

Chapter 27. The Electric Field

Electric fields are responsible for the electric currents that flow through your computer and the nerves in your body. Electric fields also line up polymer molecules to form the images in a liquid crystal display (LCD).



Chapter Goal: To learn how to calculate and use the electric field.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Chapter 27. The Electric Field

Topics:

- Electric Field Models
- The Electric Field of Multiple Point Charges
- The Electric Field of a Continuous Charge Distribution
- The Electric Fields of Rings, Disks, Planes, and Spheres
- The Parallel-Plate Capacitor
- Motion of a Charged Particle in an Electric Field
- Motion of a Dipole in an Electric Field

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Chapter 27. Reading Quizzes

What device provides a practical way to produce a uniform electric field?

- A. A long thin resistor
- B. A Faraday cage
- C. A parallel plate capacitor
- D. A toroidal inductor
- E. An electric field uniformizer

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

What device provides a practical way to produce a uniform electric field?

- A. A long thin resistor
- B. A Faraday cage
- ✓ C. **A parallel plate capacitor**
- D. A toroidal inductor
- E. An electric field uniformizer

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

For charged particles, what is the quantity q/m called?

- A. Linear charge density
- B. Charge-to-mass ratio
- C. Charged mass density
- D. Massive electric dipole
- E. Quadrupole moment

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

For charged particles, what is the quantity q/m called?

- A. Linear charge density
- ✓ B. **Charge-to-mass ratio**
- C. Charged mass density
- D. Massive electric dipole
- E. Quadrupole moment

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Which of these charge distributions did *not* have its electric field determined in Chapter 27?

- A. A line of charge
- B. A parallel-plate capacitor
- C. A ring of charge
- D. A plane of charge
- E. They were *all* determined

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Which of these charge distributions did *not* have its electric field determined in Chapter 27?

- A. A line of charge
- B. A parallel-plate capacitor
- C. A ring of charge
- D. A plane of charge
-  **E. They were *all* determined**

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

The worked examples of charged-particle motion are relevant to

- A. a transistor.
- B. a cathode ray tube.
- C. magnetic resonance imaging.
- D. cosmic rays.
- E. lasers.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

The worked examples of charged-particle motion are relevant to

- A. a transistor.
-  **B. a cathode ray tube.**
- C. magnetic resonance imaging.
- D. cosmic rays.
- E. lasers.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Chapter 27. Basic Content and Examples

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

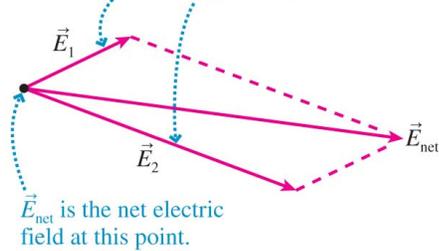
Electric Field Models

The electric field of a point charge q at the origin, $r = 0$, is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (\text{electric field of a point charge})$$

where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2$ is the permittivity constant.

Fields of source charges 1 and 2



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric Field Models

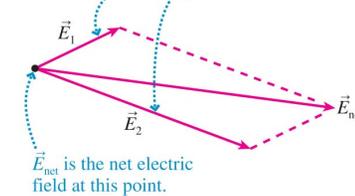
The net electric field due to a group of point charges is

$$\vec{E}_{\text{net}} = \frac{\vec{F}_{\text{on } q}}{q} = \frac{\vec{F}_{1 \text{ on } q}}{q} + \frac{\vec{F}_{2 \text{ on } q}}{q} + \dots = \vec{E}_1 + \vec{E}_2 + \dots = \sum_i \vec{E}_i$$

where E_i is the field from point charge i .

FIGURE 27.3 Electric fields obey the principle of superposition.

Fields of source charges 1 and 2



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Problem-Solving Strategy: The electric field of multiple point charges

PROBLEM-SOLVING STRATEGY 27.1 **The electric field of multiple point charges**



MODEL Model charged objects as point charges.

Problem-Solving Strategy: The electric field of multiple point charges

VISUALIZE For the pictorial representation:

- Establish a coordinate system and show the locations of the charges.
- Identify the point P at which you want to calculate the electric field.
- Draw the electric field of each charge at P.
- Use symmetry to determine if any components of \vec{E}_{net} are zero.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Problem-Solving Strategy: The electric field of multiple point charges

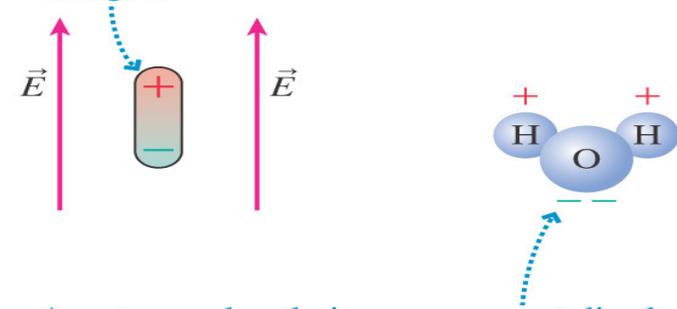
SOLVE The mathematical representation is $\vec{E}_{\text{net}} = \sum \vec{E}_i$.

- For each charge, determine its distance from P and the angle of \vec{E}_i from the axes.
- Calculate the field strength of each charge's electric field.
- Write each vector \vec{E}_i in component form.
- Sum the vector components to determine \vec{E}_{net} .
- If needed, determine the magnitude and direction of \vec{E}_{net} .

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric Dipole: Two Equal but Opposite Charges Separated by a Small Distance

This dipole is *induced*, or stretched, by the electric field acting on the + and - charges.



