Chapter 23. Ray Optics

Our everyday experience that light travels in straight lines is the basis of the ray model of light. Ray optics apply to a variety of situations, including mirrors, lenses, and shiny spoons.

Chapter Goal: To understand and apply the ray model of light.

Chapter 23. Ray Optics

Topics:

- The Ray Model of Light
- Reflection
- Refraction
- Image Formation by Refraction
- Color and Dispersion
- Thin Lenses: Ray Tracing
- Thin Lenses: Refraction Theory
- Image Formation with Spherical Mirrors

Chapter 23. Reading Quizzes

What is specular reflection?

A. The image of a specimen.
B. A reflection that separates different colors.
C. Reflection by a flat smooth object.
D. When the image is virtual and special.
E. This topic is not covered in Chapter 23.

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A. moves in a parabolic path.
B. is a ray that has been reflected from parabolic mirror.
C. is a ray that moves nearly parallel to the optical axis.
D. is a ray that moves exactly parallel to the optical axis.

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A virtual image is

A. the cause of optical illusions.
B. a point from which rays appear to diverge.
C. an image that only seems to exist.
D. the image that is left in space after you remove a viewing screen.

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The focal length of a converging lens is

A. the distance at which an image is formed.
B. the distance at which an object must be placed to form an image.
C. the distance at which parallel light rays are focused.
D. the distance from the front surface to the back surface.

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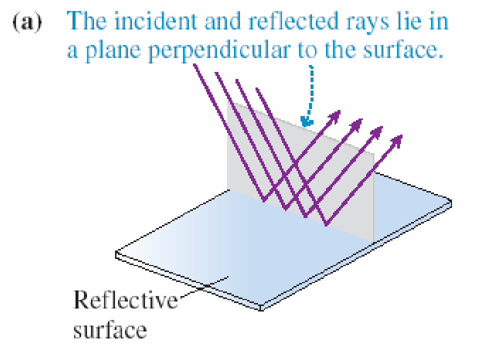
Chapter 23. Basic Content and Examples

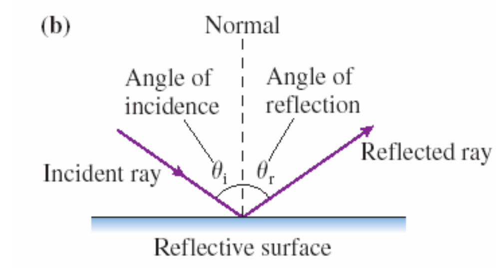
FIGURE 23.7 Specular reflection of light.



Reflection

The **law of reflection** states that

1. The incident ray and the reflected ray are in the same plane normal to the surface, and
2. The angle of reflection equals the angle of incidence: $\theta_r = \theta_i$



The Plane Mirror

Consider P, a source of rays which reflect from a mirror. The reflected rays appear to emanate from P', the same distance behind the mirror as P is in front of the mirror. That is, $s' = s$.

FIGURE 23.10 The light rays reflecting from a plane mirror.

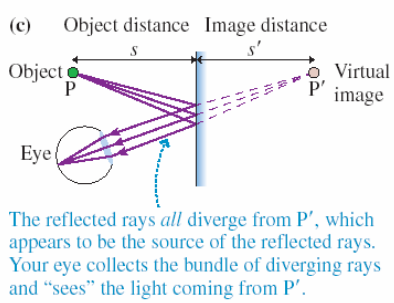


FIGURE 23.15 Refraction of light rays.

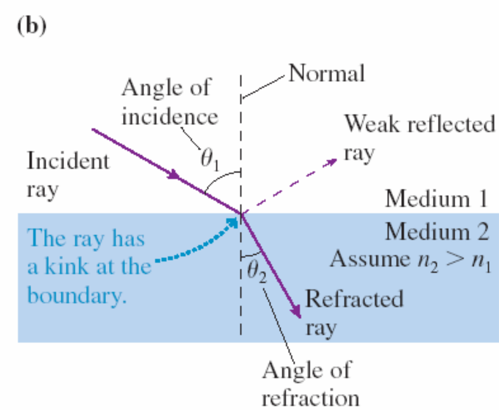
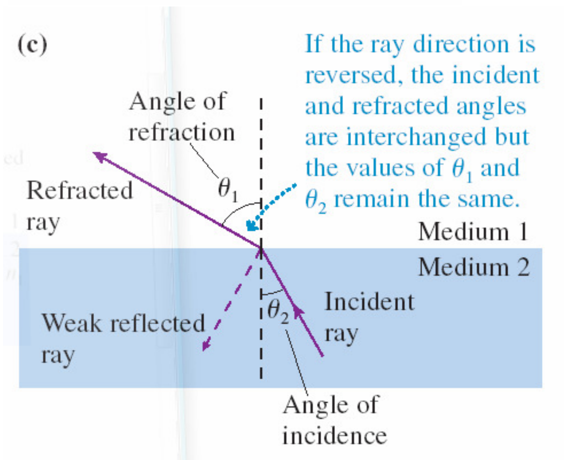


FIGURE 23.15 Refraction of light rays.



