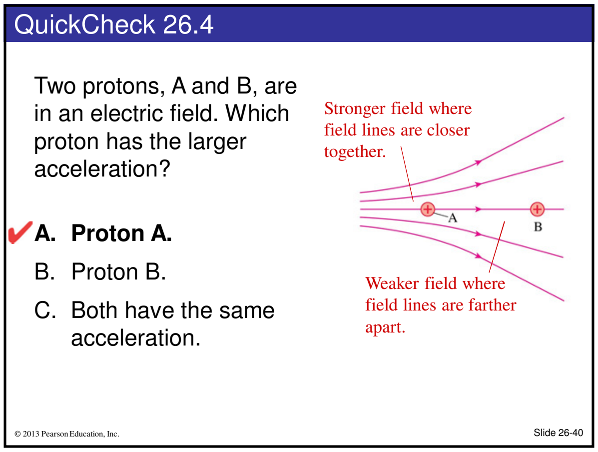
QuickCheck 26.4

Two protons, A and B, are in an electric field. Which proton has the larger acceleration?

A. Proton A.

B. Proton B.

C. Both have the same acceleration.



QuickCheck 26.4

Two protons, A and B, are in an electric field. Which proton has the larger acceleration?

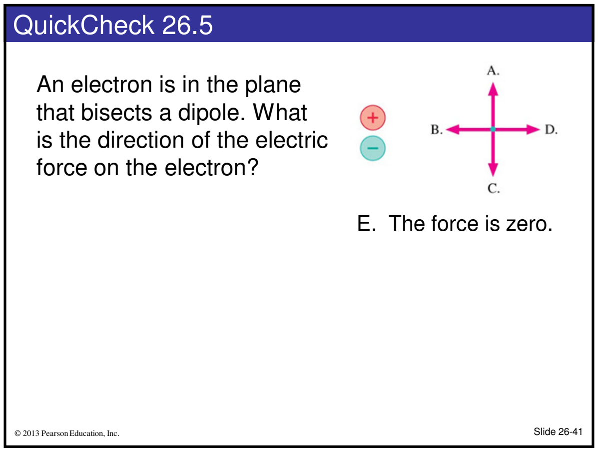
✓ **A. Proton A.**

B. Proton B.

C. Both have the same acceleration.

Stronger field where field lines are closer together.

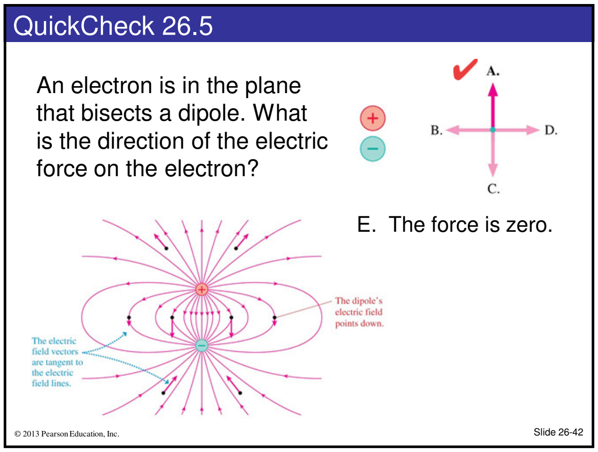
Weaker field where field lines are farther apart.



QuickCheck 26.5

An electron is in the plane that bisects a dipole. What is the direction of the electric force on the electron?

E. The force is zero.



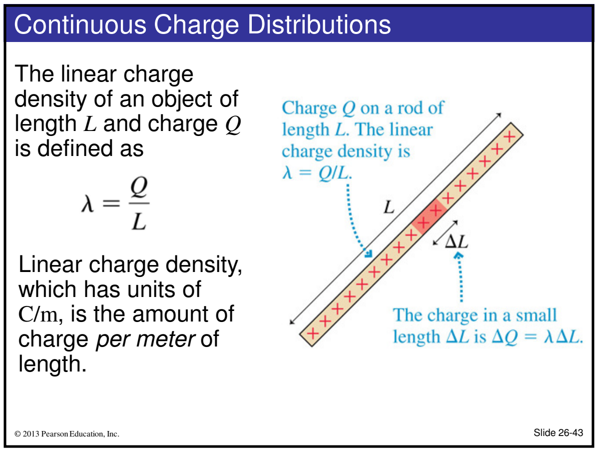
QuickCheck 26.5

An electron is in the plane that bisects a dipole. What is the direction of the electric force on the electron?

E. The force is zero.

The dipole's electric field points down.

The electric field vectors are tangent to the electric field lines.



Continuous Charge Distributions

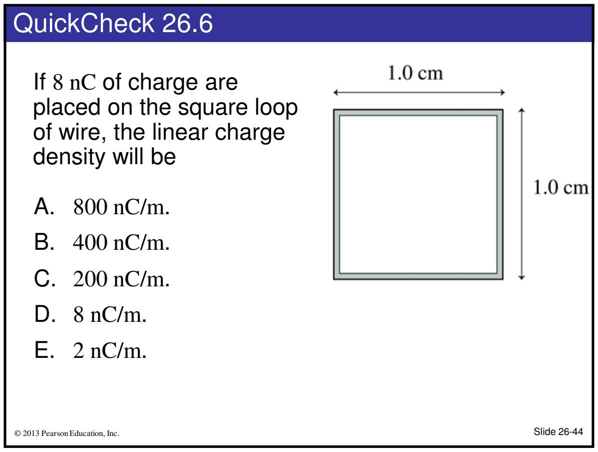
The linear charge density of an object of length L and charge Q is defined as

$$\lambda = \frac{Q}{L}$$

Linear charge density, which has units of C/m, is the amount of charge *per meter* of length.

Charge Q on a rod of length L. The linear charge density is $\lambda = Q/L$.

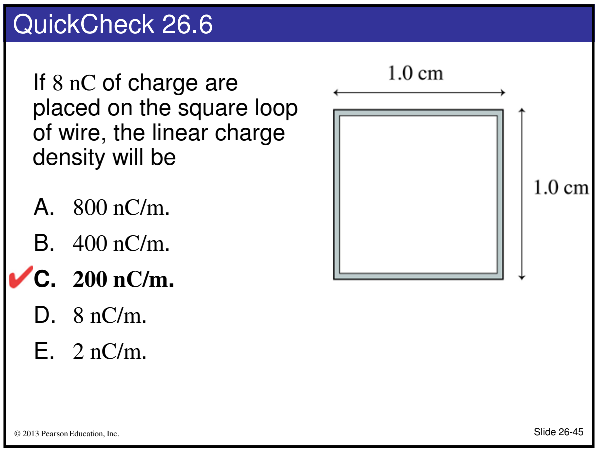
The charge in a small length ΔL is $\Delta Q = \lambda \Delta L$.



QuickCheck 26.6

If 8 nC of charge are placed on the square loop of wire, the linear charge density will be

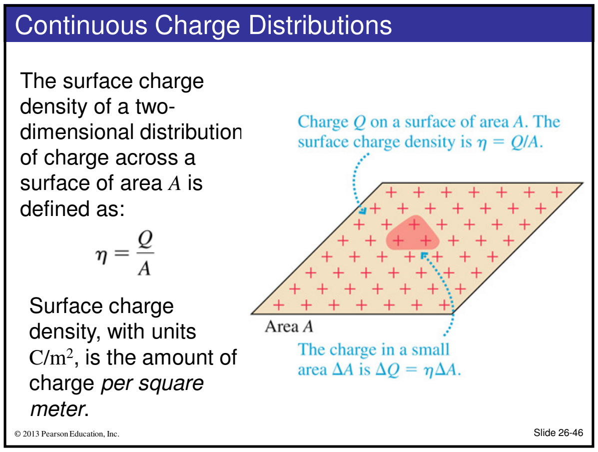
A. 800 nC/m.
B. 400 nC/m.
C. 200 nC/m.
D. 8 nC/m.
E. 2 nC/m.