A water molecule is a *permanent* dipole because the negative electrons spend more time with the oxygen atom.

Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

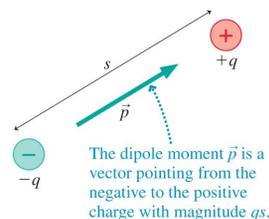
The Electric Field of a Dipole

We can represent an electric dipole by two opposite charges $\pm q$ separated by the small distance s .

The dipole moment is defined as the vector

$$\vec{p} = (qs, \text{ from the negative to the positive charge})$$

The dipole-moment magnitude $p = qs$ determines the electric field strength. The SI units of the dipole moment are C m.

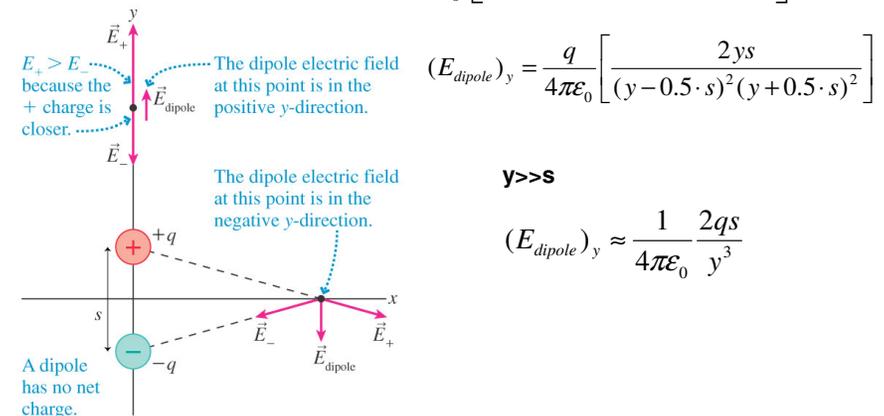


Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

The Electric Field of a Dipole

On the axis of an electric dipole

$$(E_{\text{dipole}})_y = (E_+)_{y} + (E_-)_{y} = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(y-s/2)^2} + \frac{(-q)}{(y+s/2)^2} \right]$$



$$(E_{\text{dipole}})_y = \frac{q}{4\pi\epsilon_0} \left[\frac{2ys}{(y-0.5 \cdot s)^2(y+0.5 \cdot s)^2} \right]$$

$y \gg s$

$$(E_{\text{dipole}})_y \approx \frac{1}{4\pi\epsilon_0} \frac{2qs}{y^3}$$

Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley Pearson Addison-Wesley.

The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3} \quad (\text{on the axis of an electric dipole})$$

where r is the distance measured from the *center* of the dipole.

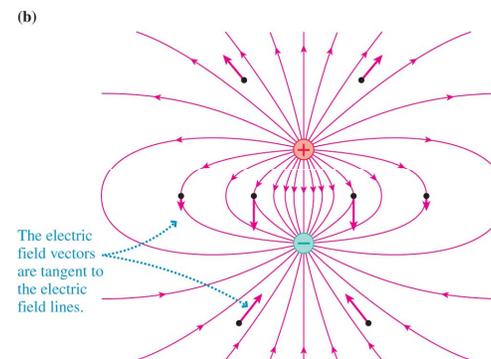
The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3} \quad (\text{perpendicular plane})$$

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

FIGURE 27.9 The electric field of a dipole.



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

EXAMPLE 27.2 The electric field of a water molecule

QUESTION:

EXAMPLE 27.2 The electric field of a water molecule

The water molecule H_2O has a permanent dipole moment of magnitude 6.2×10^{-30} C·m. What is the electric field strength 1.0 nm from a water molecule at a point on the dipole's axis?

EXAMPLE 27.2 The electric field of a water molecule

MODEL The size of a molecule is ≈ 0.1 nm. Thus $r \gg s$, and we can use Equation 27.11 for the on-axis electric field of the molecule's dipole moment.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

EXAMPLE 27.2 The electric field of a water molecule

SOLVE The on-axis electric field strength at $r = 1.0 \text{ nm}$ is

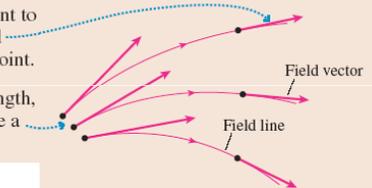
$$E \approx \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} = (9.0 \times 10^9 \text{ Nm}^2/\text{C}^2) \frac{2(6.2 \times 10^{-30} \text{ Cm})}{(1.0 \times 10^{-9} \text{ m})^3} = 1.1 \times 10^8 \text{ N/C}$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Tactics: Drawing and using electric field lines

TACTICS BOX 27.1 Drawing and using electric field lines

- Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.
- Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Tactics: Drawing and using electric field lines

- Electric field lines never cross.
- Electric field lines start from positive charges and end on negative charges.

Exercises 2–4, 12, 13

Continuous Charge Distribution: Charge Density

The total electric charge is Q .