Refraction

Snell's law states that if a ray refracts between medium 1 and medium 2, having indices of refraction n_1 an n_2, the ray angles θ_1 and θ_2 in the two media are related by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Snell's law of refraction)}$$

Notice that Snell's law does not mention which is the incident angle and which is the refracted angle.

TABLE 23.1 Indices of refraction

Medium	n
Vacuum	1.00 exactly
Air (actual)	1.0003
Air (accepted)	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.41
Silicon (infrared)	3.50

Tactics: Analyzing refraction

TACTICS BOX 23.1 **Analyzing refraction**

1. **Draw a ray diagram.** Represent the light beam with one ray.
2. **Draw a line normal to the boundary.** Do this at each point where the ray intersects a boundary.
3. **Show the ray bending in the correct direction.** The angle is larger on the side with the smaller index of refraction. This is the qualitative application of Snell's law.
4. **Label angles of incidence and refraction.** Measure all angles from the normal.
5. **Use Snell's law.** Calculate the unknown angle or unknown index of refraction.

Exercises 11–15

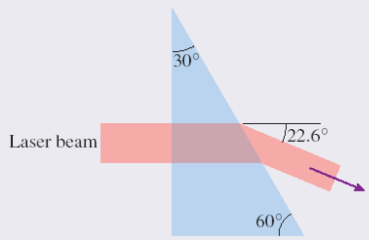
EXAMPLE 23.4 Measuring the index of refraction

QUESTION:

EXAMPLE 23.4 Measuring the index of refraction

FIGURE 23.19 shows a laser beam deflected by a 30°-60°-90° prism. What is the prism's index of refraction?

FIGURE 23.19 A prism deflects a laser beam.



EXAMPLE 23.4 Measuring the index of refraction

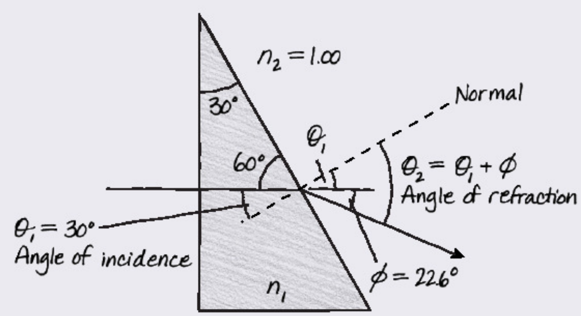
MODEL Represent the laser beam with a single ray and use the ray model of light.

EXAMPLE 23.4 Measuring the index of refraction

VISUALIZE **FIGURE 23.20** uses the steps of Tactics Box 23.1 to draw a ray diagram. The ray is incident perpendicular to the front face of the prism ($\theta_{\text{incident}} = 0°$), thus it is transmitted through the first boundary without deflection. At the second boundary it is especially important to *draw the normal to the surface* at the point of incidence and to *measure angles from the normal.*

EXAMPLE 23.4 Measuring the index of refraction

FIGURE 23.20 Pictorial representation of a laser beam passing through the prism.



θ_1 and θ_2 are measured from the normal.

EXAMPLE 23.4 Measuring the index of refraction

SOLVE From the geometry of the triangle you can find that the laser's angle of incidence on the hypotenuse of the prism is $\theta_1 = 30°$, the same as the apex angle of the prism. The ray exits the prism at angle θ_2 such that the deflection is $\phi = \theta_2 - \theta_1 = 22.6°$. Thus $\theta_2 = 52.6°$. Knowing both angles and $n_2 = 1.00$ for air, we can use Snell's law to find n_1:

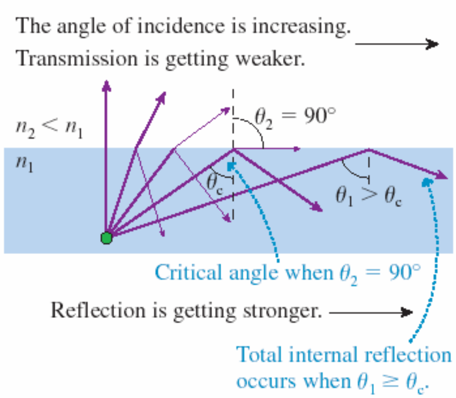
$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{1.00 \sin 52.6°}{\sin 30°} = 1.59$$

EXAMPLE 23.4 Measuring the index of refraction

ASSESS Referring to the indices of refraction in Table 23.1, we see that the prism is made of plastic.

Total Internal Reflection

FIGURE 23.22 Refraction and reflection of rays as the angle of incidence increases.



