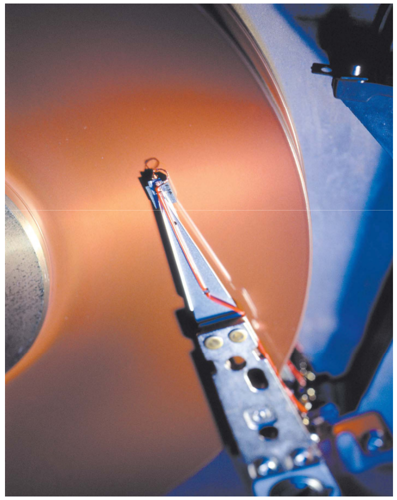
Chapter 33. The Magnetic Field

Digital information is stored on a hard disk as microscopic patches of magnetism. Just what is magnetism? How are magnetic fields created? What are their properties? These are the questions we will address.

Chapter Goal: To learn how to calculate and use the magnetic field.



Chapter 33. The Magnetic Field

Topics:

- Magnetism
- The Discovery of the Magnetic Field
- The Source of the Magnetic Field: Moving Charges
- The Magnetic Field of a Current
- Magnetic Dipoles
- Ampère's Law and Solenoids
- The Magnetic Force on a Moving Charge
- Magnetic Forces on Current-Carrying Wires
- Forces and Torques on Current Loops
- Magnetic Properties of Matter

Chapter 33. Reading Quizzes

What is the SI unit for the strength of the magnetic field?

A. Gauss
B. Henry
C. Tesla
D. Becquerel
E. Bohr magneton

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D. Becquerel
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The magnetic field of a point charge is given by

A. Biot-Savart's law.
B. Faraday's law.
C. Gauss's law.
D. Ampère's law.
E. Einstein's law.

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D. Ampère's law.
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The magnetic field of a straight, current-carrying wire is

A. parallel to the wire.
B. inside the wire.
C. perpendicular to the wire.
D. around the wire.
E. zero.

The magnetic field of a straight, current-carrying wire is

A. parallel to the wire.
B. inside the wire.
C. perpendicular to the wire.
✔ **D. around the wire.**
E. zero.

Chapter 33. Basic Content and Examples

FIGURE 33.1 The earth is a large magnet.

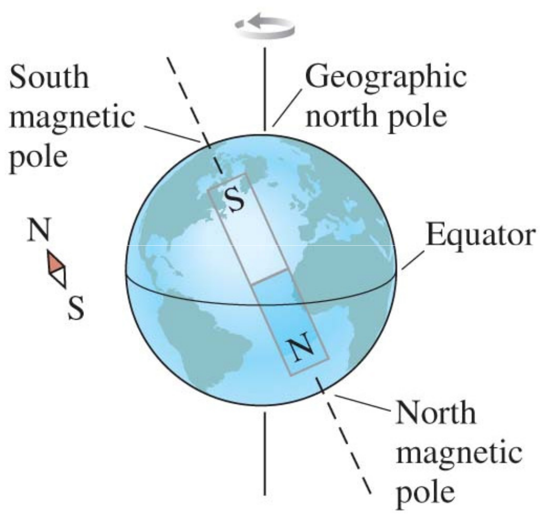


FIGURE 33.4 The magnetic field exerts forces on the poles of a compass, causing the needle to align with the field.

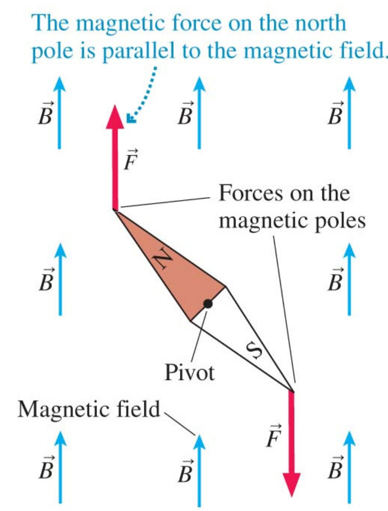
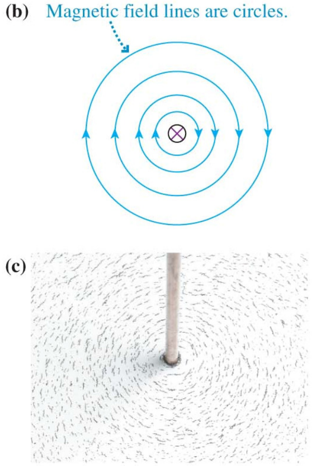


FIGURE 33.5 The magnetic field around a current-carrying wire.

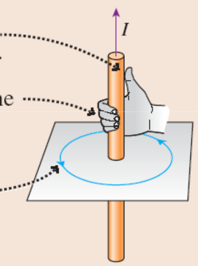


The magnetic field is revealed by the pattern of iron filings around the current-carrying wire.

