Experiment 9

Capacitors in AC Circuits

Preparation
Prepare for this week's experiment by reviewing past material and reading about AC circuits, capacitive reactance, and RC filters.

Principles
In an AC circuit the direction of the current changes sinusoidally with some angular frequency, $\omega = 2\pi f$. Consider a circuit consisting of a resistor and capacitor wire in series with an AC power supply or signal generator.

Assume that the current is

$$I(t) = I_o \sin(\omega t).$$

Then the voltage across the resistor will also vary sinusoidally can be expressed as

$$V_R = I_o R \sin(\omega t).$$

The resistor voltage rises and falls at the same time as the current. Its magnitude will be $I_o R$. It is said to be in phase with the current. This is not the case with the capacitor.

Remember, the capacitor voltage is a function of the charge on the plates, not of the current. However, charge and current are related as

$$I = \frac{dq}{dt}.$$

Since

$$V_C = \frac{q}{C},$$

then

$$\frac{dV_C}{dt} = \frac{1}{C} \frac{dq}{dt} = \frac{1}{C} I = \frac{1}{C} I_o \sin(\omega t).$$

Rearrange and integrate this equation to find $V_C$. The result is

$$V_C = \frac{-I_o}{\omega C} \cos(\omega t) = \frac{-I_o}{2\pi f C} \cos(\omega t) = \frac{-I_o}{2\pi f C} \sin\left(\omega t - \frac{\pi}{2}\right).$$

Note that $V_C$ rises and drops with the same frequency as the current, but not at the same time. The capacitor voltage will be at its maximum positive or negative value when the current is zero. The capacitor voltage will be zero when the current is the greatest. There is said to be a phase shift between the current and the capacitor voltage.
The quantity $\frac{1}{\omega C} = \frac{1}{2\pi f C}$ is called the \textbf{capacitive reactance} of the capacitor and is represented by $X_C$. It is a function of both the frequency and the capacitance. The reactance is a measure of how the capacitor impedes the flow of current in the circuit. Its unit is ohms. The magnitude of $V_C$ will be $I_O X_C$.

Kirchhoff’s rules also apply to AC circuits. At any instant the sum of the resistor and capacitor voltages will equal the signal generator voltage, $V_Z$. However, since these two values rise and fall at different times you cannot simply add their magnitudes to find the magnitude of the generator voltage. They can be added like two perpendicular vectors,

$$V_Z = \sqrt{V_C^2 + V_R^2}.$$

All the components in the circuit work together to impede the flow of current. This is analogous to the equivalent resistance in simple resistor circuits. The total resistance of the circuit is replaced by the \textbf{impedance}, $Z$, such that

$$V_Z = I_O Z.$$

Since the amplitudes of all the voltages in the circuit are proportional to the amplitude of the current, $I_O$, we can divide out the current to find an expression for the impedance.

$$V_Z = \sqrt{V_C^2 + V_R^2}$$

$$I_O Z = \sqrt{I_O^2 X_C^2 + I_O^2 R^2}$$

$$Z = \sqrt{X_C^2 + R^2}$$

Since $I_O$ varies with frequency, the magnitudes of $V_C$ and $V_R$ must also be frequency dependent. $X_C$ decreases as frequency increases, consequently $Z$ decreases and $I_O$ and $V_R$ increase. The capacitor voltage will decrease as frequency rises.

The generator voltage will not be in phase with either of the voltages. It will differ from the current by a \textbf{phase angle} $\phi$ where

$$\tan \phi = \frac{-X_C}{R}.$$

An RC circuit can be used as a filter. The signal generator voltage is your \textbf{input voltage}, $V_{in}$. If you measure the output voltage, $V_{out}$, across the resistor you will have a \textbf{high pass filter} since at high frequencies the voltage will be larger. If you measure $V_{out}$ across the capacitor you will have a \textbf{low pass filter} since at low frequencies the voltage will be larger. There are many practical applications for filter circuits.
Figure 1. Series RC circuits and phasor diagram.