$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Color

Different colors are associated with light of different wavelengths. The longest wavelengths are perceived as red light and the shortest as violet light. Table 23.2 is a brief summary of the *visible spectrum* of light.

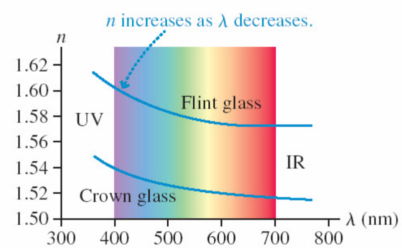
TABLE 23.2 A brief summary of the visible spectrum of light

Color	Approximate wavelength
Deepest red	700 nm
Red	650 nm
Green	550 nm
Blue	450 nm
Deepest violet	400 nm

Dispersion

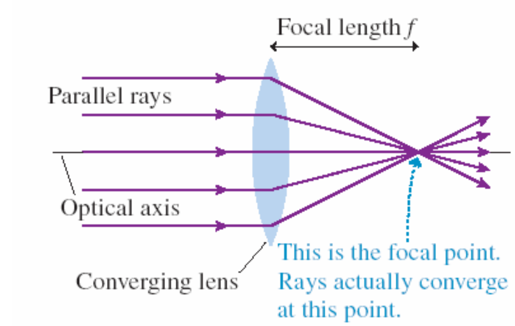
The slight variation of index of refraction with wavelength is known as **dispersion**. Shown is the dispersion curves of two common glasses. Notice that ***n* is larger when the wavelength is shorter**, thus violet light refracts more than red light.

FIGURE 23.29 Dispersion curves show how the index of refraction varies with wavelength.



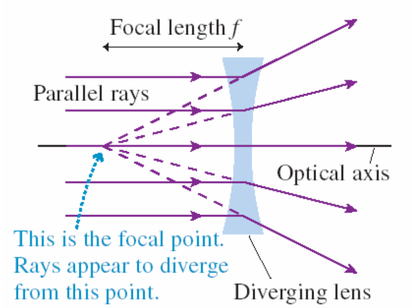
Thin Lenses: Ray Tracing

FIGURE 23.34 The focal point and focal length of converging and diverging lenses.



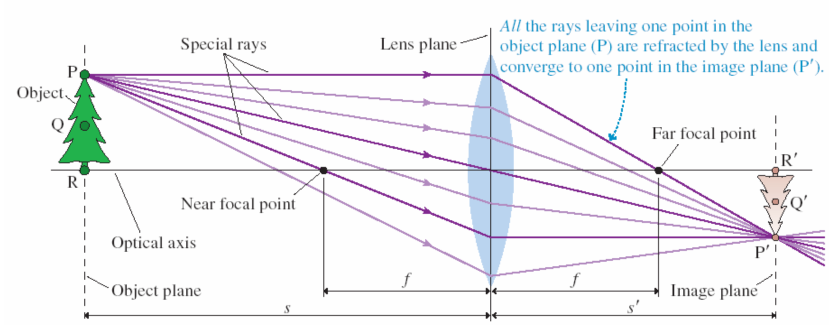
Thin Lenses: Ray Tracing

FIGURE 23.34 The focal point and focal length of converging and diverging lenses.



Thin Lenses: Ray Tracing

FIGURE 23.36 Rays from an object point P are refracted by the lens and converge to a real image at point P′.



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

Tactics: Ray tracing for a converging lens

TACTICS BOX 23.2 **Ray tracing for a converging lens**

1. **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.
2. **Center the lens on the axis.** Mark and label the focal points at distance f on either side.
3. **Represent the object with an upright arrow at distance s.** It's usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the lens.

Tactics: Ray tracing for a converging lens

4. **Draw the three "special rays" from the tip of the arrow.** Use a straight-edge.
   - **a.** A ray parallel to the axis refracts through the far focal point.
   - **b.** A ray that enters the lens along a line through the near focal point emerges parallel to the axis.
   - **c.** A ray through the center of the lens does not bend.

Tactics: Ray tracing for a converging lens

5. **Extend the rays until they converge.** This is the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.
6. **Measure the image distance s'.** Also, if needed, measure the image height relative to the object height.

Exercises 22–27

Lateral Magnification

The image can be either larger or smaller than the object, depending on the location and focal length of the lens. The **lateral magnification** m is defined as

$$m = -\frac{s'}{s}$$

1. A positive value of m indicates that the image is upright relative to the object. A negative value of m indicates that the image is inverted relative to the object.
2. The absolute value of m gives the size ratio of the image and object: $h'/h = |m|$.

EXAMPLE 23.10 Magnifying a flower

QUESTION:

EXAMPLE 23.10 Magnifying a flower
To see a flower better, a naturalist holds a 6.0-cm-focal-length magnifying glass 4.0 cm from the flower. What is the magnification?