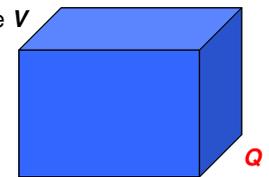
Linear, length L

Amount of charge in a small volume dV :

$$dq = \frac{Q}{L} dl = \lambda dl \quad \lambda = \frac{Q}{L}$$

Linear charge density

Volume V



Amount of charge in a small volume dV :

$$dq = \frac{Q}{V} dV = \rho dV \quad \rho = \frac{Q}{V}$$

Volume charge density

Surface, area A



Amount of charge in a small volume dA :

$$dq = \frac{Q}{A} dA = \sigma dA \quad \sigma = \frac{Q}{A}$$

Surface charge density

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

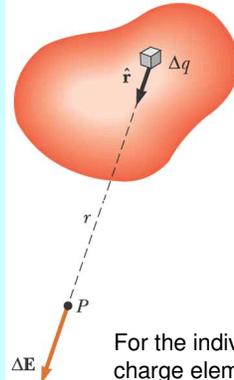
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric Field: Continuous Charge Distribution

Procedure:

- Divide the charge distribution into small elements, each of which contains Δq
- Calculate the electric field due to one of these elements at point P
- Evaluate the total field by summing the contributions of all the charge elements

Symmetry: take advantage of any symmetry to simplify calculations



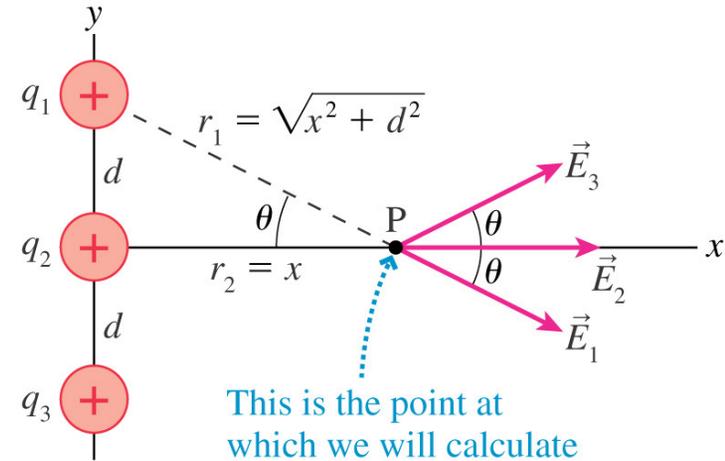
For the individual charge elements
 $\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$

Because the charge distribution is continuous

$$\mathbf{E} = k_e \lim_{\Delta q_i \rightarrow 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric field due to three equal point charges?



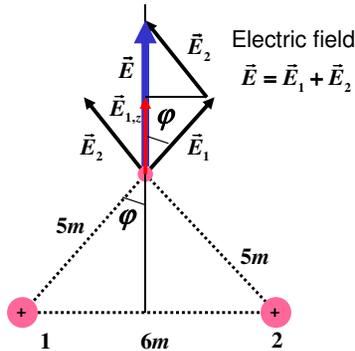
This is the point at which we will calculate the electric field.

Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley
 Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric Field: Symmetry

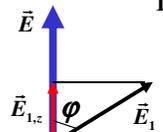
$$\vec{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

$$q_1 = 10 \mu\text{C} \quad q_2 = 10 \mu\text{C}$$



$$E = 2E_1 \cos \phi$$

$$E = 2E_{1,z} = 2E_{2,z}$$



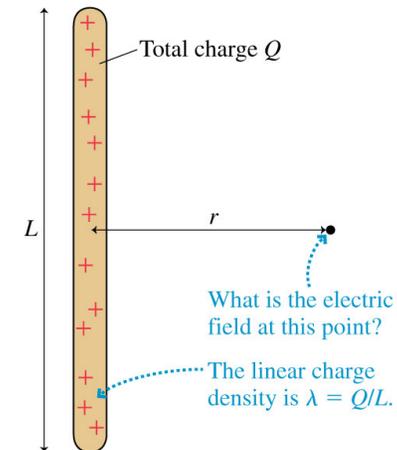
$$E_{1,z} = E_1 \cos \phi$$

$$E = 4E_{1,z} = 4E_2 \cos \phi$$

The symmetry gives us the direction of resultant electric field

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

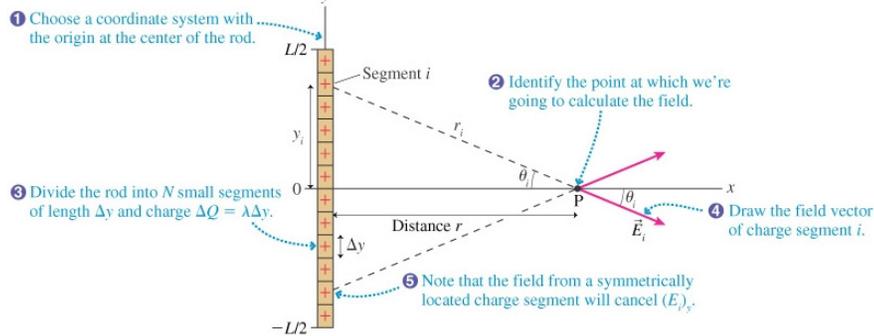
The Electric Field of a Continuous Charge Distribution at a point in the plane that bisects the rod



Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

The Electric Field of a Continuous Charge Distribution



$$(E_i)_x = (E_i)_x \cos \theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

The Electric Field of a Continuous Charge Distribution on a Thin Rod

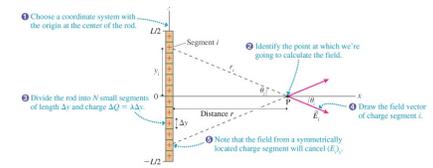
$$(E_i)_x = (E_i)_x \cos \theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$

$$(E_i)_x = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{y_i^2 + r^2} \frac{r}{\sqrt{y_i^2 + r^2}}$$

$$(E_i)_x = \frac{1}{4\pi\epsilon_0} \frac{r \Delta Q}{(y_i^2 + r^2)^{3/2}}$$

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \sum_i \frac{r \Delta y}{(y_i^2 + r^2)^{3/2}}$$

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{r dy}{(y^2 + r^2)^{3/2}}$$



$$\Delta Q = \lambda \Delta y = (Q/L) \Delta y$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

How to evaluate this integral?