Tactics: Right-hand rule for fields

TACTICS BOX 33.1 **Right-hand rule for fields**

1. Point your *right* thumb in the direction of the current.
2. Curl your fingers around the wire to indicate a circle.
3. Your fingers point in the direction of the magnetic field lines around the wire.



Exercises 6–8

The Source of the Magnetic Field: Moving Charges

The magnetic field of a charged particle q moving with velocity $\boldsymbol{v}$ is given by the **Biot-Savart law**:

$$\vec{B}_{\text{point charge}} = \left(\frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}, \text{direction given by the right-hand rule}\right)$$

where r is the distance from the charge and θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{r}$.

The Biot-Savart law can be written in terms of the cross product as

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad \text{(magnetic field of a point charge)}$$

EXAMPLE 33.1 The magnetic field of a proton

QUESTION:

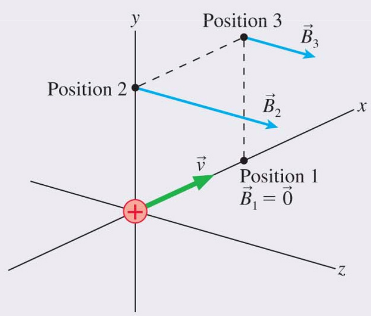
EXAMPLE 33.1 **The magnetic field of a proton**

A proton moves along the x-axis with velocity $v_x = 1.0 \times 10^7$ m/s. As it passes the origin, what is the magnetic field at the (x, y, z) positions (1 mm, 0 mm, 0 mm), (0 mm, 1 mm, 0 mm), and (1 mm, 1 mm, 0 mm)?

EXAMPLE 33.1 The magnetic field of a proton

VISUALIZE **FIGURE 33.8** shows the geometry. The first point is on the x-axis, directly in front of the proton, with $\theta_1 = 0°$. The second point is on the y-axis, with $\theta_2 = 90°$, and the third is in the xy-plane.

FIGURE 33.8 The magnetic field of Example 33.1.



EXAMPLE 33.1 The magnetic field of a proton

SOLVE Position 1, which is along the line of motion, has $\theta = 0°$. Thus $\vec{B}_1 = \vec{0}$. Position 2 (at 0 mm, 1 mm, 0 mm) is at distance $r_2 = 1\text{ mm} = 0.001\text{ m}$. Equation 33.1, the Biot-Savart law, gives us the magnetic field strength at this point as

$$
\begin{aligned}
B &= \frac{\mu_0}{4\pi}\frac{qv\sin\theta_2}{r_2^2} \\
&= \frac{4\pi \times 10^{-7}\,\text{T m/A}}{4\pi}\,\frac{(1.60 \times 10^{-19}\,\text{C})(1.0 \times 10^{7}\,\text{m/s})\sin 90°}{(0.0010\,\text{m})^2} \\
&= 1.60 \times 10^{-13}\,\text{T}
\end{aligned}
$$

EXAMPLE 33.1 The magnetic field of a proton

According to the right-hand rule, the field points in the positive z-direction. Thus

$$\vec{B}_2 = 1.60 \times 10^{-13}\,\hat{k}\,\text{T}$$

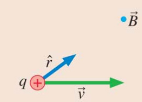
where $\hat{k}$ is the unit vector in the positive z-direction. The field at position 3, at (1 mm, 1 mm, 0 mm), also points in the z-direction, but it is weaker than at position 2 both because r is larger *and* because θ is smaller. From geometry we know $r_3 = \sqrt{2}\text{ mm} = 0.00141\text{ m}$ and $\theta_3 = 45°$. Another calculation using Equation 33.1 gives

$$\vec{B}_3 = 0.57 \times 10^{-13}\,\hat{k}\,\text{T}$$

General Principles

Magnetic Fields

The Biot-Savart law



- A **point charge,** $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v} \times \hat{r}}{r^2}$
- A **short current element,** $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta\vec{s} \times \hat{r}}{r^2}$

To find the magnetic field of a current

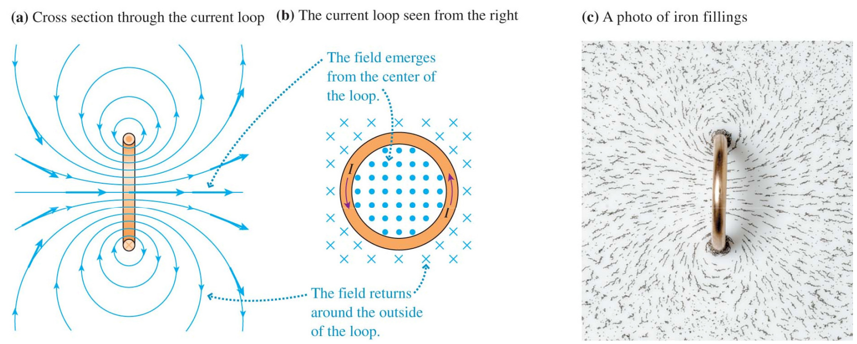
- Divide the wire into many short segments.
- Find the field of each segment Δs.
- Find $\vec{B}$ by summing the fields of all Δs, usually as an integral.