QuickCheck 26.6

If 8 nC of charge are placed on the square loop of wire, the linear charge density will be

A. 800 nC/m.
B. 400 nC/m.
✓**C. 200 nC/m.**
D. 8 nC/m.
E. 2 nC/m.



Continuous Charge Distributions

The surface charge density of a two-dimensional distribution of charge across a surface of area A is defined as:

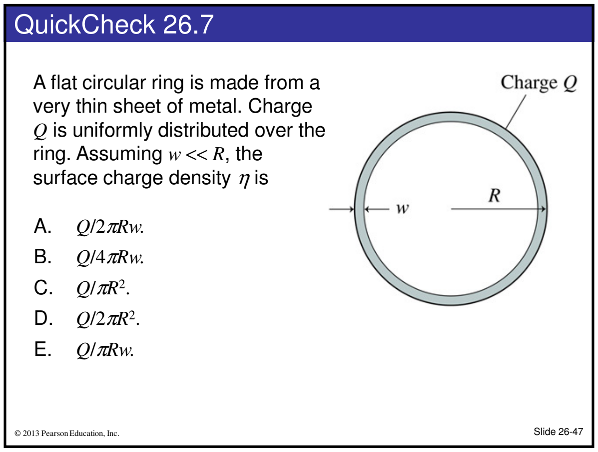
$$\eta = \frac{Q}{A}$$

Surface charge density, with units C/m^2, is the amount of charge *per square meter.*

Charge Q on a surface of area A. The surface charge density is $\eta = Q/A$.

The charge in a small area ΔA is $\Delta Q = \eta \Delta A$.

Slide 26-46

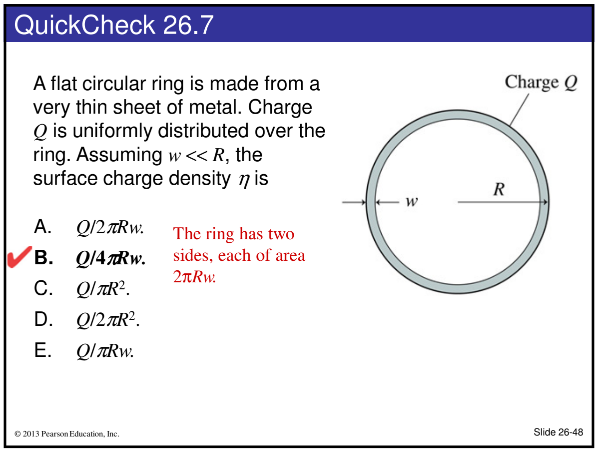


QuickCheck 26.7

A flat circular ring is made from a very thin sheet of metal. Charge Q is uniformly distributed over the ring. Assuming $w << R$, the surface charge density η is

A. $Q/2\pi Rw$.
B. $Q/4\pi Rw$.
C. $Q/\pi R^2$.
D. $Q/2\pi R^2$.
E. $Q/\pi Rw$.

Slide 26-47



QuickCheck 26.7

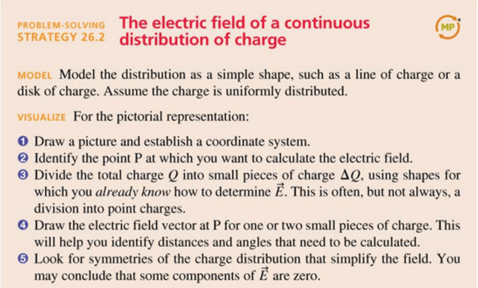
A flat circular ring is made from a very thin sheet of metal. Charge Q is uniformly distributed over the ring. Assuming $w << R$, the surface charge density η is

A. $Q/2\pi Rw$.
✓ **B. $Q/4\pi Rw$.**
C. $Q/\pi R^2$.
D. $Q/2\pi R^2$.
E. $Q/\pi Rw$.

The ring has two sides, each of area $2\pi Rw$.

Slide 26-48

Problem-Solving Strategy: The Electric Field of a Continuous Distribution of Charge



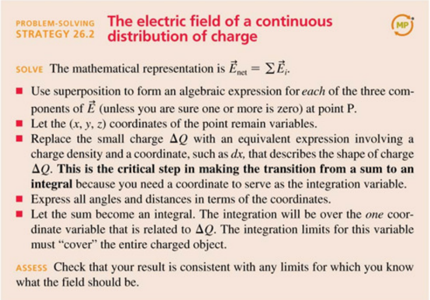
PROBLEM-SOLVING STRATEGY 26.2 **The electric field of a continuous distribution of charge**

MODEL Model the distribution as a simple shape, such as a line of charge or a disk of charge. Assume the charge is uniformly distributed.

VISUALIZE For the pictorial representation:

❶ Draw a picture and establish a coordinate system.
❷ Identify the point P at which you want to calculate the electric field.
❸ Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine $\vec{E}$. This is often, but not always, a division into point charges.
❹ Draw the electric field vector at P for one or two small pieces of charge. This will help you identify distances and angles that need to be calculated.
❺ Look for symmetries of the charge distribution that simplify the field. You may conclude that some components of $\vec{E}$ are zero.

Problem-Solving Strategy: The Electric Field of a Continuous Distribution of Charge



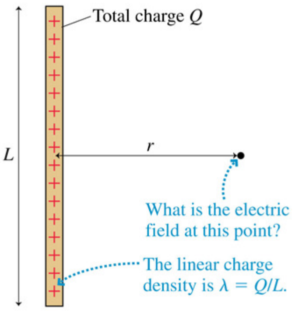
PROBLEM-SOLVING STRATEGY 26.2 **The electric field of a continuous distribution of charge**

SOLVE The mathematical representation is $\vec{E}_{\text{net}} = \sum \vec{E}_i$.

- Use superposition to form an algebraic expression for *each* of the three components of $\vec{E}$ (unless you are sure one or more is zero) at point P.
- Let the (x, y, z) coordinates of the point remain variables.
- Replace the small charge ΔQ with an equivalent expression involving a charge density and a coordinate, such as dx, that describes the shape of charge ΔQ. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- Express all angles and distances in terms of the coordinates.
- Let the sum become an integral. The integration will be over the *one* coordinate variable that is related to ΔQ. The integration limits for this variable must "cover" the entire charged object.

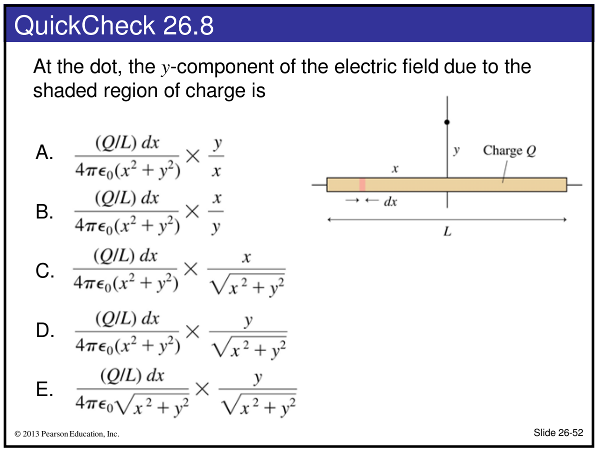
ASSESS Check that your result is consistent with any limits for which you know what the field should be.

The Electric Field of a Finite Line of Charge



Example 26.3 in the text uses integration to find the electric field strength at a radial distance r in the plane that bisects a rod of length L with total charge Q:

$$E_{\text{rod}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$



QuickCheck 26.8

At the dot, the y-component of the electric field due to the shaded region of charge is

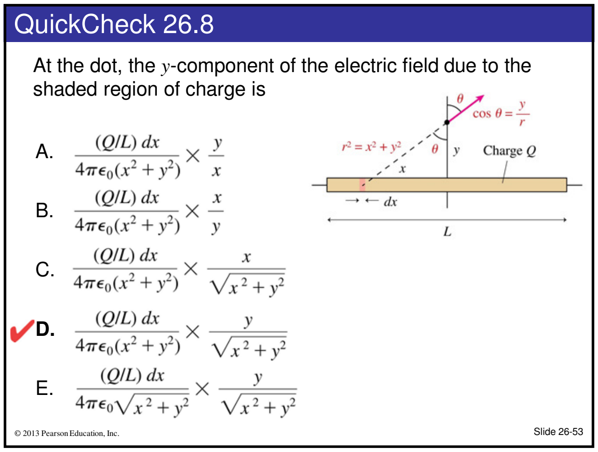
A. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{y}{x}$

B. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{x}{y}$

C. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{x}{\sqrt{x^2+y^2}}$

D. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{y}{\sqrt{x^2+y^2}}$

E. $\frac{(Q/L)\,dx}{4\pi\epsilon_0\sqrt{x^2+y^2}} \times \frac{y}{\sqrt{x^2+y^2}}$



QuickCheck 26.8

At the dot, the y-component of the electric field due to the shaded region of charge is

