Chapter 29. The Electric Potential

At any time, millions of light bulbs are transforming electric energy into light and thermal energy. Just as electric fields allowed us to understand electric forces, *Electric Potential* allows us to understand electric energy.

Chapter Goal: To calculate and use the electric potential and electric potential energy.

Chapter 29. The Electric Potential

Topics:

- Electric Potential Energy
- The Potential Energy of Point Charges
- The Potential Energy of a Dipole
- The Electric Potential
- The Electric Potential Inside a Parallel-Plate Capacitor
- The Electric Potential of a Point Charge
- The Electric Potential of Many Charges

Chapter 29. Reading Quizzes

What are the units of *potential difference*?

A. Amperes
B. Potentiometers
C. Farads
D. Volts
E. Henrys

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✔ **D. Volts**
E. Henrys

New units of the electric field were introduced in this chapter. They are:

A. V/C.
B. N/C.
C. V/m.
D. J/m^2.
E. Ω/m.

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✔ **C. V/m.**
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E. Ω/m.

The electric potential inside a capacitor

A. is constant.
B. increases linearly from the negative to the positive plate.
C. decreases linearly from the negative to the positive plate.
D. decreases inversely with distance from the negative plate.
E. decreases inversely with the square of the distance from the negative plate.

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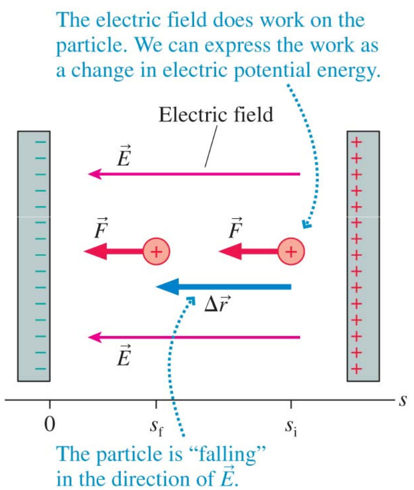
Chapter 29. Basic Content and Examples

FIGURE 29.4 The electric field does work on the charged particle.



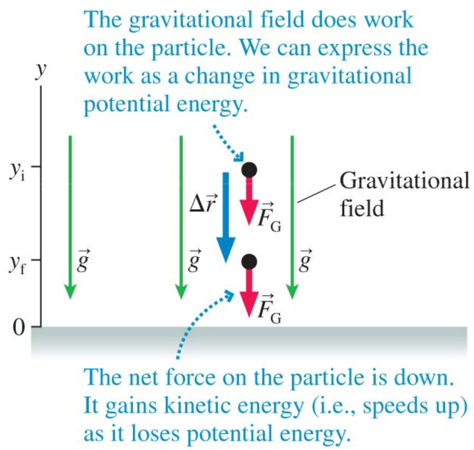
Electric Potential Energy

The **electric potential energy** of charge q in a uniform electric field is

$$U_{elec} = U_0 + qEs$$

where s is measured from the negative plate and U_0 is the potential energy at the negative plate ($s = 0$). It will often be convenient to choose $U_0 = 0$, but the choice has no physical consequences because it doesn't affect ΔU_{elec}, the *change* in the electric potential energy. Only the *change* is significant.

FIGURE 29.3 Potential energy is transformed into kinetic energy as a particle moves in a gravitational field.



The Potential Energy of Point Charges

Consider two point charges, q_1 and q_2, separated by a distance r. The electric potential energy is

$$U_{\text{elec}} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

This is explicitly the energy of *the system*, not the energy of just q_1 or q_2.
Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

FIGURE 29.9 The potential-energy diagrams for two like charges and two opposite charges.