An alternative method for fields with a high degree of symmetry is Ampère's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

where I_{through} is the current through the area bounded by the integration path.

FIGURE 33.18 The magnetic field of a current loop.



Tactics: Finding the magnetic field direction of a current loop

TACTICS BOX 33.2 **Finding the magnetic field direction of a current loop**

Use either of the following methods to find the magnetic field direction:

1. Point your right thumb in the direction of the current at any point on the loop and let your fingers curl through the center of the loop. Your fingers are then pointing in the direction in which $\vec{B}$ leaves the loop.
2. Curl the fingers of your right hand around the loop in the direction of the current. Your thumb is then pointing in the direction in which $\vec{B}$ leaves the loop.

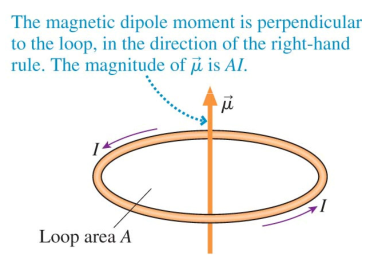
Exercises 18–20

Magnetic Dipoles

The **magnetic dipole moment** of a current loop enclosing an area A is defined as

$$\vec{\mu} = (AI, \text{from the south pole to the north pole})$$

FIGURE 33.20 The magnetic dipole moment of a circular current loop.



The SI units of the magnetic dipole moment are A m^2. The on-axis field of a magnetic dipole is

$$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3} \quad \text{(on the axis of a magnetic dipole)}$$

EXAMPLE 33.7 The field of a magnetic dipole

QUESTIONS:

EXAMPLE 33.7 The field of a magnetic dipole

a. The on-axis magnetic field strength 10 cm from a magnetic dipole is 1.0×10^{-5} T. What is the size of the magnetic dipole moment?

b. If the magnetic dipole is created by a 4.0-mm-diameter current loop, what is the current?

EXAMPLE 33.7 The field of a magnetic dipole

SOLVE a. If $z \gg R$, the size of the magnetic dipole moment is

$$\mu = \frac{4\pi}{\mu_0}\frac{z^3 B}{2}$$

$$= \frac{4\pi}{4\pi \times 10^{-7}\,\mathrm{T\,m/A}} \frac{(0.10\,\mathrm{m})^3(1.0 \times 10^{-5}\,\mathrm{T})}{2} = 0.050\,\mathrm{A\,m^2}$$

b. The magnetic dipole moment of a current loop is $\mu = AI$, so the necessary current is

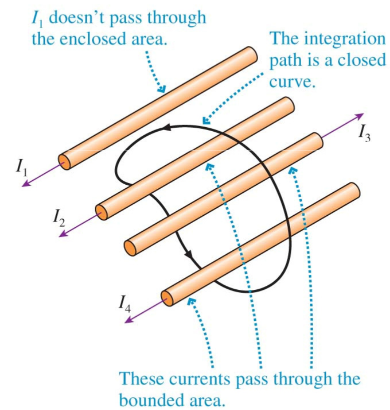
$$I = \frac{\mu}{\pi R^2} = \frac{0.050\,\mathrm{A\,m^2}}{\pi(0.0020\,\mathrm{m})^2} = 4000\,\mathrm{A}$$

Ampère's law

Whenever total current I_{through} passes through an area bounded by a *closed curve,* the line integral of the magnetic field around the curve is given by Ampère's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

FIGURE 33.24 Using Ampère's law.



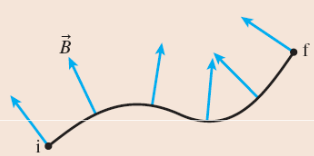
Tactics: Evaluating line integrals

TACTICS BOX 33.3 **Evaluating line integrals**

MP

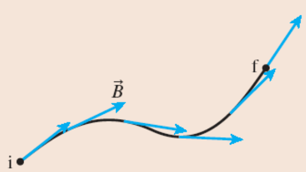
❶ If $\vec{B}$ is everywhere perpendicular to a line, the line integral of $\vec{B}$ is

$$\int_i^f \vec{B} \cdot d\vec{s} = 0$$



❷ If $\vec{B}$ is everywhere tangent to a line of length l *and* has the same magnitude B at every point, then

$$\int_i^f \vec{B} \cdot d\vec{s} = Bl$$



Exercises 23–24

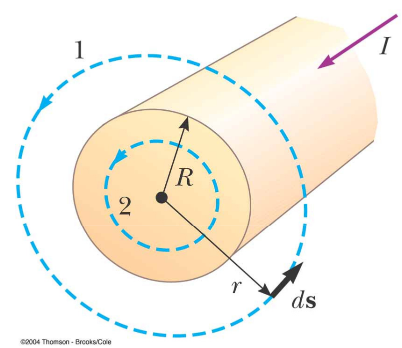
Field due to a long Straight Wire

- The magnitude of magnetic field depends only on distance ***r*** from the center of a wire.