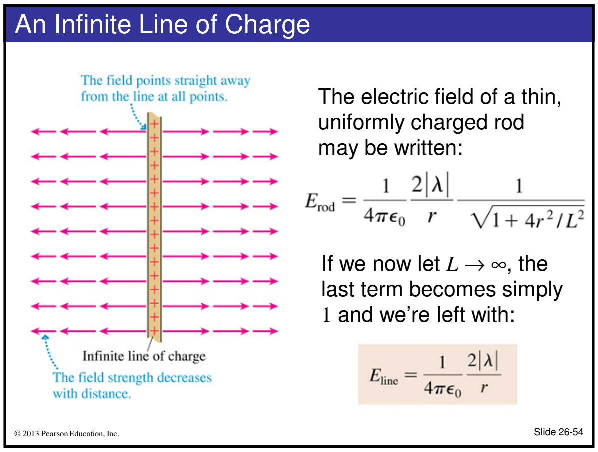
A. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{y}{x}$

B. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{x}{y}$

C. $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{x}{\sqrt{x^2+y^2}}$

✓**D.** $\frac{(Q/L)\,dx}{4\pi\epsilon_0(x^2+y^2)} \times \frac{y}{\sqrt{x^2+y^2}}$

E. $\frac{(Q/L)\,dx}{4\pi\epsilon_0\sqrt{x^2+y^2}} \times \frac{y}{\sqrt{x^2+y^2}}$



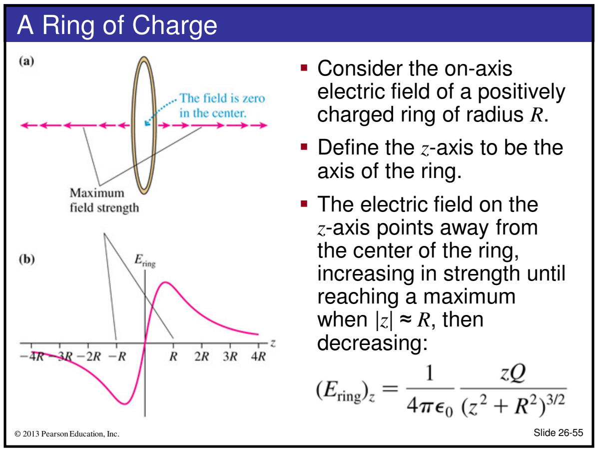
An Infinite Line of Charge

The electric field of a thin, uniformly charged rod may be written:

$$E_{\text{rod}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1+4r^2/L^2}}$$

If we now let $L \to \infty$, the last term becomes simply 1 and we're left with:

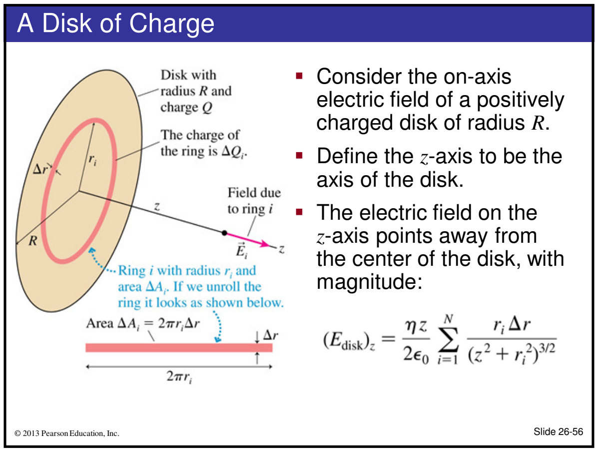
$$E_{\text{line}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}$$



A Ring of Charge

- Consider the on-axis electric field of a positively charged ring of radius R.
- Define the z-axis to be the axis of the ring.
- The electric field on the z-axis points away from the center of the ring, increasing in strength until reaching a maximum when $|z| \approx R$, then decreasing:

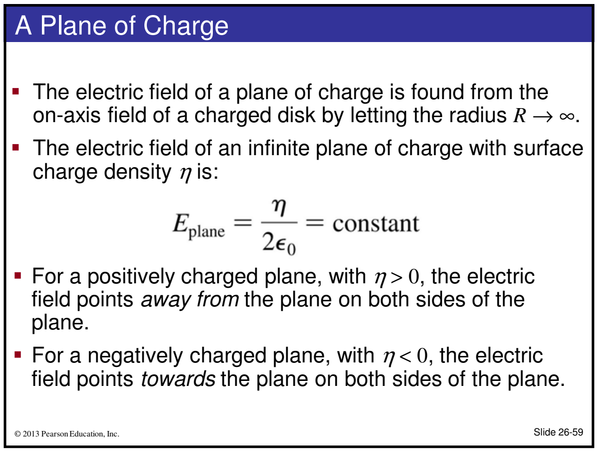
$$(E_{\text{ring}})_z = \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$



A Disk of Charge

- Consider the on-axis electric field of a positively charged disk of radius R.
- Define the z-axis to be the axis of the disk.
- The electric field on the z-axis points away from the center of the disk, with magnitude:

$$(E_{\text{disk}})_z = \frac{\eta z}{2\epsilon_0}\sum_{i=1}^{N}\frac{r_i\,\Delta r}{(z^2+r_i^2)^{3/2}}$$

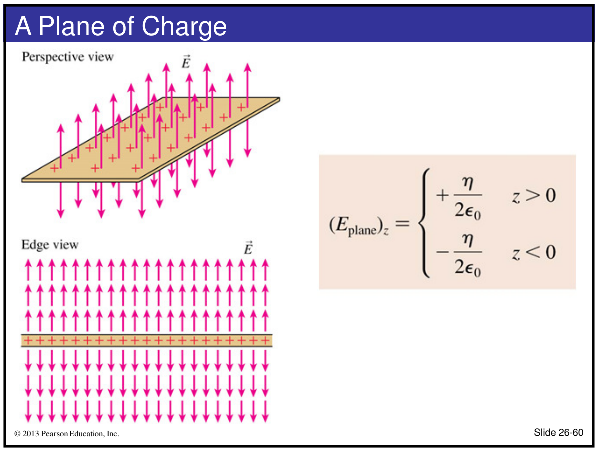


A Plane of Charge

- The electric field of a plane of charge is found from the on-axis field of a charged disk by letting the radius $R \to \infty$.
- The electric field of an infinite plane of charge with surface charge density η is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

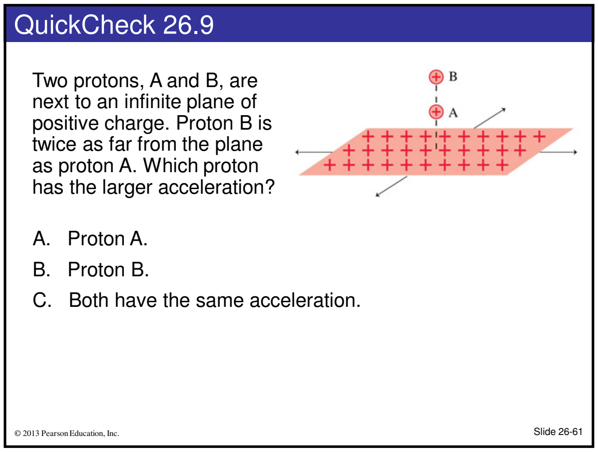
- For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
- For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.



A Plane of Charge

$$(E_{\text{plane}})_z = \begin{cases} +\dfrac{\eta}{2\epsilon_0} & z > 0 \\ -\dfrac{\eta}{2\epsilon_0} & z < 0 \end{cases}$$

Slide 26-60

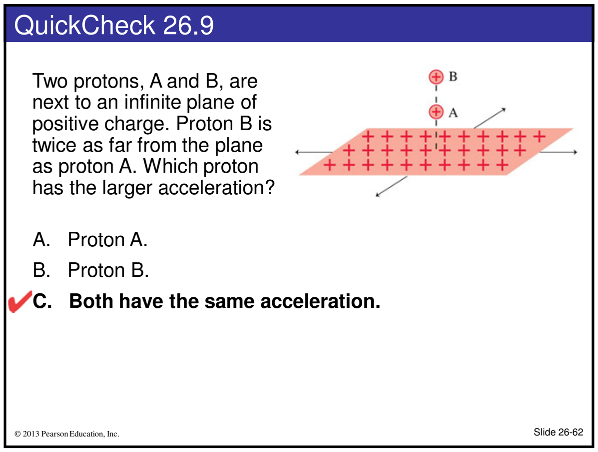


QuickCheck 26.9

Two protons, A and B, are next to an infinite plane of positive charge. Proton B is twice as far from the plane as proton A. Which proton has the larger acceleration?