-Trigonometric Substitution

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{r dy}{(y^2 + r^2)^{3/2}}$$

http://en.wikipedia.org/wiki/Trigonometric_substitution

From Wikipedia, the free encyclopedia

In **mathematics**, **trigonometric substitution** is the substitution of trigonometric functions for other expressions. One may use the **trigonometric identities** to simplify certain **integrals** containing the radical expressions:

$$1 - \sin^2 \theta = \cos^2 \theta \text{ for } \sqrt{a^2 - x^2}$$

$$1 + \tan^2 \theta = \sec^2 \theta \text{ for } \sqrt{a^2 + x^2}$$

$$\sec^2 \theta - 1 = \tan^2 \theta \text{ for } \sqrt{x^2 - a^2}$$

In the expression $a^2 - x^2$, the substitution of $a \sin(\theta)$ for x makes it possible to use the identity $1 - \sin^2 \theta = \cos^2 \theta$.

In the expression $a^2 + x^2$, the substitution of $a \tan(\theta)$ for x makes it possible to use the identity $\tan^2 \theta + 1 = \sec^2 \theta$.

Similarly, in $x^2 - a^2$, the substitution of $a \sec(\theta)$ for x makes it possible to use the identity $\sec^2 \theta - 1 = \tan^2 \theta$.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric Field due to a Thin Rod

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{r dy}{(y^2 + r^2)^{3/2}}$$

Use:

$$\int \frac{1}{(a^2 + x^2)^{3/2}} dx = \frac{x}{a^2 \sqrt{a^2 + x^2}}$$

<http://integrals.wolfram.com/index.jsp>

(to evaluate, $x = a \tan \theta$, $dx = a \sec^2 \theta d\theta$
 $\sin \theta = x / (x^2 + a^2)^{1/2}$)

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \frac{y}{r \sqrt{y^2 + r^2}} \Big|_{-L/2}^{L/2} = \frac{Q/L}{4\pi\epsilon_0} \left[\frac{L/2}{r \sqrt{(L/2)^2 + r^2}} - \frac{-L/2}{r \sqrt{(-L/2)^2 + r^2}} \right]$$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{r \sqrt{(L/2)^2 + r^2}}$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

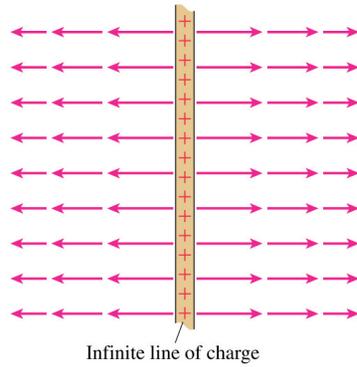
An infinite Line of Charge

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{r\sqrt{(L/2)^2 + r^2}}$$

$$E_{line} = \lim_{L \rightarrow \infty} \frac{1}{4\pi\epsilon_0} \frac{Q}{r\sqrt{(L/2)^2 + r^2}}$$

$$E_{line} = \frac{1}{4\pi\epsilon_0} \frac{Q}{rL/2}$$

$$E_{line} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}$$

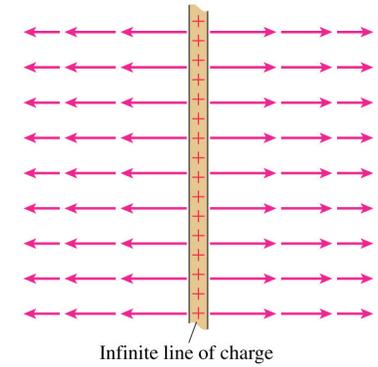


Infinite line of charge
Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

An infinite Line of Charge

$$E_{line} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}$$

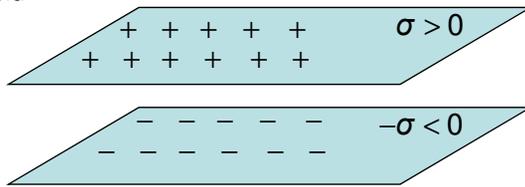
Unlike a point charge, for which the field decreases as $1/r^2$, the field of an infinitely long charged wire decreases more slowly – as only $1/r$



Infinite line of charge
Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

Parallel-Plate Capacitor

Find electric field

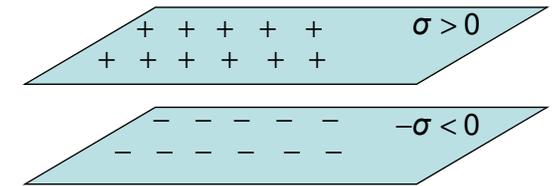


$$E_+ = \frac{\sigma}{2\epsilon_0}$$

$$E_- = \frac{\sigma}{2\epsilon_0}$$

Parallel-Plate Capacitor

Find electric field



$$E_+ = \frac{\sigma}{2\epsilon_0} \quad E_- = \frac{\sigma}{2\epsilon_0}$$

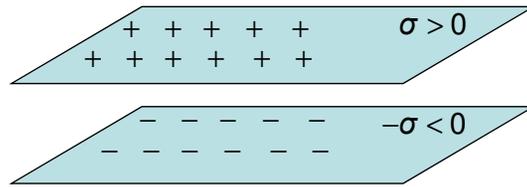
$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$E = E_+ - E_- = 0$$

$$E = E_+ + E_- = \frac{\sigma}{\epsilon_0}$$

$$E = E_+ - E_- = 0$$

Parallel-Plate Capacitor



$$\begin{array}{c}
 E = 0 \qquad \qquad \qquad \sigma > 0 \\
 \hline
 E = \frac{\sigma}{\epsilon_0} \quad \vec{E} \downarrow \\
 \hline
 E = 0 \qquad \qquad \qquad -\sigma < 0
 \end{array}$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