- Outside of the wire, ***r > R***

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_o I$$

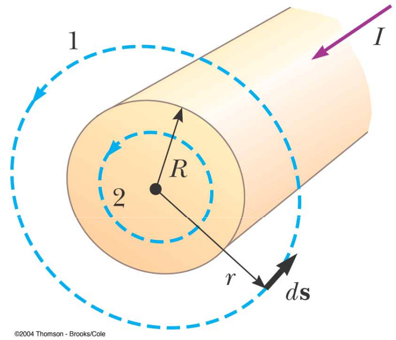
$$B = \frac{\mu_o I}{2\pi r}$$



$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o I$$

Field due to a long Straight Wire

- The magnitude of magnetic field depends only on distance ***r*** from the center of a wire.



- Inside the wire, we need I', the current inside the amperian circle

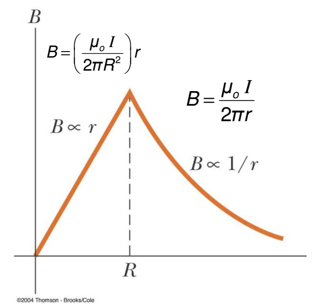
$$\oint \mathbf{B}\cdot d\mathbf{s} = B(2\pi r) = \mu_o I' \rightarrow I' = \frac{r^2}{R^2} I$$

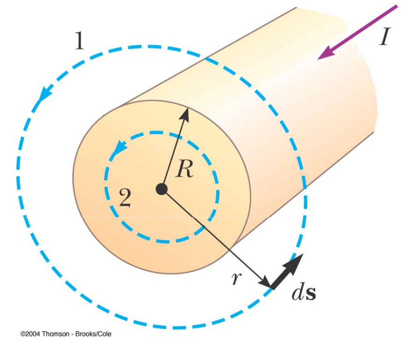
$$B = \left(\frac{\mu_o I}{2\pi R^2}\right) r$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_o I$$

Field due to a long Straight Wire

- The field is proportional to ***r*** inside the wire
- The field varies as **1/*r*** outside the wire
- Both equations are equal at ***r* = *R***

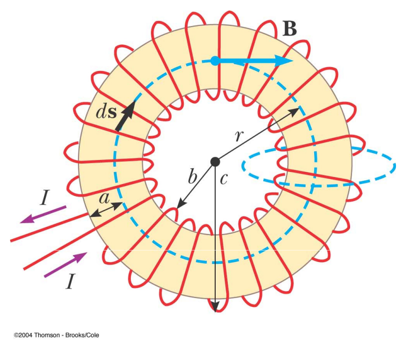




$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_o I$$

Magnetic Field of a Toroid

- Find the field at a point at distance ***r*** from the center of the toroid
- The toroid has ***N*** turns of wire

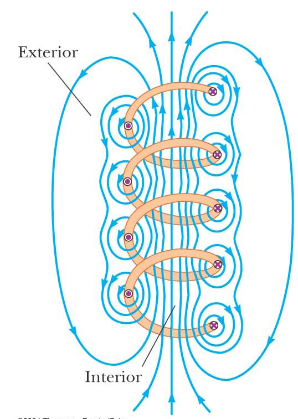


$$\oint \mathbf{B}\cdot d\mathbf{s} = B(2\pi r) = \mu_o N I$$

$$B = \frac{\mu_o N I}{2\pi r}$$

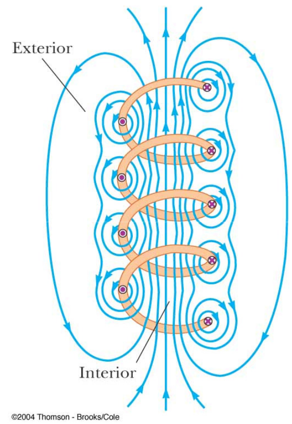
Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix
- A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire



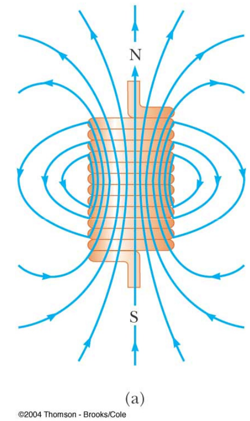
Magnetic Field of a Solenoid

- The field lines in the interior are
  - approximately parallel to each other
  - uniformly distributed
  - close together
- This indicates the field is strong and almost uniform



