1. Waves and Particles
2. Interference of Waves
3. Wave Nature of Light

1. Double-Slit Experiment – reading: Chapter 22
2. Single-Slit Diffraction – reading: Chapter 22
3. Diffraction Grating – reading: Chapter 22

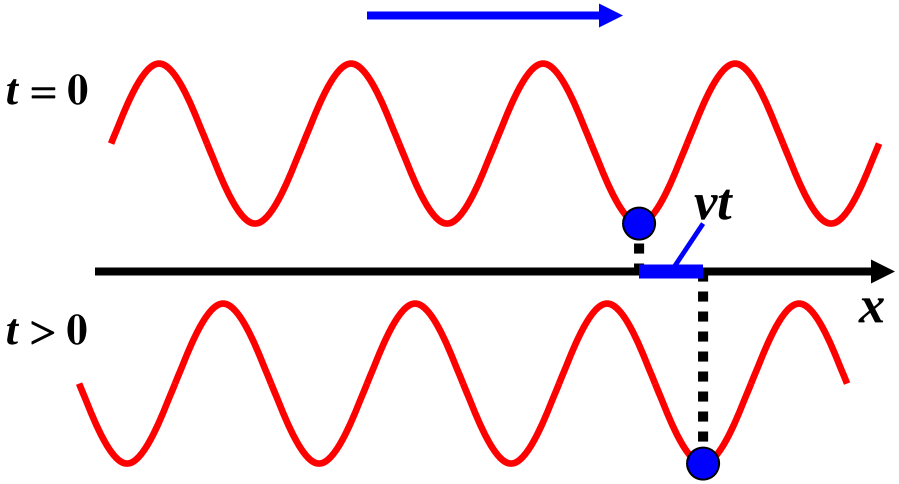
Chapter 20

Traveling Waves

Waves and Particles

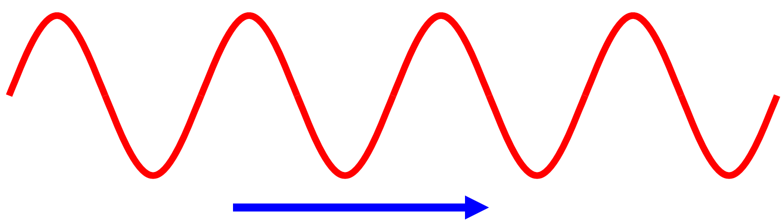
Wave – periodic oscillations in space and in time of something

It is moving as a whole with some velocity *v*

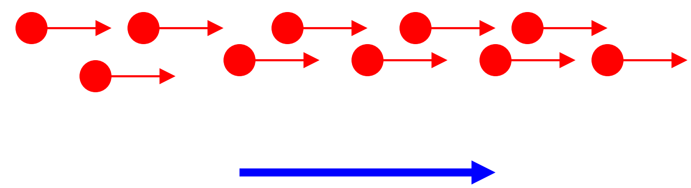


Particle and Waves

Sinusoidal Wave

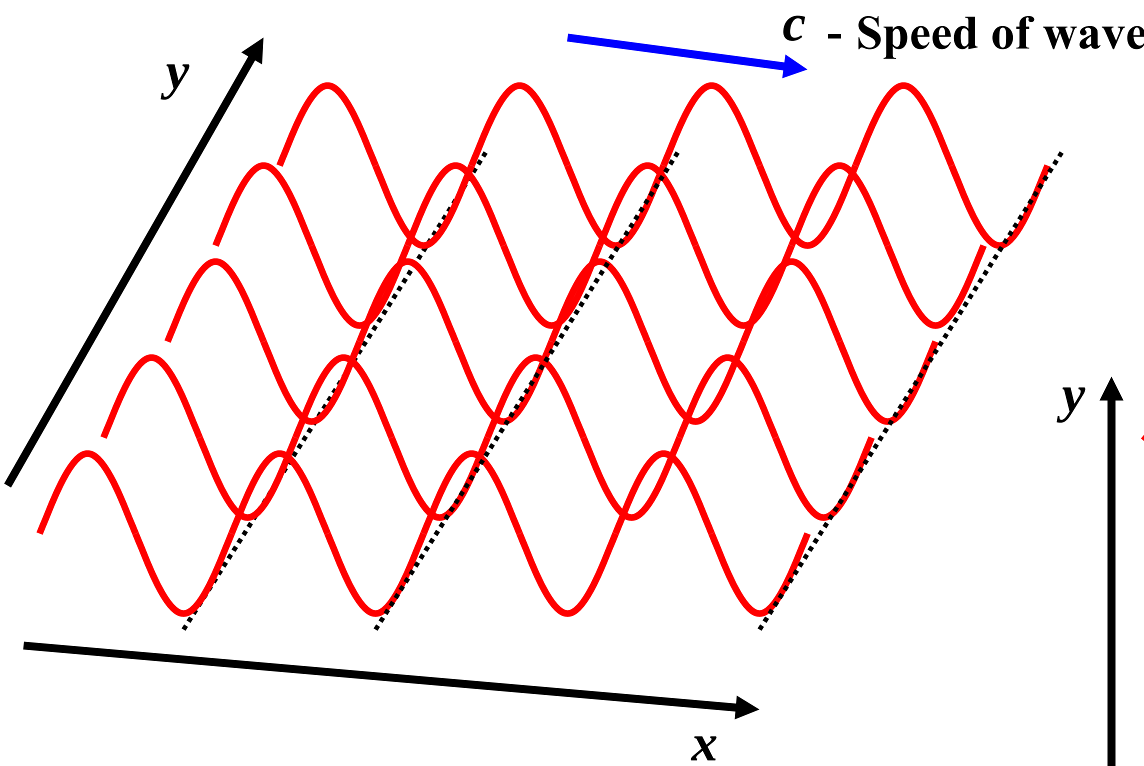


Particles

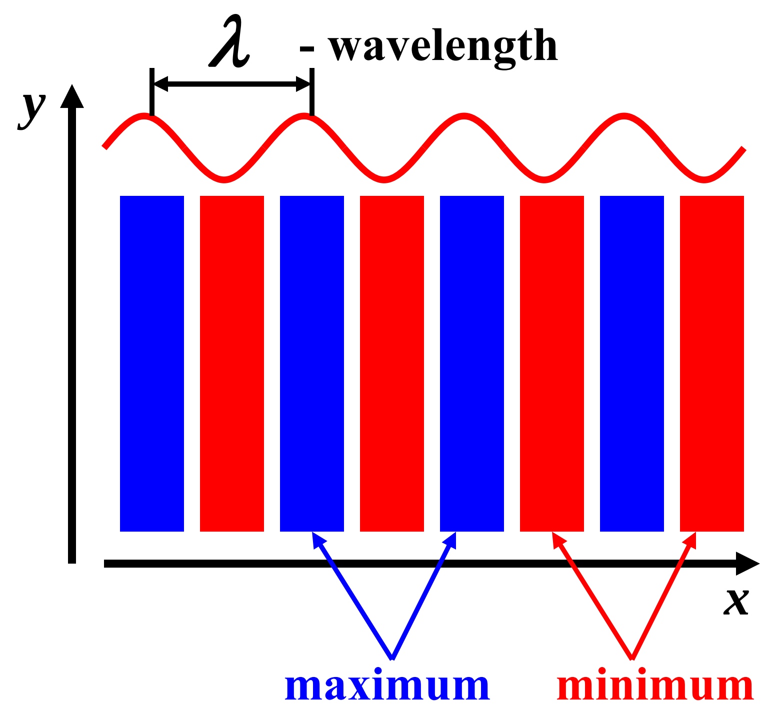


Sinusoidal Wave

Plane wave



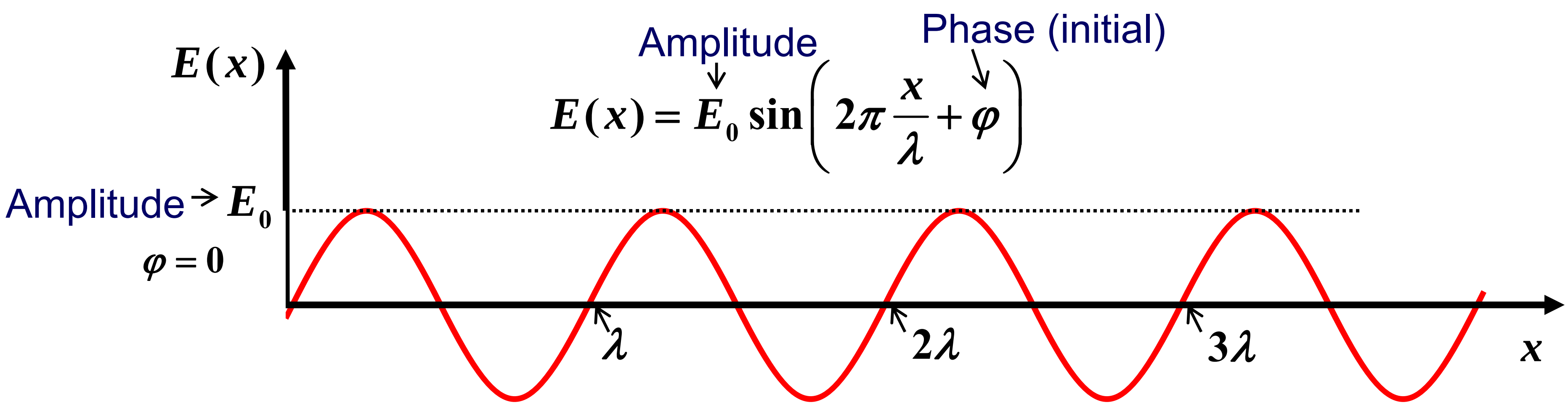
changes only along one direction



Period of "oscillation" – $T = \frac{\lambda}{c}$
(time to travel a distance of wavelength)

Frequency of wave $f = \frac{1}{T} = \frac{c}{\lambda}$

Sin-function



Sinusoidal (Basic) Wave

$$E(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi ft\right) = E_0 \sin\left(2\pi \frac{x}{\lambda} + \omega t\right)$$

Distribution of some Field in space and in time with frequency *f*

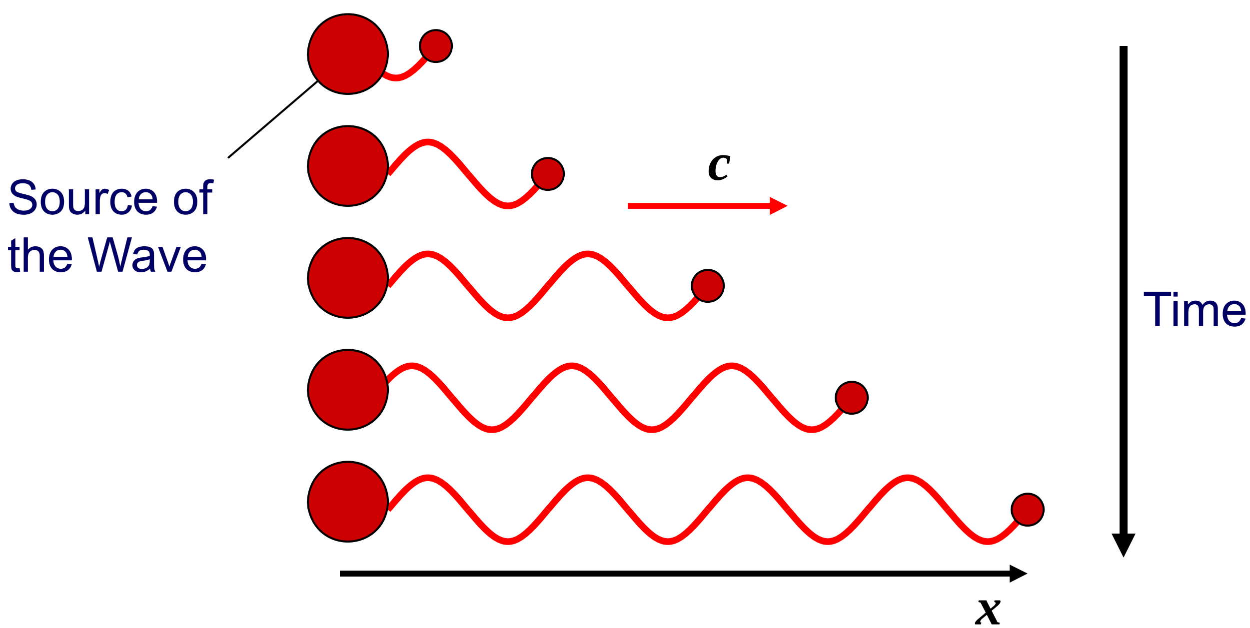
$$f = \frac{1}{T} = \frac{c}{\lambda}$$

Distribution of Electric Field in space at different time

$$E(x) = E_0 \sin\left(2\pi \frac{x}{\lambda} + \varphi_t\right)$$

- usual *sin*-function with initial phase, depending on ***t*** as

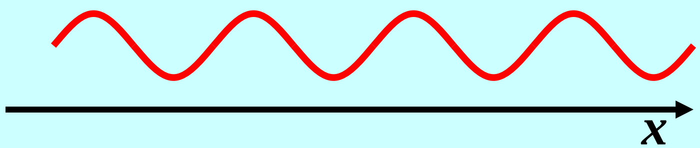
$$\varphi_t = \omega t$$



Sinusoidal Wave

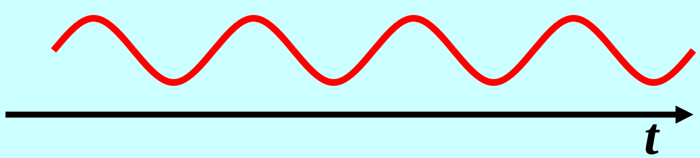
$$E(x,t) = E_0 \sin\left(2\pi\frac{x}{\lambda} + 2\pi ft\right) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \omega t\right)$$

Distribution of some Field in space and in time with frequency *f*



At a given time ***t*** we have sin-function of ***x*** with "initial" phase, depending on ***t***

$$E(x) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \varphi_t\right) \qquad \varphi_t = 2\pi f\, t = \omega t$$



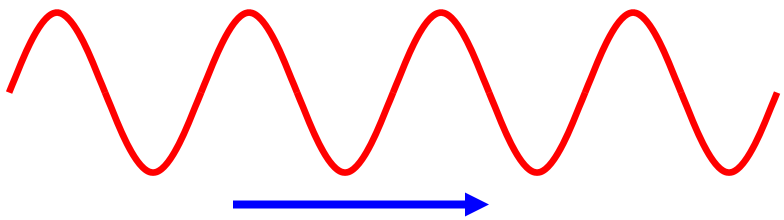
At a given space point ***x*** we have sin-function of ***t*** with "initial" phase, depending on ***x***

$$E(x) = E_0 \sin\left(\omega t + \varphi_x\right) \qquad \varphi_x = 2\pi\frac{x}{\lambda}$$

Particle and Waves

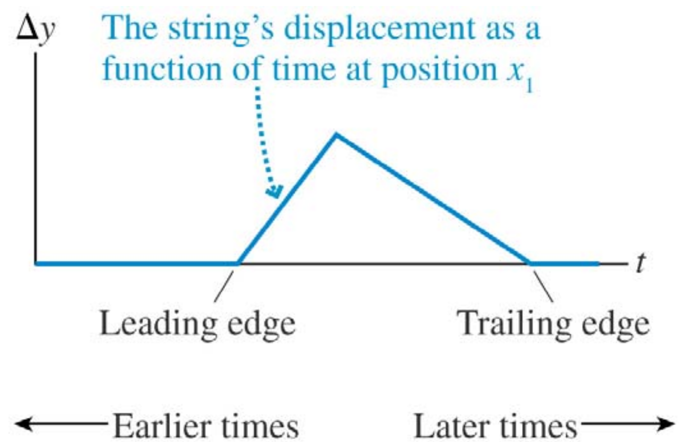
We can take the sum of many sinusoidal waves (with different wavelengths, amplitudes) = wave pack

Sum

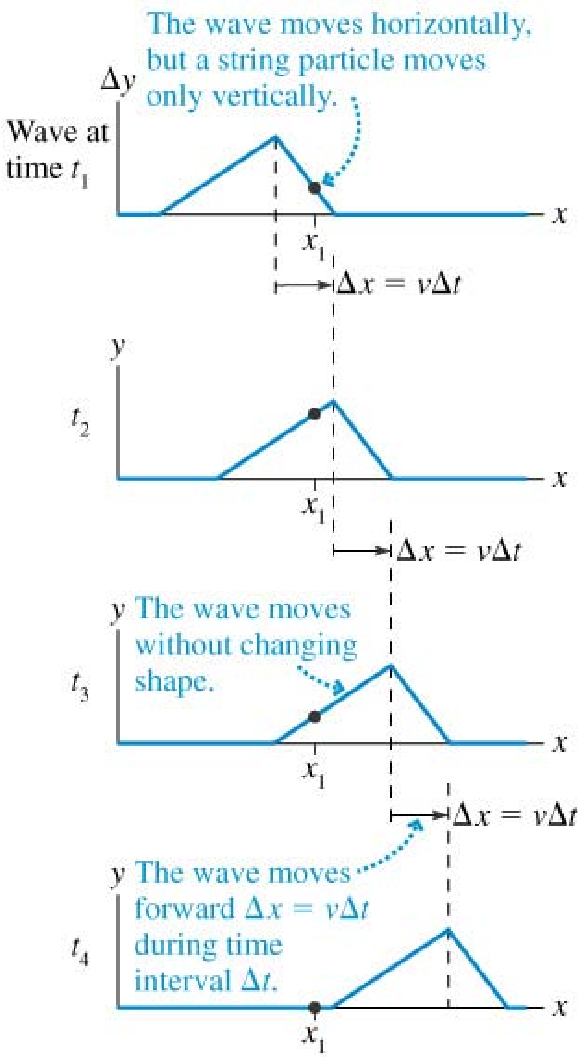


=

Any shape which is moving as a whole with constant velocity



Wave Pack

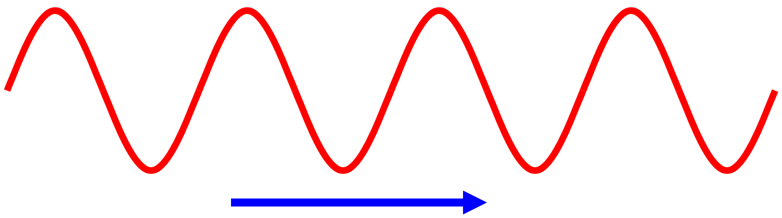


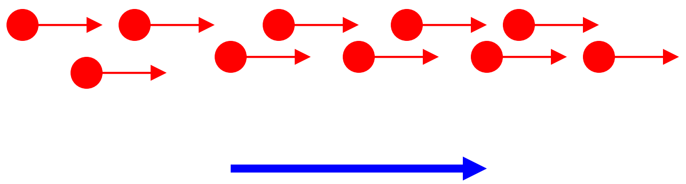
Wave pack can be considered as a particle

Particle and Waves

How can we distinguish between particles and waves?