EXAMPLE 27.7 The electric field inside a capacitor

QUESTIONS:

EXAMPLE 27.7 The electric field inside a capacitor

Two $1.0 \text{ cm} \times 2.0 \text{ cm}$ rectangular electrodes are 1.0 mm apart. What charge must be placed on each electrode to create a uniform electric field of strength $2.0 \times 10^6 \text{ N/C}$? How many electrons must be moved from one electrode to the other to accomplish this?

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

EXAMPLE 27.7 The electric field inside a capacitor

MODEL The electrodes can be modeled as a parallel-plate capacitor because the spacing between them is much smaller than their lateral dimensions.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

EXAMPLE 27.7 The electric field inside a capacitor

SOLVE The electric field strength inside the capacitor is $E = Q/\epsilon_0 A$. Thus the charge to produce a field of strength E is

$$\begin{aligned}
 Q &= (8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(2.0 \times 10^{-4} \text{ m}^2)(2.0 \times 10^6 \text{ N/C}) \\
 &= 3.5 \times 10^{-9} \text{ C} = 3.5 \text{ nC}
 \end{aligned}$$

The positive plate must be charged to $+3.5 \text{ nC}$ and the negative plate to -3.5 nC . In practice, the plates are charged by using a *battery* to move electrons from one plate to the other. The number of electrons in 3.5 nC is

$$N = \frac{Q}{e} = \frac{3.5 \times 10^{-9} \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 2.2 \times 10^{10} \text{ electrons}$$

Thus 2.2×10^{10} electrons are moved from one electrode to the other. Note that the capacitor *as a whole* has no net charge.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

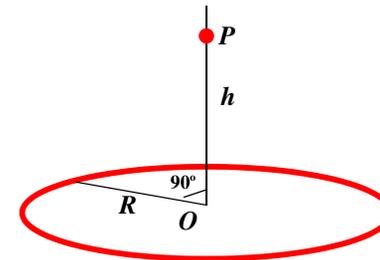
EXAMPLE 27.7 The electric field inside a capacitor

ASSESS The plate spacing does not enter the result. As long as the spacing is much smaller than the plate dimensions, as is true in this example, the field is independent of the spacing.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Electric Field: Continuous Charge Distribution on a ring

What is the electric field at point P ?

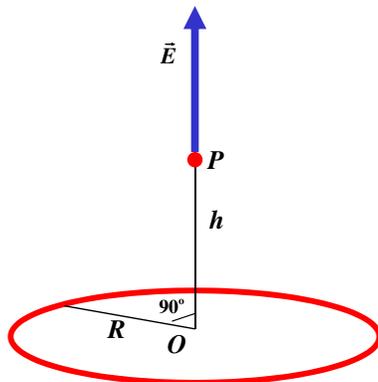


λ - linear charge density

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

What is the electric field at point P ? λ - linear charge density

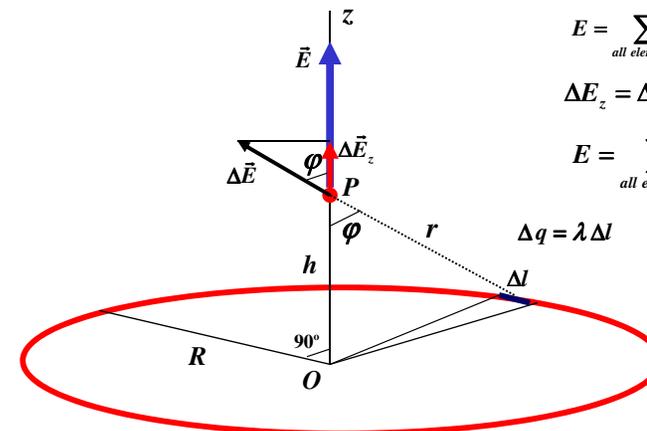
1. **Symmetry** determines the direction of the electric field.



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

What is the electric field at point P ? λ - linear charge density

2. Divide the charge distribution into small elements, each of which contains Δq



$$\vec{E} = \sum_{\text{all elements}} \Delta \vec{E} = \sum_{\text{all elements}} \Delta \vec{E}_z$$

$$E = \sum_{\text{all elements}} \Delta E_z$$

$$\Delta E_z = \Delta E \cos \phi = k_e \frac{\Delta q}{r^2} \cos \phi$$

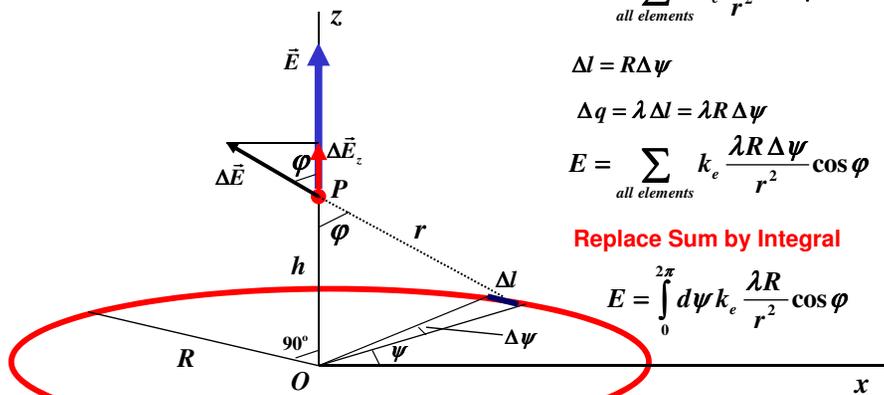
$$E = \sum_{\text{all elements}} k_e \frac{\Delta q}{r^2} \cos \phi$$

$$\Delta q = \lambda \Delta l$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