A. Proton A.
B. Proton B.
C. Both have the same acceleration.

Slide 26-61



QuickCheck 26.9

Two protons, A and B, are next to an infinite plane of positive charge. Proton B is twice as far from the plane as proton A. Which proton has the larger acceleration?

A. Proton A.
B. Proton B.
✓**C. Both have the same acceleration.**

Slide 26-62

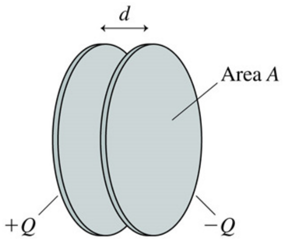
A Sphere of Charge

A sphere of charge Q and radius R, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere that is exactly the same as that of a point charge Q located at the center of the sphere:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r \geq R$$

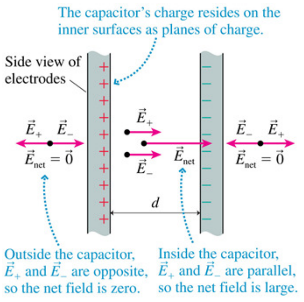
The Parallel-Plate Capacitor

- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance d apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**
- Capacitors play important roles in many electric circuits.



The Parallel-Plate Capacitor

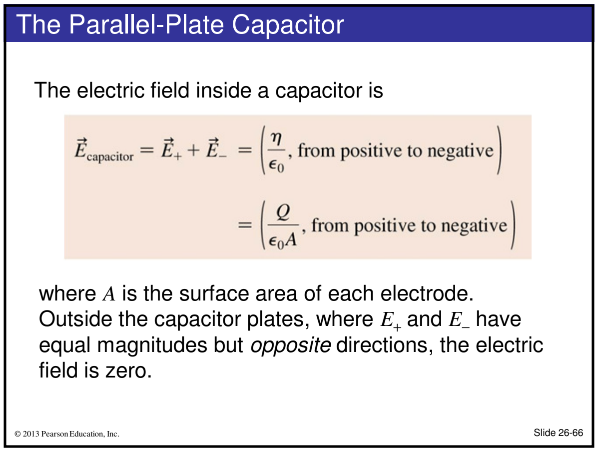
- The figure shows two capacitor plates, seen from the side.
- Because opposite charges attract, all of the charge is on the *inner* surfaces of the two plates.
- Inside the capacitor, the net field points toward the negative plate.
- Outside the capacitor, the net field is zero.



The capacitor's charge resides on the inner surfaces as planes of charge.

Outside the capacitor, $\vec{E}_+$ and $\vec{E}_-$ are opposite, so the net field is zero.

Inside the capacitor, $\vec{E}_+$ and $\vec{E}_-$ are parallel, so the net field is large.

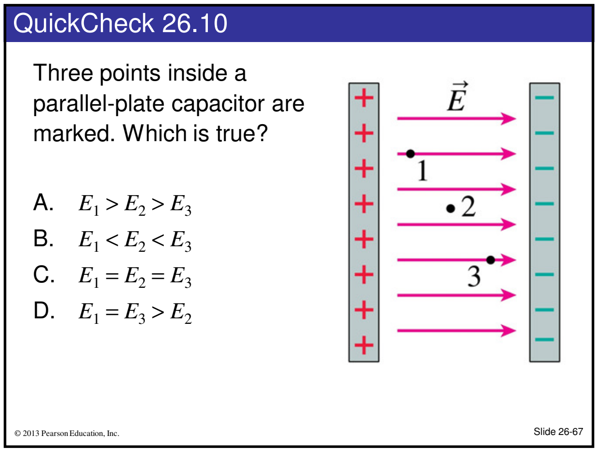


The Parallel-Plate Capacitor

The electric field inside a capacitor is

$$\vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$
$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

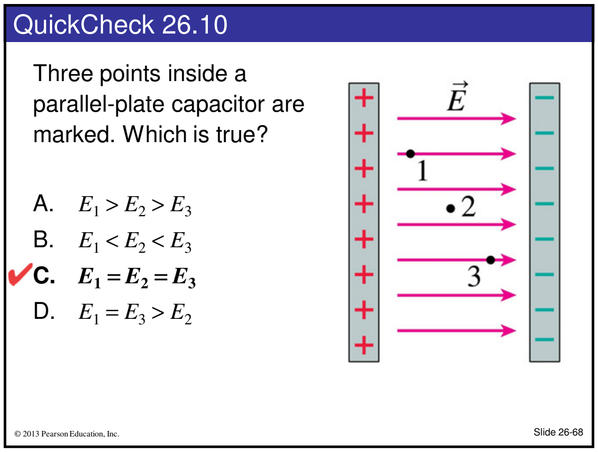
where A is the surface area of each electrode. Outside the capacitor plates, where E_+ and E_- have equal magnitudes but *opposite* directions, the electric field is zero.



QuickCheck 26.10

Three points inside a parallel-plate capacitor are marked. Which is true?

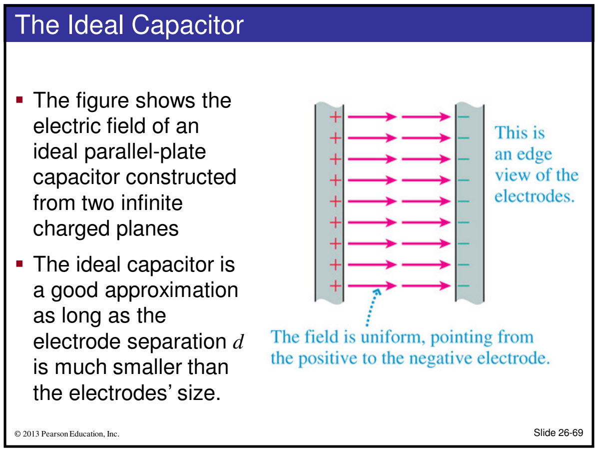
A. $E_1 > E_2 > E_3$
B. $E_1 < E_2 < E_3$
C. $E_1 = E_2 = E_3$
D. $E_1 = E_3 > E_2$



QuickCheck 26.10

Three points inside a parallel-plate capacitor are marked. Which is true?

A. $E_1 > E_2 > E_3$
B. $E_1 < E_2 < E_3$
✓**C.** $\boldsymbol{E_1 = E_2 = E_3}$
D. $E_1 = E_3 > E_2$

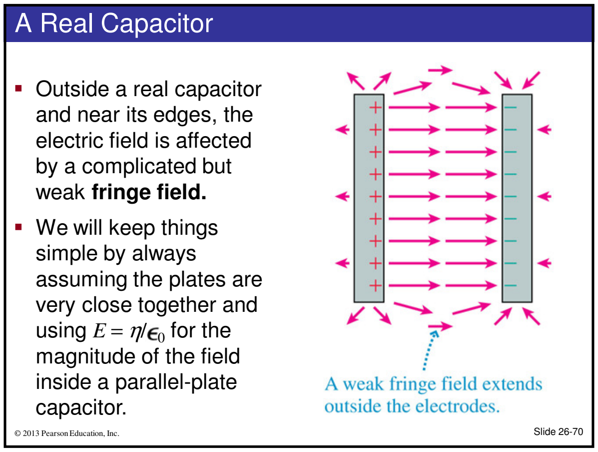


The Ideal Capacitor

- The figure shows the electric field of an ideal parallel-plate capacitor constructed from two infinite charged planes
- The ideal capacitor is a good approximation as long as the electrode separation d is much smaller than the electrodes' size.

This is an edge view of the electrodes.

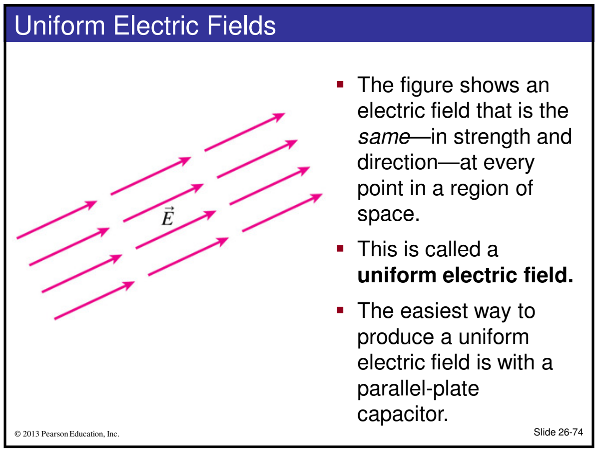
The field is uniform, pointing from the positive to the negative electrode.



