Chapter 1 Representing Motion

Chapter Goal: To introduce the fundamental concepts of motion and to review related basic mathematical principles.

Chapter 1 Preview
Looking Ahead: Describing Motion

• This series of images of a skier clearly shows his motion. Such visual depictions are a good first step in describing motion.

• You’ll learn to make motion diagrams that provide a simplified view of the motion of an object.

Chapter 1 Preview
Looking Ahead: Numbers and Units

• Quantitative descriptions involve numbers, and numbers require units. This speedometer gives speed in mph and km/h.

• You’ll learn the units used in science, and you’ll learn to convert between these and more familiar units.
Chapter 1 Preview
Looking Ahead

Chapter 1 Preview
Looking Back: Trigonometry

• In a previous course, you learned mathematical relationships among the sides and the angles of triangles.

• In this course you’ll use these relationships to analyze motion and other problems.

Reading Question 1.1
What is the difference between speed and velocity?

A. Speed is an average quantity while velocity is not.
B. Velocity contains information about the direction of motion while speed does not. ✔
C. Speed is measured in mph, while velocity is measured in m/s.
D. The concept of speed applies only to objects that are neither speeding up nor slowing down, while velocity applies to every kind of motion.
E. Speed is used to measure how fast an object is moving in a straight line, while velocity is used for objects moving along curved paths.
Reading Question 1.2
The quantity $2.67 \times 10^3$ has how many significant figures?

A. 1
B. 2
C. 3
D. 4
E. 5

Reading Question 1.3
The correct SI units for distance and mass are

A. Feet, pounds.
B. Centimeters, grams.
C. Meters, grams.
D. Meters, kilograms.

Correct Answer:

C. 3

D. Meters, kilograms.
**Reading Question 1.4**

If Sam walks 100 m to the right, then 200 m to the left, his net displacement vector

A. Points to the right.
B. Points to the left.
C. Has zero length.
D. Cannot tell without more information.

**Reading Question 1.5**

Velocity vectors point

A. In the same direction as displacement vectors.
B. In the opposite direction as displacement vectors.
C. Perpendicular to displacement vectors.
D. In the same direction as acceleration vectors.
E. Velocity is not represented by a vector.
Section 1.1 Motion: A First Look

Types of Motion

- **Motion** is the change of an object’s position or orientation with time.

  - The path along which an object moves is called the object’s **trajectory**.

Making a Motion Diagram

- These motion diagrams in one dimension show objects moving at constant speed (skateboarder), speeding up (runner) and slowing down (car).

Making a Motion Diagram

- This motion diagram shows motion in two dimensions with changes in both speed and direction.
QuickCheck 1.1

Motion diagrams are made of two cars. Both have the same time interval between photos. Which car, A or B, is going slower?

Car A

Car B

QuickCheck 1.1

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The Particle Model

• The particle model of motion is a simplification in which we treat a moving object as if all of its mass were concentrated at a single point

(a) Motion diagram of a car stopping

(b) Same motion diagram using the particle model

The same amount of time elapses between each frame and the next.

0 1 2 3

Numbers show the order in which the frames were taken. A single dot is used to represent the object.

Section 1.2 Position and Time: Putting Numbers on Nature
Position and Coordinate Systems

- To specify **position** we need a reference point (the **origin**), a **distance** from the origin, and a **direction** from the origin.

  The post office defines the zero, or origin, of the coordinate system.

  ![Coordinate System Diagram](image)

  - This cow is at position -5 miles.
  - Your car is at position +4 miles.

- The combination of an origin and an **axis** marked in both the positive and negative directions makes a **coordinate system**.

Time

- For a complete motion diagram we need to label each frame with its corresponding time (symbol \( t \)) as read off a clock.

  If we’re interested in the entire motion of the car, we assign this point the time \( t = 0 \) s.

  ![Motion Diagram](image)

  - If we’re interested in only the braking part of the motion, we assign \( t = 0 \) s here.

Changes in Position and Displacement

- A **change** of position is called a **displacement**.

  ![Displacement Diagram](image)

  - Displacement is the **difference** between a final position and an initial position:
    \[
    \Delta x = x_f - x_i = 150 \text{ ft} - 50 \text{ ft} = 100 \text{ ft}
    \]
Change in Time

- In order to quantify motion, we’ll need to consider changes in time, which we call time intervals.

A time interval $\Delta t$ measures the elapsed time as an object moves from an initial position $x_i$ at time $t_i$ to a final position $x_f$ at time $t_f$. $\Delta t$ is always positive.

QuickCheck 1.4

Maria is at position $x = 23$ m. She then undergoes a displacement $\Delta x = -50$ m. What is her final position?

- A. $-27$ m
- B. $-50$ m
- C. $23$ m
- D. $73$ m

Correct Answer: A. $-27$ m

Section 1.3 Velocity
Velocity and Speed

- Motion at a constant speed in a straight line is called **uniform motion**.

Example Problem

Jane walks to the right at a constant rate, moving 3 m in 3 s. At $t = 0$ s she passes the $x = 1$ m mark. Draw her motion diagram from $t = -1$ s to $t = 4$ s.

