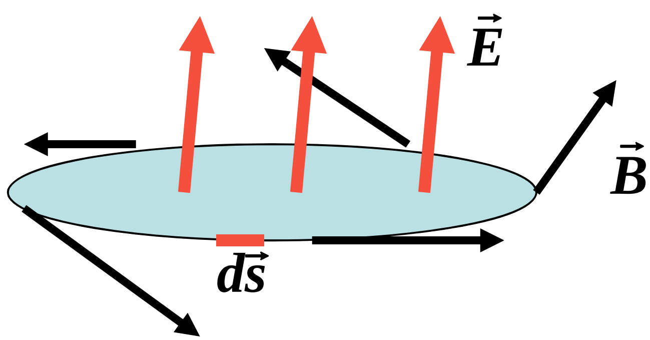
Chapter 31

Faraday’s Law

Ampere's law

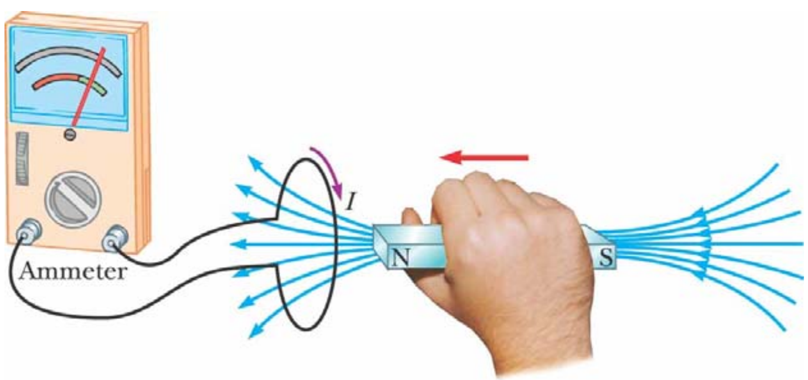
- **Magnetic field is produced by time variation of electric field**

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \left(I + I_d \right) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



Induction

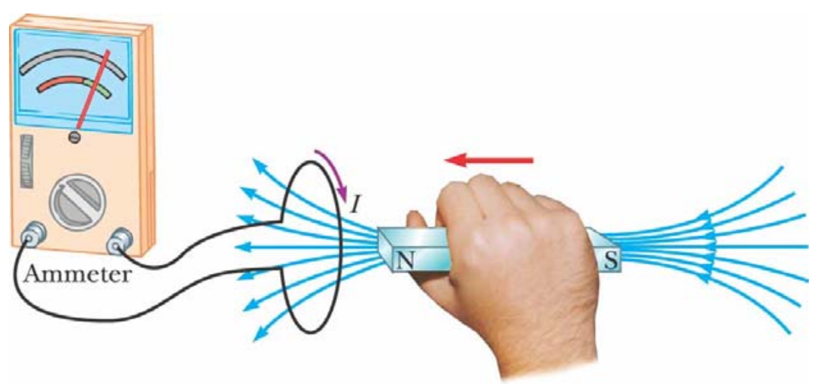
- A loop of wire is connected to a sensitive ammeter
- When a magnet is moved toward the loop, the ammeter deflects



(a)

Induction

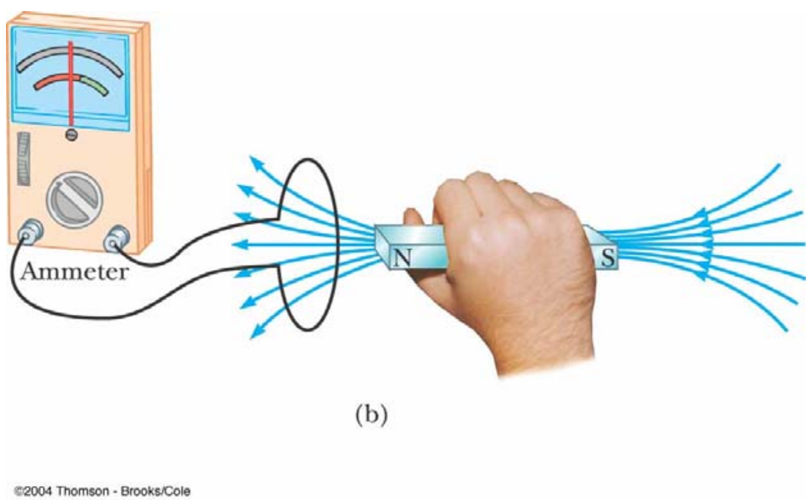
- An ***induced current*** is produced by a changing magnetic field
- There is an ***induced emf*** associated with the induced current
- A current can be produced without a battery present in the circuit
- Faraday's law of induction describes the induced emf



(a)

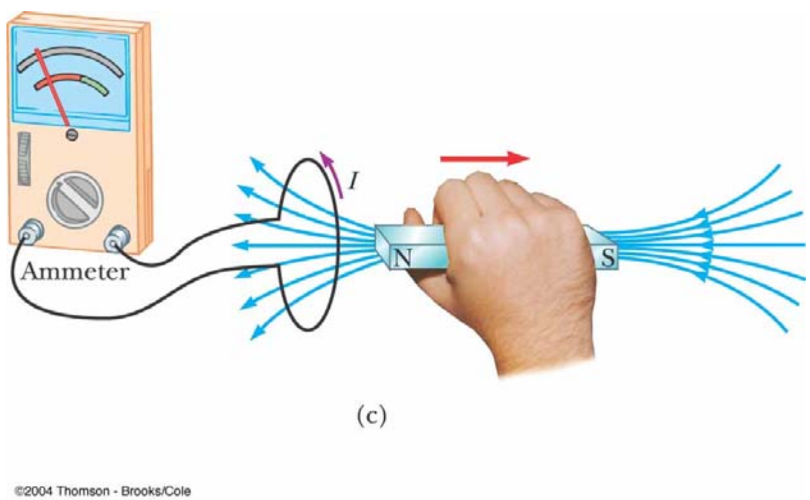
Induction

- When the magnet is held stationary, there is no deflection of the ammeter
- Therefore, there is no induced current
  - Even though the magnet is in the loop