A Real Capacitor

- Outside a real capacitor and near its edges, the electric field is affected by a complicated but weak **fringe field.**
- We will keep things simple by always assuming the plates are very close together and using $E = \eta/\epsilon_0$ for the magnitude of the field inside a parallel-plate capacitor.

A weak fringe field extends outside the electrodes.



Uniform Electric Fields

- The figure shows an electric field that is the *same*—in strength and direction—at every point in a region of space.
- This is called a **uniform electric field.**
- The easiest way to produce a uniform electric field is with a parallel-plate capacitor.

Example 26.7 Charge Density on a Cell Wall

EXAMPLE 26.7 **Charge density on a cell wall**

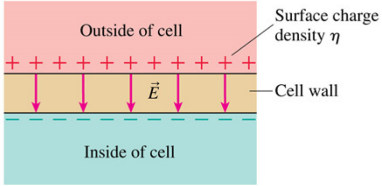
Example 25.7 noted that the electric field strength in the cell wall of a neuron is typically 1.0×10^7 N/C. This electric field is established because the outer surface of the cell wall is positive and the inner surface negative. What is a typical surface charge density on the surface of a cell wall?

MODEL Although cells are roughly spherical, the wall thickness is much less than the radius of the cell. Locally, at a point inside the cell wall, the curvature is negligible, so we can model the cell wall as a parallel-plate capacitor.

Example 26.7 Charge Density on a Cell Wall

EXAMPLE 26.7 **Charge density on a cell wall**

VISUALIZE The figure below shows a section of the cell wall. The charges are due to ions, not electrons, but that doesn't affect our analysis.

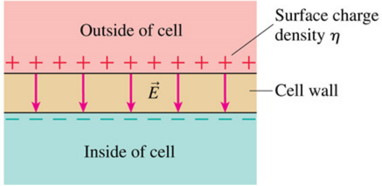


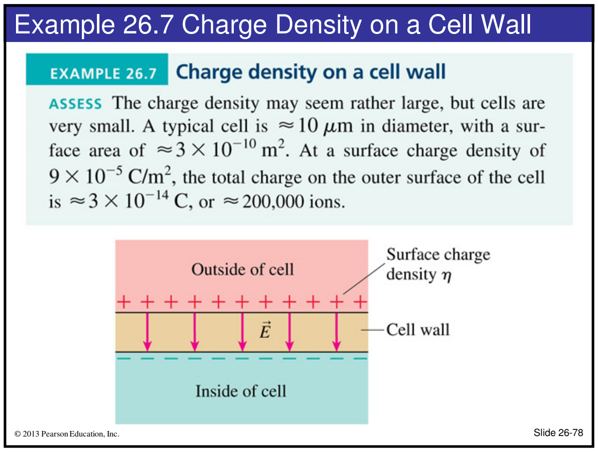
Example 26.7 Charge Density on a Cell Wall

EXAMPLE 26.7 **Charge density on a cell wall**

SOLVE The electric field strength inside a capacitor is $E = \eta/\epsilon_0$. The surface charge density needed to produce a known field is

$$\eta = \epsilon_0 E = (8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2})(1.0 \times 10^7\ \mathrm{N/C})$$
$$= 8.9 \times 10^{-5}\ \mathrm{C/m^2}$$

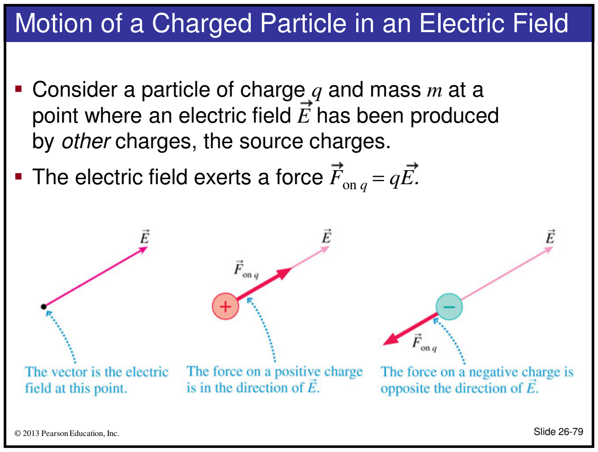




Example 26.7 Charge Density on a Cell Wall

EXAMPLE 26.7 **Charge density on a cell wall**

ASSESS The charge density may seem rather large, but cells are very small. A typical cell is $\approx 10\ \mu\text{m}$ in diameter, with a surface area of $\approx 3 \times 10^{-10}\ \text{m}^2$. At a surface charge density of $9 \times 10^{-5}\ \text{C/m}^2$, the total charge on the outer surface of the cell is $\approx 3 \times 10^{-14}$ C, or $\approx$ 200,000 ions.



Motion of a Charged Particle in an Electric Field

- Consider a particle of charge q and mass m at a point where an electric field $\vec{E}$ has been produced by *other* charges, the source charges.
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$.

The vector is the electric field at this point.

The force on a positive charge is in the direction of $\vec{E}$.

The force on a negative charge is opposite the direction of $\vec{E}$.

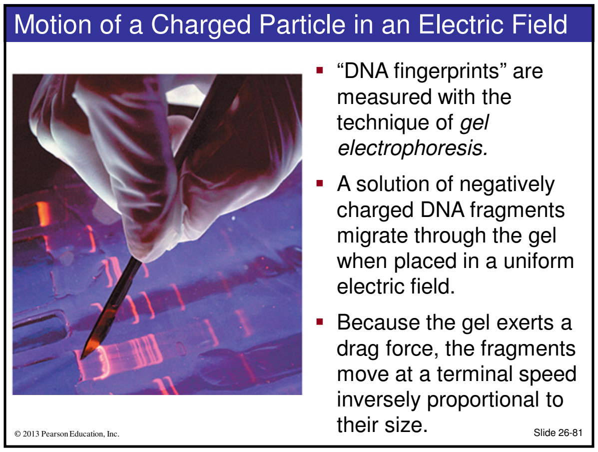
Motion of a Charged Particle in an Electric Field

- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$ on a charged particle.
- If this is the only force acting on q, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

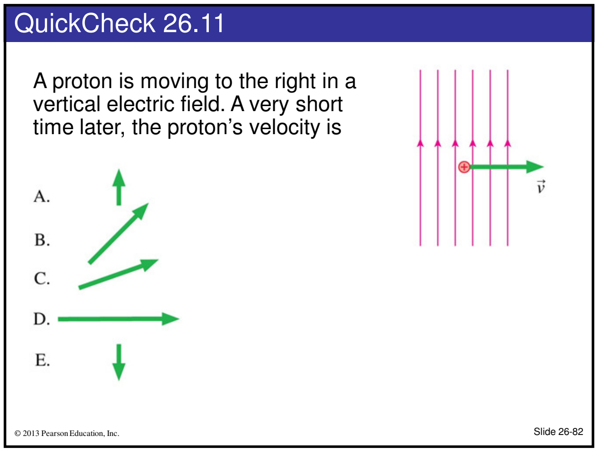
- In a uniform field, the acceleration is constant:

$$a = \frac{qE}{m} = \text{constant}$$



Motion of a Charged Particle in an Electric Field

- “DNA fingerprints” are measured with the technique of *gel electrophoresis.*
- A solution of negatively charged DNA fragments migrate through the gel when placed in a uniform electric field.
- Because the gel exerts a drag force, the fragments move at a terminal speed inversely proportional to their size.



QuickCheck 26.11

A proton is moving to the right in a vertical electric field. A very short time later, the proton’s velocity is

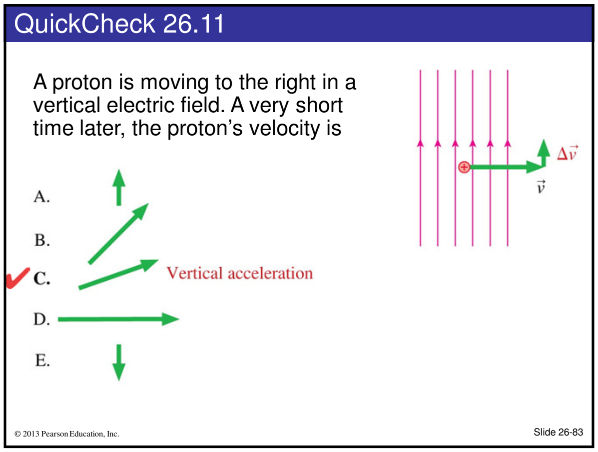
A.