For waves we have interference, for particles – not!

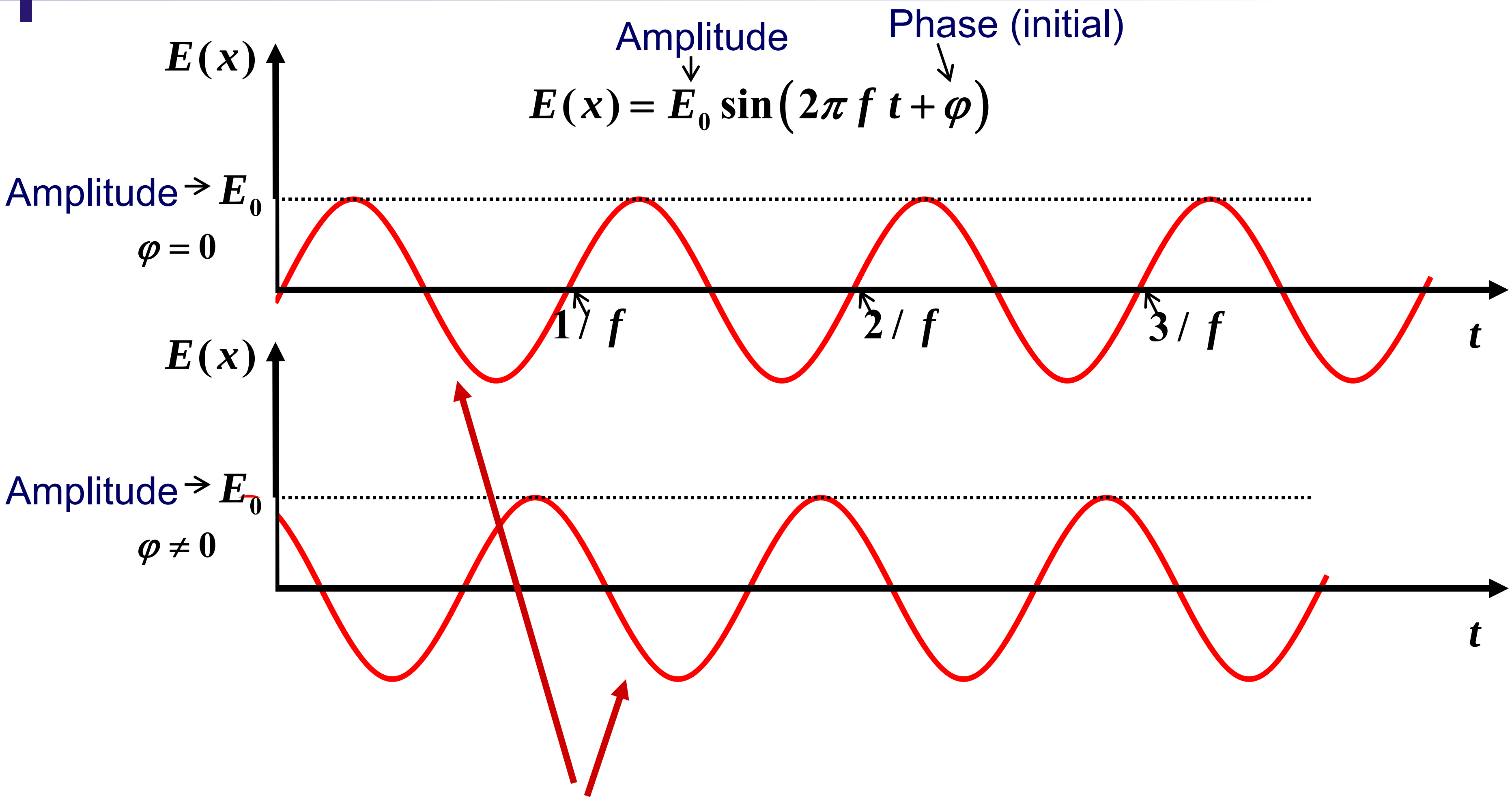




Chapter 21

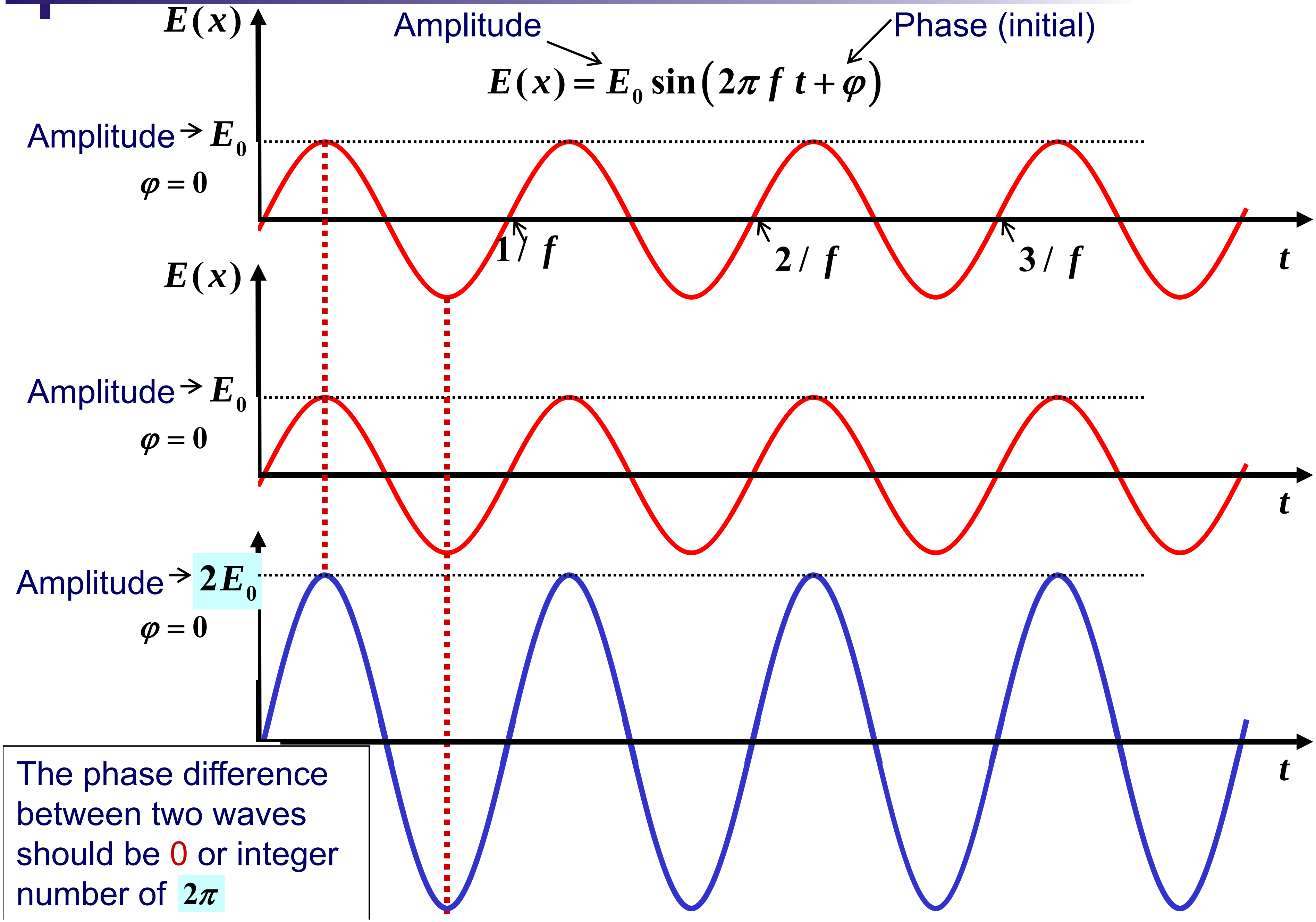
Interference of Waves

Sin-function



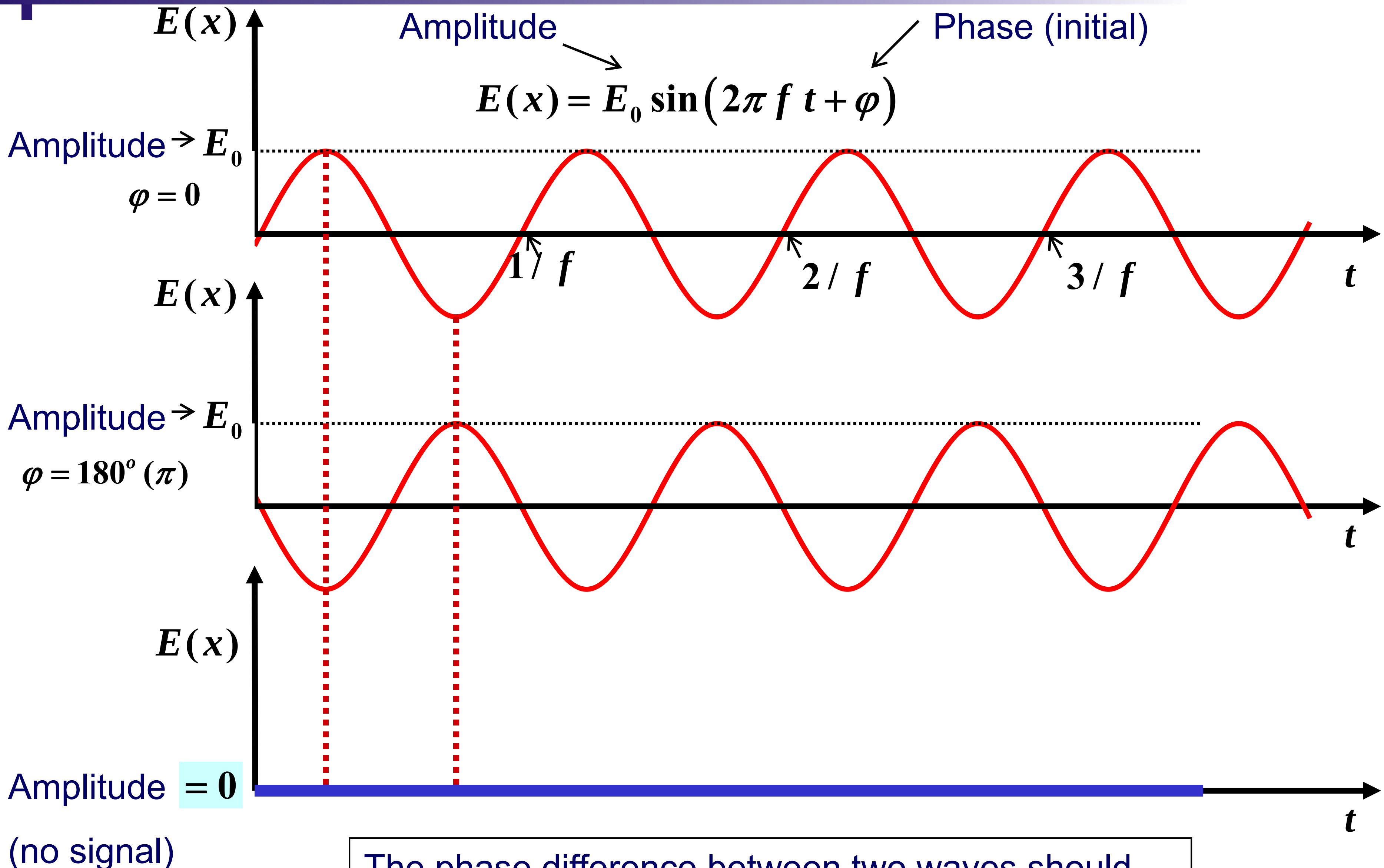
Interference: THE SUM OF TWO SIGNALS (WAVES)

Sin-function: Constructive Interference



The phase difference between two waves should be 0 or integer number of 2π

Sin-function: Destructive Interference

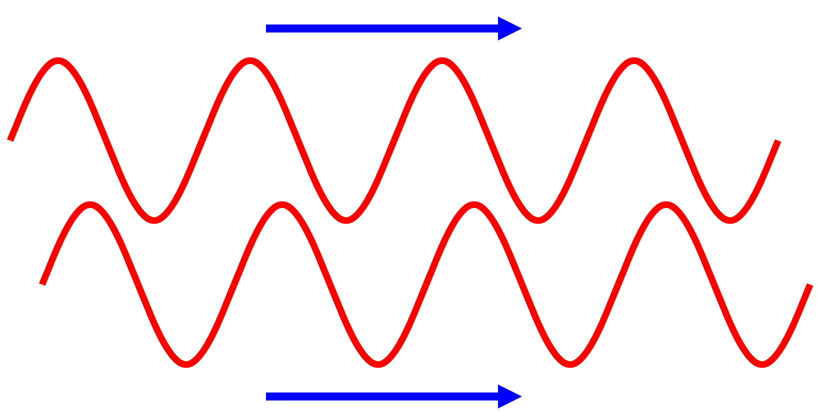


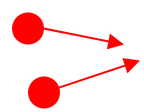
The phase difference between two waves should be π or π plus integer number of 2π

Waves and Particles

Interference of waves: THE SUM OF TWO WAVES

Analog of Interference for particles: Collision of two particles





The difference between the interference of waves and collision of particles is the following: THE INTERFERENCE AFFECTS MUCH LARGER REGION OF SPACE THAN COLLISION DOES

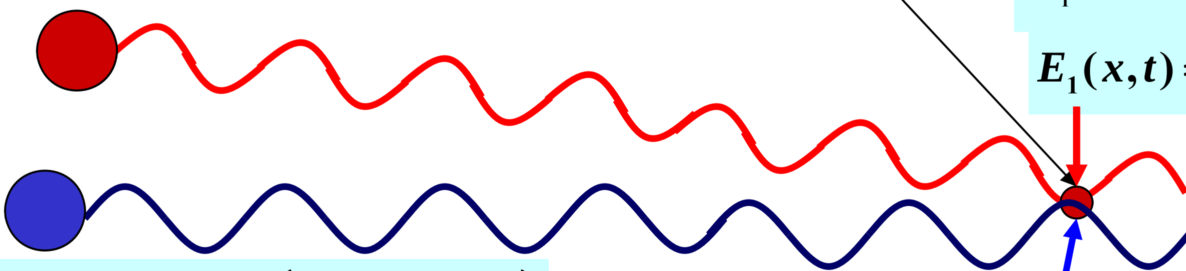
Waves: Interference

$$E_1(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi ft\right)$$

Interference – sum of two waves

$$\varphi_{x_1} = 2\pi \frac{x_1}{\lambda}$$

$$E_1(x,t) = E_0 \sin\left(2\pi ft + \varphi_{x_1}\right)$$



$$E_2(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi ft\right)$$

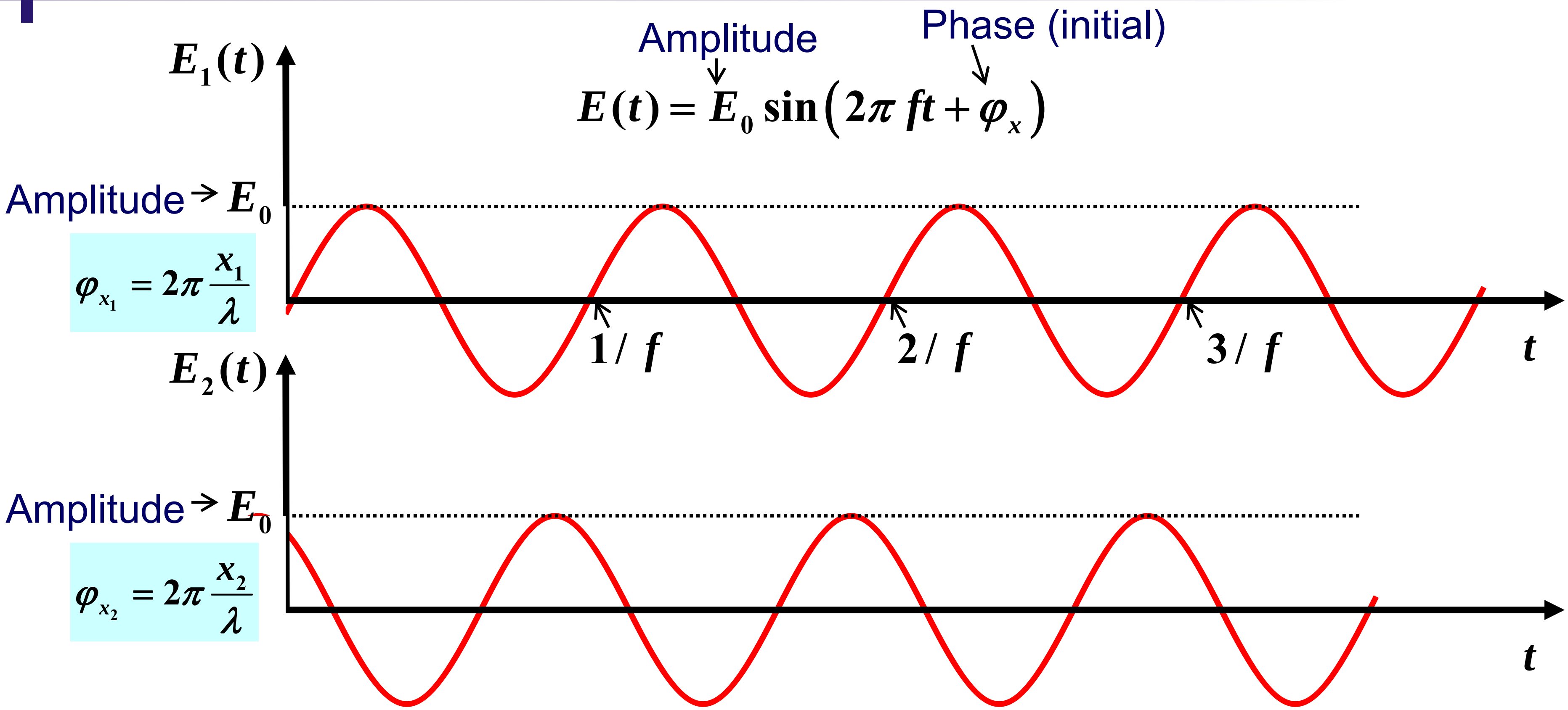
$$E_2(x,t) = E_0 \sin\left(2\pi ft + \varphi_{x_2}\right)$$

$$\varphi_{x_2} = 2\pi \frac{x_2}{\lambda}$$

- In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave

- In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave

Waves: Interference



Constructive Interference: The phase difference between two waves should be 0 or integer number of 2π

$$\varphi_{x_1} - \varphi_{x_2} = 2\pi m \qquad m = 0, \pm 1, \pm 2...$$

Destructive Interference: The phase difference between two waves should be π or π integer number of 2π

$$\varphi_{x_1} - \varphi_{x_2} = \pi + 2\pi m \qquad m = 0, \pm 1, \pm 2...$$

Conditions for Interference

To observe interference the following two conditions must be met:

1) The sources must be **coherent**

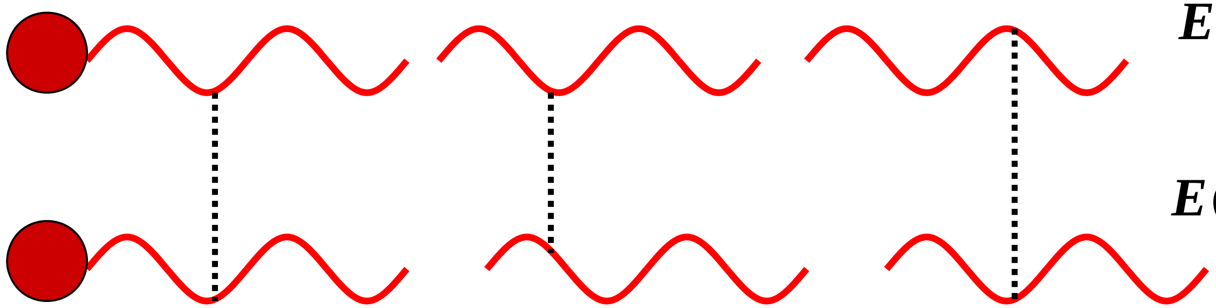
- They must maintain a constant phase with respect to each other

2) The sources should be **monochromatic**

- Monochromatic means they have a single (the same) wavelength

Conditions for Interference: Coherence

coherent

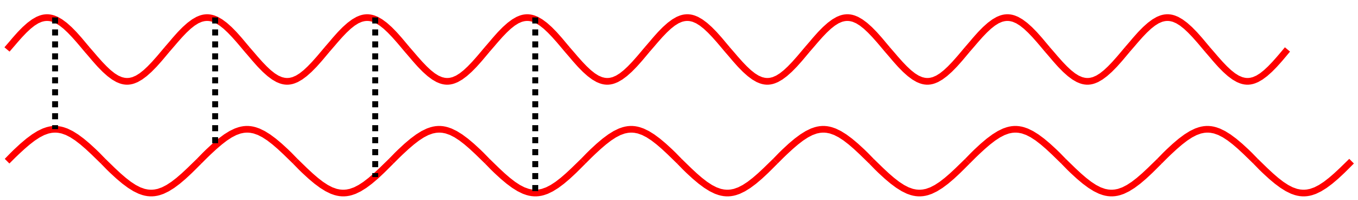


$$E(x) = E_0 \sin(\omega t + \varphi_1)$$

$$E(x) = E_0 \sin(\omega t + \varphi_2)$$

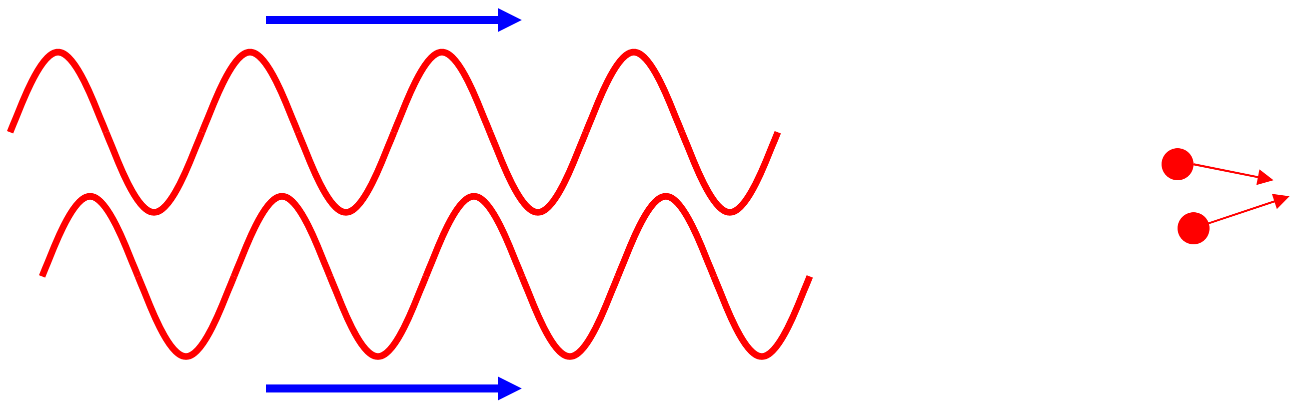
The sources should be monochromatic (have the same frequency)

$$E(x) = E_0 \sin(\omega_1 t + \varphi)$$



$$E(x) = E_0 \sin(\omega_2 t + \varphi)$$

Waves and Particles



The difference between the interference of waves and collision of particles is the following: THE INTERFERENCE AFFECTS MUCH LARGER REGION OF SPACE THAN COLLISION AND FOR A MUCH LONGER TIME

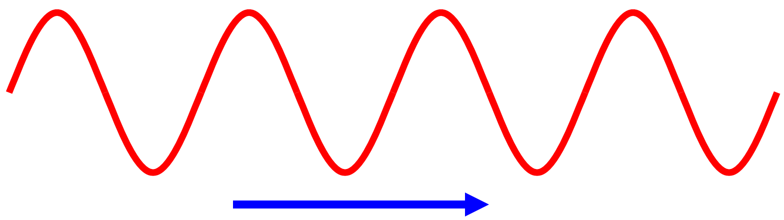
If we are looking at the region of space that is much larger than the wavelength of wave (or the size of the wave) than the “wave” can be considered as a particle

Chapter 22

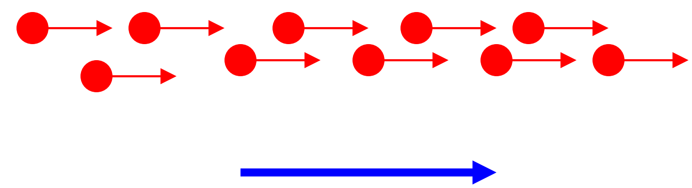
Light as a Wave:
Wave Optics

The Nature of Light – Particle or Waves?

WAVE?



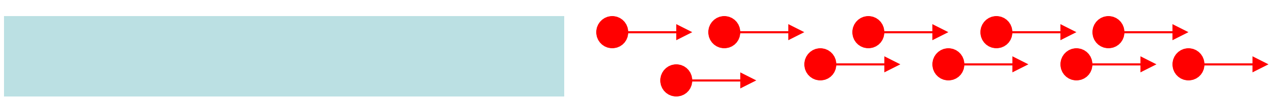
PARTICLES?



The Nature of Light – Particle or Waves?

- **Before the beginning of the nineteenth century, light was considered to be a stream of particles**
- **Newton was the chief architect of the particle theory of light**
  - **He believed the particles left the object and stimulated the sense of sight upon entering the eyes**

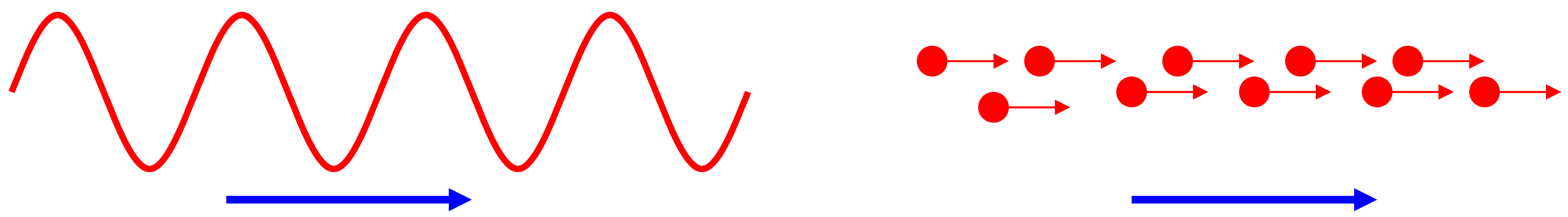
But he was wrong. LIGHT IS A WAVE.



The Nature of Light – Particle or Waves?

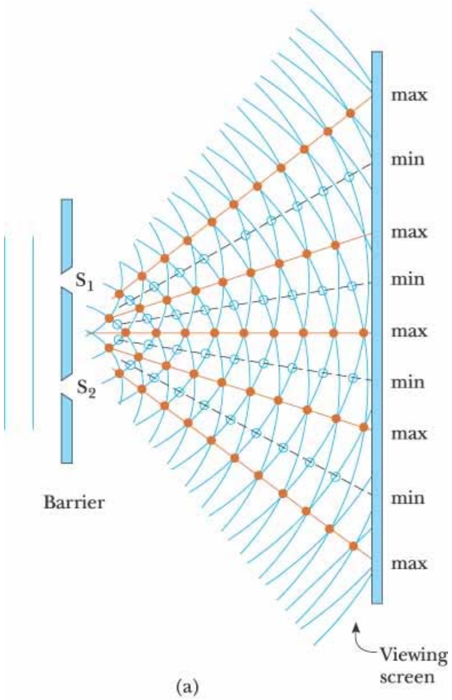
How can we distinguish between particles and waves?

For waves we have interference, for particles – not!



The Nature of Light – Wave Theory?

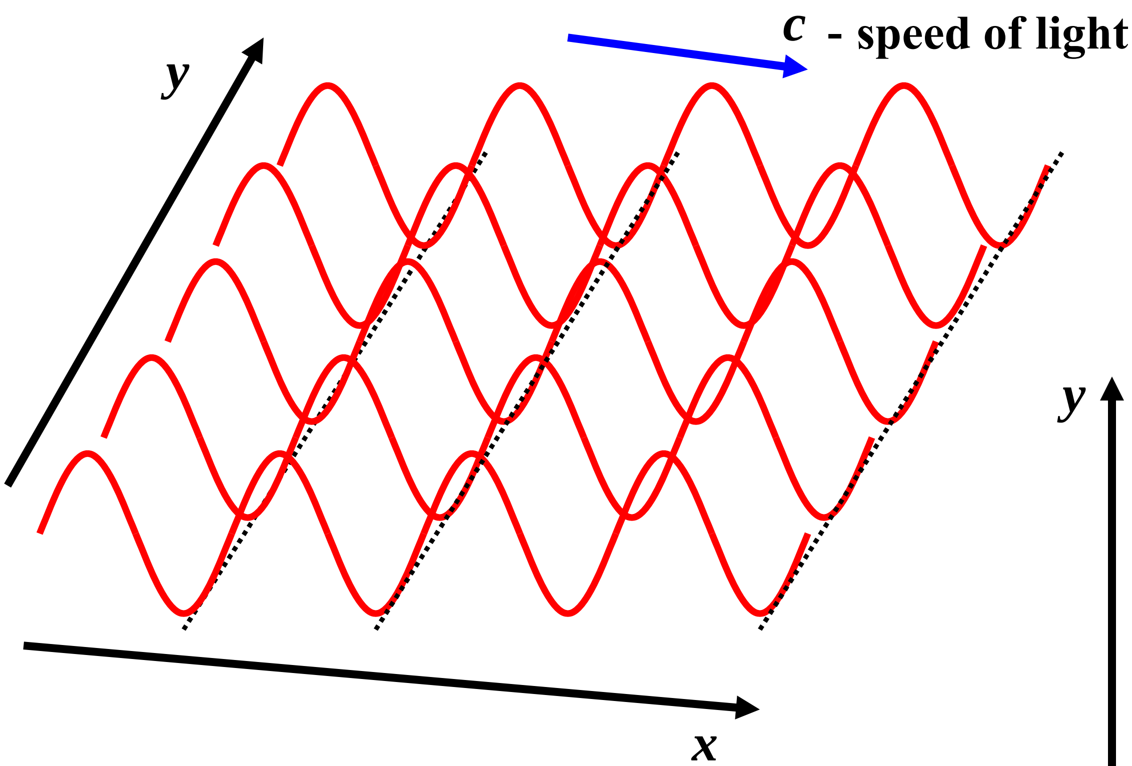
- Christian Huygens argued that light might be some sort of a wave motion
- Thomas Young (1801) provided the first clear demonstration of the wave nature of light
  - He showed that light rays interfere with each other
  - Such behavior could not be explained by particles



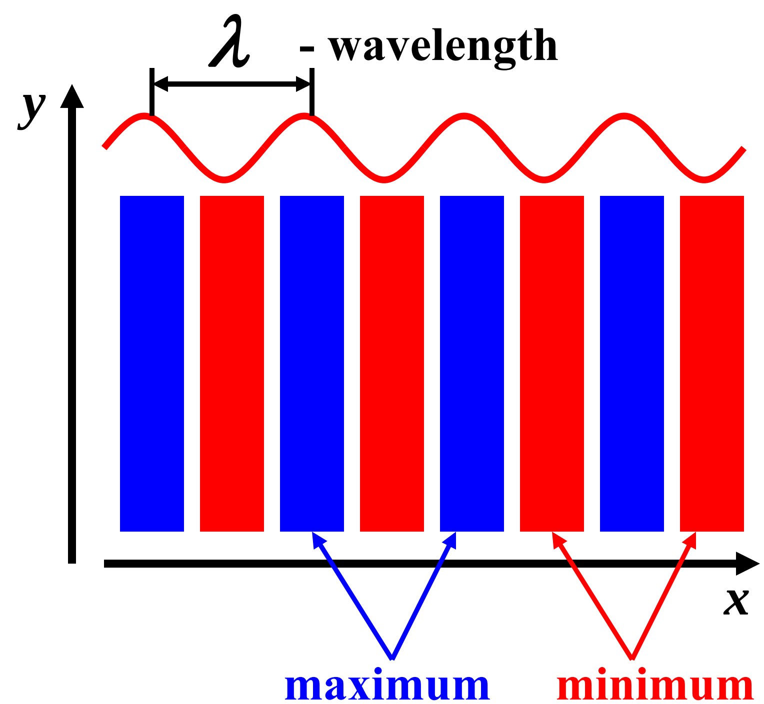
During the nineteenth century, other developments led to the general acceptance of the wave theory of light

Light as a Wave

Plane wave



changes only along one direction



Period of "oscillation" – $T = \frac{\lambda}{c}$
(time to travel distance of wavelength)

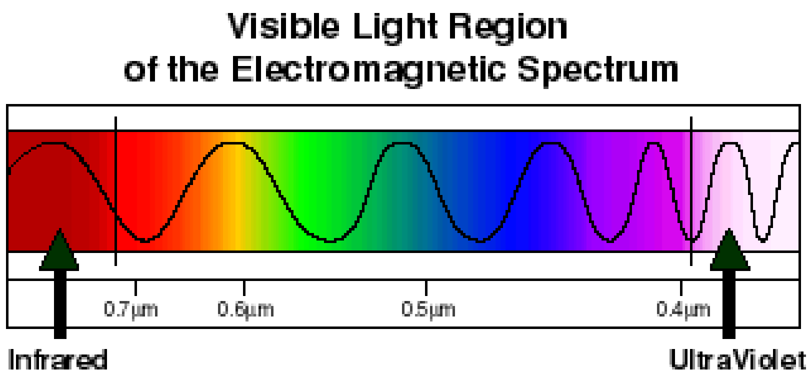
Frequency of light $f = \frac{1}{T} = \frac{c}{\lambda}$

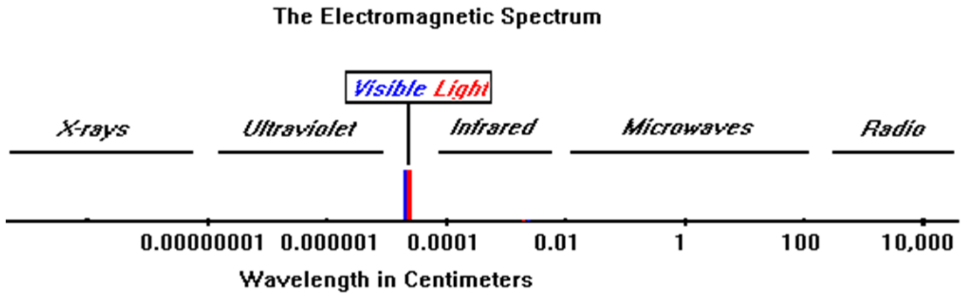
Light as a Wave

Light is characterized by

- its speed c and
- wavelength λ (or frequency f)

Different frequency (wavelength) – different color of light

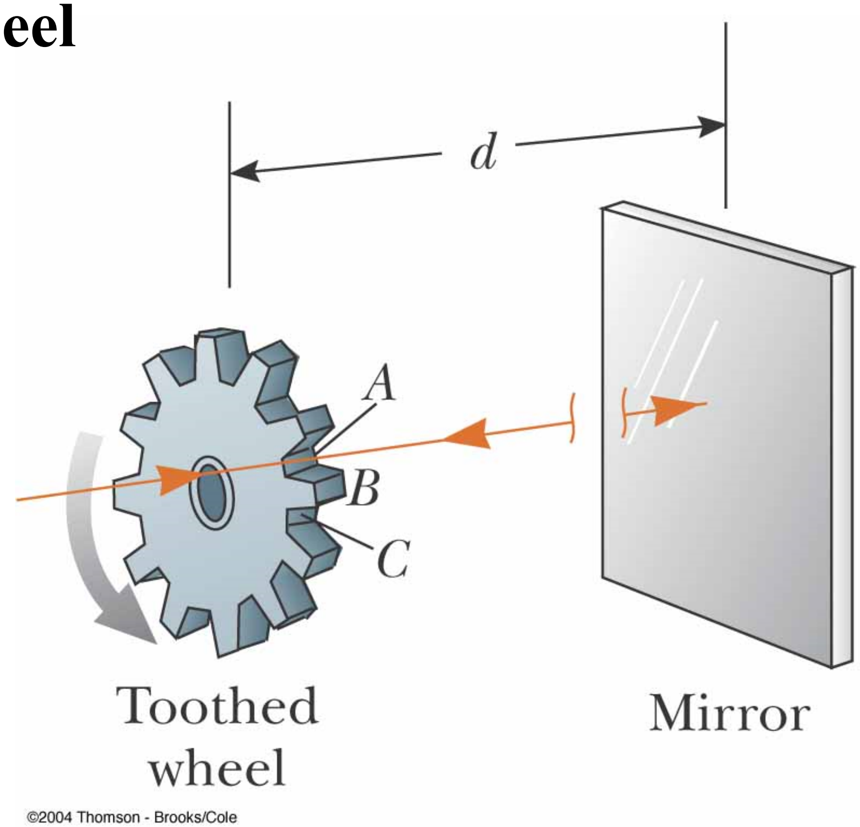




What is the speed of light?