Induction

- The magnet is moved away from the loop
- The ammeter deflects in the opposite direction



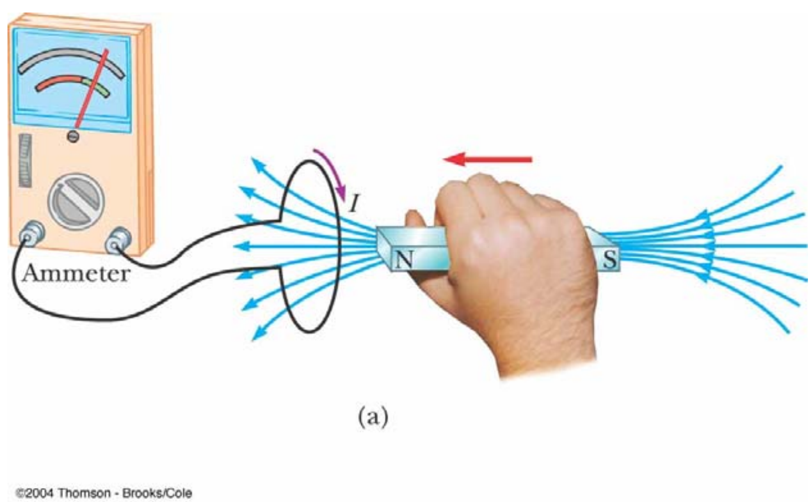
Induction

- The ammeter deflects when the **magnet is moving toward or away from the loop**
- The ammeter also deflects when **the loop is moved toward or away from the magnet**
- Therefore, the loop detects that the magnet is moving relative to it
  - We relate this detection to a change in the magnetic field
  - This is the induced current that is produced by an **induced emf**

Faraday's law

- Faraday's law of induction states that "the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit"
- Mathematically,

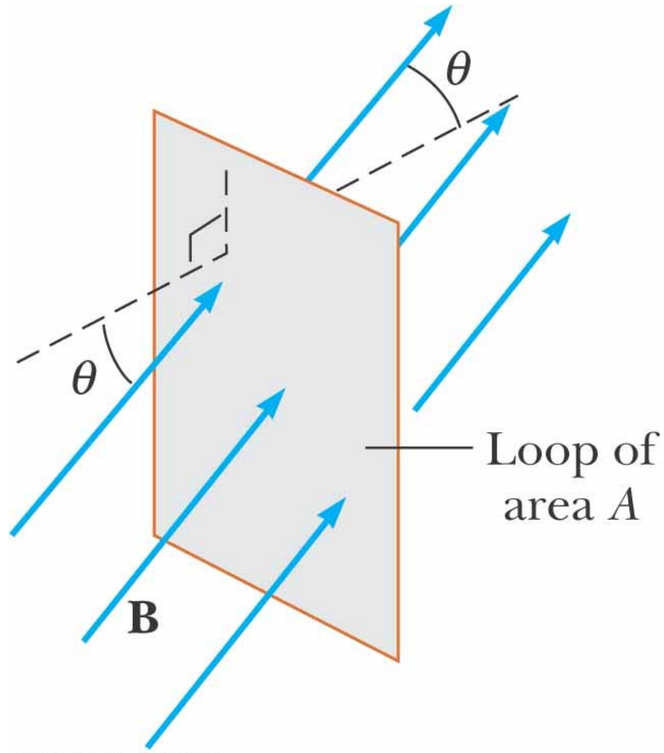
$$\varepsilon = -\frac{d\Phi_B}{dt}$$



Faraday's law

- Assume a loop enclosing an area *A* lies in a uniform magnetic field **B**
- The magnetic flux through the loop is $\Phi_B = BA \cos \theta$
- The induced emf is

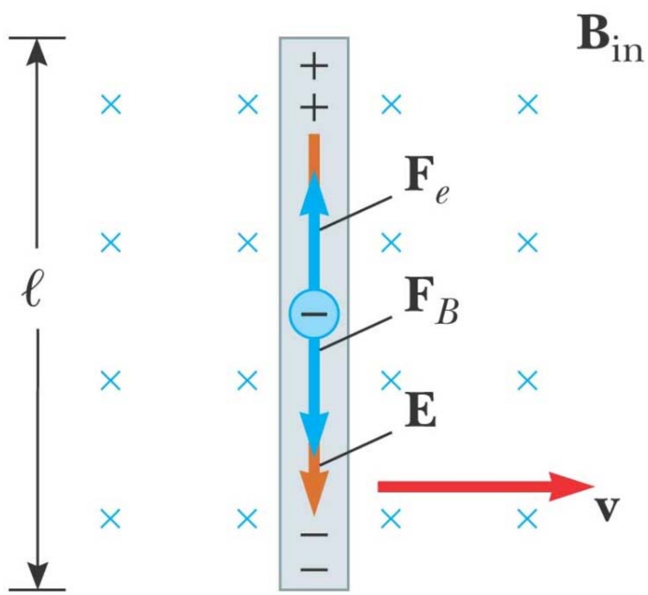
$$\varepsilon = -\frac{d(BA\cos\theta)}{dt}$$