EXAMPLE 23.10 Magnifying a flower

MODEL The flower is in the object plane. Use ray tracing to locate the image.

EXAMPLE 23.10 Magnifying a flower

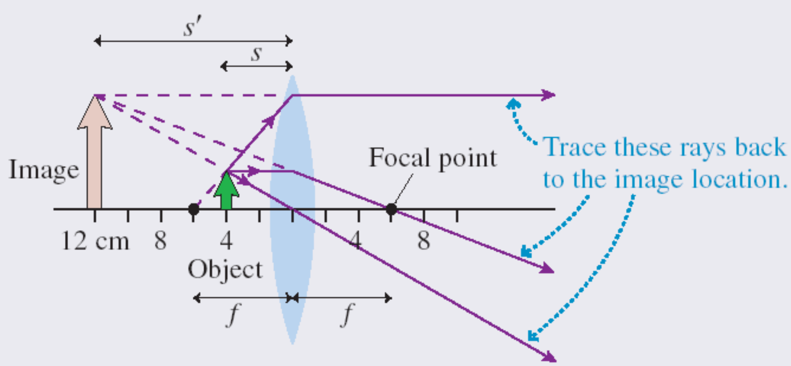
VISUALIZE FIGURE 23.41 shows the ray-tracing diagram. The three special rays diverge from the lens, but we can use a straightedge to extend the rays backward to the point from which they diverge. This point, the image point, is seen to be 12 cm to the left of the lens. Because this is a virtual image, the image distance is $s' = -12$ cm. Thus the magnification is

$$m = -\frac{s'}{s} = -\frac{-12\text{ cm}}{4.0\text{ cm}} = 3.0$$

The image is three times as large as the object and, because m is positive, upright.

EXAMPLE 23.10 Magnifying a flower

FIGURE 23.41 Ray-tracing diagram for Example 23.10.



Tactics: Ray tracing for a diverging lens

TACTICS BOX 23.3 **Ray tracing for a diverging lens**

❶–❸ **Follow steps 1 through 3 of Tactics Box 23.2.**

❹ **Draw the three "special rays" from the tip of the arrow.** Use a straight-edge.

a. A ray parallel to the axis diverges along a line through the near focal point.
b. A ray along a line toward the far focal point emerges parallel to the axis.
c. A ray through the center of the lens does not bend.

❺ **Trace the diverging rays backward.** The point from which they are diverging is the image point, which is always a virtual image.

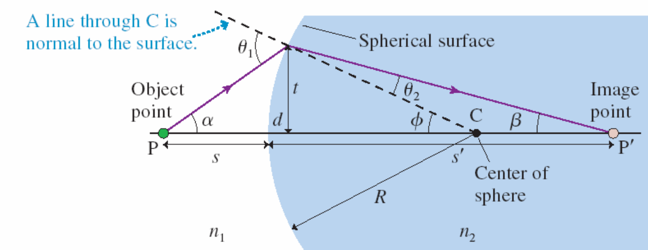
❻ **Measure the image distance s'.** This will be a negative number.

Exercise 28

Thin Lenses: Refraction Theory

Consider a spherical boundary between two transparent media with indices of refraction n_1 and n_2. The sphere has radius of curvature R and is centered at point C.

FIGURE 23.44 Image formation due to refraction at a spherical surface. The angles are exaggerated.



Thin Lenses: Refraction Theory

If an object is located at distance s from a spherical refracting surface, an image will be formed at distance s' given by

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

TABLE 23.3 Sign convention for refracting surfaces

	Positive	Negative
R	Convex toward the object	Concave toward the object
s'	Real image, opposite side from object	Virtual image, same side as object

A plane can be thought of as a sphere with $R \to \infty$. Then we find

$$s' = \frac{n_2}{n_1} s$$

EXAMPLE 23.13 A goldfish in a bowl

QUESTION:

EXAMPLE 23.13 **A goldfish in a bowl**

A goldfish lives in a spherical fish bowl 50 cm in diameter. If the fish is 10 cm from the near edge of the bowl, where does the fish appear when viewed from the outside?

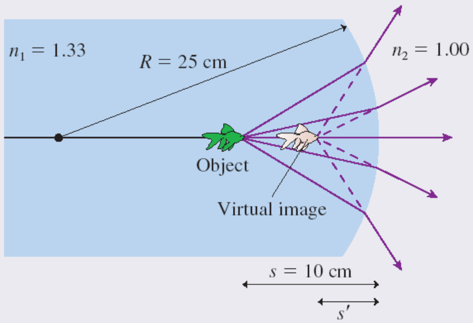
EXAMPLE 23.13 A goldfish in a bowl

MODEL Model the fish as a point source and consider the paraxial rays that refract from the water into the air. The thin glass wall has little effect and will be ignored.