**Equipment**

1. protoboard
1. multimeter and leads
1. oscilloscope
1. signal generator with power cord
2. three prong adapters
3. BNC connectors
3. small wires
1. 330 Ω resistor
1. 0.05 µF capacitor
3. 18" banana wires, one red, two black
2. 24" banana wires, one red, one black
2. spade lugs
Using the Oscilloscope

The oscilloscope is used to display time-varying voltages. The horizontal (x) axis represents time and the vertical axis (y) represents voltage. The following list will familiarize you with the features of the oscilloscope and will also tell you procedures you should use every time you use the scope.

**Sweep**
This determines the horizontal scale for the display. The scale is read in the upper white window. Its units are seconds/division. The divisions on the screen are the blocks that are about 1 cm square. Along the center axes are five subdivisions, each 0.2 divisions wide.

**Horizontal Position**
This enables you to move the signal back and forth along the x-axis. In this way, the value of the signal at the origin can be determined.

**Sweep Calibration**
This enables you to change the horizontal scale. Unless this knob is turned all the way in the direction of the arrow on the knob, the scope is not calibrated and your data is worthless. Turn this knob until it clicks and check it frequently as you take data.

**Sensitivity**
This determines the vertical scale. It is read in the left hand white window. The units are volts/division.

**Vertical Position**
This knob controls the vertical position of the trace. You will find it very convenient when you are setting or reading voltages.

**Sensitivity Calibration**
This knob is used to change the vertical scale. If it is not turned all the way in the direction of the arrow on the knob, the scope is not calibrated and your data is worthless. Turn this knob until it clicks and check it frequently as you take data.

**Channel Select**
Most oscilloscopes are dual trace, which means that they can display two signals at once. This is the reason that there are two signal ports and two sensitivity controls. If you are only using one channel for an experiment, push the button for channel 1. Push “both” to display two channels at one time.
**Signal ports**
There is one signal port for each channel. A BNC (banana to coaxial) connector is used to connect banana wires to the signal port.

**AC/DC Select**
When this is set to AC, the DC part of the signal is filtered out by a capacitor placed in series between the signal input and the scope. When the selector is set to ground, the beam will move to zero volts. When the selector is set to DC, the entire signal will be displayed on the scope. Always keep the switch on DC, even if you are looking at an AC signal.
AC/DC Select
When this is set to AC, the DC part of the signal is filtered out by a capacitor placed in series between the signal input and the scope. When the selector is set to ground, the beam will move to zero volts. When the selector is set to DC, the entire signal will be displayed on the scope. Always keep the switch on DC, even if you are looking at an AC signal.

Triggering Source and Mode
You will use the scope to observe signals that repeat frequently. The scope must start the sweep at the same point on the waveform every time in order to produce a stable image on the screen. This function is called triggering. Put the source switch on internal. This lets the scope decide when to trigger. Set the mode switch to auto.

Level
This sets an internal voltage which is compared to the voltage of the input signal. When the input signal voltage is equal to the trigger voltage, the scope will trigger. If you get an image that seems to be a superposition of many waves, turn the level knob back and forth slowly until you get a stable image.

Slope
Usually the signal voltage will equal the triggering voltage twice, once as the signal voltage is increasing and once as it is decreasing. This button enables you to select which of these the scope will trigger on.

Focus
Rotate this button until the trace is sharp.

Intensity
Adjust the brightness of the trace until you can just see all the details of the waveform. If the trace is too bright, you will not get the best data, your eyes will get very tired, and you could damage the scope.

On/Off
Remember; do not use the wall plug as an on/off switch.

Procedure
The impedance bridge can be used to measure capacitances to at least three and sometimes four significant digits.
1. Use the impedance bridge to measure the capacitance of the capacitor. Use the DMM to measure the resistance of the resistor. Use ground isolators (three to two prong adapters) on the oscilloscope and the signal generator.