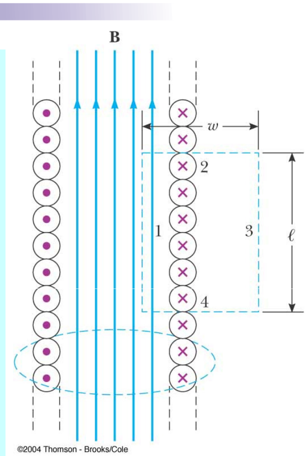
Magnetic Field of a Solenoid

- The field distribution is similar to that of a bar magnet
- As the length of the solenoid increases
  - the interior field becomes more uniform
  - the exterior field becomes weaker



Magnetic Field of a Solenoid

- An *ideal solenoid* is approached when:
  - the turns are closely spaced
  - the length is much greater than the radius of the turns
- Consider a rectangle with side ℓ parallel to the interior field and side w perpendicular to the field
- The side of length ℓ inside the solenoid contributes to the field
  - This is path 1 in the diagram



Magnetic Field of a Solenoid

- Applying Ampere's Law gives

$$\oint \mathbf{B}\cdot d\mathbf{s} = \int_{\text{path1}} \mathbf{B}\cdot d\mathbf{s} = B\int_{\text{path1}} ds = B\ell$$

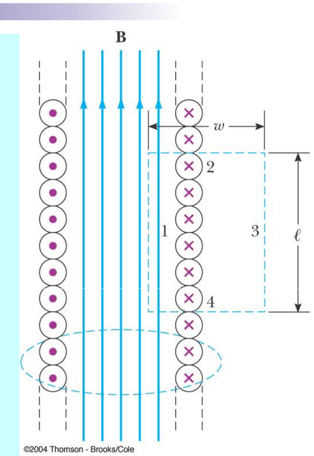
- The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$\oint \mathbf{B}\cdot d\mathbf{s} = B\ell = \mu_o NI$$

- Solving Ampere's law for the magnetic field is

$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

  - $n = N/\ell$ is the number of turns per unit length
- This is valid only at points near the center of a very long solenoid



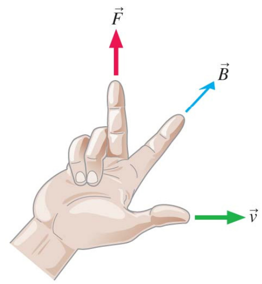
The Magnetic Force on a Moving Charge

The magnetic force on a charge q as it moves through a magnetic field $\boldsymbol{B}$ with velocity $\boldsymbol{v}$ is

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

where α is the angle between $\boldsymbol{v}$ and $\boldsymbol{B}$.

FIGURE 33.34 The right-hand rule for magnetic forces.

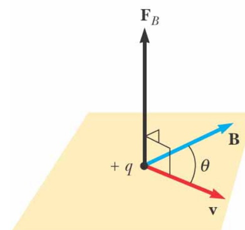


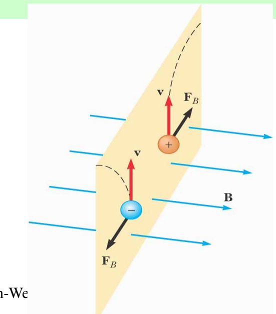
The Magnetic Force on a Moving Charge

- The properties can be summarized in a vector equation:

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$$

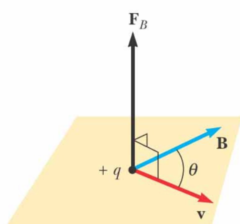
  - $\mathbf{F}_B$ is the magnetic force
  - q is the charge
  - $\mathbf{v}$ is the velocity of the moving charge
  - $\mathbf{B}$ is the magnetic field

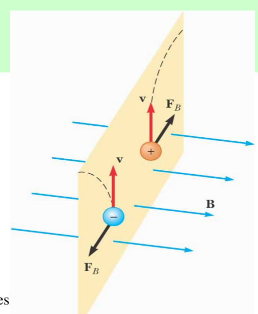




The Magnetic Force on a Moving Charge

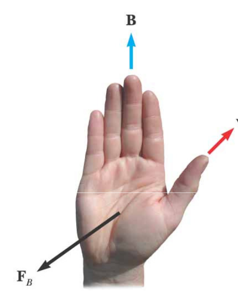
- The magnitude of the magnetic force on a charged particle is $F_B = |q|\, vB \sin\theta$
  - ➢ θ is the smallest angle between **v** and **B**
  - F_B is zero when **v** and **B** are parallel or antiparallel
    - ➢ $\theta = 0$ or 180°
  - F_B is a maximum when **v** and **B** are perpendicular
    - ➢ $\theta = 90^{\circ}$