What is the electric field at point P ? λ - linear charge density

3. Evaluate the total field by summing the contributions of all the charge elements Δq



$$E = \sum_{\text{all elements}} k_e \frac{\Delta q}{r^2} \cos \varphi$$

$$\Delta l = R \Delta \psi$$

$$\Delta q = \lambda \Delta l = \lambda R \Delta \psi$$

$$E = \sum_{\text{all elements}} k_e \frac{\lambda R \Delta \psi}{r^2} \cos \varphi$$

Replace Sum by Integral

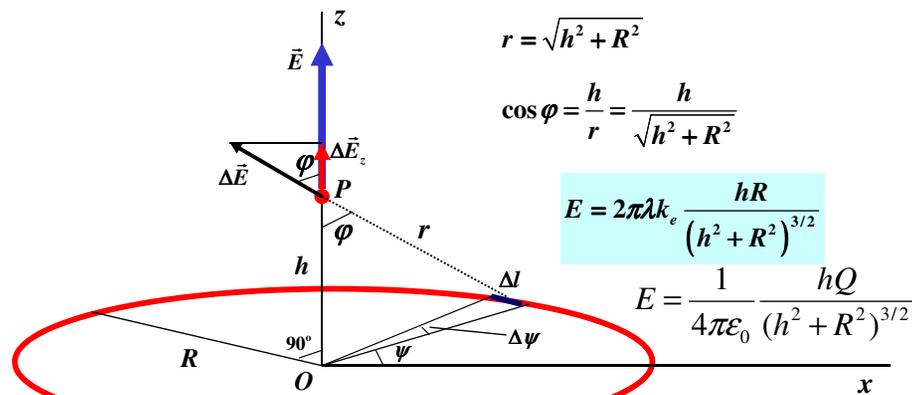
$$E = \int_0^{2\pi} d\psi k_e \frac{\lambda R}{r^2} \cos \varphi$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

What is the electric field at point P ? λ - linear charge density

4. Evaluate the integral

$$E = \int_0^{2\pi} d\psi k_e \frac{\lambda R}{r^2} \cos \varphi = k_e \frac{\lambda R}{r^2} \cos \varphi \int_0^{2\pi} d\psi = 2\pi k_e \frac{\lambda R}{r^2} \cos \varphi$$



$$r = \sqrt{h^2 + R^2}$$

$$\cos \varphi = \frac{h}{r} = \frac{h}{\sqrt{h^2 + R^2}}$$

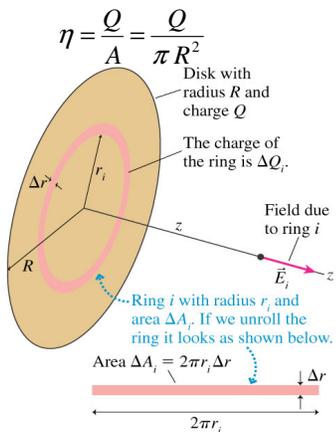
$$E = 2\pi \lambda k_e \frac{hR}{(h^2 + R^2)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{hQ}{(h^2 + R^2)^{3/2}}$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

A Circular Disk of Charge

Electric field due to a ring: $E = \frac{1}{4\pi\epsilon_0} \frac{hQ}{(h^2 + R^2)^{3/2}}$



$$(E_i)_z = \frac{1}{4\pi\epsilon_0} \frac{z \Delta Q_i}{(z^2 + r_i^2)^{3/2}}$$

$$(E_{\text{disk}})_z = \sum_{i=1}^N (E_i)_z = \frac{z}{4\pi\epsilon_0} \sum_{i=1}^N \frac{\Delta Q_i}{(z^2 + r_i^2)^{3/2}}$$

$$(E_{\text{disk}})_z = \frac{\eta z}{2\epsilon_0} \sum_{i=1}^N \frac{r_i \Delta r}{(z^2 + r_i^2)^{3/2}}$$

$$(E_{\text{disk}})_z = \frac{\eta z}{2\epsilon_0} \int_0^R \frac{r dr}{(z^2 + r^2)^{3/2}}$$

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right]$$

Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

Quiz: If $z \gg R$, then $E = ?$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Motion of a Charged Particle

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

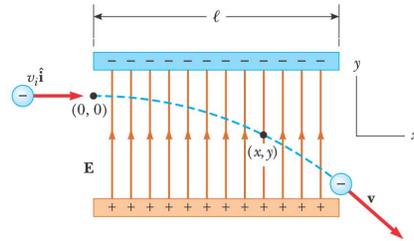
Motion of Charged Particle

- When a charged particle is placed in an electric field, it experiences an electrical force
- If this is the only force on the particle, it must be the net force
- The net force will cause the particle to accelerate according to Newton's second law

$$\vec{F} = q\vec{E} \quad \text{- Coulomb's law}$$

$$\vec{F} = m\vec{a} \quad \text{- Newton's second law}$$

$$\vec{a} = \frac{q}{m}\vec{E}$$



©2004 Thomson - Brooks/Cole

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Chapter 27. Summary Slides

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Motion of Charged Particle

What is the final velocity?