- **Ways of inducing emf:**
- The magnitude of **B** can change with time
- The area ***A*** enclosed by the loop can change with time
- The angle θ can change with time
- Any combination of the above can occur

Motional emf

- A **motional emf** is one induced in a conductor moving through a constant magnetic field
- The electrons in the conductor experience a force, $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ that is directed along ℓ

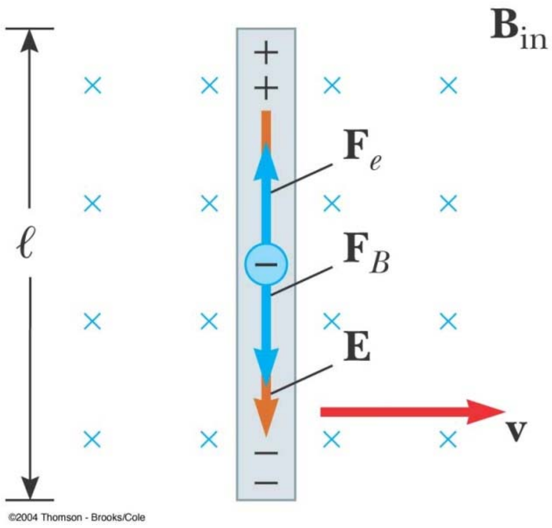


Motional emf

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

- Under the influence of the force, the electrons move to the lower end of the conductor and accumulate there
- As a result, an electric field **E** is produced inside the conductor
- The charges accumulate at both ends of the conductor until they are in equilibrium with regard to the electric and magnetic forces

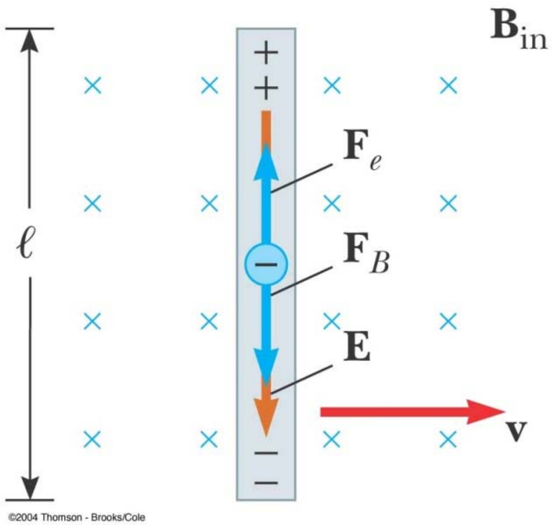
$$qE = qvB \quad \text{or} \quad E = vB$$



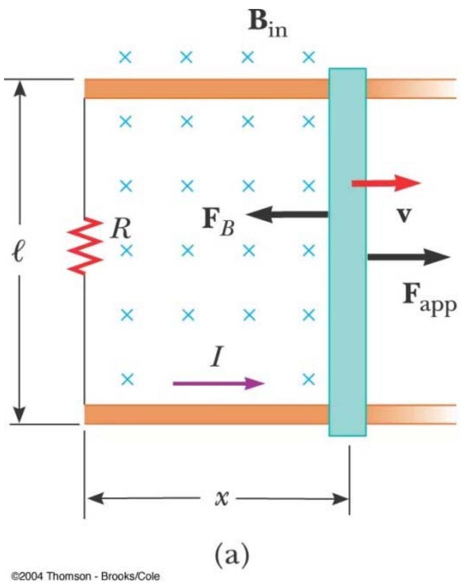
Motional emf

$$E = vB$$

- A potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field
- If the direction of the motion is reversed, the polarity of the potential difference is also reversed



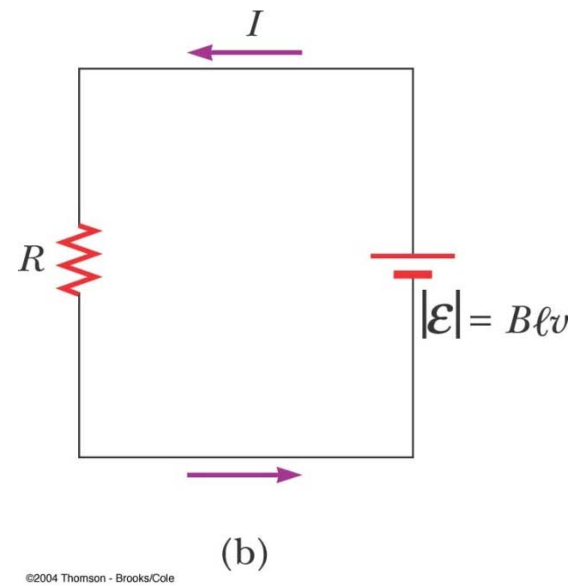
Example: Sliding Conducting Bar



(a)