Velocity and Speed

- Speed measures only how fast an object moves, but velocity tells us both an object’s speed and its direction.

- The velocity defined by Equation 1.2 is called the **average velocity**.

**Section 1.4 A Sense of Scale:**
Significant Figures, Scientific Notation, and Units
Measurements and Significant Figures

• When we measure any quantity we can do so with only a certain precision.

• We state our knowledge of a measurement through the use of significant figures: digits that are reliably known.

QuickCheck 1.7

Rank in order, from the most to the least, the number of significant figures in the following numbers. For example, if b has more than c, c has the same number as a, and a has more than d, you would give your answer as b > c = a > d.

a. 8200  
   b. 0.0052  
   c. 0.430  
   d. 4.321 \times 10^{-10}

A. d > c > b = a
B. a = b = d > c
C. b = d > c > a
D. d > c > a > b
E. a = d > c > b

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Scientific Notation

- Writing very large (much greater than 1) and very small (much less than 1) numbers is cumbersome and does not make clear how many significant figures are involved.

Units

- Scientists use a system of units called *le Système International d’Unités*, commonly referred to as SI Units.

**TABLE 1.1** Common SI units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
</tbody>
</table>

Unit Conversions

**TACTICS BOX 1.2**

- **Making a unit conversion**
  1. Start with the quantity you wish to convert.
  2. Multiply by the appropriate conversion factor. Because this conversion factor is equal to 1, multiplying by it does not change the value of the quantity—only its units.
  3. You can cancel the original unit (here, miles) because it appears in both the numerator and the denominator.
  4. Calculate the answer; it is in the desired units. Remember, 60 mi and 96.54 km are the same distance; they are simply in different units.
  5. Remember to count your final answer to the correct number of significant figures!
Estimation

- A one-significant-figure estimate or calculation is called an order-of-magnitude estimate.
- An order-of-magnitude estimate is indicated by the symbol ~, which indicates even less precision than the “approximately equal” symbol ≈.

**Example 1.5 How fast do you walk?**

Estimate how fast you walk, in meters per second.

**PREPARE** In order to compute speed, we need a distance and a time. If you walked a mile to campus, how long would this take? You’d probably say 30 minutes or so—half an hour. Let’s use this rough number in our estimate.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>SI unit</th>
<th>Approximate conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kg</td>
<td>1 kg ~ 2 lb</td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>1 m ~ 3 ft</td>
</tr>
<tr>
<td></td>
<td>cm</td>
<td>3 cm ~ 1 in</td>
</tr>
<tr>
<td></td>
<td>km</td>
<td>5 km ~ 3 mi</td>
</tr>
<tr>
<td>Speed</td>
<td>m/s</td>
<td>1 m/s = 2 mph</td>
</tr>
<tr>
<td></td>
<td>km/h</td>
<td>10 km/h ~ 6 mph</td>
</tr>
</tbody>
</table>

**SOLVE** Given this estimate, we compute your speed as

\[
\text{speed} = \frac{\text{distance}}{\text{time}} \sim \frac{1 \text{ mile}}{1/2 \text{ hour}} = \frac{2 \text{ mi}}{\text{h}}
\]

But we want the speed in meters per second. Since our calculation is only an estimate, we use an approximate conversion factor from Table 1.4:

\[
\frac{1 \text{ mi}}{\text{h}} \sim 0.5 \frac{\text{m}}{\text{s}}
\]

This gives an approximate walking speed of 1 m/s.

**ASSESS** Is this a reasonable value? Let’s do another estimate. Your stride is probably about 1 yard long—about 1 meter. And you take about one step per second; next time you are walking, you can count and see. So a walking speed of 1 meter per second sounds pretty reasonable.
Scalars and Vectors

- When a physical quantity is described by a single number (with a unit), we call it a **scalar quantity**.
- A **vector quantity** is a quantity that has both a size (How far? or How fast?) and a direction (Which way?).
- The size or length of a vector is called its **magnitude**.
- We graphically represent a vector as an **arrow**.

Displacement Vectors

- The displacement vector represents the distance and direction of an object’s motion.
- An object’s displacement vector is drawn from the object’s initial position to its final position, regardless of the actual path followed between these two points.

Vector Addition

- The net displacement for a trip with two legs is the sum of the two displacements that made it up.
QuickCheck 1.6

Given vectors \( \vec{p} \) and \( \vec{q} \), what is \( \vec{p} + \vec{q} \)?

A. 
B. 
C. 
D.

Example 1.7 How far away is Anna?

Anna walks 90 m due east and then 50 m due north. What is her displacement from her starting point?

**PREPARE** Let's start with the sketch in FIGURE 1.25a. We set up a coordinate system with Anna’s original position as the origin, and then we drew her two subsequent motions as the two displacement vectors \( \vec{d}_1 \) and \( \vec{d}_2 \).

**SOLVE** We drew the two vector displacements with the tail of one vector starting at the head of the previous one—exactly what is needed to form a vector sum. The vector \( \vec{d}_{\text{net}} \) in FIGURE 1.25a is the vector sum of the successive displacements and thus represents Anna’s net displacement from the origin.
Example 1.7 How far away is Anna? (cont.)