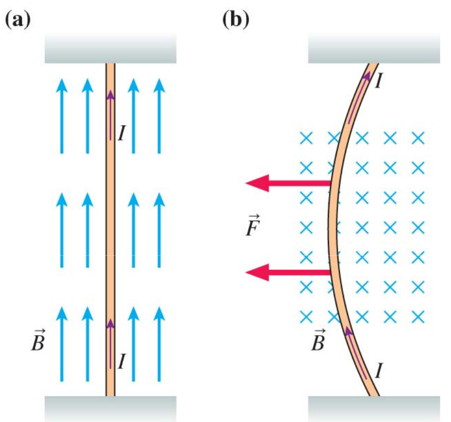
Direction of Magnetic Force

- Thumb is in the direction of **v**
- Fingers are in the direction of **B**
- Palm is in the direction of $\mathbf{F}_B$
  - On a positive particle
  - You can think of this as your hand pushing the particle



(b)

FIGURE 33.43 Magnetic force on a current-carrying wire.



There's no force on a current parallel to a magnetic field.

A current perpendicular to the field experiences a force in the direction of the right-hand rule.

Magnetic Forces on Current-Carrying Wires

Consider a segment of wire of length l carrying current I in the direction of the vector $\boldsymbol{l}$. The wire exists in a constant magnetic field $\boldsymbol{B}$. The magnetic force on the wire is

$$F_{\text{wire}} = Il \times B = (IlB\sin\alpha, \text{ direction of right-hand rule})$$

where α is the angle between the direction of the current and the magnetic field.

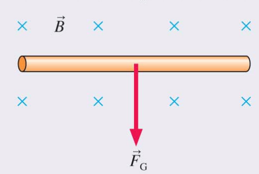
EXAMPLE 33.13 Magnetic Levitation

QUESTION:

EXAMPLE 33.13 Magnetic Levitation

The 0.10 T uniform magnetic field of **FIGURE 33.45** is horizontal, parallel to the floor. A straight segment of 1.0-mm-diameter copper wire, also parallel to the floor, is perpendicular to the magnetic field. What current through the wire, and in which direction, will allow the wire to "float" in the magnetic field?

FIGURE 33.45 Magnetic levitation.



EXAMPLE 33.13 Magnetic Levitation

MODEL The wire will float in the magnetic field if the magnetic force on the wire points upward and has magnitude mg, allowing it to balance the downward gravitational force.

EXAMPLE 33.13 Magnetic Levitation

SOLVE We can use the right-hand rule to determine which current direction experiences an upward force. With $\vec{B}$ pointing away from us, the direction of the current needs to be from left to right. The forces will balance when

$$F = IlB = mg = \rho(\pi r^2 l)g$$

where $\rho = 8920 \text{ kg/m}^3$ is the density of copper. The length of the wire cancels, leading to

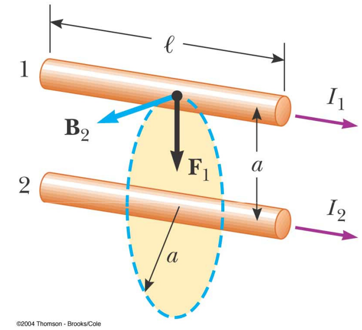
$$I = \frac{\rho \pi r^2 g}{B} = \frac{(8920 \text{ kg/m}^3)\pi(0.00050 \text{ m})^2(9.80 \text{ m/s}^2)}{0.10 \text{ T}}$$
$$= 0.69 \text{ A}$$

A 0.69 A current from left to right will levitate the wire in the magnetic field.

Magnetic Force between two parallel conductors

- Two parallel wires each carry a steady current
- The field $\mathbf{B}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$
- Substituting the equation for B_2 gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a}\ell$$

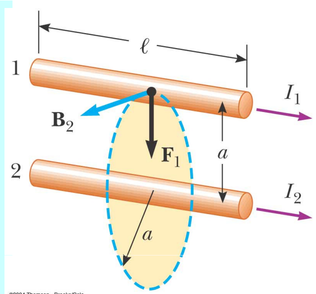


Magnetic Force between two parallel conductors

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a}\ell$$

- Parallel conductors carrying currents in the same direction attract each other
- Parallel conductors carrying current in opposite directions repel each other
- The result is often expressed as the magnetic force *between* the two wires, F_B
- This can also be given as the *force per unit length*:

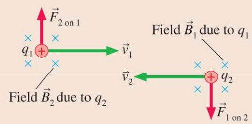
$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$



Chapter 33. Summary Slides

General Principles

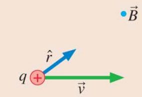
At its most fundamental level, magnetism is an interaction between moving charges. The magnetic field of one moving charge exerts a force on another moving charge.



General Principles

Magnetic Fields

The Biot-Savart law



- A **point charge,** $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$
- A **short current element,** $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta\vec{s}\times\hat{r}}{r^2}$

To find the magnetic field of a current

- Divide the wire into many short segments.
- Find the field of each segment Δs.
- Find $\vec{B}$ by summing the fields of all Δs, usually as an integral.