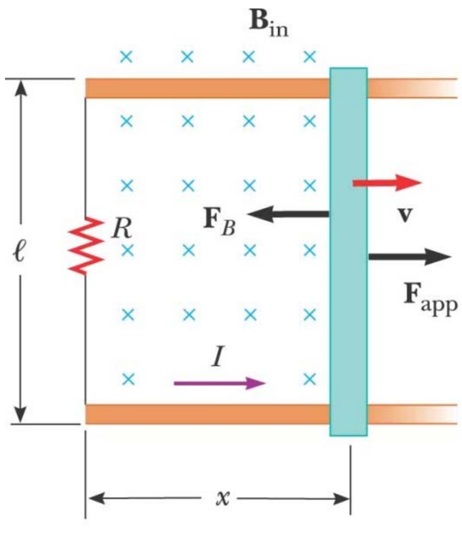
$$E = vB$$

$$\varepsilon = El = Blv$$

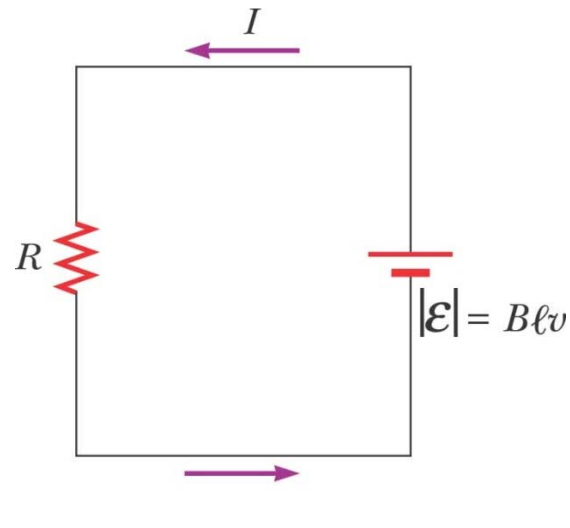


(b)

Example: Sliding Conducting Bar



(a)



(b)

- The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

Lenz’s law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

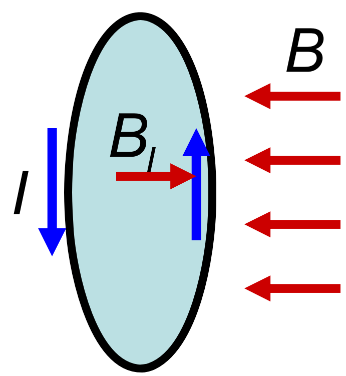
- Faraday’s law indicates that the induced emf and the change in flux have opposite algebraic signs
- This has a physical interpretation that has come to be known as **Lenz’s law**
- **Lenz’s law**: *the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop*
- The induced current tends to keep the original magnetic flux through the circuit from changing

Lenz's law

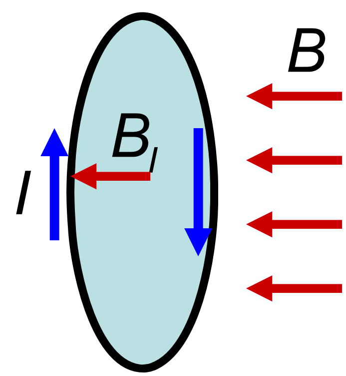
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- **Lenz's law**: *the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop*
- The induced current tends to keep the original magnetic flux through the circuit from changing

B is increasing with time

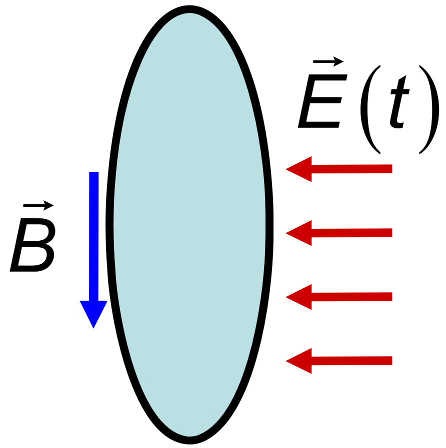


B is decreasing with time

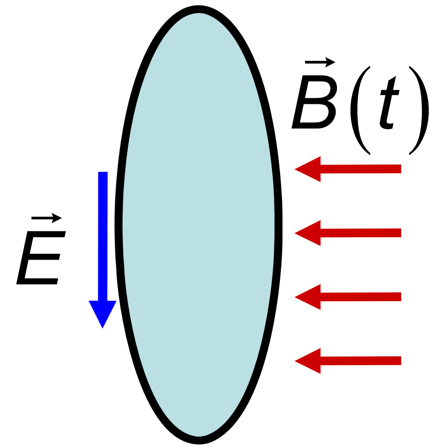


Electric and Magnetic Fields

Ampere-Maxwell law



Faraday's law

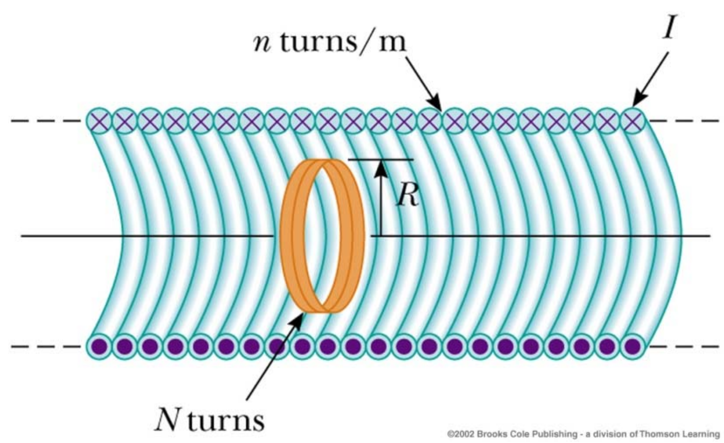


Example 1

.A long solenoid has ***n*** turns per meter and carries a current $I = I_{max}\left(1 - e^{-\alpha t}\right)$
Inside the solenoid and coaxial with it is a coil that has a radius ***R*** and consists of a total of ***N*** turns of fine wire.
What emf is induced in the coil by the changing current?