(a) Like charges

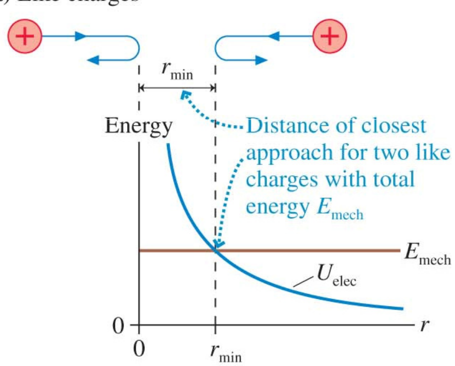
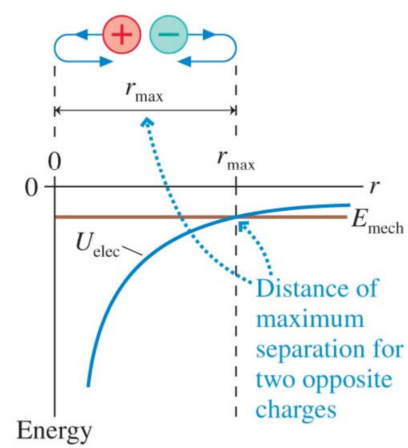


FIGURE 29.9 The potential-energy diagrams for two like charges and two opposite charges.

(b) Opposite charges



EXAMPLE 29.2 Approaching a charged sphere

QUESTION:

EXAMPLE 29.2 Approaching a charged sphere
A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

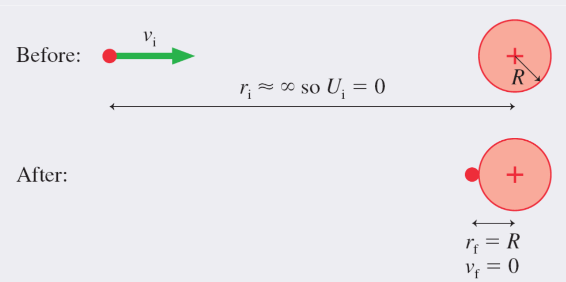
EXAMPLE 29.2 Approaching a charged sphere

MODEL Energy is conserved. The glass sphere can be treated as a charged particle, so the potential energy is that of two point charges. The proton starts "far away," which we interpret as sufficiently far to make $U_i \approx 0$.

EXAMPLE 29.2 Approaching a charged sphere

VISUALIZE FIGURE 29.12 shows the before-and-after pictorial representation. To "just reach" the glass sphere means that the proton comes to rest, $v_f = 0$, as it reaches $r_f = 0.50$ mm, the *radius* of the sphere.

FIGURE 29.12 A proton approaching a glass sphere.



EXAMPLE 29.2 Approaching a charged sphere

SOLVE Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + \frac{Kq_p q_{sphere}}{r_f} = \frac{1}{2}mv_i^2 + 0$$

The proton charge is $q_p = e$. With this, we can solve for the proton's initial speed:

$$v_i = \sqrt{\frac{2Keq_{sphere}}{mr_f}} = 1.86 \times 10^7 \text{ m/s}$$

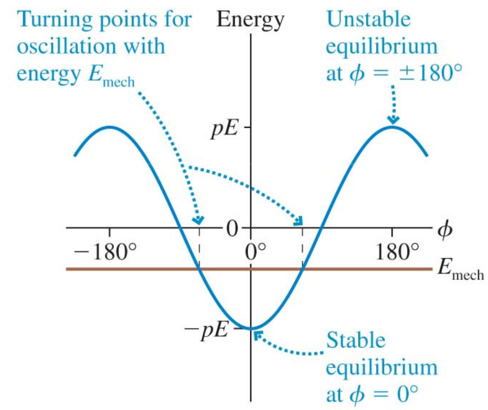
The Potential Energy of a Dipole

The potential energy of an electric dipole p in a uniform electric field E is

$$U_{\text{dipole}} = -pE\cos\phi = -\vec{p}\cdot E$$

The potential energy is minimum at $\phi = 0°$ where the dipole is aligned with the electric field.

FIGURE 29.16 The energy of a dipole in an electric field.



EXAMPLE 29.5 Rotating a molecule

QUESTION:

EXAMPLE 29.5 Rotating a molecule

The water molecule is a permanent electric dipole with dipole moment 6.2×10^{-30} Cm. A water molecule is aligned in an electric field with field strength 1.0×10^{7} N/C. How much energy is needed to rotate the molecule 90°?

EXAMPLE 29.5 Rotating a molecule

MODEL The molecule is at the point of minimum energy. It won't spontaneously rotate 90°. However, an external force that supplies energy, such as a collision with another molecule, can cause the water molecule to rotate.