EXAMPLE 23.13 A goldfish in a bowl

VISUALIZE **FIGURE 23.46** shows the rays refracting *away* from the normal as they move from the water into the air. We expect to find a virtual image at a distance less than 10 cm.

FIGURE 23.46 The curved surface of a fish bowl produces a virtual image of the fish.



EXAMPLE 23.13 A goldfish in a bowl

SOLVE The object is in the water, so $n_1 = 1.33$ and $n_2 = 1.00$. The inner surface is concave (you can remember "concave" because it's like looking into a cave), so $R = -25$ cm. The object distance is $s = 10$ cm. Thus Equation 23.21 is

$$\frac{1.33}{10\text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.33}{-25\text{ cm}} = \frac{0.33}{25\text{ cm}}$$

Solving for the image distance s' gives

$$\frac{1.00}{s'} = \frac{0.33}{25\text{ cm}} - \frac{1.33}{10\text{ cm}} = -0.12\text{ cm}^{-1}$$

$$s' = \frac{1.00}{-0.12\text{ cm}^{-1}} = -8.3\text{ cm}$$

EXAMPLE 23.13 A goldfish in a bowl

ASSESS The image is virtual, located to the left of the boundary. A person looking into the bowl will see a fish that appears to be 8.3 cm from the edge of the bowl.

The Thin Lens Equation

The object distance s is related to the image distance s' by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

where f is the focal length of the lens, which can be found from

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{(lens maker's equation)}$$

where R_1 is the radius of curvature of the first surface, and R_2 is the radius of curvature of the second surface, and the material surrounding the lens has $n = 1$.

EXAMPLE 23.15 Designing a lens

QUESTION:

EXAMPLE 23.15 Designing a lens
The objective lens of a microscope uses a planoconvex glass lens with the flat side facing the specimen. A real image is formed 160 mm behind the lens when the lens is 8.0 mm from the specimen. What is the radius of the lens's curved surface?

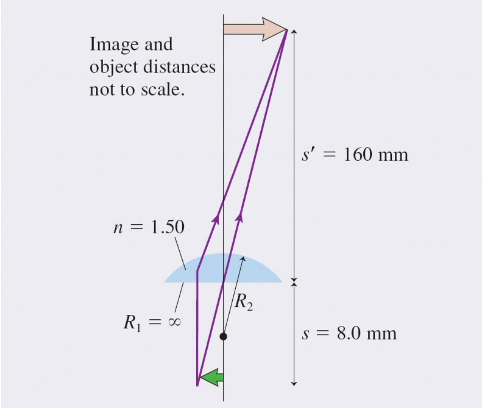
EXAMPLE 23.15 Designing a lens

MODEL Treat the lens as a thin lens. Its focal length is given by the lens maker's equation.

EXAMPLE 23.15 Designing a lens

VISUALIZE **FIGURE 23.49** clarifies the shape of the lens and defines R_2. The index of refraction was taken from Table 23.1.

FIGURE 23.49 A planoconvex microscope lens.



EXAMPLE 23.15 Designing a lens

SOLVE We can use the lens maker's equation to solve for R_2 if we know the lens's focal length. Because we know both the object and image distances, we can use the thin-lens equation to find

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{8.0\text{ mm}} + \frac{1}{160\text{ mm}} = 0.131\text{ mm}^{-1}$$

The focal length is $f = 1/(0.131\text{ mm}^{-1}) = 7.6$ mm, but $1/f$ is all we need for the lens maker's equation. The front surface of the lens is planar, which we can consider a portion of a sphere with $R_1 \to \infty$. Consequently $1/R_1 = 0$.

EXAMPLE 23.15 Designing a lens

With this, we can solve the lens maker's equation for R_2:

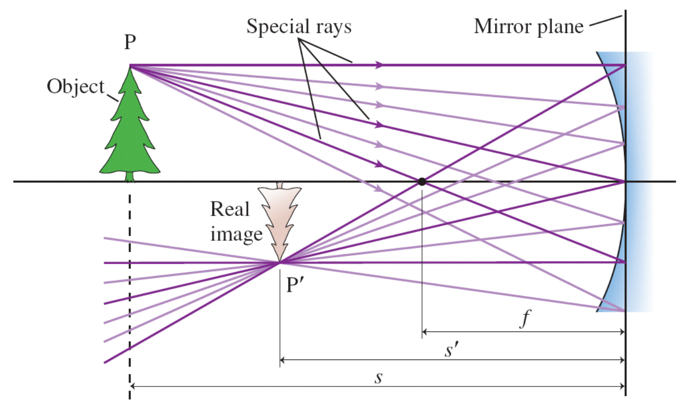
$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{n-1}\frac{1}{f} = 0 - \left(\frac{1}{1.50 - 1}\right)(0.131\text{ mm}^{-1})$$
$$= -0.262\text{ mm}^{-1}$$
$$R_2 = -3.8\text{ mm}$$

The minus sign appears because the curved surface is concave toward the object. Physically, the radius of the curved surface is 3.8 mm.