$$B(t) = \mu_0 n I(t)$$

$$\Phi(t) = \pi R^2 N B(t) = \mu_0 \pi R^2 N n I(t)$$



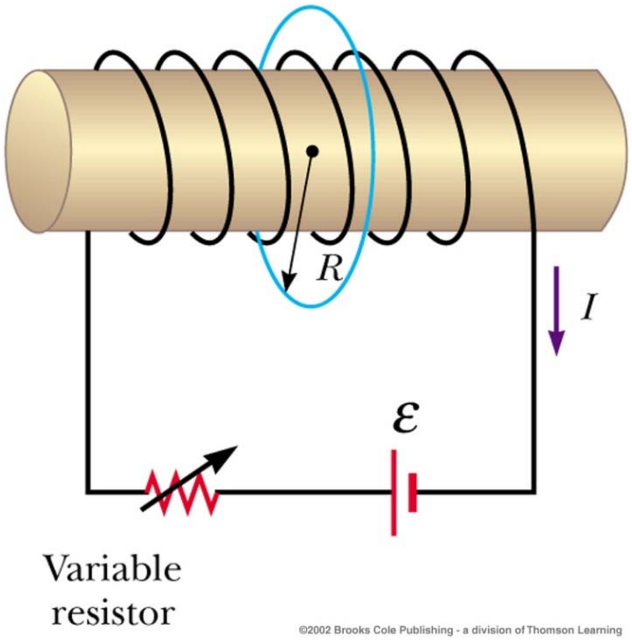
$$\mathcal{E} = -\frac{d\Phi(t)}{dt} = -\mu_0 \pi R^2 N n \frac{dI(t)}{dt} = \mu_0 \pi R^2 N n \alpha I_{max} e^{-\alpha t}$$

Example 2

A single-turn, circular loop of radius ***R*** is coaxial with a long solenoid of radius ***r*** and length ***ℓ*** and having ***N*** turns. The variable resistor is changed so that the solenoid current decreases linearly from I_1 to I_2 in an interval **Δ***t*. Find the induced emf in the loop.

$$B(t) = \mu_0 \frac{N}{l} I(t)$$

$$\Phi(t) = \pi r^2 B(t) = \mu_0 \pi r^2 \frac{N}{l} I(t)$$



$$\varepsilon = -\frac{d\Phi(t)}{dt} = -\mu_0 \pi r^2 \frac{N}{l} \frac{dI(t)}{dt} = -\mu_0 \pi r^2 \frac{N}{l} \frac{I_2 - I_1}{\Delta t}$$

Example 3

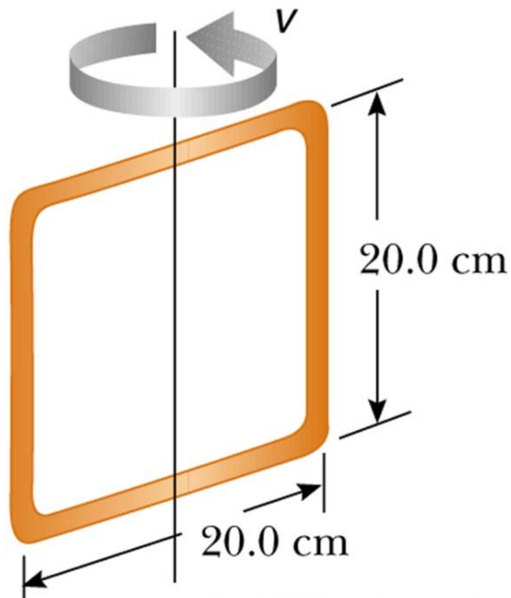
A square coil (20.0 cm × 20.0 cm) that consists of 100 turns of wire rotates about a vertical axis at 1 500 rev/min. The horizontal component of the Earth's magnetic field at the location of the coil is 2.00 × 10^{-5} T. Calculate the maximum emf induced in the coil by this field.

$$\Phi = BA\cos\theta$$

$$\varepsilon = -\frac{d(BA\cos\theta)}{dt} \qquad \theta = \omega t$$

$$\varepsilon = -BA\frac{d(\cos\omega t)}{dt} = BA\omega\sin\omega t$$

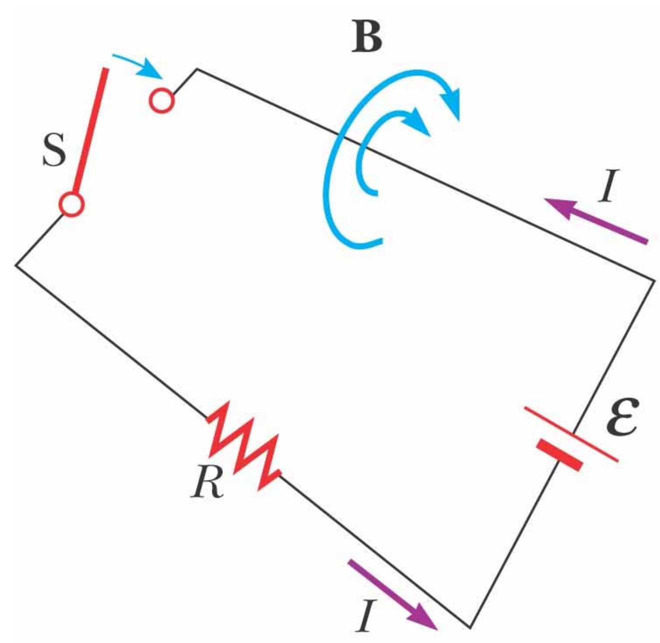
$$\varepsilon_{\max} = BA\omega = 12.6mV$$