EXAMPLE 29.5 Rotating a molecule

SOLVE The molecule starts at $\phi_i = 0°$ and ends at $\phi_f = 90°$. The increase in potential energy is

$$\Delta U_{dipole} = U_f - U_i = -pE\cos 90° - (-pE\cos 0°)$$
$$= pE = 6.2 \times 10^{-23}\ \text{J}$$

This is the energy needed to rotate the molecule 90°.

EXAMPLE 29.5 Rotating a molecule

ASSESS ΔU_{dipole} is significantly less than $k_B T$ at room temperature. Thus collisions with other molecules can easily supply the energy to rotate the water molecules and keep them from staying aligned with the electric field.

The Electric Potential

We define the electric potential V (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

Charge q is used as a probe to determine the electric potential, but the value of V is *independent of q*. **The electric potential, like the electric field, is a property of the source charges.**

The unit of electric potential is the joule per coulomb, which is called the volt V:

$$1\ \text{volt} = 1\ \text{V} \equiv 1\ \text{J/C}$$

The Electric Potential Inside a Parallel-Plate Capacitor

The electric potential inside a parallel-plate capacitor is

$$V = Es \qquad \text{(electric potential inside a parallel-plate capacitor)}$$

where ***s* is the distance from the *negative* electrode.**

The electric potential, like the electric field, exists at *all points* inside the capacitor.

The electric potential is created by the source charges on the capacitor plates and exists whether or not charge q is inside the capacitor.

EXAMPLE 29.7 A proton in a capacitor

QUESTIONS:

EXAMPLE 29.7 A proton in a capacitor

A parallel-plate capacitor is constructed of two 2.0-cm-diameter disks spaced 2.0 mm apart. It is charged to a potential difference of 500 V.

a. What is the electric field strength inside?
b. How much charge is on each plate?
c. A proton is shot through a small hole in the negative plate with a speed of 2.0×10^5 m/s. Does it reach the other side? If not, where is the turning point?

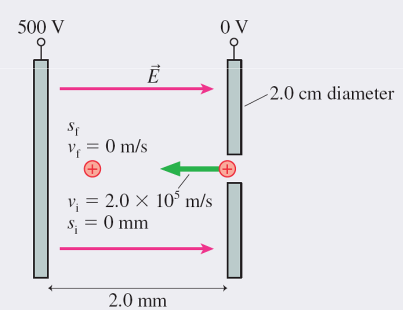
EXAMPLE 29.7 A proton in a capacitor

MODEL Energy is conserved. The proton's potential energy inside the capacitor can be found from the capacitor's electric potential.

EXAMPLE 29.7 A proton in a capacitor

VISUALIZE **FIGURE 29.24** is a before-and-after pictorial representation of the proton in the capacitor. Notice the *terminal symbols* where the potential is applied to the capacitor plates.

FIGURE 29.24 A proton moving in a capacitor.



EXAMPLE 29.7 A proton in a capacitor

SOLVE a. The electric field strength inside the capacitor is

$$E = \frac{\Delta V_C}{d} = \frac{500 \text{ V}}{0.0020 \text{ m}} = 2.5 \times 10^5 \text{ V/m}$$

b. Because $E = \eta/\epsilon_0$ for a parallel-plate capacitor, with $\eta = Q/A = Q/\pi R^2$, we find

$$Q = \pi R^2 \epsilon_0 E = 7.0 \times 10^{-10} \text{ C} = 0.70 \text{ nC}$$

EXAMPLE 29.7 A proton in a capacitor

c. The proton has charge $q = e$, and its potential energy at a point where the capacitor's potential is V is $U = eV$. It will gain potential energy $\Delta U = e\Delta V_C$ if it moves all the way across the capacitor. The increase in potential energy comes at the expense of kinetic energy, so the proton has sufficient kinetic energy to make it all the way across only if

$$K_i \geq e\Delta V_C$$

EXAMPLE 29.7 A proton in a capacitor

We can calculate that $K_i = 3.3 \times 10^{-17}$ J and that $e\Delta V_C = 8.0 \times 10^{-17}$ J. The proton does *not* have sufficient kinetic energy to be able to gain 8.0×10^{-17} J of potential energy, so it will not make it across. Instead, the proton will reach a turning point and reverse direction.