B.

C.

D.

E.



QuickCheck 26.11

A proton is moving to the right in a vertical electric field. A very short time later, the proton’s velocity is

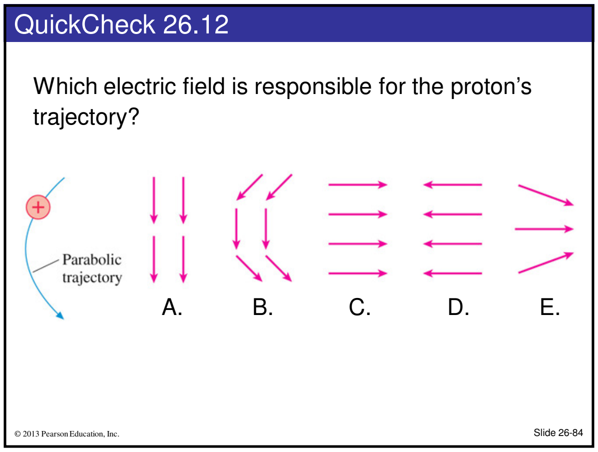
A.

B.

✓ **C.** Vertical acceleration

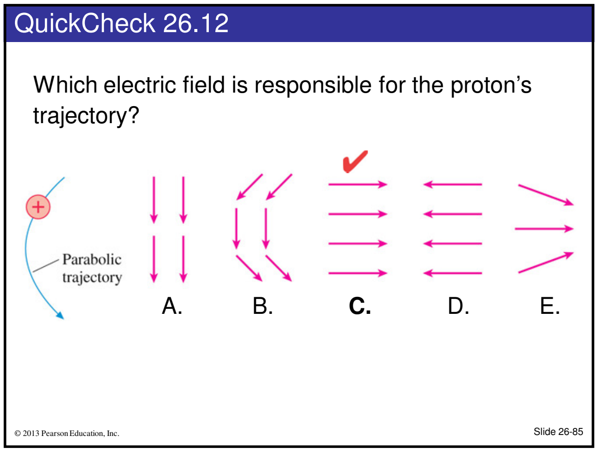
D.

E.



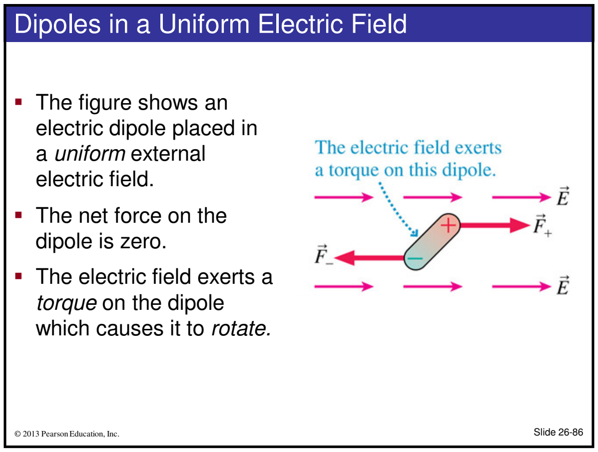
QuickCheck 26.12

Which electric field is responsible for the proton's trajectory?



QuickCheck 26.12

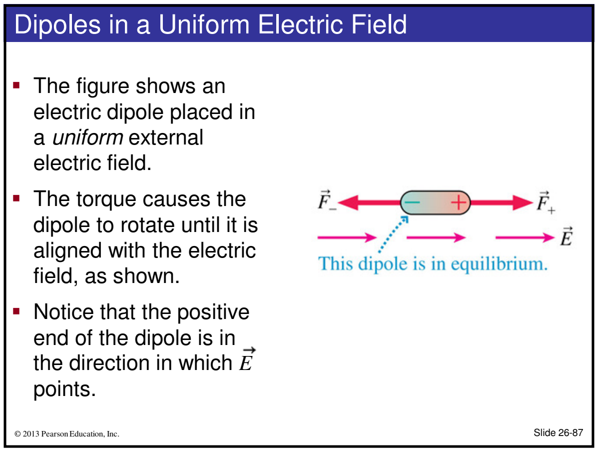
Which electric field is responsible for the proton's trajectory?



Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The net force on the dipole is zero.
- The electric field exerts a *torque* on the dipole which causes it to *rotate.*

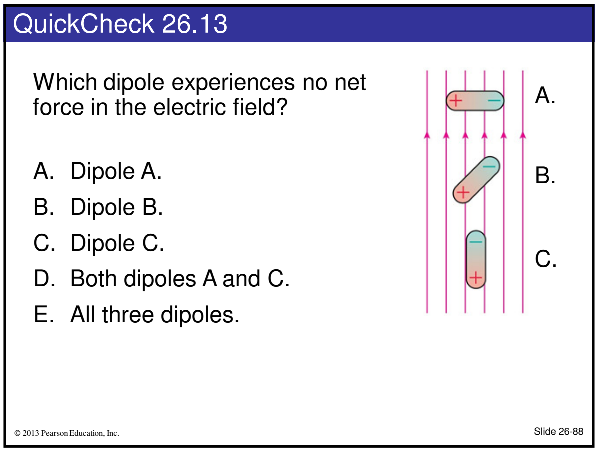
The electric field exerts a torque on this dipole.



Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The torque causes the dipole to rotate until it is aligned with the electric field, as shown.
- Notice that the positive end of the dipole is in the direction in which $\vec{E}$ points.

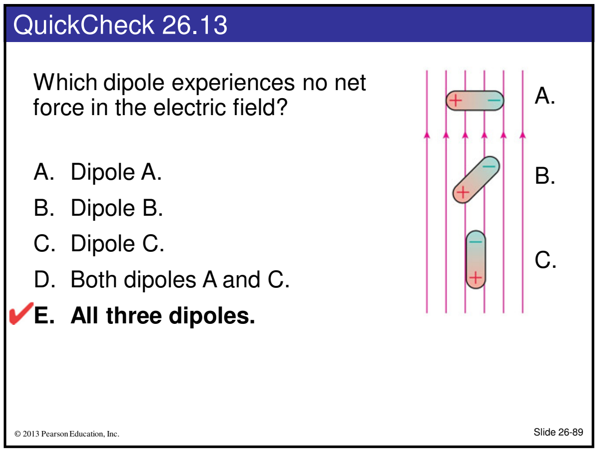
This dipole is in equilibrium.



QuickCheck 26.13

Which dipole experiences no net force in the electric field?

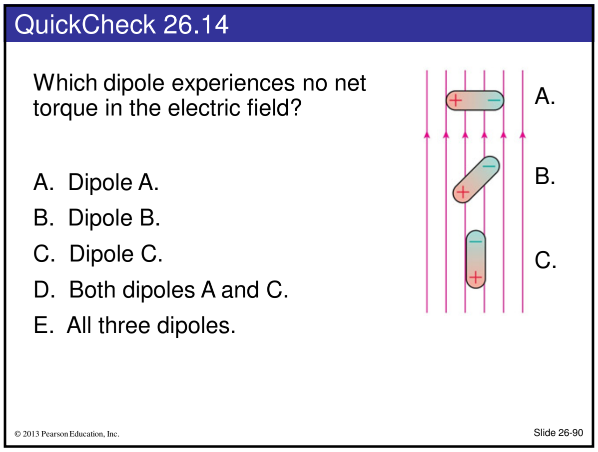
A. Dipole A.
B. Dipole B.
C. Dipole C.
D. Both dipoles A and C.
E. All three dipoles.



QuickCheck 26.13

Which dipole experiences no net force in the electric field?

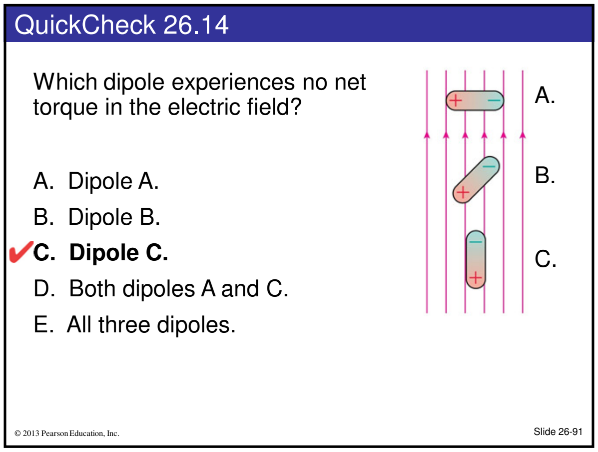
A. Dipole A.
B. Dipole B.
C. Dipole C.
D. Both dipoles A and C.
✓**E. All three dipoles.**



QuickCheck 26.14

Which dipole experiences no net torque in the electric field?