EXAMPLE 23.15 Designing a lens

ASSESS The actual thickness of the lens is much less than R_2, probably no more than 1.0 mm. This thickness is significantly less than the object and image distances, so the thin-lens approximation is justified.

FIGURE 23.52 A real image formed by a concave mirror.



Tactics: Ray tracing for a spherical mirror

TACTICS BOX 23.4 Ray tracing for a spherical mirror

1. **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.
2. **Center the mirror on the axis.** Mark and label the focal point at distance f from the mirror's surface.
3. **Represent the object with an upright arrow at distance s.** It's usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the mirror.

Tactics: Ray tracing for a spherical mirror

4. **Draw the three "special rays" from the tip of the arrow.** Use a straightedge.
   - **a.** A ray parallel to the axis reflects through (concave) or away from (convex) the focal point.
   - **b.** An incoming ray passing through (concave) or heading toward (convex) the focal point reflects parallel to the axis.
   - **c.** A ray that strikes the center of the mirror reflects at an equal angle on the opposite side of the optical axis.

Tactics: Ray tracing for a spherical mirror

5. **Extend the rays forward or backward until they converge.** This is the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.
6. **Measure the image distance s'.** Also, if needed, measure the image height relative to the object height.

Exercises 32–33

The Mirror Equation

For a spherical mirror with negligible thickness, the object and image distances are related by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-mirror equation)}$$

where the focal length f is related to the mirror's radius of curvature by

$$f = \frac{R}{2}$$

TABLE 23.5 Sign convention for spherical mirrors

	Positive	Negative
R and f	Concave toward the object	Convex toward the object
s'	Real image, same side as object	Virtual image, opposite side from object

Chapter 23. Summary Slides

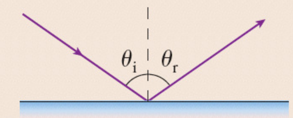
General Principles

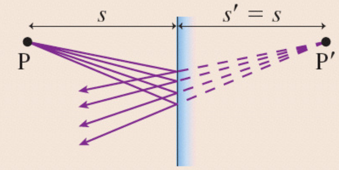
Reflection

Law of reflection: $\theta_r = \theta_i$

Reflection can be **specular** (mirror-like) or **diffuse** (from rough surfaces).

Plane mirrors: A virtual image is formed at P' with $s' = s$.



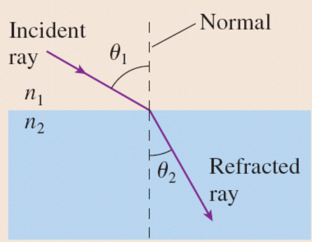


General Principles

Refraction

Snell's law of refraction:
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

Index of refraction is $n = c/v$. The ray is closer to the normal on the side with the larger index of refraction.



If $n_2 < n_1$, total internal reflection (TIR) occurs when the angle of incidence $\theta_1 \geq \theta_c = \sin^{-1}(n_2/n_1)$.

Important Concepts

The ray model of light

Light travels along straight lines, called **light rays,** at speed $v = c/n$.

A light ray continues forever unless an interaction with matter causes it to reflect, refract, scatter, or be absorbed.

Light rays come from **objects.** Each point on the object sends rays in all directions.

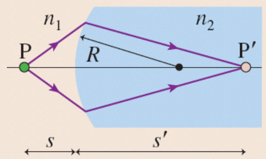
The eye sees an object (or an image) when diverging rays are collected by the pupil and focused on the retina.

► Ray optics is valid when lenses, mirrors, and apertures are larger than ≈ 1 mm.

Important Concepts

Image formation

If rays diverge from P and interact with a lens or mirror so that the refracted/reflected rays *diverge* from P′ and appear to come from P′, then P′ is a virtual image of P.



If rays diverge from P and interact with a lens or mirror so that the refracted rays *converge* at P′, then P′ is a real image of P.

Spherical surface: Object and image distances are related by

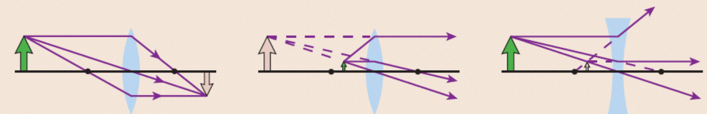
$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

Plane surface: $R \to \infty$, so $|s'/s| = n_2/n_1$.

Applications

Ray tracing

3 special rays in 3 basic situations:



Converging lens Real image — Converging lens Virtual image — Diverging lens Virtual image

Magnification $m = -\frac{s'}{s}$

m is + for an upright image, − for inverted.