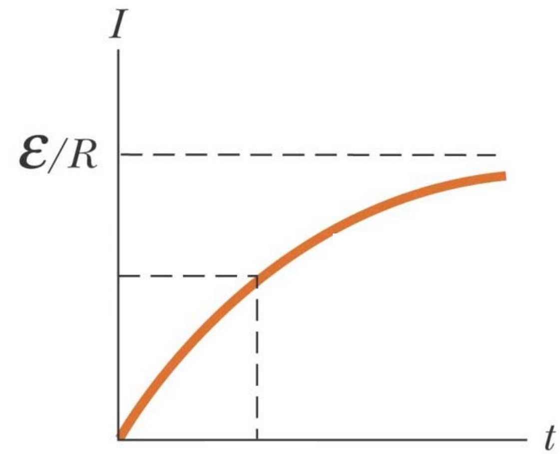


Induction

Self-Inductance

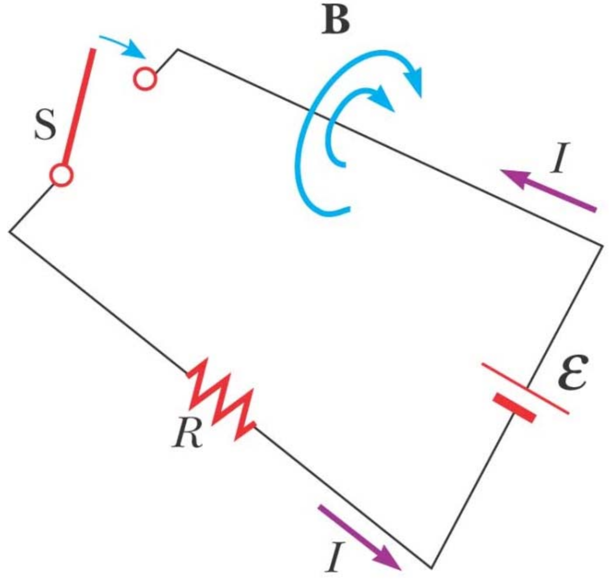
- When the switch is closed, the current does not immediately reach its maximum value
- Faraday’s law can be used to describe the effect
- As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time
- This corresponding flux due to this current also increases
- This increasing flux creates an induced emf in the circuit

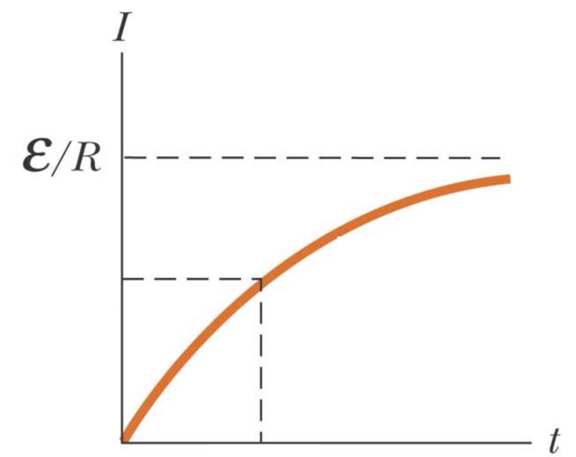




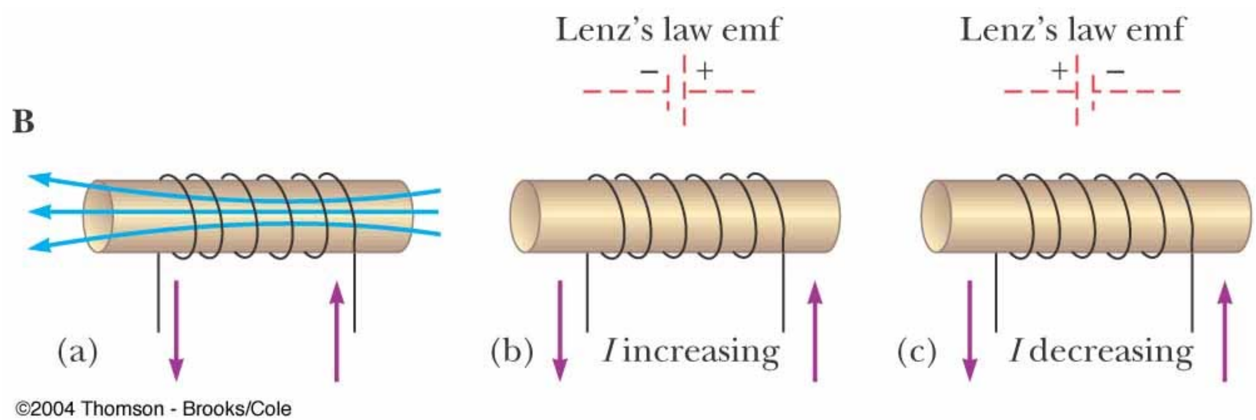
Self-Inductance

- **Lenz Law:** The direction of the induced emf is such that it would cause an induced current in the loop which would establish a magnetic field opposing the change in the original magnetic field
- The direction of the induced emf is opposite the direction of the emf of the battery
- This results in a ***gradual*** increase in the current to its final equilibrium value
- This effect is called **self-inductance**
- The emf ε_L is called a **self-induced emf**



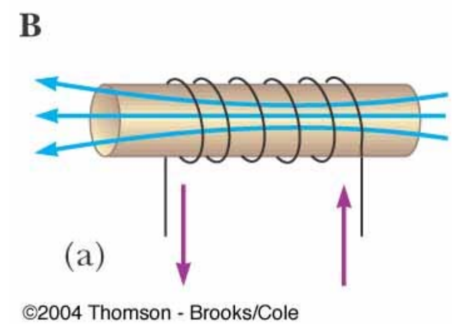


Self-Inductance: Coil Example



- A current in the coil produces a magnetic field directed toward the left
- If the current increases, the increasing flux creates an induced emf of the polarity shown in (b)
- The polarity of the induced emf reverses if the current decreases