An alternative method for fields with a high degree of symmetry is Ampère's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}}$$

where I_{through} is the current through the area bounded by the integration path.

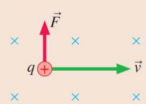
General Principles

Magnetic Forces

The magnetic force on a moving charge is

$$\vec{F} = q\vec{v}\times\vec{B}$$

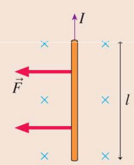
The force is perpendicular to $\vec{v}$ and $\vec{B}$.



The magnetic force on a current-carrying wire is

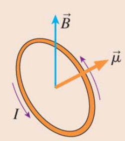
$$\vec{F} = I\vec{l}\times\vec{B}$$

$\vec{F} = \vec{0}$ for a charge or current moving parallel to $\vec{B}$.



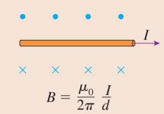
The magnetic torque on a magnetic dipole is

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

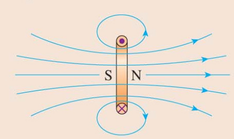


Applications

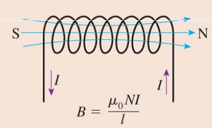
Wire



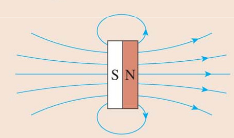
Loop



Solenoid



Flat magnet



Right-hand rule

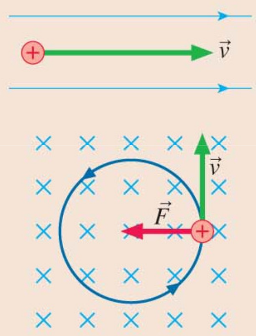
Point your right thumb in the direction of I. Your fingers curl in the direction of $\vec{B}$. For a dipole, $\vec{B}$ emerges from the side that is the north pole.

Applications

Charged-particle motion

No force if $\vec{v}$ is parallel to $\vec{B}$.

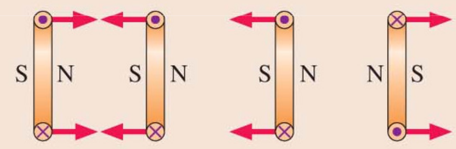
Circular motion at the cyclotron frequency $f_{cyc} = qB/2\pi m$ if $\vec{v}$ is perpendicular to $\vec{B}$.

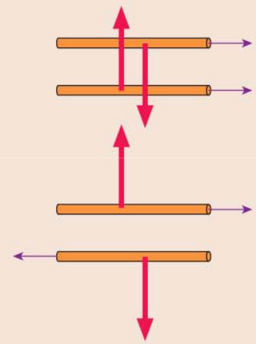


Applications

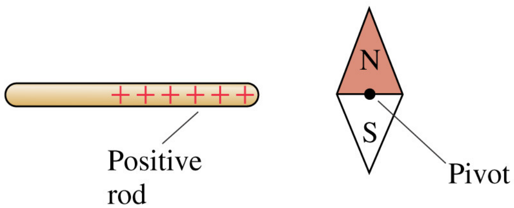
Parallel wires and current loops

Parallel currents attract.
Opposite currents repel.



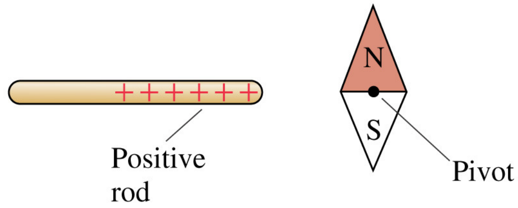


Chapter 33. Questions



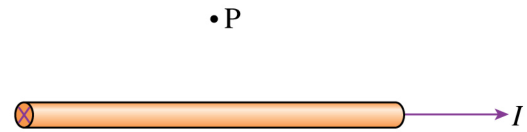
Does the compass needle rotate clockwise (cw), counterclockwise (ccw) or not at all?

A. Clockwise
B. Counterclockwise
C. Not at all



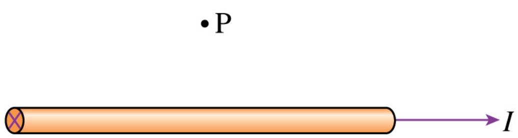
Does the compass needle rotate clockwise (cw), counterclockwise (ccw) or not at all?

A. Clockwise
B. Counterclockwise
C. Not at all



The magnetic field at the position P points

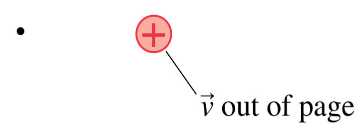
A. Into the page.
B. Up.
C. Down.
D. Out of the page.



The magnetic field at the position P points

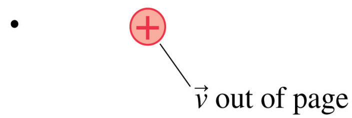
A. Into the page.
B. Up.
C. Down.
D. Out of the page.