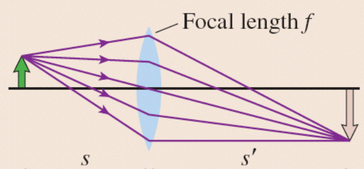
The height ratio is $h'/h = |m|$.

Applications

Thin lenses

The image and object distances are related by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$



where the focal length is given by the lens maker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

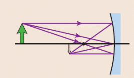
R	+ for surface convex toward object	− for concave
f	+ for a converging lens	− for diverging
s'	+ for a real image	− for virtual

Applications

Spherical mirrors

The image and object distances are related by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

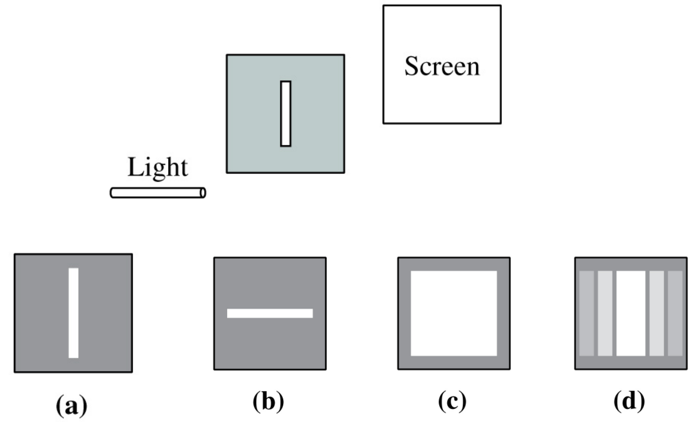


R, f	+ for concave mirror	− for convex
s'	+ for a real image	− for virtual

Focal length $f = R/2$

Chapter 23. Clicker Questions

A long, thin light bulb illuminates a vertical aperture. Which pattern of light do you see on a viewing screen behind the aperture?



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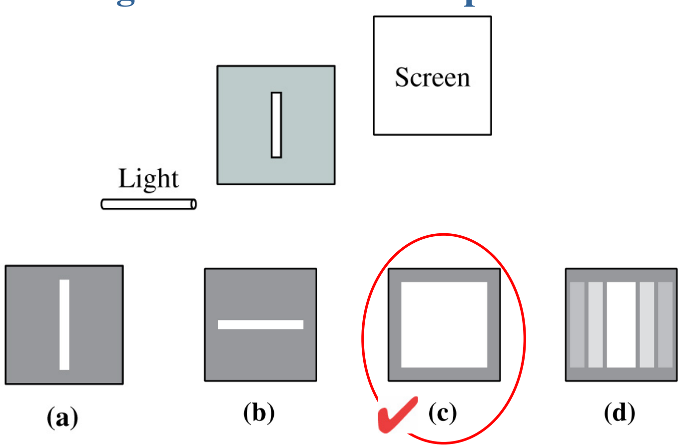
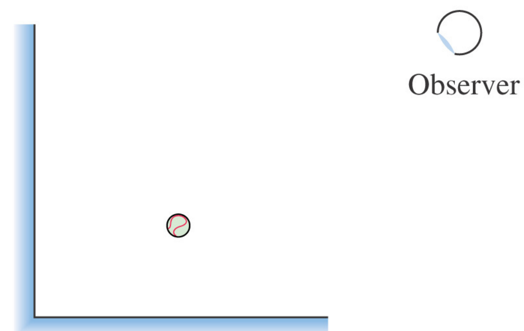


Two plane mirrors form a right angle. How many images of the ball can you see in the mirrors?

A. 1
B. 2
C. 3
D. 4



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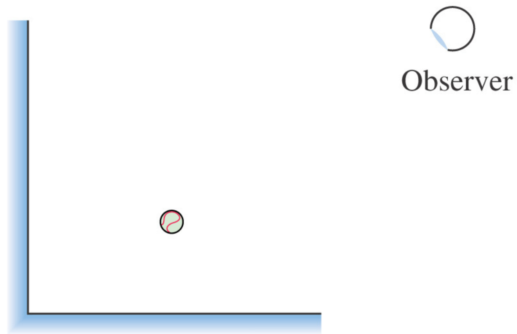


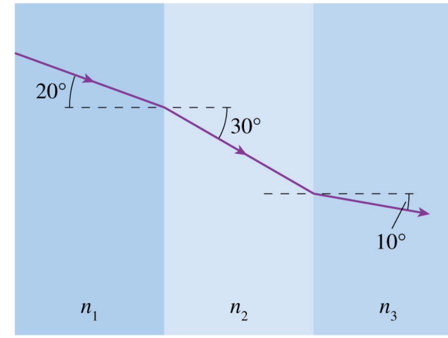
A. 1

B. 2

✔ **C. 3**

D. 4

A light ray travels from medium 1 to medium 3 as shown. For these media,



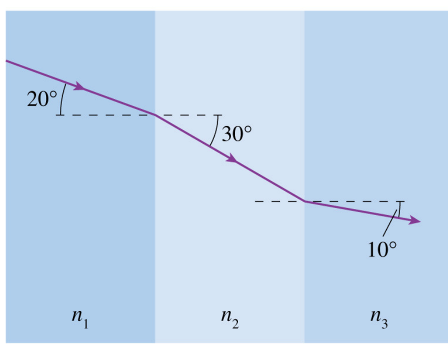
A. $n_3 = n_1$.