The proton starts at the negative plate, where $s_i = 0$ mm. Let the turning point be at s_f. The potential inside the capacitor is given by $V = (s/d)\Delta V_C$ with $d = 0.0020$ m and $\Delta V_C = 500$ V.

EXAMPLE 29.7 A proton in a capacitor

Conservation of energy requires $K_f + eV_f = K_i + eV_i$. This is

$$0 + e\frac{s}{d}\Delta V_C = \frac{1}{2}mv_i^2 + 0$$

where we used $V_i = 0$ V at the negative plate. The solution for the turning point is

$$s_f = \frac{mdv_i^2}{2e\Delta V_C} = 0.84 \text{ mm}$$

The proton travels less than halfway across before being turned back.

EXAMPLE 29.7 A proton in a capacitor

ASSESS We were able to use the electric potential inside the capacitor to determine the proton's potential energy. Notice that we used V/m as the electric field units.

The Electric Potential of a Point Charge

Let q be the source charge, and let a second charge q', a distance r away, probe the electric potential of q. The potential energy of the two point charges is

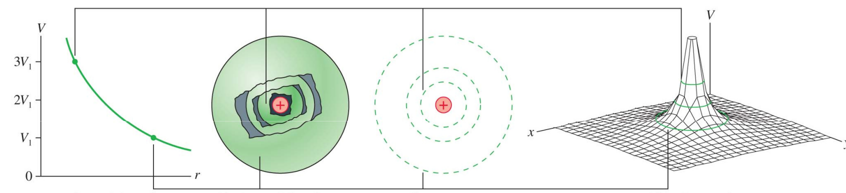
$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r}$$

By definition, the electric potential of charge q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(electric potential of a point charge)}$$

The potential extends through all of space, showing the influence of charge q, but it weakens with distance as $1/r$. This expression for V assumes that we have chosen $V = 0$ to be at $r = \infty$.

FIGURE 29.27 Four graphical representations of the electric potential of a point charge.



EXAMPLE 29.8 Calculating the potential of a point charge

QUESTIONS:

EXAMPLE 29.8 Calculating the potential of a point charge

What is the electric potential 1.0 cm from a +1.0 nC charge? What is the potential difference between a point 1.0 cm away and a second point 3.0 cm away?

EXAMPLE 29.8 Calculating the potential of a point charge

SOLVE The potential at $r = 1.0$ cm is

$$V_{1\,\text{cm}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = (9.0 \times 10^9\ \text{N m}^2/\text{C}^2)\frac{1.0 \times 10^{-9}\ \text{C}}{0.010\ \text{m}}$$
$$= 900\ \text{V}$$

We can similarly calculate $V_{3\,\text{cm}} = 300$ V. Thus the potential difference between these two points is $\Delta V = V_{1\,\text{cm}} - V_{3\,\text{cm}} = 600$ V.

EXAMPLE 29.8 Calculating the potential of a point charge

ASSESS 1 nC is typical of the electrostatic charge produced by rubbing, and you can see that such a charge creates a fairly large potential nearby. Why are we not shocked and injured when working with the “high voltages” of such charges? The sensation of being shocked is a result of current, not potential. Some high-potential sources simply do not have the ability to generate much current. We will look at this issue in Chapter 32.

The Electric Potential of a Charged Sphere

In practice, you are more likely to work with a charged sphere, of radius R and total charge Q, than with a point charge. Outside a uniformly charged sphere, the electric potential is identical to that of a point charge Q at the center. That is,

$$V = \frac{1}{4\pi\epsilon}\frac{Q}{r} \qquad \text{(sphere of charge, } r \geq R\text{)}$$

Or, in a more useful form, the potential outside a sphere that is charged to potential V_0 is

$$V = \frac{R}{r}V_0 \qquad \text{(sphere charged to potential } V_0\text{)}$$

The Electric Potential of Many Charges

The electric potential V at a point in space is the sum of the potentials due to each charge:

$$V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i}$$

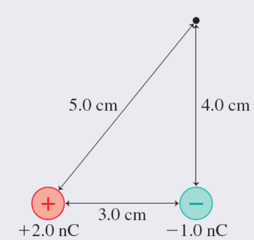
where r_i is the distance from charge q_i to the point in space where the potential is being calculated.
In other words, **the electric potential, like the electric field, obeys the principle of superposition.**

EXAMPLE 29.10 The potential of two charges

QUESTION:

EXAMPLE 29.10 The potential of two charges
What is the electric potential at the point indicated in **FIGURE 29.30**?