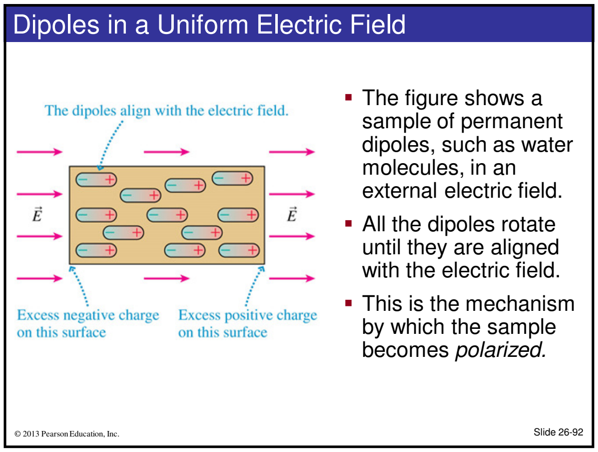
A. Dipole A.
B. Dipole B.
C. Dipole C.
D. Both dipoles A and C.
E. All three dipoles.



QuickCheck 26.14

Which dipole experiences no net torque in the electric field?

A. Dipole A.
B. Dipole B.
✓**C. Dipole C.**
D. Both dipoles A and C.
E. All three dipoles.



Dipoles in a Uniform Electric Field

The dipoles align with the electric field.

Excess negative charge on this surface

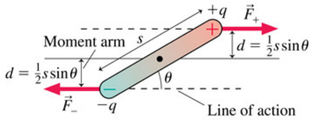
Excess positive charge on this surface

- The figure shows a sample of permanent dipoles, such as water molecules, in an external electric field.
- All the dipoles rotate until they are aligned with the electric field.
- This is the mechanism by which the sample becomes *polarized.*

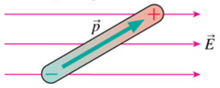
The Torque on a Dipole

The torque on a dipole placed in a uniform external electric field is

$$\tau = 2 \times dF_{+} = 2(\tfrac{1}{2}s\sin\theta)(qE) = pE\sin\theta$$



In terms of vectors, $\vec{\tau} = \vec{p} \times \vec{E}$.



Example 26.10 The Angular Acceleration of a Dipole Dumbbell

EXAMPLE 26.10 The angular acceleration of a dipole dumbbell

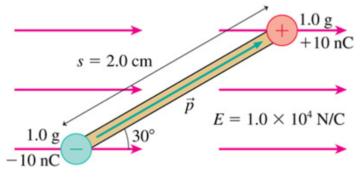
Two 1.0 g balls are connected by a 2.0-cm-long insulating rod of negligible mass. One ball has a charge of +10 nC, the other a charge of −10 nC. The rod is held in a 1.0×10^4 N/C uniform electric field at an angle of 30° with respect to the field, then released. What is its initial angular acceleration?

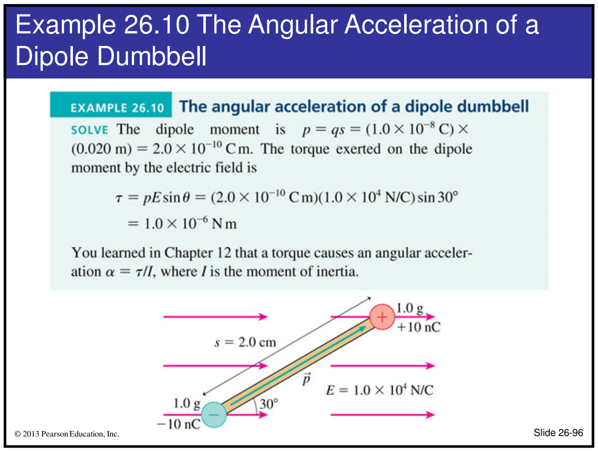
MODEL The two oppositely charged balls form an electric dipole. The electric field exerts a torque on the dipole, causing an angular acceleration.

Example 26.10 The Angular Acceleration of a Dipole Dumbbell

EXAMPLE 26.10 The angular acceleration of a dipole dumbbell

VISUALIZE The figure below shows the dipole in the electric field.





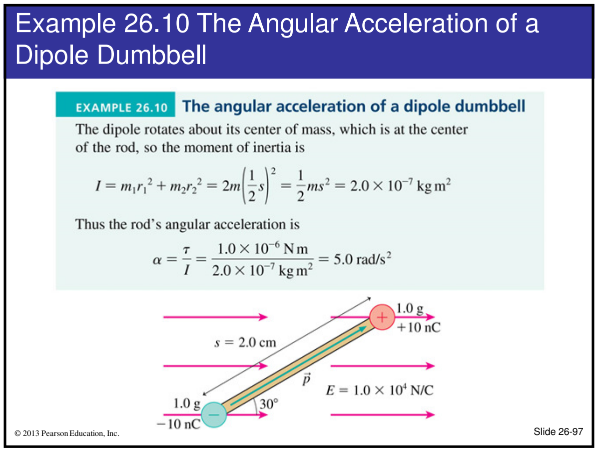
Example 26.10 The Angular Acceleration of a Dipole Dumbbell

EXAMPLE 26.10 **The angular acceleration of a dipole dumbbell**

SOLVE The dipole moment is $p = qs = (1.0 \times 10^{-8}\text{ C}) \times (0.020\text{ m}) = 2.0 \times 10^{-10}\text{ C m}$. The torque exerted on the dipole moment by the electric field is

$$\tau = pE\sin\theta = (2.0 \times 10^{-10}\text{ C m})(1.0 \times 10^{4}\text{ N/C})\sin 30^\circ$$
$$= 1.0 \times 10^{-6}\text{ N m}$$

You learned in Chapter 12 that a torque causes an angular acceleration $\alpha = \tau/I$, where I is the moment of inertia.



Example 26.10 The Angular Acceleration of a Dipole Dumbbell

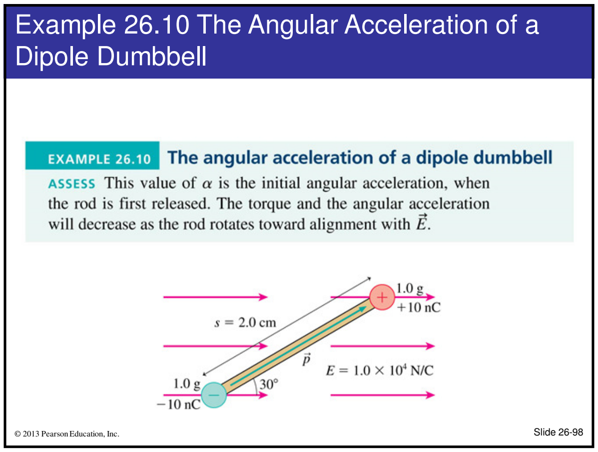
EXAMPLE 26.10 **The angular acceleration of a dipole dumbbell**

The dipole rotates about its center of mass, which is at the center of the rod, so the moment of inertia is

$$I = m_1 r_1{}^2 + m_2 r_2{}^2 = 2m\left(\frac{1}{2}s\right)^2 = \frac{1}{2}ms^2 = 2.0 \times 10^{-7}\text{ kg m}^2$$

Thus the rod's angular acceleration is

$$\alpha = \frac{\tau}{I} = \frac{1.0 \times 10^{-6}\text{ N m}}{2.0 \times 10^{-7}\text{ kg m}^2} = 5.0\text{ rad/s}^2$$

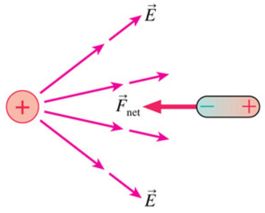


Example 26.10 The Angular Acceleration of a Dipole Dumbbell

EXAMPLE 26.10 **The angular acceleration of a dipole dumbbell**

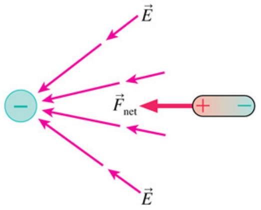
ASSESS This value of α is the initial angular acceleration, when the rod is first released. The torque and the angular acceleration will decrease as the rod rotates toward alignment with $\vec{E}$.