The positive charge is moving straight out of the page. What is the direction of the magnetic field at the position of the dot?



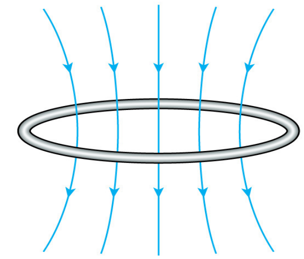
A. Left
B. Right
C. Down
D. Up

The positive charge is moving straight out of the page. What is the direction of the magnetic field at the position of the dot?



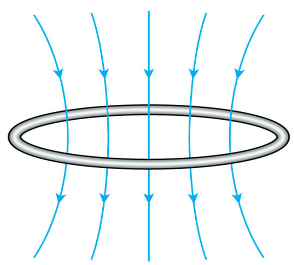
A. Left
B. Right
C. Down
D. Up

What is the current direction in this loop? And which side of the loop is the north pole?

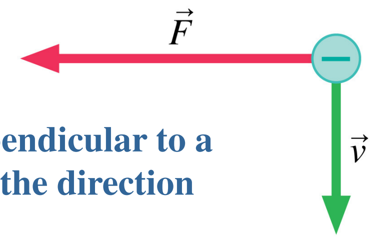


A. Current counterclockwise, north pole on bottom
B. Current clockwise; north pole on bottom
C. Current counterclockwise, north pole on top
D. Current clockwise; north pole on top

What is the current direction in this loop? And which side of the loop is the north pole?

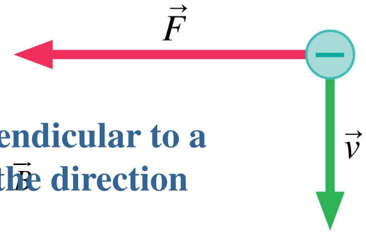


A. Current counterclockwise, north pole on bottom
B. Current clockwise; north pole on bottom
C. Current counterclockwise, north pole on top
D. Current clockwise; north pole on top



An electron moves perpendicular to a magnetic field. What is the direction of $\vec{B}$?

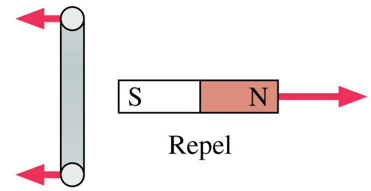
A. Left
B. Into the page
C. Out of the page
D. Up
E. Down



An electron moves perpendicular to a magnetic field. What is the direction of $\vec{B}$?

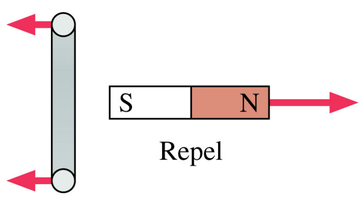
A. Left
B. Into the page
C. Out of the page
D. Up
E. Down

What is the current direction in the loop?



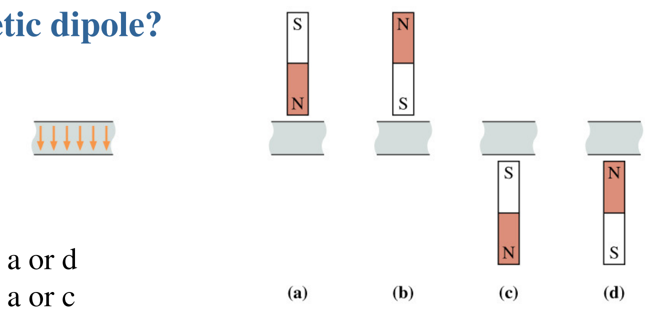
A. Out of the page at the top of the loop, into the page at the bottom.
B. Out of the page at the bottom of the loop, into the page at the top.

What is the current direction in the loop?



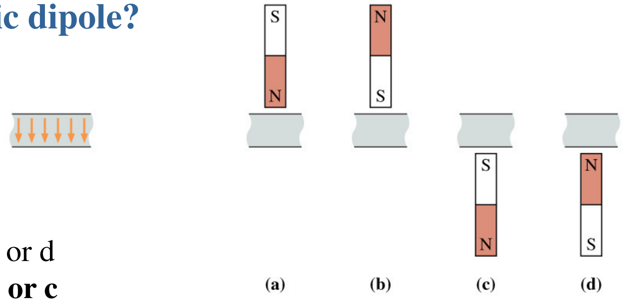
A. Out of the page at the top of the loop, into the page at the bottom.
B. Out of the page at the bottom of the loop, into the page at the top.

Which magnet or magnets produced this induced magnetic dipole?



A. a or d
B. a or c
C. b or d
D. b or c
E. any of a, b, c or d

Which magnet or magnets produced this induced magnetic dipole?



A. a or d
B. a or c
C. b or d
D. b or c
E. any of a, b, c or d