Anna’s distance from the origin is the length of this vector \( \vec{d}_{\text{net}} \).

FIGURE 1.25b shows that this vector is the hypotenuse of a right triangle with sides 50 m (because Anna walked 50 m north) and 90 m (because she walked 90 m east). We can compute the magnitude of this vector, her net displacement, using the Pythagorean theorem (the square of the length of the hypotenuse of a triangle is equal to the sum of the squares of the lengths of the sides):

\[
\vec{d}_{\text{net}} = (50\text{ m})^2 + (90\text{ m})^2
\]

\[
\vec{d}_{\text{net}} = \sqrt{(50\text{ m})^2 + (90\text{ m})^2} = 103\text{ m} \approx 100\text{ m}
\]

ASSESS We can use our drawing to assess our result. If the two sides of the triangle are 50 m and 90 m, a length of 100 m for the hypotenuse seems about right. The angle is certainly less than 45°, but not too much less, so 29° seems reasonable.

Example 1.7 How far away is Anna? (cont.)

We have rounded off to the appropriate number of significant figures, giving us 100 m for the magnitude of the displacement vector. How about the direction? Figure 1.25b identifies the angle that gives the angle north of east of Anna’s displacement. In the right triangle, 50 m is the opposite side and 90 m is the adjacent side, so the angle is given by

\[
\theta = \tan^{-1}\left(\frac{30\text{ m}}{90\text{ m}}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 29°
\]

Putting it all together, we get a net displacement of

\[
\vec{d}_{\text{net}} = (100\text{ m}, 29° \text{ north of east})
\]

Velocity Vectors

- We represent the velocity of an object by a velocity vector that points in the direction of the object’s motion, and whose magnitude is the object’s speed.
Example 1.8 Drawing a ball’s motion diagram

Jake hits a ball at a 60° angle from the horizontal. It is caught by Jim. Draw a motion diagram of the ball that shows velocity vectors rather than displacement vectors.

The motion diagram of a ball traveling from Jake to Jim

Summary and Organization of Chapters

- This chapter has been an introduction to some of the fundamental ideas about motion and some of the basic techniques that you will use.
- Each new chapter depends on those that preceded it.
- Each chapter begins with a chapter preview that will let you know which topics are especially important to review.
- The last element in each chapter will be an integrated example that brings together the principles and techniques you have just learned with those you learned previously.

Summary: Important Concepts

Motion Diagrams

The particle model represents a moving object as if all its mass were concentrated at a single point. Using this model, we can represent motion with a motion diagram, where dots indicate the object’s positions at successive times. In a motion diagram, the time interval between successive dots is always the same.

Each dot represents the position of the object. Each position is labeled with the time at which the dot was there.

The time interval between successive positions is the same.

Scalars and Vectors

Scalar quantities have only a magnitude and can be represented by a single number. Temperature, time, and mass are scalars.

A vector is a quantity described by both a magnitude and a direction. Velocity and displacement are vectors.

Velocity vectors can be drawn on a motion diagram by connecting successive points with a vector.

The length of a vector is proportional to its magnitude.

Velocity vectors span successive points in a motion diagram.

The velocity vectors are getting longer, so the object is speeding up.
**Summary: Important Concepts**

### Describing Motion

**Position** locates an object with respect to a chosen coordinate system. It is described by a coordinate. The coordinate is the variable used to describe the position.

A change in position is called a displacement. For motion along a line, a displacement is a signed quantity. The displacement from $x_i$ to $x_f$ is $\Delta x = x_f - x_i$.

**Time** is measured from a particular instant to which we assign $t = 0$. A time interval is the elapsed time between two specific instants $t_i$ and $t_f$. It is given by $\Delta t = t_f - t_i$.

**Velocity** is the ratio of the displacement of an object to the time interval during which this displacement occurs:

$$v = \frac{\Delta x}{\Delta t}$$

**Units**

Every measurement of a quantity must include a unit. The standard system of units used in science is the SI system. Common SI units include:

- Length: meters (m)
- Time: seconds (s)
- Mass: kilograms (kg)

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**Summary: Applications**

### Working with Numbers

In scientific notation, a number is expressed as a decimal number between 1 and 10 multiplied by a power of ten. In scientific notation, the diameter of the earth is $1.27 \times 10^7$ m.

A prefix can be used before a unit to indicate a multiple of 10 or 1/10. Thus we can write the diameter of the earth as 12,700 km, where the k in km denotes 1000.

We can perform a unit conversion to convert the diameter of the earth to a different unit, such as miles. We do so by multiplying by a conversion factor equal to 1, such as $1 = 1$ mi/1.61 km.

**Significant figures** are reliably known digits. The number of significant figures for:

- Multiplication, division, and powers is set by the value with the fewest significant figures.
- Addition and subtraction is set by the value with the smallest number of decimal places.

An order-of-magnitude estimate is an estimate that has an accuracy of about one significant figure. Such estimates are usually made using rough numbers from everyday experience.
IMPORTANT CONCEPTS

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Applications

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