FIGURE 29.30 Finding the potential of two charges.



EXAMPLE 29.10 The potential of two charges

MODEL The potential is the sum of the potentials due to each charge.

EXAMPLE 29.10 The potential of two charges

SOLVE The potential at the indicated point is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

$$= (9.0 \times 10^9\ \mathrm{N\,m^2/C^2})\left(\frac{2.0 \times 10^{-9}\ \mathrm{C}}{0.050\ \mathrm{m}} + \frac{-1.0 \times 10^{-9}\ \mathrm{C}}{0.040\ \mathrm{m}}\right)$$

$$= 135\ \mathrm{V}$$

EXAMPLE 29.10 The potential of two charges

ASSESS The potential is a *scalar,* so we found the net potential by adding two numbers. We don't need any angles or components to calculate the potential.

Chapter 29. Summary Slides

General Principles

Sources of *V*

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

General Principles

Consequences of *V*

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + U_f = K_i + U_i$$

The potential energy of **two point charges** separated by distance r is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

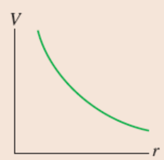
The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

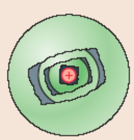
$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Applications

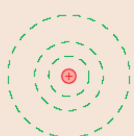
Graphical representations of the potential:



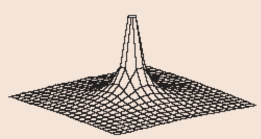
Potential graph



Equipotential surfaces



Contour map

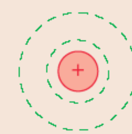


Elevation graph

Applications

Sphere of charge *Q*

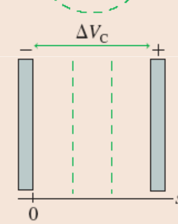
Same as a point charge if $r \geq R$



Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \frac{\Delta V_C}{d}$$



Applications

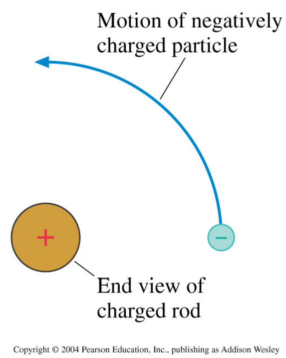
Units

Electric potential: 1 V = 1 J/C

Electric field: 1 V/m = 1 N/C

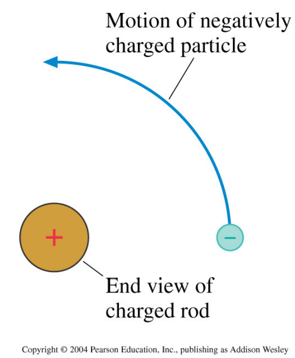
Chapter 29. Questions

The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod's electric field positive, negative or zero?



A. Positive
B. Negative
C. Zero

The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod's electric field positive, negative or zero?



A. Positive
B. Negative
✔ **C. Zero**

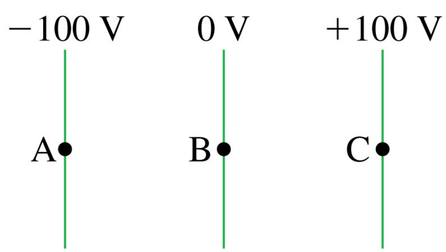
Rank in order, from largest to smallest, the potential energies U_a to U_d of these four pairs of charges. Each + symbol represents the same amount of charge.

A. $U_a = U_b > U_c = U_d$
B. $U_b = U_d > U_a = U_c$
C. $U_a = U_c > U_b = U_d$
D. $U_d > U_c > U_b > U_a$
E. $U_d > U_b = U_c > U_a$

Rank in order, from largest to smallest, the potential energies U_a to U_d of these four pairs of charges. Each + symbol represents the same amount of charge.