2. Wire the series RC circuit shown in Figure 21.

3. Set the signal generator to "sine wave" and set the generator voltage to 4 \( V_{pp} \). Be sure that the calibration knobs remain off. If your signal generator does not have a built in frequency counter, wire a frequency counter in parallel with the generator and set it to "LPF". The actual value of the signal generator voltage does not matter, but keeping it constant throughout the experiment will make your analysis simpler. Check the generator voltage throughout the experiment as it may change when the impedance changes. Be sure that the calibration knobs remain off, otherwise the data will be worthless. Record the frequency and the peak-to-peak voltages (\( V_{pp} \)) across the resistor and capacitor for the following frequencies

\[
\begin{align*}
1 \text{ kHz} & \quad \text{to} \quad 10 \text{ kHz} \quad \text{at} \quad 1 \text{ kHz intervals} \\
10 \text{ kHz} & \quad \text{to} \quad 20 \text{ kHz} \quad \text{at} \quad 2 \text{ kHz intervals} \\
20 \text{ kHz} & \quad \text{to} \quad 70 \text{ kHz} \quad \text{at} \quad 10 \text{ kHz intervals}
\end{align*}
\]

The easiest way to take your data is to follow these steps:

1. Set the frequency and record its value.
2. Check the calibration knobs on the 'scope.
3. Check the signal generator voltage and adjust it if necessary.
4. Measure and record \( V_R \).
5. Measure and record \( V_C \).
6. Go back to step 1.

4. Wire the capacitor and the resistance substitution box in series as shown in Figure 3. Be sure that the resistor box is not set to zero resistance. Calculate the resistor value that will make \( \varphi = \pi/4 \) and set the sub box to that value. In other words, set

\[
R = \frac{I}{2\pi fC}.
\]

5. Set the 'scope to "both" and wire the circuit across the entire circuit and channel 2 measures voltage across the variable resistor.
is no need for a second ground wire. Set the frequency to about 5000 Hz. Adjust the 'scope so that one complete wave covers 4 divisions on the screen. Adjust the frequency if necessary. This means that, on the screen, 4 divisions equals a phase shift of $2\pi$ radians. Two divisions equal $\pi$ radians, etc.

6. Observe the phase shift on the screen. The two traces should be separated by half a division. Set the box to a smaller value and record the phase angle. Set the box to a larger value and record the phase angle.

7. Have your instructor check your data before you put your equipment away.

**Data**

Data for the first part of the experiment should consist of the measured capacitance and resistance, the signal generator voltage, the frequencies, and the capacitor and resistor voltages. For the second part it should consist of the frequency, the substitution box settings, and the phase angles.

**Analysis**

1. Calculate the magnitude of current for each frequency by dividing the resistor voltage by the resistance

   \[ I_0 = \frac{V_R}{R}. \]

2. Calculate the theoretical and experimental capacitive reactance for each frequency. Reactance and capacitance can be found experimentally if the current and voltages are known. The experimental value for the reactance can be found from

   \[ I_0 = \frac{V_R}{R} = \frac{V_C}{X_C}. \]

3. How good is the agreement between theoretical and experimental values? Find the percent error in each case.

4. Calculate the theoretical phase shift for each frequency. Use the measured values for your components and your measured frequencies.

5. Calculate the expected phase shift for each of the three resistances. Were the theoretical and experimental phase shifts in good agreement?

6. On semilog graph paper, graph the experimental values for $V_{out}/V_{in}$ as a function of frequency for the high and low pass filters. Frequency goes on the log axis. Draw the best fitting curve for each set of results.
**Questions**

1. Draw neat schematics of the circuits built for this experiment. Explain how you took the data.

2. Use your results to discuss the effect of changing the frequency on the current in the circuit where the resistor value is kept constant. Use the data from the first part of the experiment to discuss the effect changing the frequency in the circuit has on the phase angle.

3. How do the phase angle and current change as the resistance changes in a circuit where the frequency stays constant? How do these results differ from those in the case where the resistance stays the same and the frequency changes?

4. Explain why the two lines on your graph should always cross at the frequency where $X_C = R$. Find this frequency from your component values and compare it to your graph. What should $\frac{V_{out}}{V_{in}}$ equal for each filter at that point?

If it applies to you, write "I have not cheated on this lab report" and sign your name.

**Grading**

4 pts Data and Analysis.

3 pts For questions 1

4 pts Each for questions 2 and 3.

5 pts For question 5.