$$\vec{F} = q\vec{E} \quad \text{- Coulomb's law}$$

$$\vec{F} = m\vec{a} \quad \text{- Newton's second law}$$

$$a_y = -\frac{|q|}{m}E$$

Motion in x - with constant velocity v_0

Motion in x - with constant acceleration

©2004 Thomson - Brooks/Cole

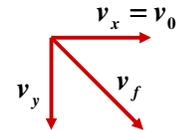
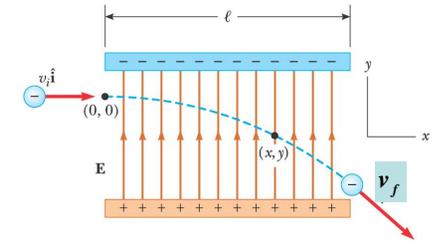
$$a_y = -\frac{|q|}{m}E$$

$$t = \frac{l}{v_0} \quad \text{- travel time}$$

After time t the velocity in y direction becomes

$$v_y = a_y t = -\frac{|q|}{m}Et \quad \text{then} \quad v_f = \sqrt{v_0^2 + \left(\frac{q}{m}Et\right)^2}$$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.



General Principles

Sources of \vec{E}

Electric fields are created by charges.

Two major tools for calculating \vec{E} are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$

Continuous distribution of charge

- Divide the charge into segments ΔQ for which you already know the field.
- Find the field of each ΔQ .
- Find \vec{E} by summing the fields of all ΔQ .

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

General Principles

Consequences of \vec{E}

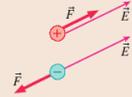
The electric field exerts a force on a charged particle:

$$\vec{F} = q\vec{E}$$

The force causes acceleration:

$$\vec{a} = (q/m)\vec{E}$$

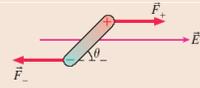
Trajectories of charged particles are calculated with kinematics.



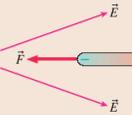
The electric field exerts a torque on a dipole:

$$\tau = pE \sin \theta$$

The torque tends to align the dipoles with the field.



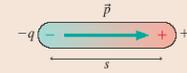
In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Applications

Electric dipole



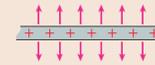
The electric dipole moment is

$$\vec{p} = (qs, \text{ from negative to positive})$$

$$\text{Field on axis: } \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$$

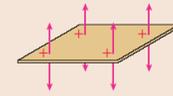
$$\text{Field in bisecting plane: } \vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3}$$

Infinite line of charge with linear charge density λ



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}, \text{ perpendicular to line} \right)$$

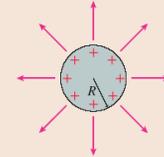
Infinite plane of charge with surface charge density η



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{ perpendicular to plane} \right)$$

Sphere of charge

Same as a point charge Q for $r > R$



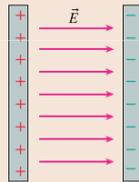
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Applications

Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field**:

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative} \right)$$



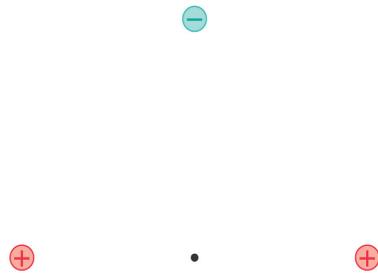
A real capacitor has a weak **fringe field** around it.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Chapter 27. Questions

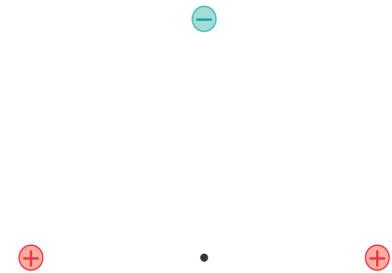
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

At the position of the dot, the electric field points

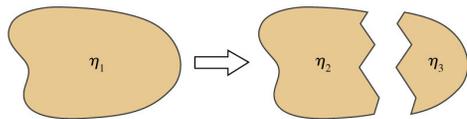


- A. Up.
- B. Down.
- C. Left.
- D. Right.
- E. The electric field is zero.

At the position of the dot, the electric field points

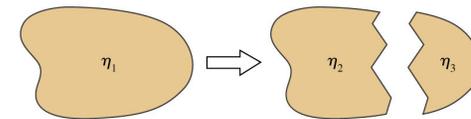


- A. Up.
- B. Down.
- C. Left.
- D. Right.
- E. The electric field is zero.



A piece of plastic is uniformly charged with surface charge density η_1 . The plastic is then broken into a large piece with surface charge density η_2 and a small piece with surface charge density η_3 . Rank in order, from largest to smallest, the surface charge densities η_1 to η_3 .

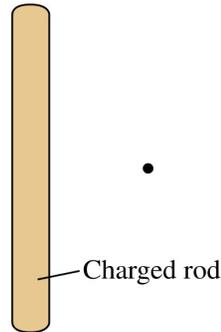
- A. $\eta_2 = \eta_3 > \eta_1$
- B. $\eta_1 > \eta_2 > \eta_3$
- C. $\eta_1 > \eta_2 = \eta_3$
- D. $\eta_3 > \eta_2 > \eta_1$
- E. $\eta_1 = \eta_2 = \eta_3$



A piece of plastic is uniformly charged with surface charge density η_1 . The plastic is then broken into a large piece with surface charge density η_2 and a small piece with surface charge density η_3 . Rank in order, from largest to smallest, the surface charge densities η_1 to η_3 .