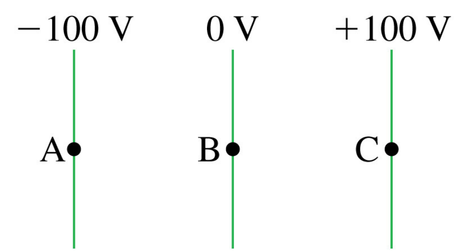
A. $U_a = U_b > U_c = U_d$
✔ **B. $U_b = U_d > U_a = U_c$**
C. $U_a = U_c > U_b = U_d$
D. $U_d > U_c > U_b > U_a$
E. $U_d > U_b = U_c > U_a$

A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton



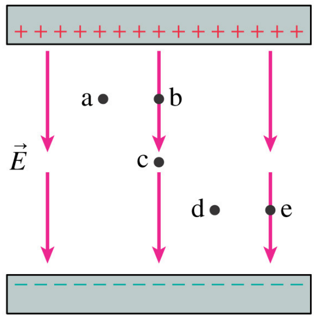
A. moves toward A with a steady speed.
B. moves toward A with an increasing speed.
C. moves toward C with a steady speed.
D. moves toward C with an increasing speed.
E. remains at rest at B.

A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton



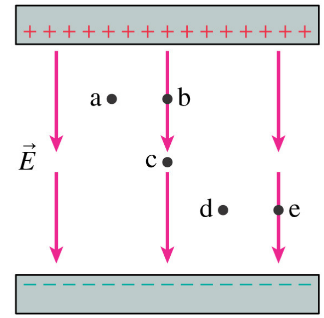
A. moves toward A with a steady speed.
✔ **B. moves toward A with an increasing speed.**
C. moves toward C with a steady speed.
D. moves toward C with an increasing speed.
E. remains at rest at B.

Rank in order, from largest to smallest, the potentials V_a to V_e at the points a to e.



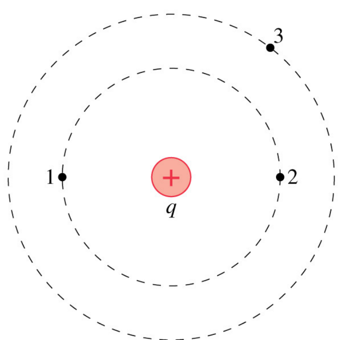
A. $V_d = V_e > V_c > V_a = V_b$
B. $V_b = V_c = V_e > V_a = V_d$
C. $V_a = V_b = V_c = V_d = V_e$
D. $V_a = V_b > V_c > V_d = V_e$
E. $V_a = V_b = V_d = V_e > V_c$

Rank in order, from largest to smallest, the potentials V_a to V_e at the points a to e.



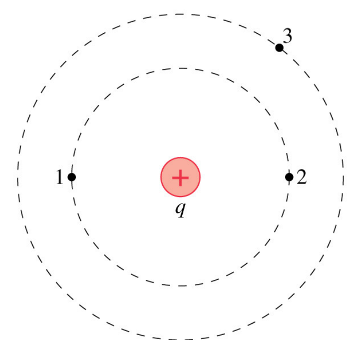
A. $V_d = V_e > V_c > V_a = V_b$
B. $V_b = V_c = V_e > V_a = V_d$
C. $V_a = V_b = V_c = V_d = V_e$
✔ **D. $V_a = V_b > V_c > V_d = V_e$**
E. $V_a = V_b = V_d = V_e > V_c$

Rank in order, from largest to smallest, the potential differences ΔV_{12}, ΔV_{13}, and ΔV_{23} between points 1 and 2, points 1 and 3, and points 2 and 3.



A. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$
B. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$
C. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$
D. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$
E. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$

Rank in order, from largest to smallest, the potential differences ΔV_{12}, ΔV_{13}, and ΔV_{23} between points 1 and 2, points 1 and 3, and points 2 and 3.



A. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$
✔ **B. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$**
C. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$
D. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$
E. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$