Measurements of the Speed of Light – Fizeau's Method (1849)

- ***d*** **is the distance between the wheel and the mirror**
- **Δt is the time for one round trip**
- **Then $c = 2d / \Delta t$**
- **Fizeau found a value of**
  $c = 3.1 \times 10^8$ m/s



$c = 3.00 \times 10^8$ m/s **- Speed in Vacuum!**

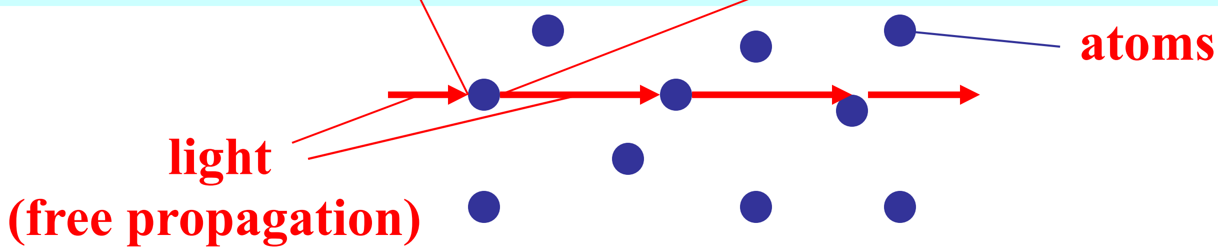
Speed of Light

What is the speed of light in a medium?

The speed of light in a medium is smaller than the speed in vacuum.

To understand this you can think about it in a following way:

- The medium consists of atoms (or molecules), which can absorb light and then emit it,
- so the propagation of light through the medium can be considered as a process of absorption and subsequent emission (AFTER SOME TIME Δt)



very schematic picture

Speed of Light

$$v = \frac{c}{n}$$ - The speed of light in the medium

The properties of the medium is characterized by one dimensionless constant – *n*, (it is called index of refraction, we will see later why)

- **which is equal to 1 for vacuum (and very close to 1 for air),**
- **greater then 1 for all other media**

Table 35.1

Indices of Refraction[a]

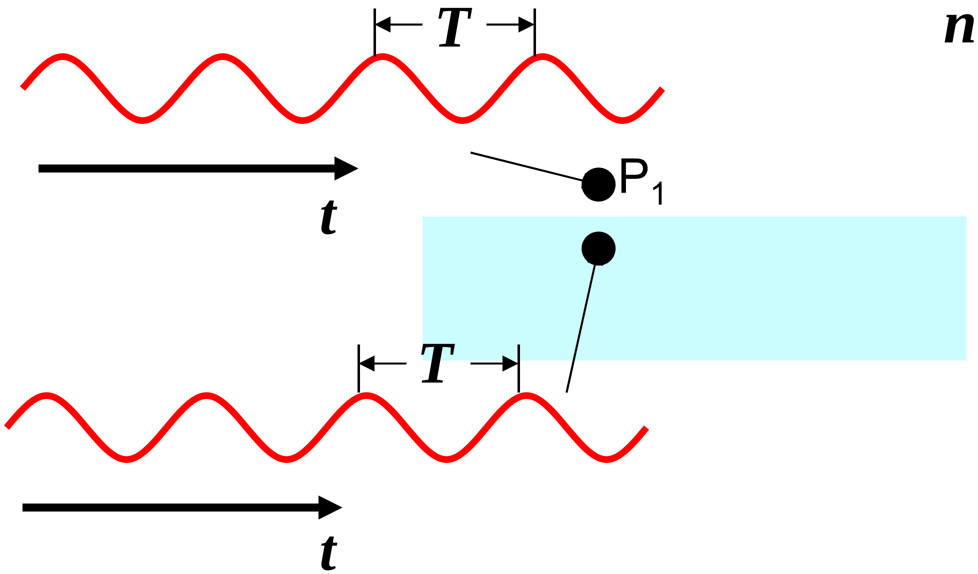
Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		*Liquids at 20°C*	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF_2)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO_2)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H_2O)	1.309	*Gases at 0°C, 1 atm*	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

[a] All values are for light having a wavelength of 589 nm in vacuum.

Light in the Media

E at a given point

$$v = \frac{c}{n}$$ - The speed of light in the medium



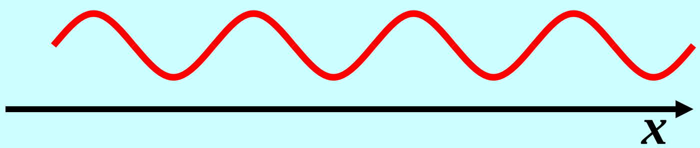
The same period (frequency) in all media, then

$$\lambda_n = \frac{c}{n} T = \frac{\lambda_{air}}{n}$$

Light as a Wave

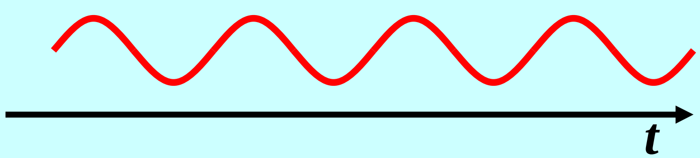
$$E(x,t) = E_0 \sin\left(2\pi\frac{x}{\lambda} + 2\pi f t\right) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \omega t\right)$$

Distribution of some Field inside the wave of frequency *f*



At a given time ***t*** we have sin-function of ***x*** with "initial" phase, depending on ***t***

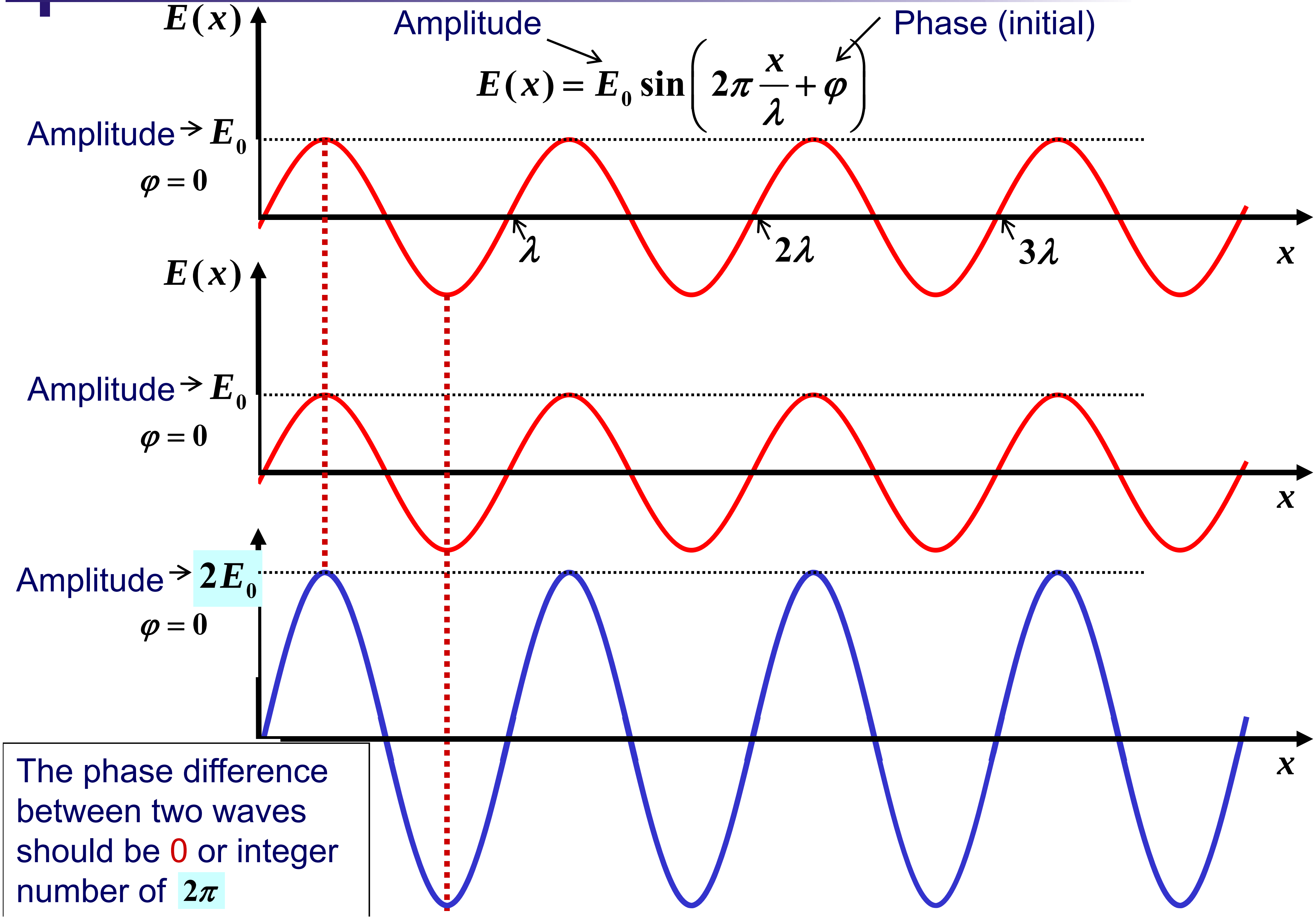
$$E(x) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \varphi_t\right) \qquad \varphi_t = 2\pi f t = \omega t$$



At a given space point ***x*** we have sin-function of ***t*** with "initial" phase, depending on ***x***

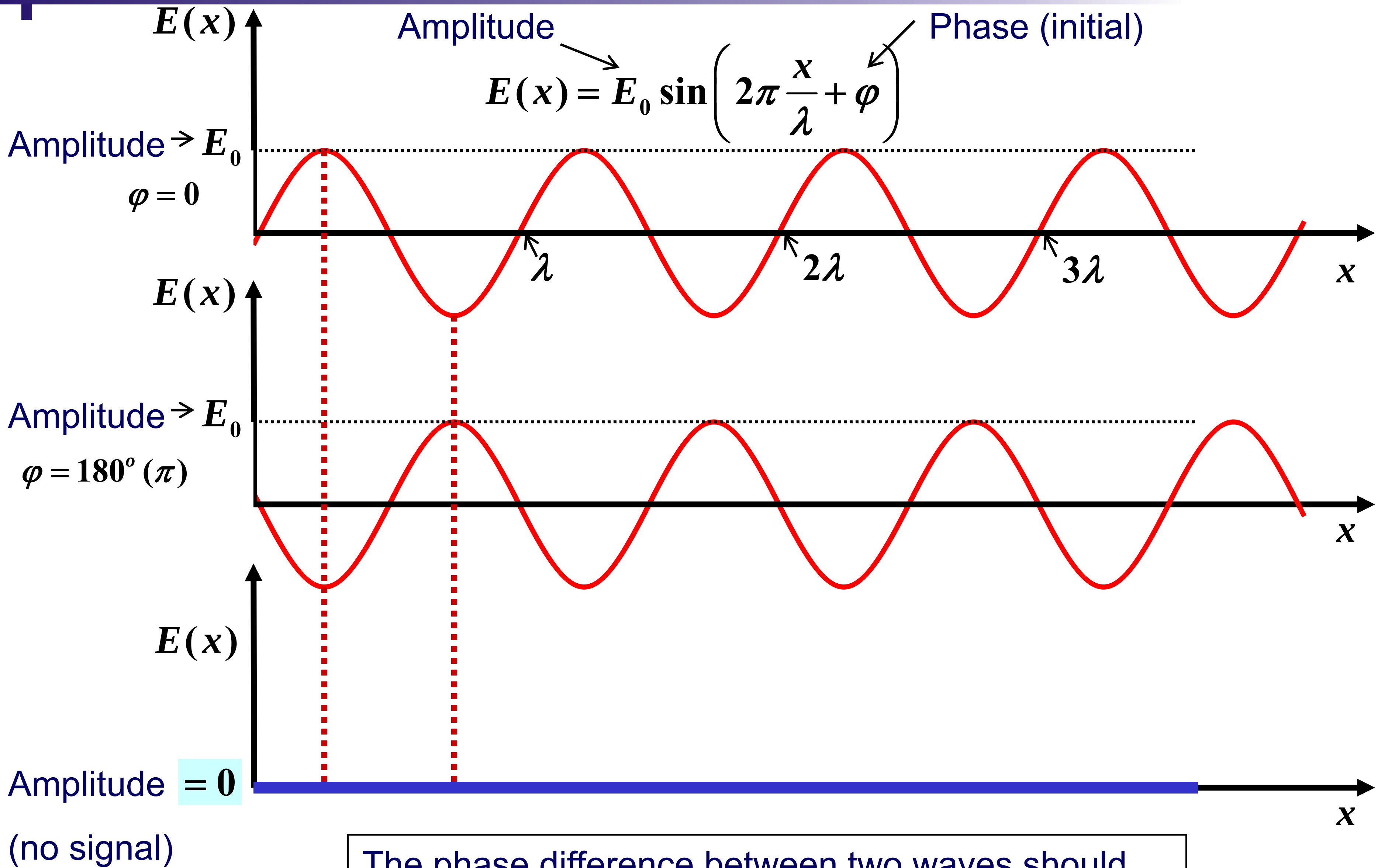
$$E(x) = E_0 \sin\left(\omega t + \varphi_x\right) \qquad \varphi_x = 2\pi\frac{x}{\lambda}$$

Sin-function: Constructive Interference



The phase difference between two waves should be 0 or integer number of 2π

Sin-function: Destructive Interference



The phase difference between two waves should be π or π plus integer number of 2π

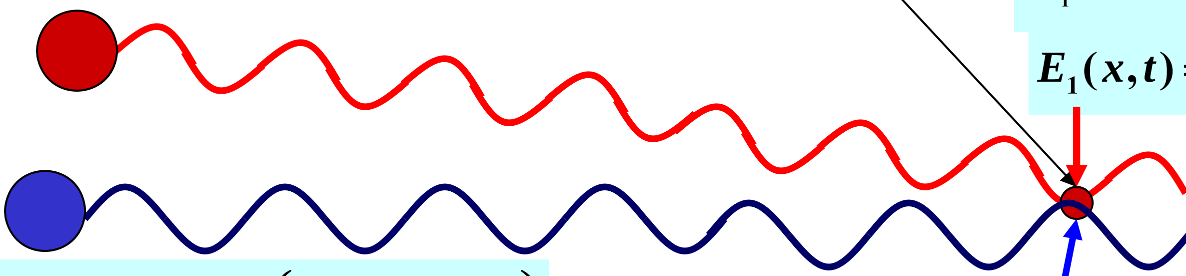
Waves: Interference

$$E_1(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi ft\right)$$

Interference – sum of two waves

$$\varphi_{x_1} = 2\pi \frac{x_1}{\lambda}$$

$$E_1(x,t) = E_0 \sin\left(2\pi ft + \varphi_{x_1}\right)$$



$$E_2(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi ft\right)$$

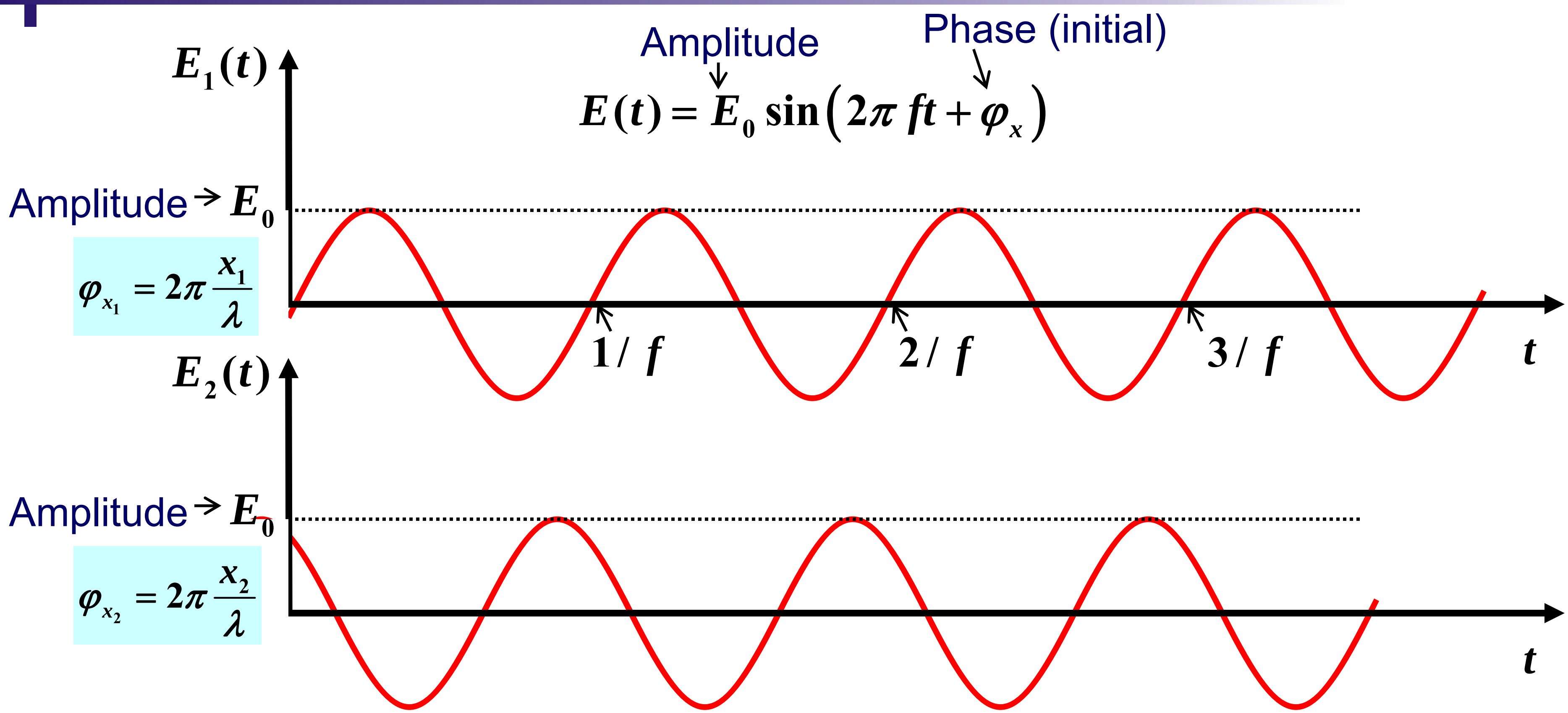
$$E_2(x,t) = E_0 \sin\left(2\pi ft + \varphi_{x_2}\right)$$

$$\varphi_{x_2} = 2\pi \frac{x_2}{\lambda}$$

- In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave

- In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave

Waves: Interference



Constructive Interference: The phase difference between two waves should be 0 or integer number of 2π