- A. $\eta_2 = \eta_3 > \eta_1$
- B. $\eta_1 > \eta_2 > \eta_3$
- C. $\eta_1 > \eta_2 = \eta_3$
- D. $\eta_3 > \eta_2 > \eta_1$
- E. $\eta_1 = \eta_2 = \eta_3$

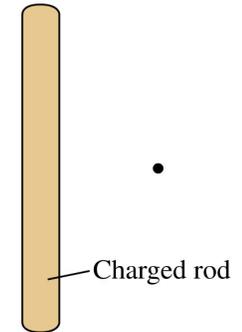
Which of the following actions will increase the electric field strength at the position of the dot?



- A. Make the rod longer without changing the charge.
- B. Make the rod fatter without changing the charge.
- C. Make the rod shorter without changing the charge.
- D. Remove charge from the rod.
- E. Make the rod narrower without changing the charge.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

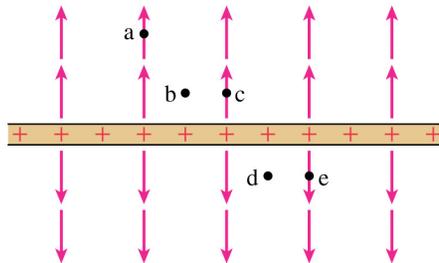
Which of the following actions will increase the electric field strength at the position of the dot?



- A. Make the rod longer without changing the charge.
- B. Make the rod fatter without changing the charge.
- ✓ C. **Make the rod shorter without changing the charge.**
- D. Remove charge from the rod.
- E. Make the rod narrower without changing the charge.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.

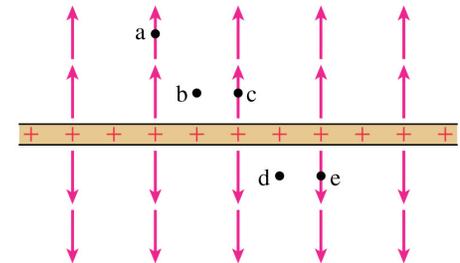


Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

- A. $E_a > E_c > E_b > E_e > E_d$
- B. $E_a = E_b = E_c = E_d = E_e$
- C. $E_a > E_b = E_c > E_d = E_e$
- D. $E_b = E_c = E_d = E_e > E_a$
- E. $E_c > E_d > E_c > E_b > E_a$

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.

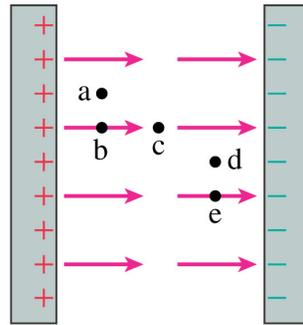


Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

- A. $E_a > E_c > E_b > E_e > E_d$
- ✓ B. **$E_a = E_b = E_c = E_d = E_e$**
- C. $E_a > E_b = E_c > E_d = E_e$
- D. $E_b = E_c = E_d = E_e > E_a$
- E. $E_c > E_d > E_c > E_b > E_a$

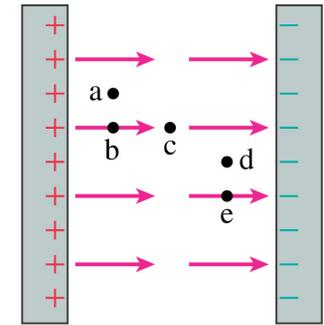
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Rank in order, from largest to smallest, the forces F_a to F_e a proton would experience if placed at points a – e in this parallel-plate capacitor.



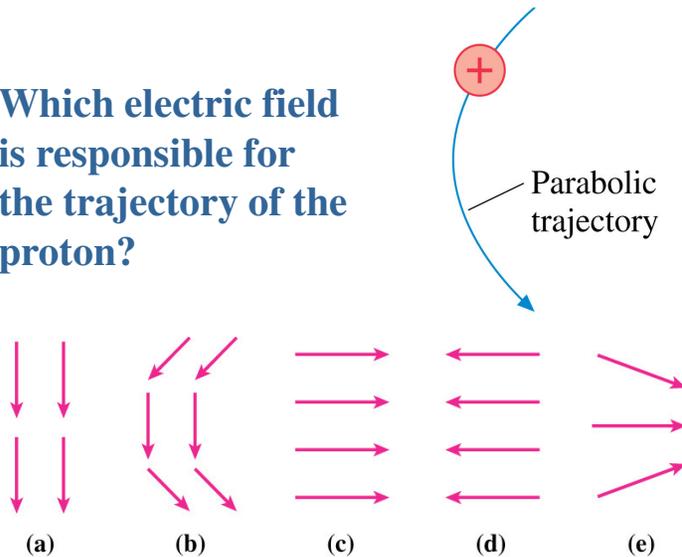
- A. $F_a = F_b = F_d = F_e > F_c$
- B. $F_a = F_b > F_c > F_d = F_e$
- C. $F_a = F_b = F_c = F_d = F_e$
- D. $F_e = F_d > F_c > F_a = F_b$
- E. $F_e > F_d > F_c > F_b > F_a$

Rank in order, from largest to smallest, the forces F_a to F_e a proton would experience if placed at points a – e in this parallel-plate capacitor.



- A. $F_a = F_b = F_d = F_e > F_c$
- B. $F_a = F_b > F_c > F_d = F_e$
- C. $F_a = F_b = F_c = F_d = F_e$
- D. $F_e = F_d > F_c > F_a = F_b$
- E. $F_e > F_d > F_c > F_b > F_a$

Which electric field is responsible for the trajectory of the proton?



Which electric field is responsible for the trajectory of the proton?