B. $n_3 > n_1$.

C. $n_3 < n_1$.

D. We can't compare n_1 to n_3 without knowing n_2.

A light ray travels from medium 1 to medium 3 as shown. For these media,



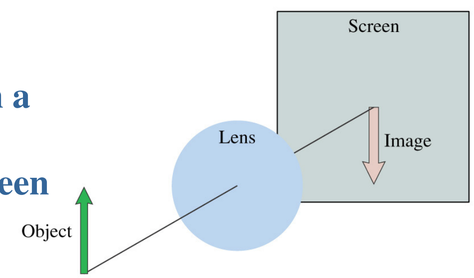
A. $n_3 = n_1$.

✔ **B. $n_3 > n_1$.**

C. $n_3 < n_1$.

D. We can't compare n_1 to n_3 without knowing n_2.

A lens produces a sharply-focused, inverted image on a screen. What will you see on the screen if the lens is removed?



A. The image will be inverted and blurry.

B. The image will be as it was, but much dimmer.

C. There will be no image at all.

D. The image will be right-side-up and sharp.

E. The image will be right-side-up and blurry.

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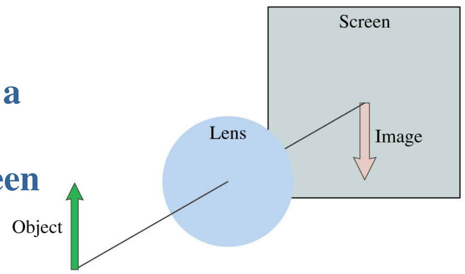
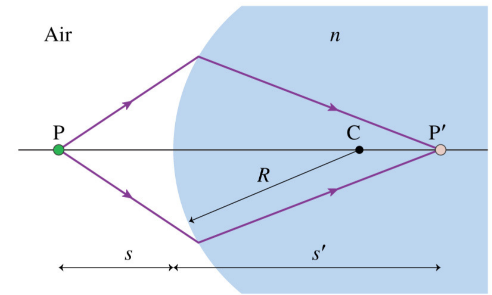


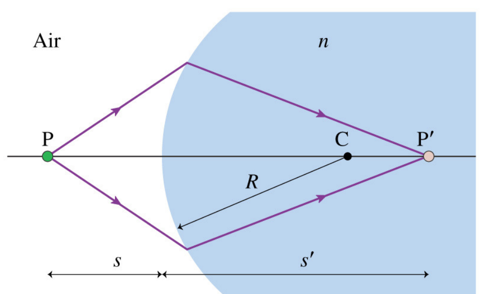
A. The image will be inverted and blurry.
B. The image will be as it was, but much dimmer.
✔ **C. There will be no image at all.**
D. The image will be right-side-up and sharp.
E. The image will be right-side-up and blurry.

Which of these actions will move the image point P´ further from the boundary?



A. Decrease the radius of curvature *R*.
B. Increase the index of refraction *n*.
C. Increase the radius of curvature *R*.
D. Increase the object distance *s*.

Which of these actions will move the image point P´ further from the boundary?



A. Decrease the radius of curvature *R*.
B. Increase the index of refraction *n*.
✔ **C. Increase the radius of curvature *R*.**
D. Increase the object distance *s*.

The image of a slide on the screen is blurry because the screen is in front of the image plane. To focus the image, should you move the lens toward the slide or away from the slide?

A. Away from the slide.
B. Toward the slide.

The image of a slide on the screen is blurry because the screen is in front of the image plane. To focus the image, should you move the lens toward the slide or away from the slide?

✔ **A. Away from the slide.**
B. Toward the slide.

A concave mirror of focal length *f* forms an image of the moon. Where is the image located?

A. Almost exactly a distance behind the mirror.
B. Almost exactly a distance in front of the mirror.
C. At a distance behind the mirror equal to the distance of the moon in front of the mirror.
D. At the mirror's surface.

A concave mirror of focal length *f* forms an image of the moon. Where is the image located?

A. Almost exactly a distance behind the mirror.
✔ **B. Almost exactly a distance in front of the mirror.**
C. At a distance behind the mirror equal to the distance of the moon in front of the mirror.
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