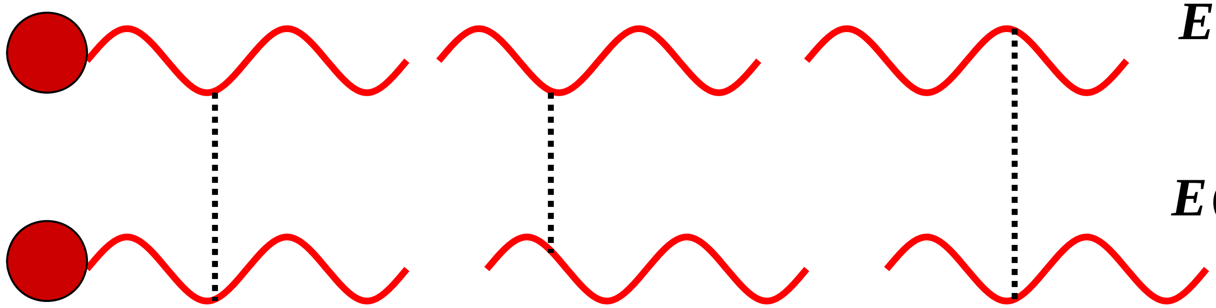
$$\varphi_{x_1} - \varphi_{x_2} = 2\pi m \qquad m = 0, \pm 1, \pm 2 \ldots$$

Destructive Interference: The phase difference between two waves should be π or π integer number of 2π

$$\varphi_{x_1} - \varphi_{x_2} = \pi + 2\pi m \qquad m = 0, \pm 1, \pm 2 \ldots$$

Conditions of Interference

coherent

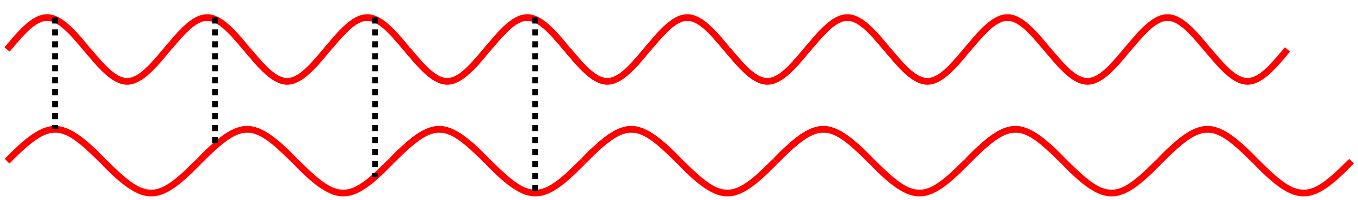


$$E(x) = E_0 \sin(\omega t + \varphi_1)$$

$$E(x) = E_0 \sin(\omega t + \varphi_2)$$

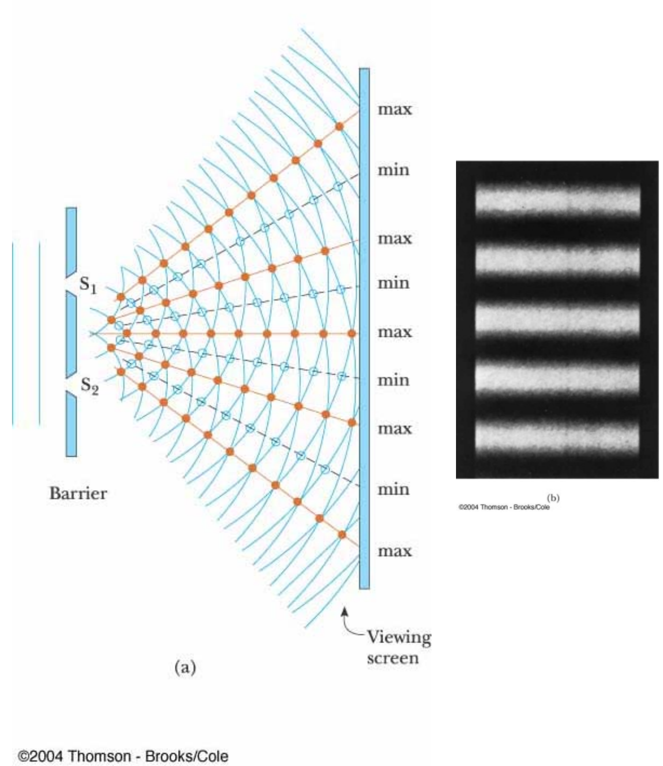
The sources should be monochromatic (have the same frequency)

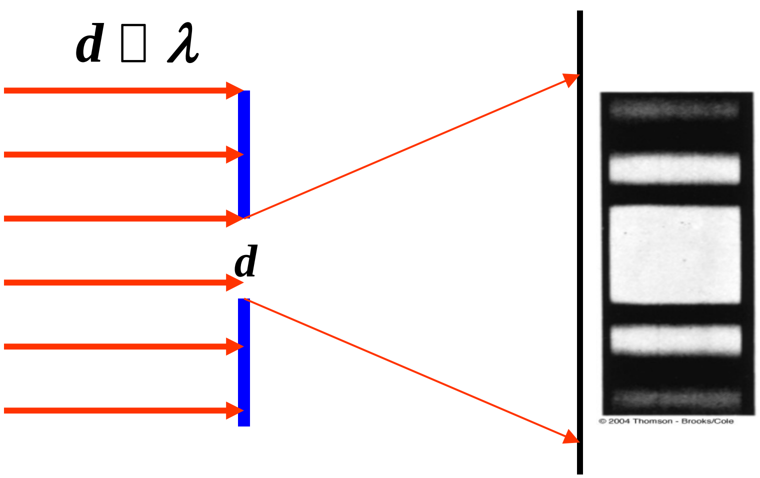
$$E(x) = E_0 \sin(\omega_1 t + \varphi)$$



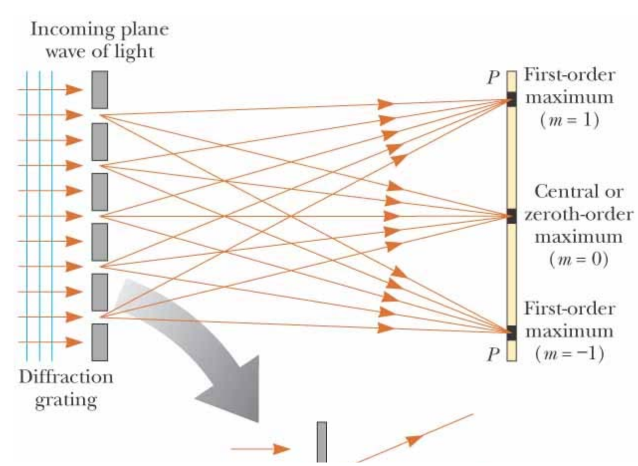
$$E(x) = E_0 \sin(\omega_2 t + \varphi)$$

1. Double-Slit Experiment (interference)





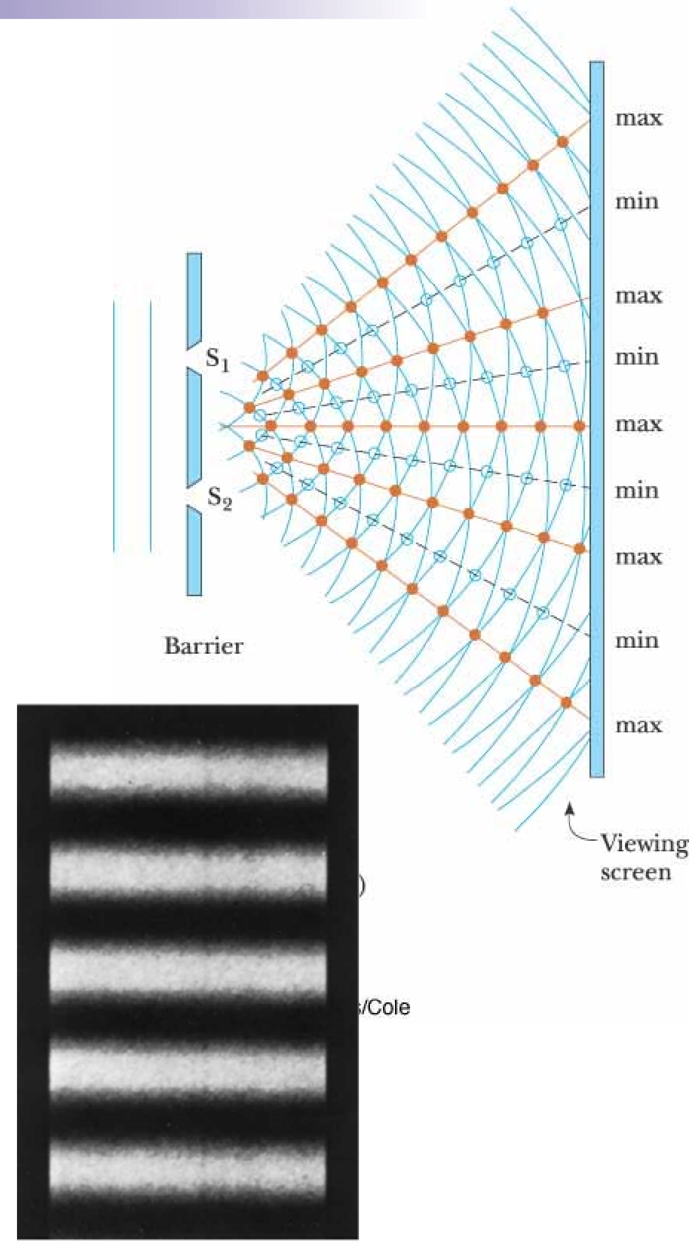
2. Single-Slit Diffraction



3. Diffraction Grating

Young's Double-Slit Experiment

- **Thomas Young first demonstrated interference in light waves from two sources in 1801**
- **The narrow slits S_1 and S_2 act as sources of waves**
- **The waves emerging from the slits originate from the *same wave front* and therefore *are always in phase***



(b)

Double-Slit Experiment: Interference

$$E(x) = E_0 \sin(\omega t + \varphi_x)$$

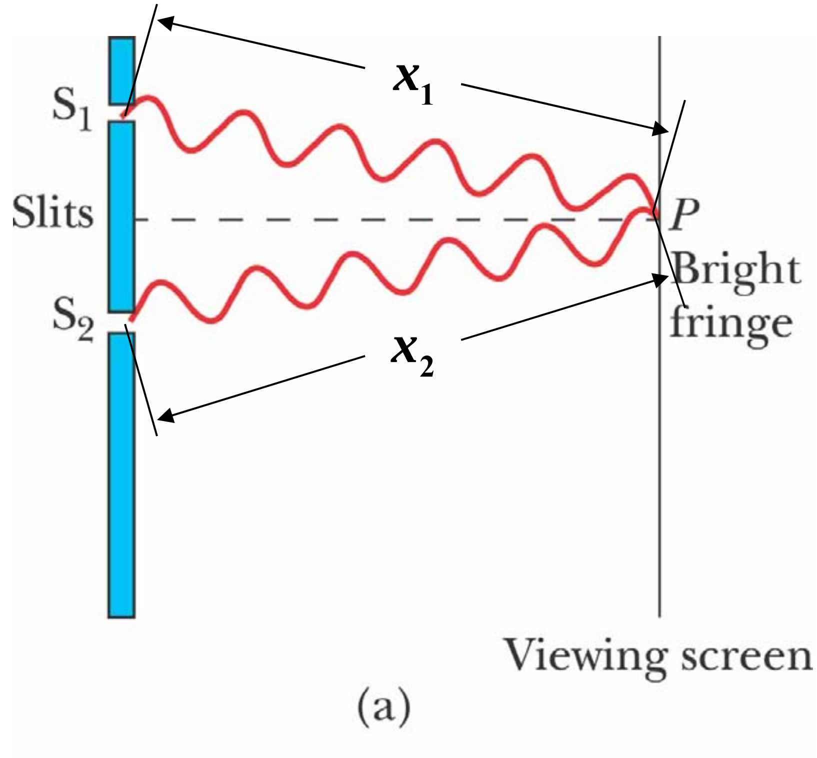
$$\varphi_x = 2\pi \frac{x}{\lambda}$$

The phase of wave 1:

$$\varphi_{x,1} = 2\pi \frac{x_1}{\lambda}$$

The phase of wave 2:

$$\varphi_{x,2} = 2\pi \frac{x_2}{\lambda}$$



Constructive Interference: $\varphi_{x,2} - \varphi_{x,1} = 2\pi n$ where n is integer
(bright fringe)

$$2\pi \frac{x_2}{\lambda} - 2\pi \frac{x_1}{\lambda} = 2\pi n \longrightarrow x_2 - x_1 = n\lambda$$

Destructive Interference: $\varphi_{x,2} - \varphi_{x,1} = \pi + 2\pi n$ where n is integer
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + n\lambda$$

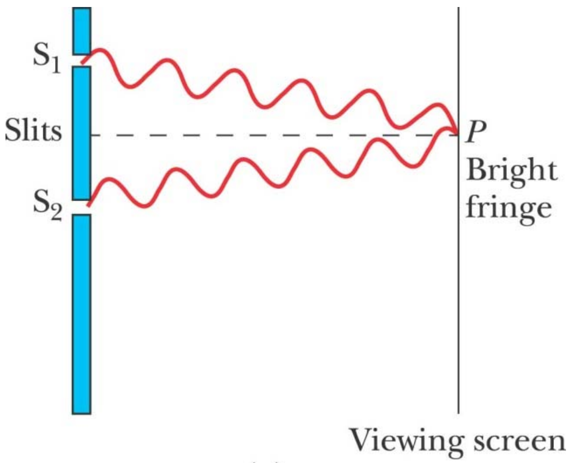
Double-Slit Experiment: Interference

Constructive Interference:
(bright fringe)

$$x_2 - x_1 = n\lambda$$

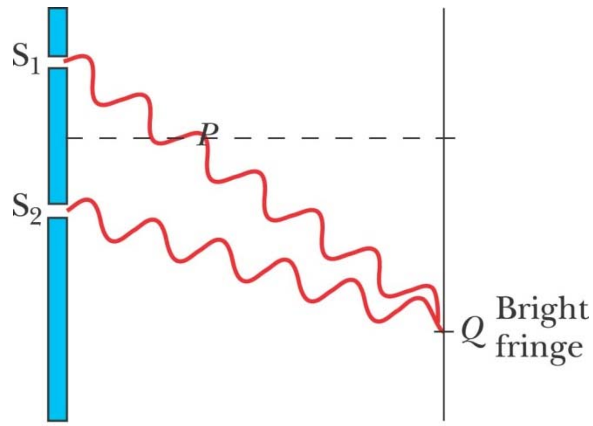
Destructive Interference:
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + n\lambda$$