Solenoid



- Assume a uniformly wound solenoid having ***N*** turns and length ***ℓ***
- The interior magnetic field is

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

- The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

- The magnetic flux through all ***N*** turns

$$\Phi_t = N\Phi_B = \mu_0 \frac{N^2 A}{\ell} I$$

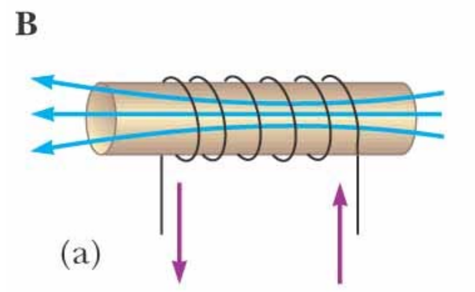
- If I depends on time then self-induced emf can found from the Faraday's law

$$\mathcal{E}_{si} = -\frac{d\Phi_t}{dt} = -\mu_0 \frac{N^2 A}{\ell} \frac{dI}{dt}$$

Solenoid

- The magnetic flux through all ***N*** turns

$$\Phi_t = \mu_0 \frac{N^2 A}{\ell} I = L I$$



- Self-induced emf:

$$\mathcal{E}_{si} = -\frac{d\Phi_t}{dt} = -\mu_0 \frac{N^2 A}{\ell} \frac{dI}{dt} = -L \frac{dI}{dt}$$

Inductance

$$\varepsilon_L = -L\frac{dI}{dt} \qquad \Phi = LI$$

- ***L*** is a constant of proportionality called the **inductance** of the coil and it depends on the geometry of the coil and other physical characteristics

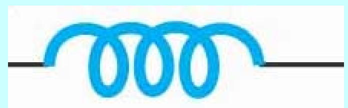
- The SI unit of inductance is the **henry** (H)

$$1\,\mathrm{H} = 1\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}$$

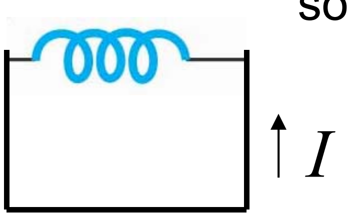
Named for Joseph Henry

Inductor

$$\varepsilon_L = -L\frac{dI}{dt} \qquad \Phi = L\,I$$

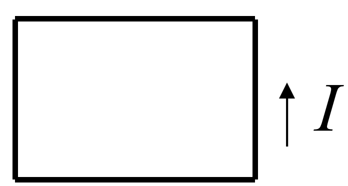
- A circuit element that has a large self-inductance is called an **inductor**
- The circuit symbol is 
- We assume the self-inductance of the rest of the circuit is negligible compared to the inductor
  - However, even without a coil, a circuit will have some self-inductance

$\Phi_1 = L_1 I$ Flux through solenoid



$L_1 >> L_2$

$\Phi_2 = L_2 I$ Flux through the loop



The effect of Inductor

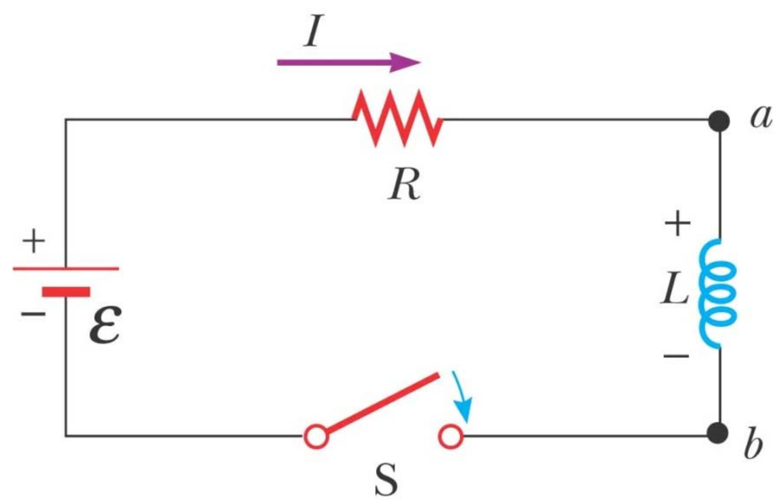
$$\varepsilon_L = -L\frac{dI}{dt} \qquad \Phi = LI$$

- The inductance results in a back emf
- Therefore, the inductor in a circuit opposes changes in current in that circuit

RL circuit

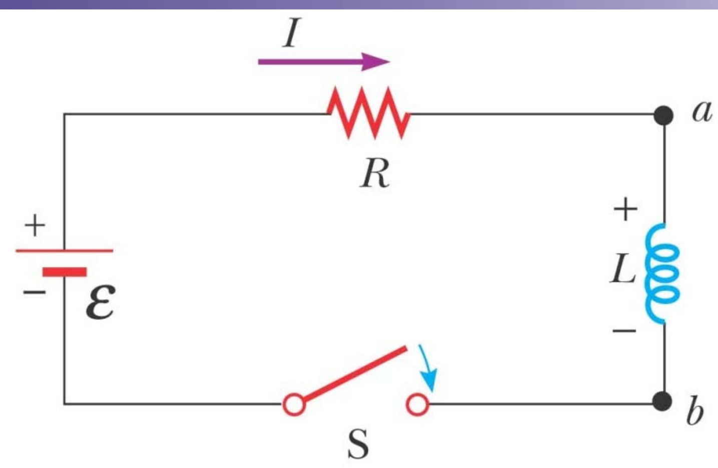
$$\mathcal{E}_L = -L\frac{dI}{dt} \qquad \Phi = LI$$