Dipoles in a Nonuniform Electric Field



- Suppose that a dipole is placed in a nonuniform electric field, such as the field of a positive point charge.
- The first response of the dipole is to rotate until it is aligned with the field.
- Once the dipole is aligned, the leftward attractive force on its negative end is slightly stronger than the rightward repulsive force on its positive end.
- This causes a net force to the *left*, toward the point charge.

Dipoles in a Nonuniform Electric Field



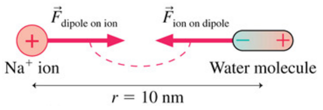
- A dipole near a negative point charge is also attracted toward the point charge.
- The net force on a dipole is toward the direction of the strongest field.
- Because field strength increases as you get closer to any finite-sized charged object, we can conclude that a **dipole will experience a net force toward any charged object.**

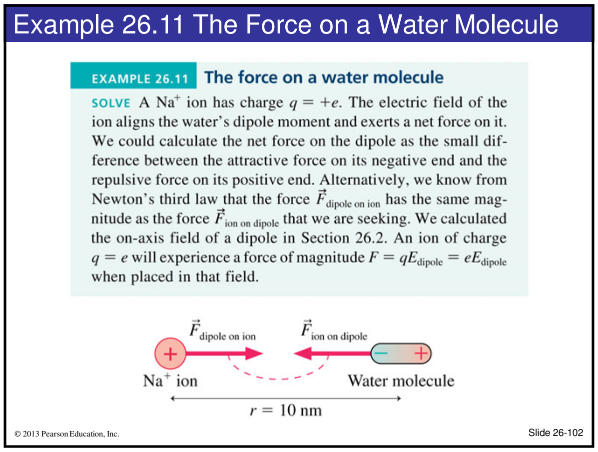
Example 26.11 The Force on a Water Molecule

EXAMPLE 26.11 **The force on a water molecule**

The water molecule H_2O has a permanent dipole moment of magnitude 6.2×10^{-30} C m. A water molecule is located 10 nm from a Na^+ ion in a saltwater solution. What force does the ion exert on the water molecule?

VISUALIZE The figure below shows the ion and the dipole. The forces are an action/reaction pair.

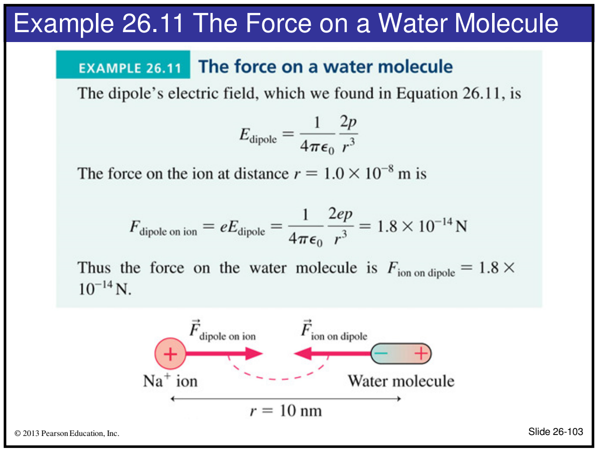




Example 26.11 The Force on a Water Molecule

EXAMPLE 26.11 **The force on a water molecule**

SOLVE A Na^+ ion has charge $q = +e$. The electric field of the ion aligns the water's dipole moment and exerts a net force on it. We could calculate the net force on the dipole as the small difference between the attractive force on its negative end and the repulsive force on its positive end. Alternatively, we know from Newton's third law that the force $\vec{F}_{\text{dipole on ion}}$ has the same magnitude as the force $\vec{F}_{\text{ion on dipole}}$ that we are seeking. We calculated the on-axis field of a dipole in Section 26.2. An ion of charge $q = e$ will experience a force of magnitude $F = qE_{\text{dipole}} = eE_{\text{dipole}}$ when placed in that field.



Example 26.11 The Force on a Water Molecule

EXAMPLE 26.11 **The force on a water molecule**

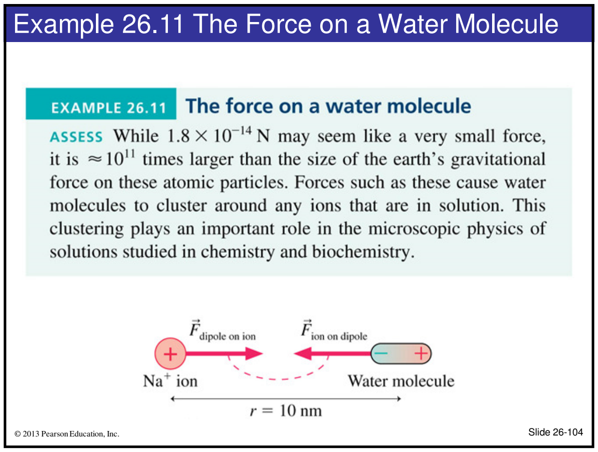
The dipole's electric field, which we found in Equation 26.11, is

$$E_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3}$$

The force on the ion at distance $r = 1.0 \times 10^{-8}$ m is

$$F_{\text{dipole on ion}} = eE_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2ep}{r^3} = 1.8 \times 10^{-14}\,\text{N}$$

Thus the force on the water molecule is $F_{\text{ion on dipole}} = 1.8 \times 10^{-14}$ N.



Example 26.11 The Force on a Water Molecule

EXAMPLE 26.11 **The force on a water molecule**

ASSESS While 1.8×10^{-14} N may seem like a very small force, it is $\approx 10^{11}$ times larger than the size of the earth's gravitational force on these atomic particles. Forces such as these cause water molecules to cluster around any ions that are in solution. This clustering plays an important role in the microscopic physics of solutions studied in chemistry and biochemistry.

Chapter 26 Summary Slides

General Principles

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$

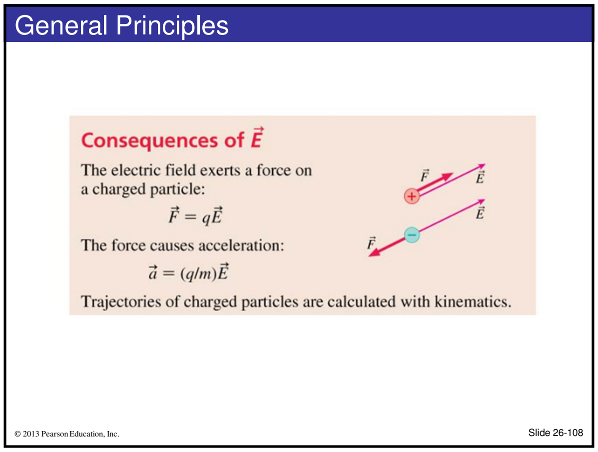
General Principles

Sources of $\vec{E}$

Continuous distribution of charge

- Divide the charge into segments ΔQ for which you already know the field.
- Find the field of each ΔQ.
- Find $\vec{E}$ by summing the fields of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.



General Principles

Consequences of $\vec{E}$

The electric field exerts a force on a charged particle:

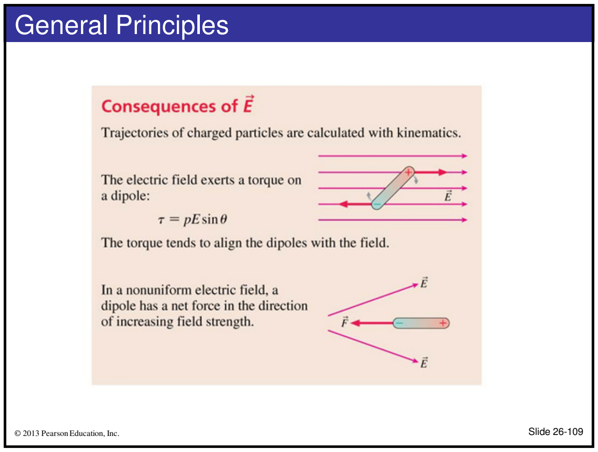
$$\vec{F} = q\vec{E}$$

The force causes acceleration:

$$\vec{a} = (q/m)\vec{E}$$

Trajectories of charged particles are calculated with kinematics.

Slide 26-108



General Principles

Consequences of $\vec{E}$

Trajectories of charged particles are calculated with kinematics.

The electric field exerts a torque on a dipole:

$$\tau = pE\sin\theta$$

The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.

Slide 26-109