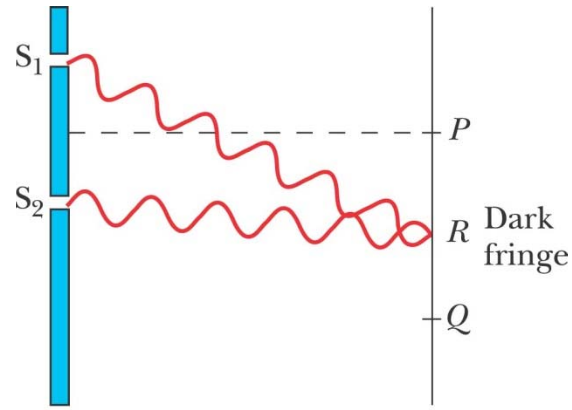
(a)

$$x_2 - x_1 = 0$$



(b)

$$x_2 - x_1 = \lambda$$

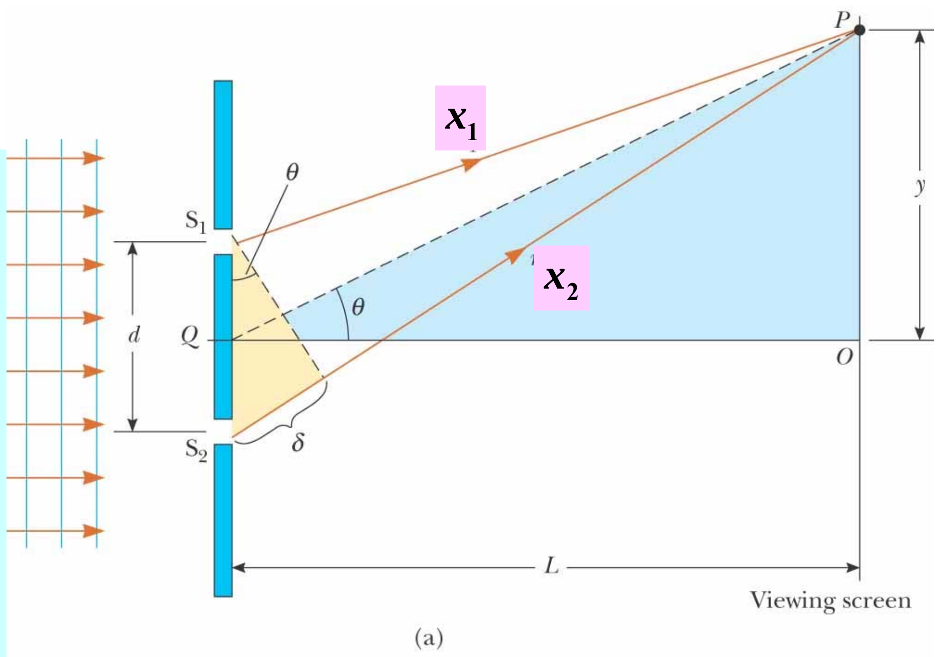


(c)

$$x_2 - x_1 = \frac{\lambda}{2}$$

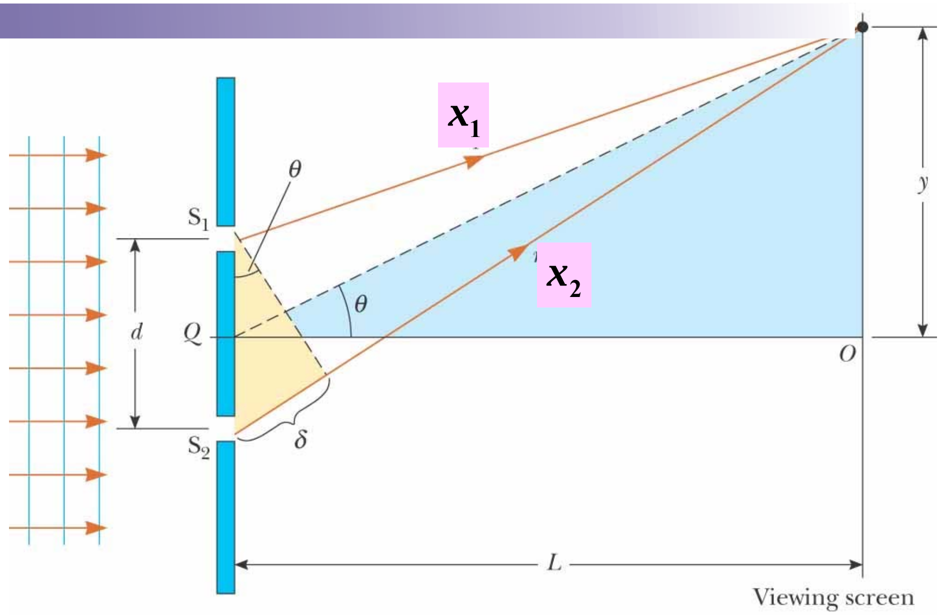
Double-Slit Experiment: Interference

- The path difference, δ, is found from the tan triangle
- $\delta = x_2 - x_1 = d \sin \theta$
  - This assumes the paths are parallel
  - Not exactly true, but a very good approximation if L is much greater than d



Double-Slit Experiment: Interference

$\delta = x_2 - x_1 = d \sin \theta$



(a)

Bright fringes (constructive interference):

$\delta = d \sin \theta = n\lambda$ $\quad$ $n = 0, \pm 1, \pm 2, \ldots$

n is called the **order number**

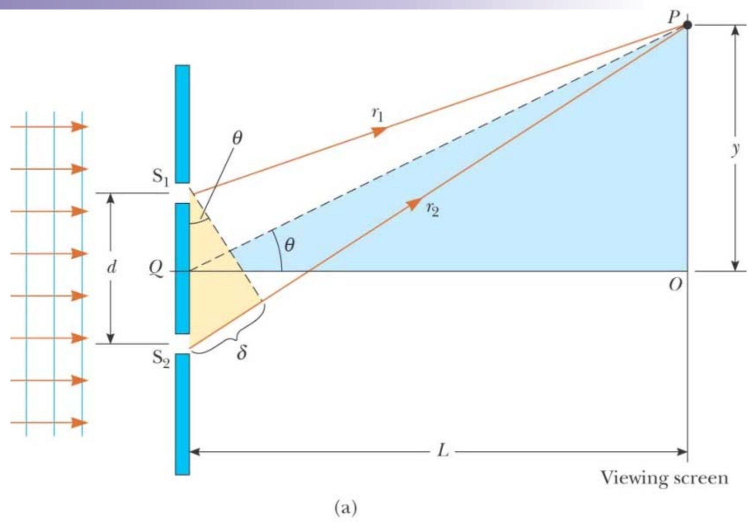
- when $n = 0$, it is the *zeroth-order maximum*
- when $n = \pm 1$, it is called the *first-order maximum*

Dark fringes (destructive interference):

$\delta = d \sin \theta = (n + ½)\lambda$ $\quad$ $n = 0, \pm 1, \pm 2, \ldots$

Double-Slit Experiment: Interference

$$\delta = x_2 - x_1 = d \sin \theta$$



The positions of the fringes can be measured vertically from the zeroth-order maximum

θ is small and therefore the small angle approximation $\tan \theta \sim \sin \theta$ can be used

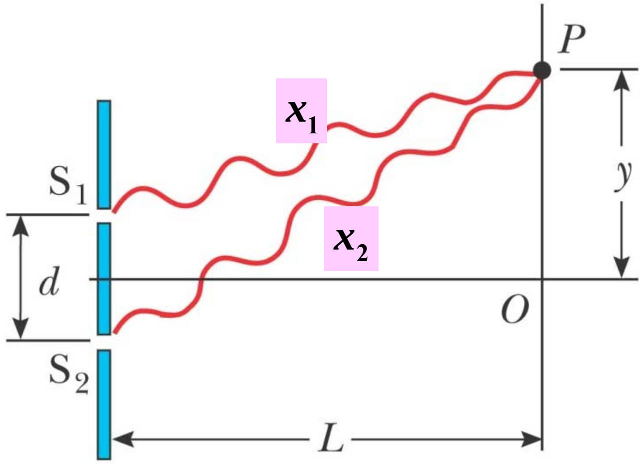
$y = L \tan \theta \approx L \sin \theta$

For bright fringes

$$y_{\text{bright}} = \frac{\lambda L}{d} n \quad (\text{n} = 0, \pm 1, \pm 2 \ldots)$$

For dark fringes

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(n + \frac{1}{2}\right) \quad (n = 0, \pm 1, \pm 2 \ldots)$$



Constructive Interference: $\varphi_{x,2} - \varphi_{x,1} = 2\pi m$ where m is integer $m = 0, \pm 1, \pm 2, ...$
(bright fringe)

$$2\pi \frac{x_2}{\lambda} - 2\pi \frac{x_1}{\lambda} = 2\pi m \longrightarrow x_2 - x_1 = m\lambda$$

$$y_{bright} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots)$$

Destructive Interference: $\varphi_{x,2} - \varphi_{x,1} = \pi + 2\pi m$ where m is integer $m = 0, \pm 1, \pm 2, ...$
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + m\lambda$$

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2 \ldots)$$

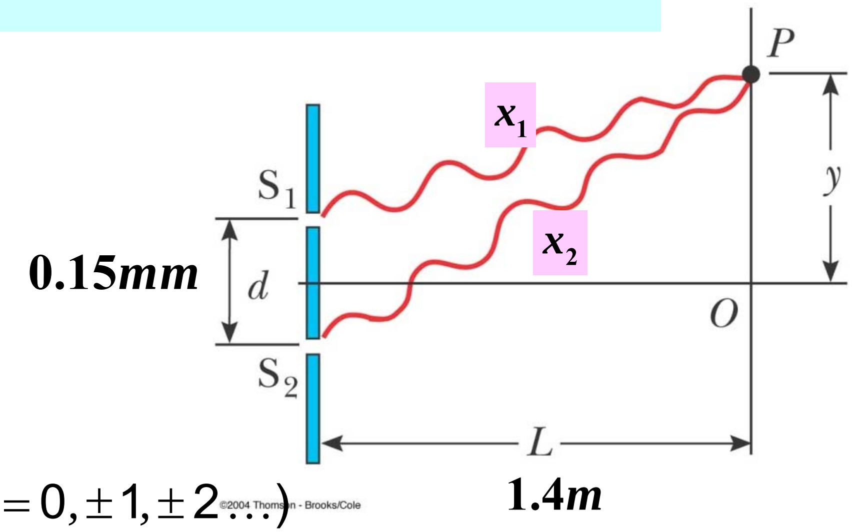
Double-Slit Experiment: Example

The two slits are separated by 0.150 mm, and the incident light includes light of wavelengths $\lambda_1 = 540nm$ and $\lambda_2 = 450nm$. At what minimal distance from the center of the screen the bright line of the λ_1 light coincides with a bright line of the λ_2 light

Bright lines:

$$y_{\text{bright},1} = \frac{\lambda_1 L}{d} m_1 \quad (m_1 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{\text{bright},2} = \frac{\lambda_2 L}{d} m_2 \quad (m_2 = 0, \pm 1, \pm 2 \ldots)$$



$$y_{\text{bright},1} = \frac{540 \cdot 10^{-9} \cdot 1.4}{0.15 \cdot 10^{-3}} m_1(\text{m}) = 5m_1(\text{mm}) \qquad (m_1 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{\text{bright},2} = \frac{450 \cdot 10^{-9} \cdot 1.4}{0.15 \cdot 10^{-3}} m_2(\text{m}) \approx 4m_2(\text{mm}) \qquad (m_2 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{\text{bright},1} = 0, 5, 10, 15, 20, 25 \ldots (\text{mm})$$

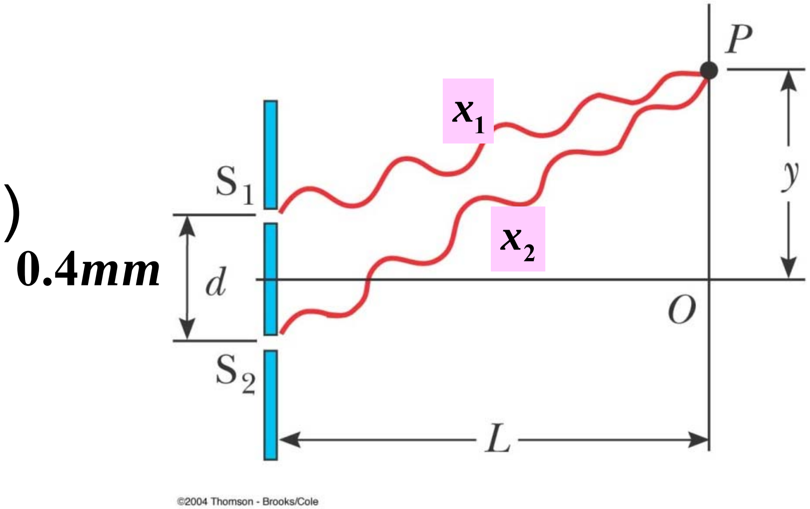
$$y_{\text{bright},1} = 0, 4, 8, 12, 16, 20 \ldots (\text{mm})$$

Double-Slit Experiment: Example

Light with a wavelength of ***442 nm*** passes through a double-slip system that has a slip separation ***d=0.4 mm***. Determine ***L*** so that the first dark fringe appears directly opposite both slits.

Dark lines:

$$y_{\text{dark},n} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2 \ldots)$$



$$y_{\text{dark},1} = \frac{d}{2}$$

$$y_{\text{dark},1} = \frac{1}{2}\frac{\lambda L}{d}$$

$$\frac{d}{2} = \frac{1}{2}\frac{\lambda L}{d}$$

$$L = \frac{d^2}{\lambda} = \frac{0.4^2 \cdot 10^{-6} m^2}{442 \cdot 10^{-9} m} = 0.36m = 36cm$$

Chapter 22

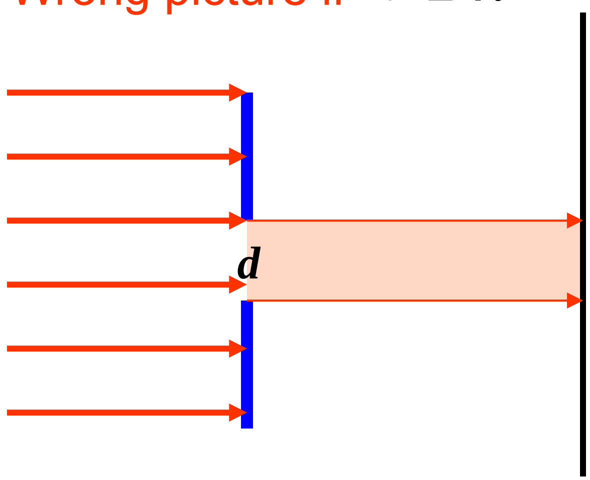
Diffraction Pattern and Interference

Diffraction

Diffraction:

Light spreads beyond the narrow path defined by the slit into regions that would be in shadow if light traveled in straight lines

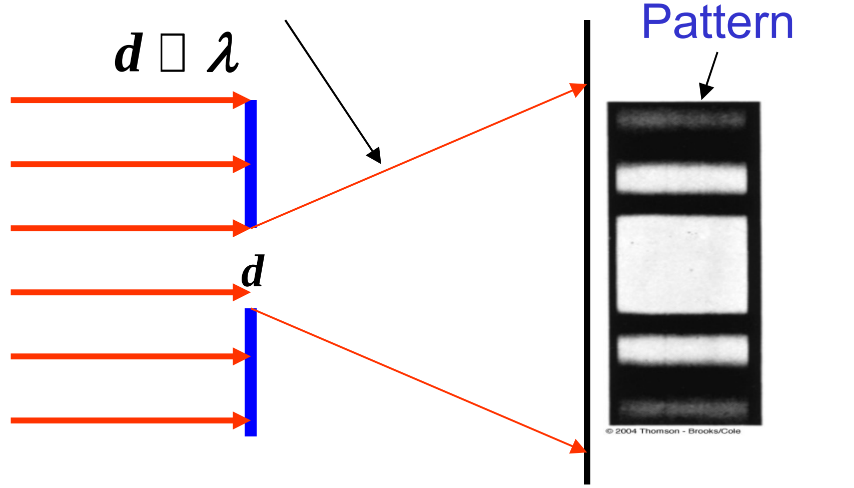
Wrong picture if $d \,\square\, \lambda$



Geometric Optics - if $d >> \lambda$

Diffraction

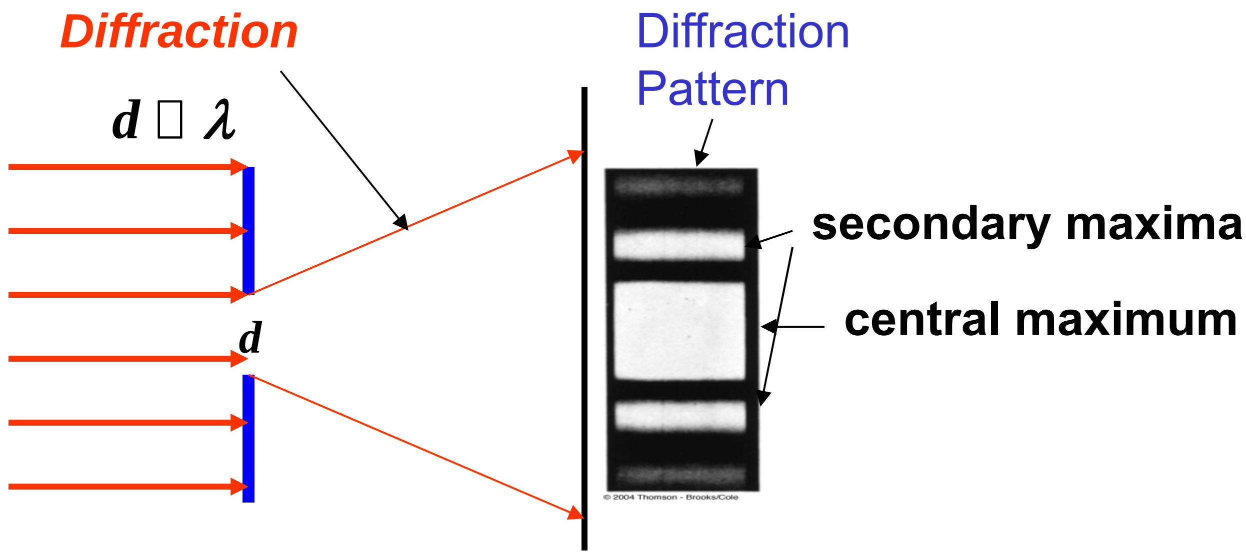
Diffraction Pattern

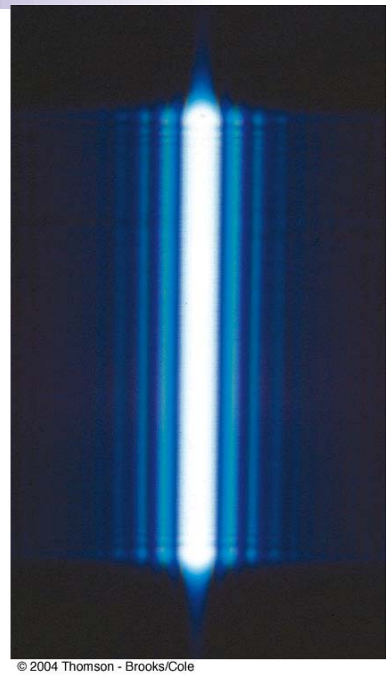


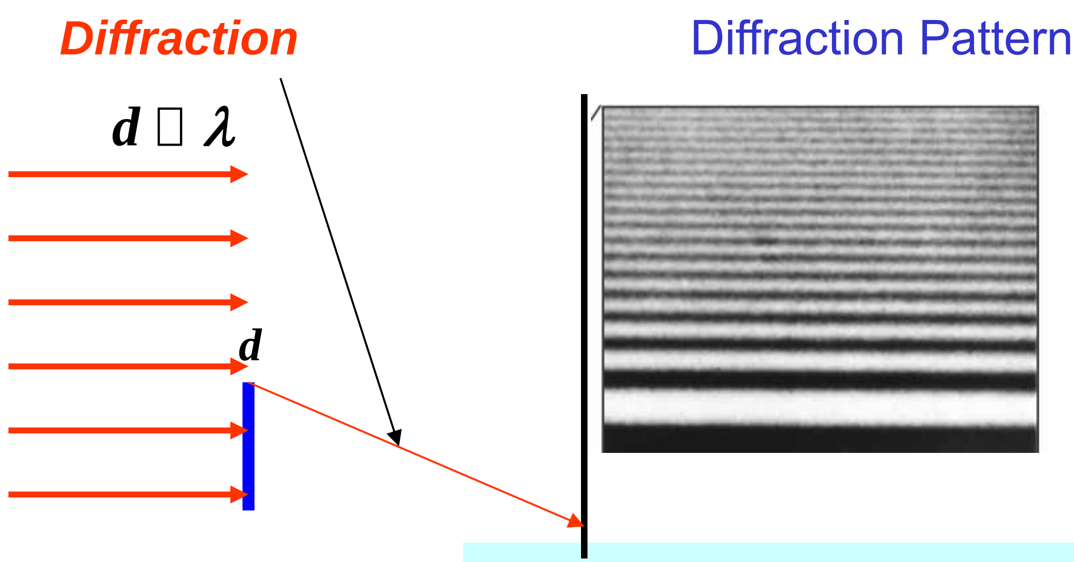
Diffraction and Interference are closely related;

Diffraction Patterns are due to Interference

Diffraction Pattern







Diffraction Pattern is similar to Interference Pattern

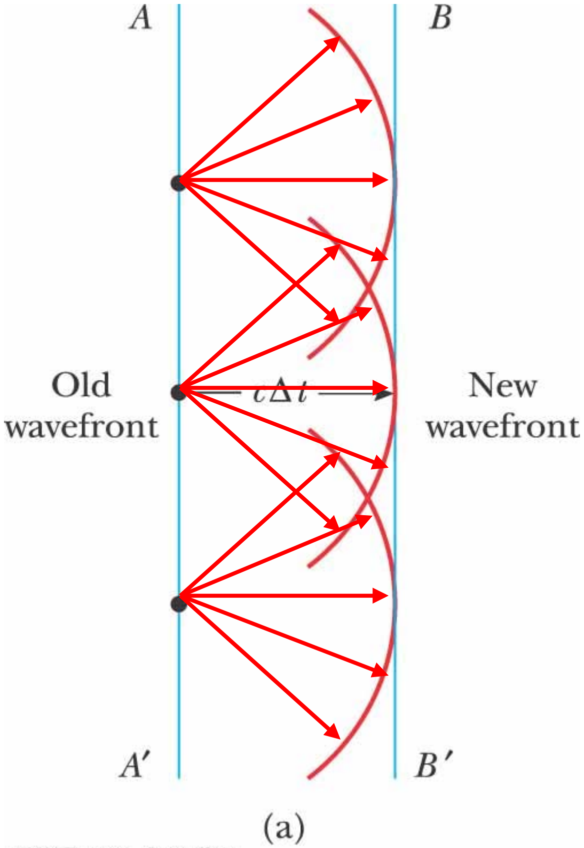
Huygens's Principle

Huygens's Principle

Huygens's Principle is a geometric construction for determining the position of a new wave at some point based on the knowledge of the wave front that preceded it

- All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outward through a medium with speeds characteristic of waves in that medium

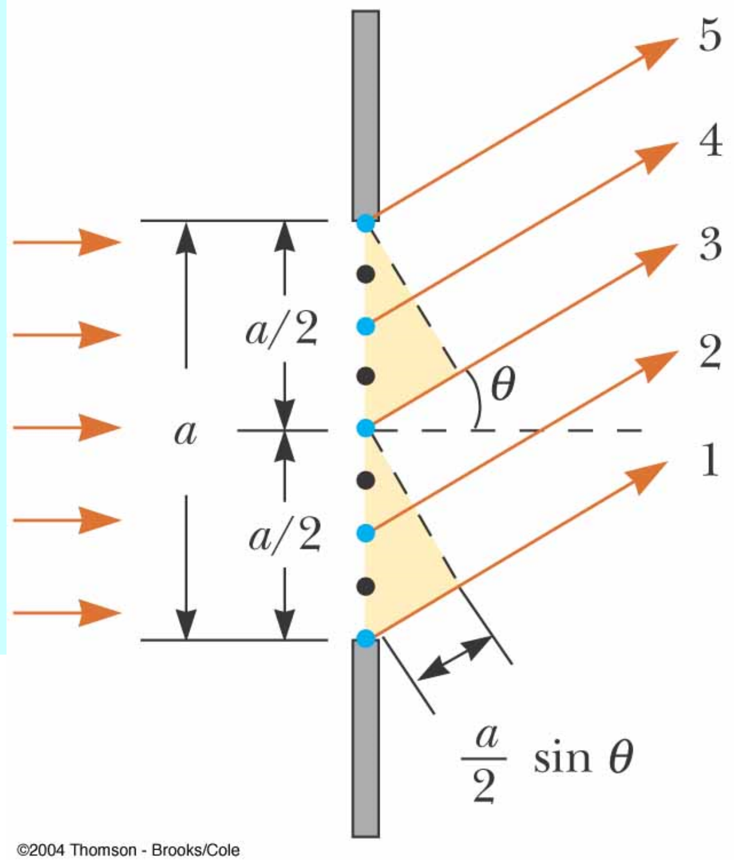
- After some time has passed, the new position of the wave front is the surface tangent to the wavelets



Single-Slip Diffraction

Single Slit Diffraction

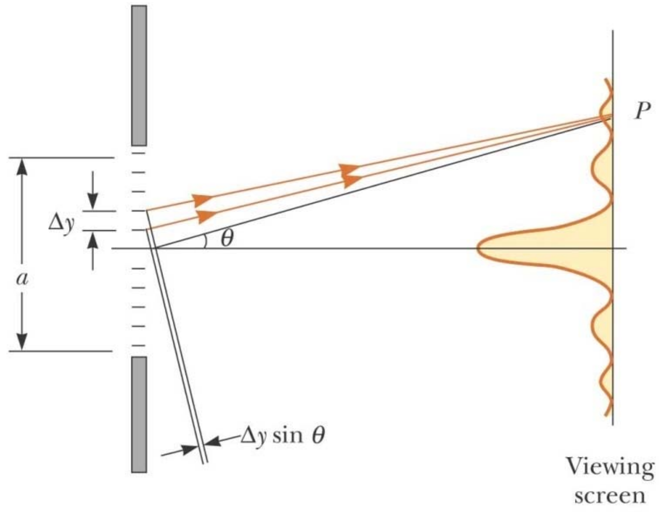
- Each portion of the slit acts as a source of light waves
- Therefore, light from one portion of the slit can *interfere* with light from another portion



Intensity of Single-Slit Diffraction Pattern

$$I(\varphi) = I_{\max}\left[\frac{\sin(\varphi/2)}{\varphi/2}\right]^2 \qquad \varphi = \frac{2\pi a\sin\theta}{\lambda}$$

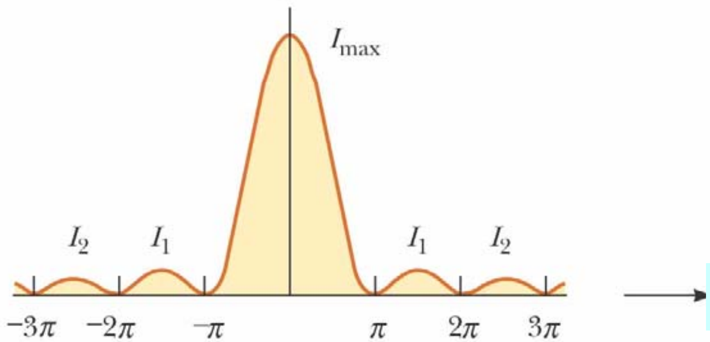
$$I(\theta) = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2$$



The first minimum occurs at

$\sin(\varphi/2) = 0$ or $\varphi = 2\pi$ or

$\sin\theta_{dark} = \lambda/a$



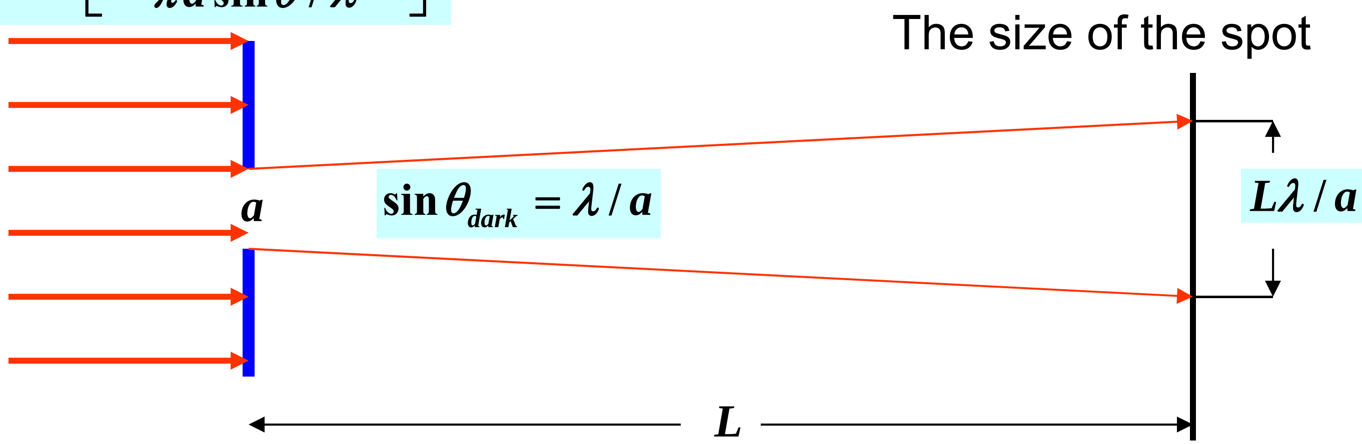
(a)

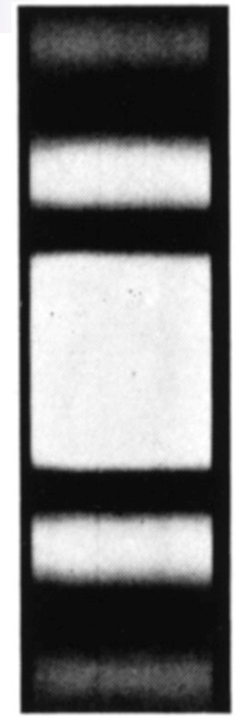
$\rightarrow \varphi/2$

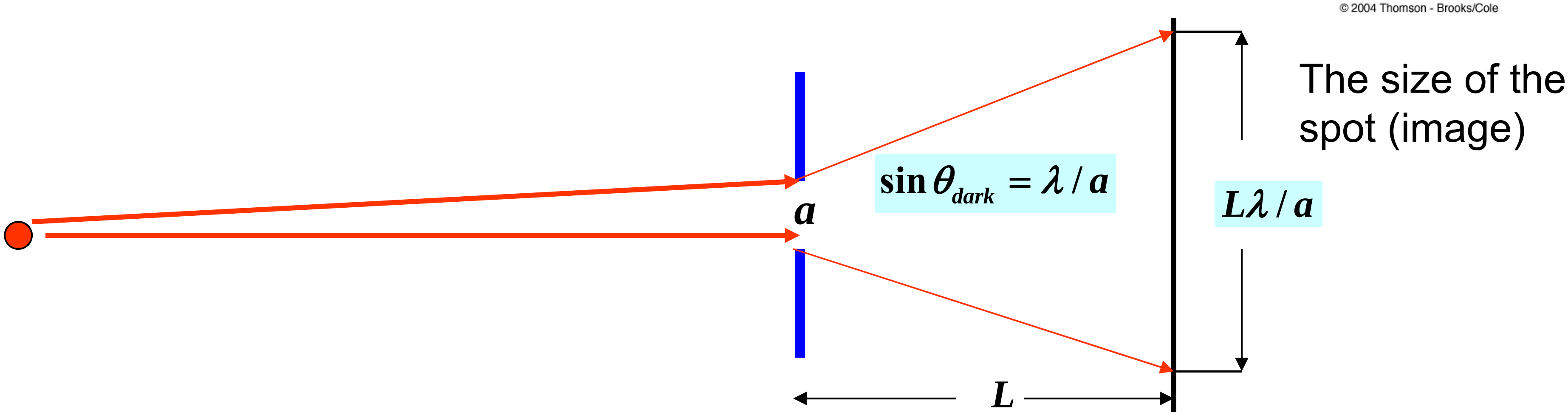
(b)

Diffraction

$$I(\theta) = I_{\max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$







Diffraction: Example

The source of the light emits the light with wavelength $\lambda = 540nm$.
The diffraction pattern is observed in the water, $n = 1.33$.

L = 10m, a=0.5 mm

What is the size of the spot, *D* ?

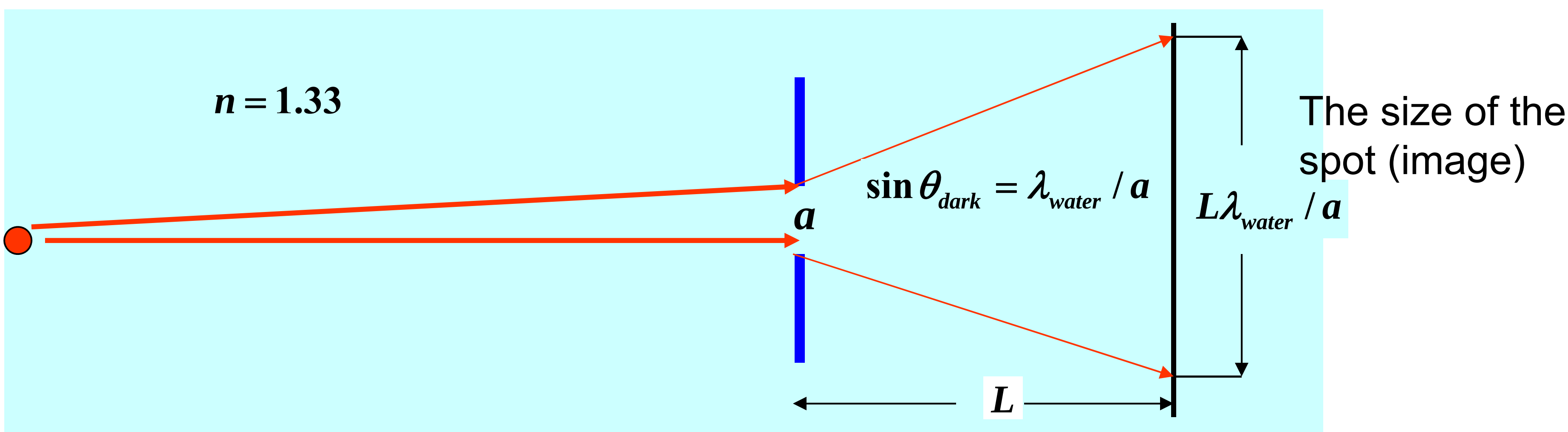
wavelength in the water

$\sin\theta_{dark} = \lambda_{water} / a$

$D = L\lambda_{water} / a$

$\lambda_{water} = \lambda / n$

$$D = \frac{L\lambda}{na} = \frac{10 \cdot 540 \cdot 10^{-9}}{1.33 \cdot 0.5 \cdot 10^{-3}} m = 8 \cdot 10^{-3} m = 8mm$$

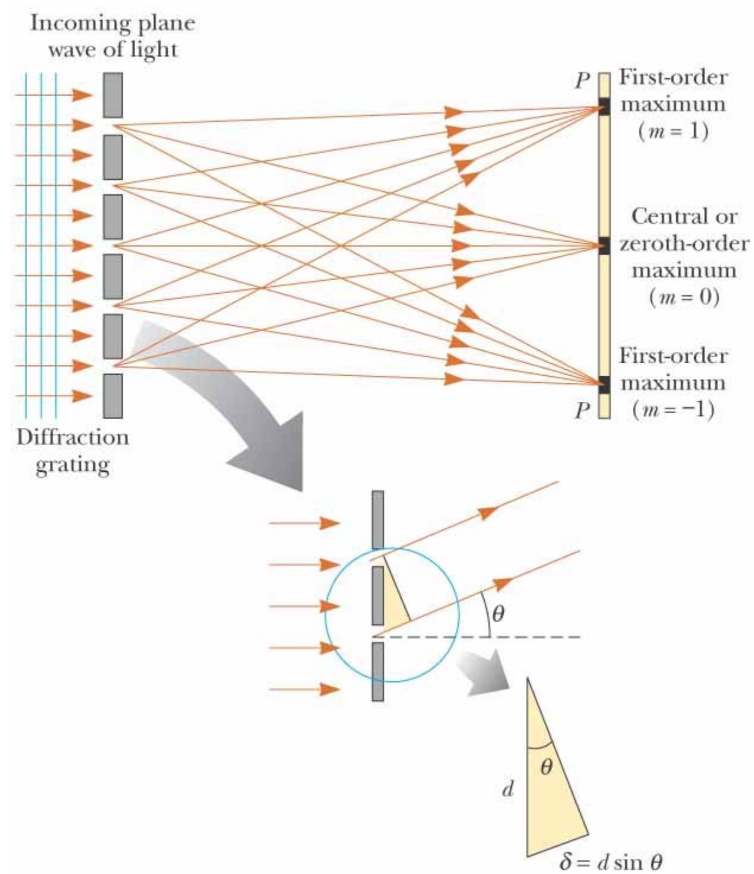


Chapter 22

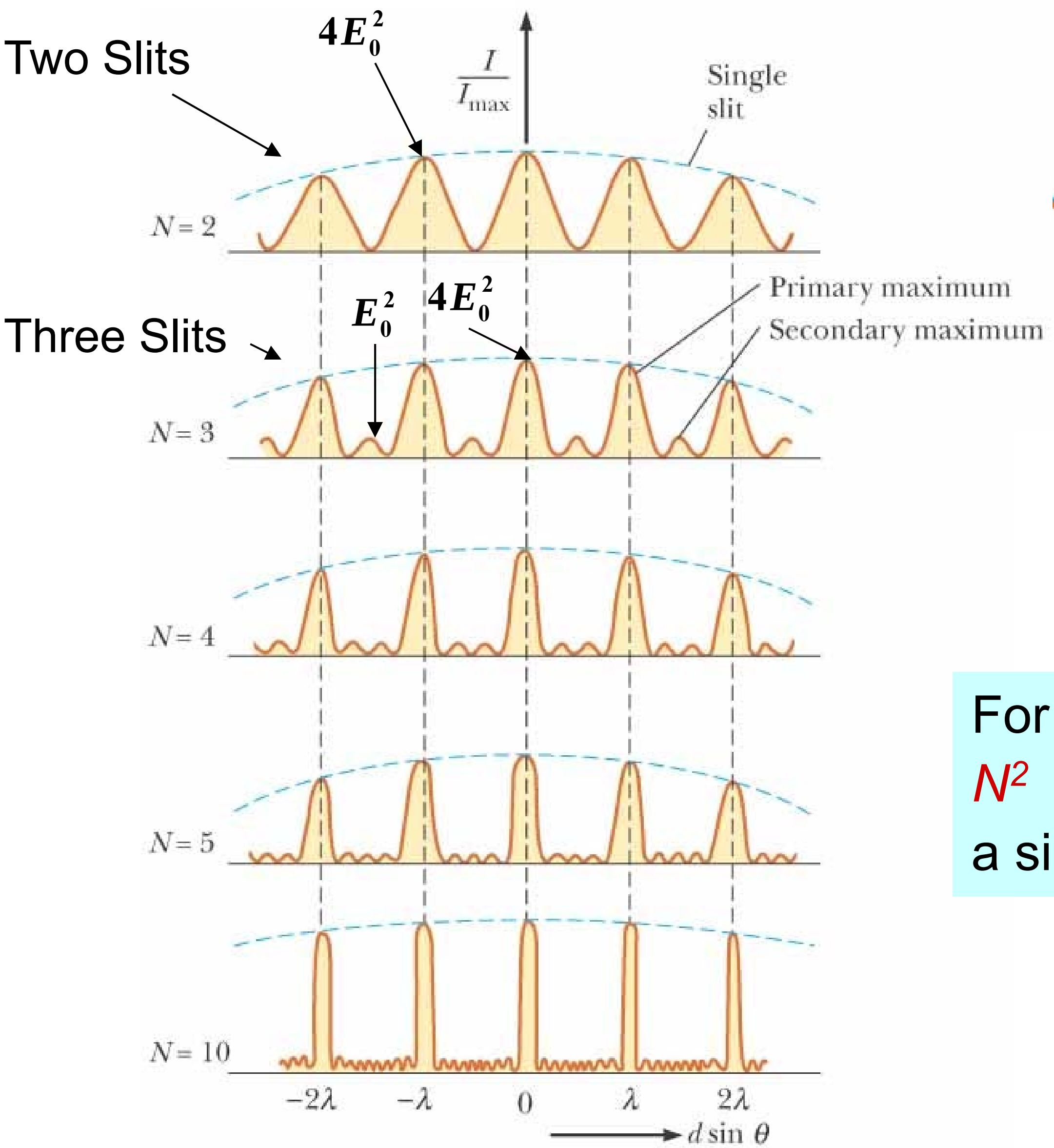
Diffraction Grating

Diffraction Grating

- The ***diffraction grating*** consists of a ***large number*** of ***equally spaced*** parallel slits
  - A typical grating contains several thousand lines per centimeter
- The intensity of the pattern on the screen is the result of the ***combined effects of interference and diffraction***
  - Each slit produces diffraction, and the diffracted beams interfere with one another to form the final pattern



N-Slit Interference: Intensity Graph



For *N* slits, the primary maxima is N^2 times greater than that due to a single slit

Diffraction Grating

The condition for *maxima* is

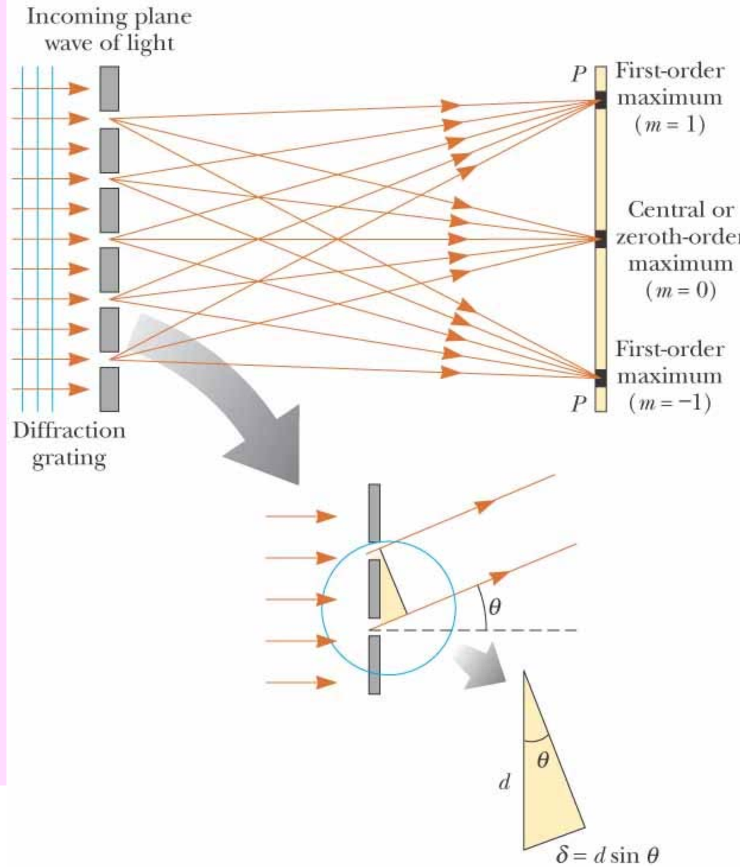
$$\Delta\varphi = 2\pi m \quad , \qquad m = 0, \pm 1, \pm 2, \ldots$$

$$\Delta\varphi = 2\pi \frac{\delta}{\lambda} = 2\pi \frac{d \sin\theta_{bright}}{\lambda}$$

then

$$d \sin\theta_{bright} = m\lambda$$

The integer ***m*** is the *order number* of the diffraction pattern

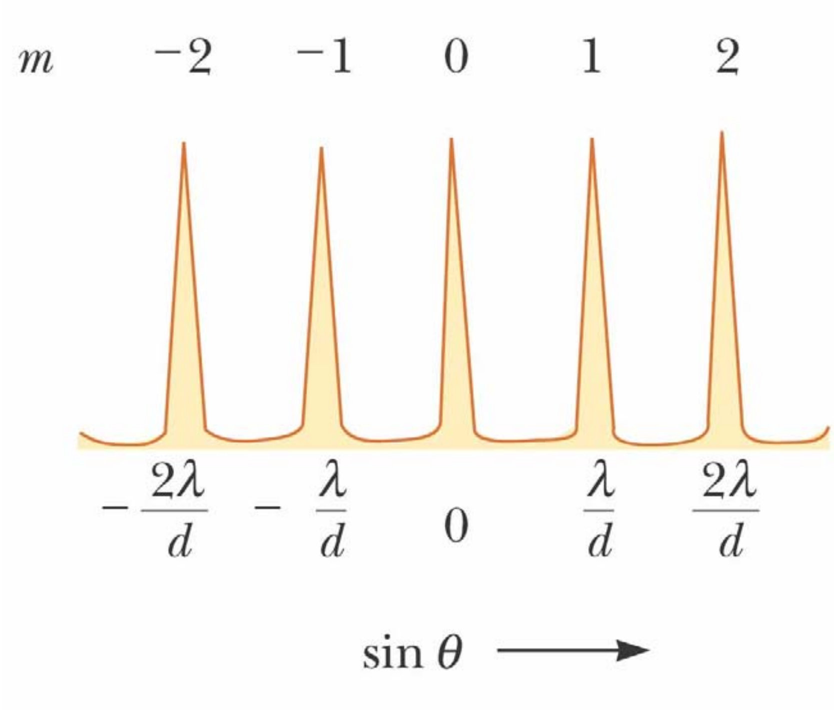


$$\Delta\varphi = 2\pi m$$

Diffraction Grating

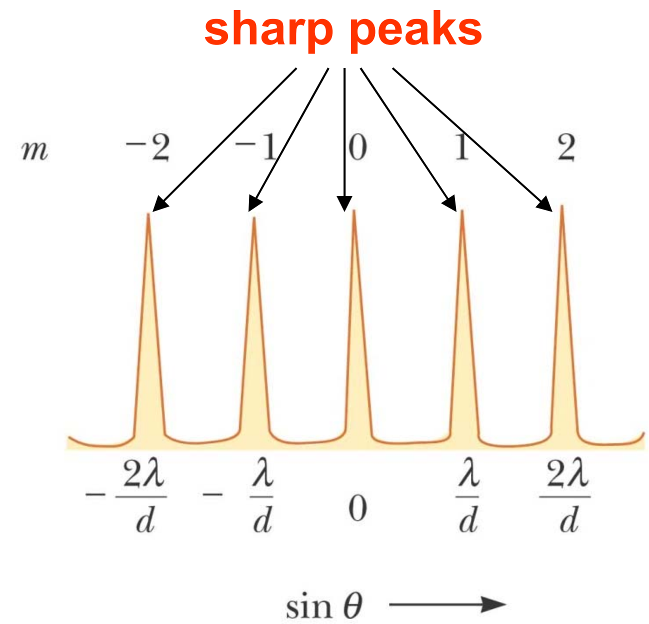
- All the wavelengths are seen at ***m* = 0**
  - This is called the zeroth-order maximum
- The first-order maximum corresponds to *m* = 1
- Note the sharpness of the principle maxima and the broad range of the dark areas

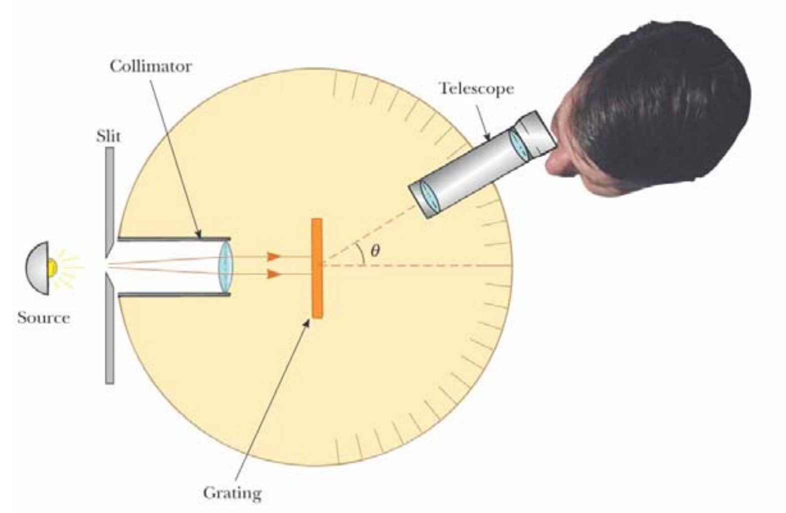
$$d \sin \theta_{bright} = m\lambda$$



Diffraction Grating Spectrometer

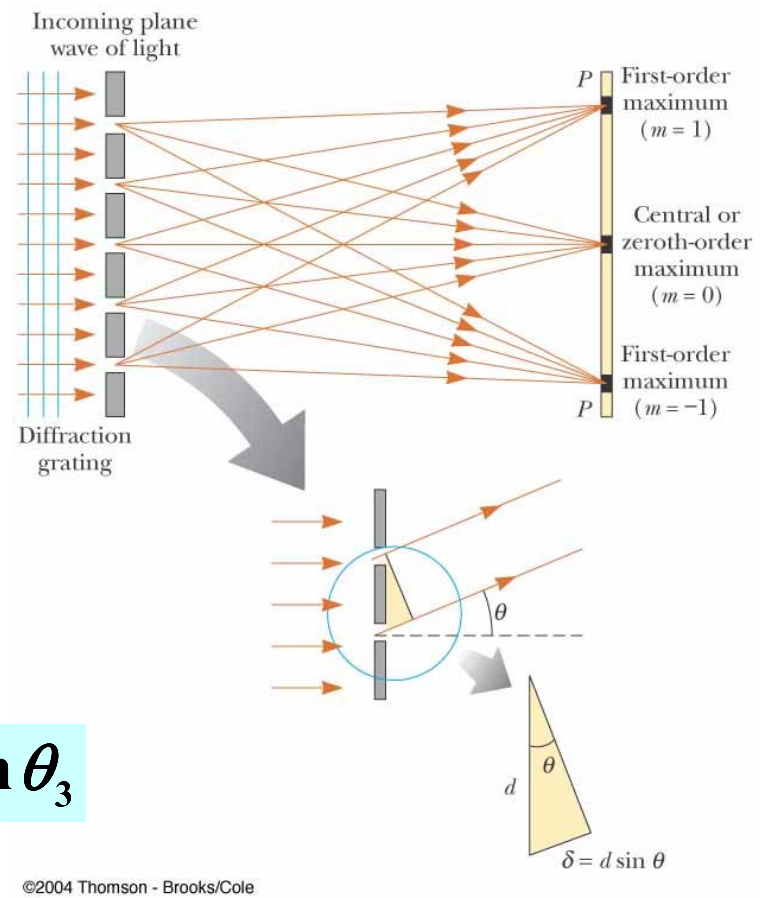
- The collimated beam is incident on the grating
- The diffracted light leaves the gratings and the telescope is used to view the image
- The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders





Diffraction Grating: Example

Three discrete spectral lines occur at angles 10.09^0, 13.71^0, and 14.77^0 in the first order spectrum of a grading spectrometer. If the grading has ***N*=**3600 slits per centimeter, what are the wavelength of the light?



$$d \sin\theta_{bright} = m\lambda$$

First order means that ***m*=1,** then

$$\lambda_1 = d\sin\theta_1 \qquad \lambda_2 = d\sin\theta_2 \qquad \lambda_3 = d\sin\theta_3$$

$$d = \frac{1cm}{N}$$

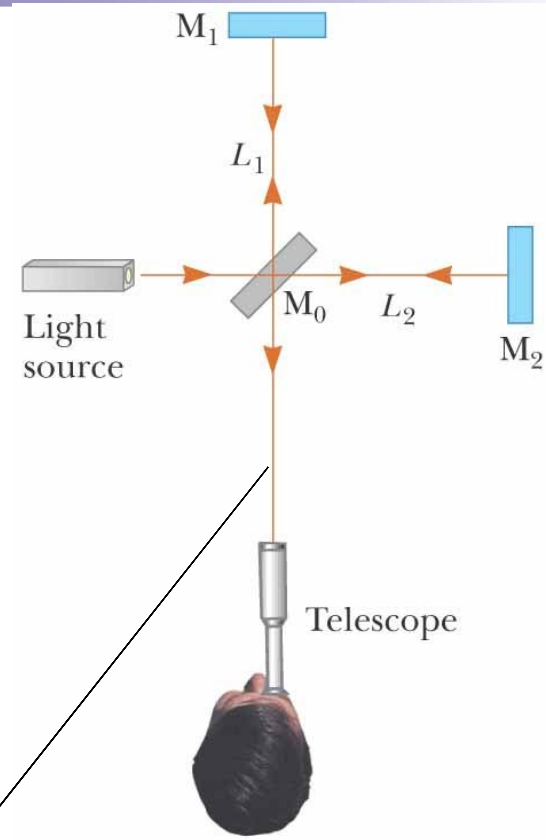
Then

$$\lambda_1 = \frac{1}{N}\sin\theta_1 = \frac{\sin 10.09}{3600}cm = 480nm \qquad \lambda_2 = \frac{\sin 13.71}{3600}cm = 658nm \qquad \lambda_3 = \frac{\sin 14.77}{3600}cm = 708nm$$

Michelson Interferometer

Michelson Interferometer

- A ray of light is split into two rays by the mirror M_o
  - The mirror is at 45° to the incident beam
  - The mirror is called a *beam splitter*
- It transmits half the light and reflects the rest
- The two rays travel separate paths L_1 and L_2



$$\Delta\varphi = 2\pi \frac{2(L_2 - L_1)}{\lambda}$$

Maximum (constructive interference):

$$\Delta\varphi = 2\pi \frac{2(L_2 - L_1)}{\lambda} = 2\pi m$$

$$2(L_2 - L_1) = \lambda m \qquad m = 0, \pm 1, \pm 2, \ldots$$

Michelson Interferometer

The maximum – with and without glass film. What is the value of *d*?

Without glass film: maximum (constructive interference):

$$2(L_2 - L_1) = m_1\lambda \qquad m_1 = 0, \pm 1, \pm 2, \ldots$$

With glass film: maximum (constructive interference):

$$\Delta\varphi = 2\pi\frac{2L_2}{\lambda} - 2\pi\frac{2(L_1 - d)}{\lambda} - 2\pi n\frac{2d}{\lambda}$$

$$\Delta\varphi = 2\pi m_2 \qquad 2[L_2 - L_1 - d(n-1)] = m_2\lambda$$

$$m_2 = 0, \pm 1, \pm 2, \ldots$$

$$2d(n-1) = (m_2 - m_1)\lambda = m_3\lambda$$

$$m_3 = 0, \pm 1, \pm 2, \ldots$$