- An *RL* circuit contains an inductor and a resistor
- When the switch is closed (at time $t = 0$), the current begins to increase
- At the same time, a back emf is induced in the inductor that opposes the original increasing current



RL circuit

$$\varepsilon_L = -L\frac{dI}{dt}$$



- Kirchhoff's loop rule:

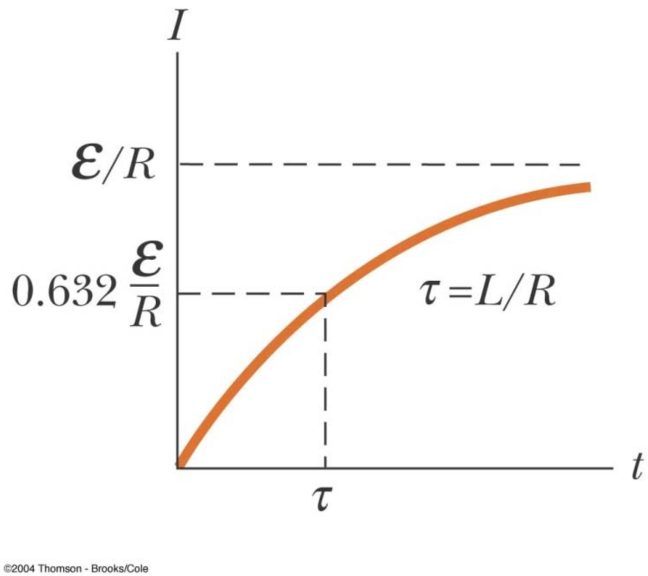
$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

- Solution of this equation:

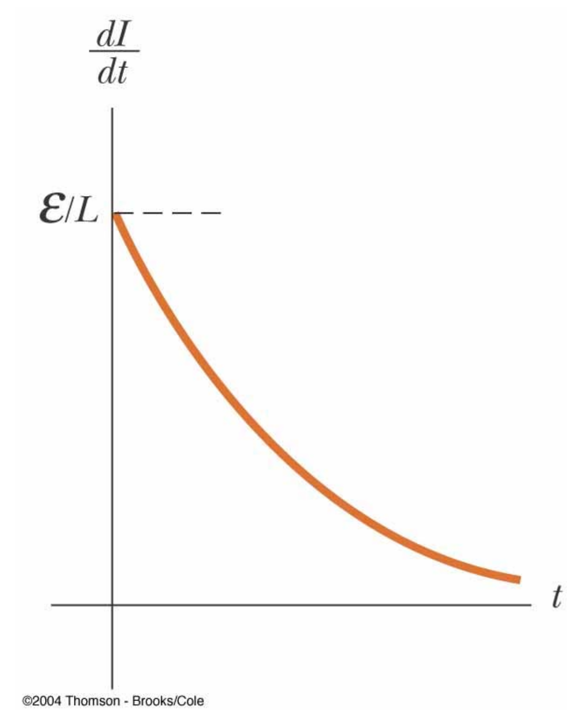
$$I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) \qquad I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

where $\tau = L/R$ - time constant

RL circuit



$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$



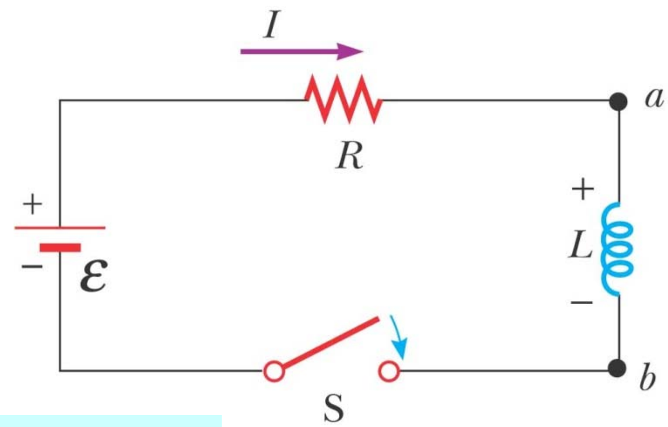
$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau}$$

Energy Density of Magnetic Field

Energy of Magnetic Field

$$\mathcal{E}_L = -L\frac{dI}{dt} \qquad \mathcal{E} = IR + L\frac{dI}{dt}$$

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$



- Let U denote the energy stored in the inductor at any time
- The rate at which the energy is stored is

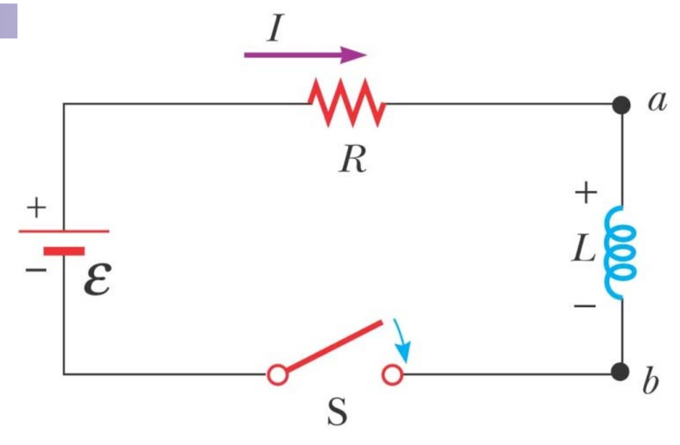
$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

- To find the total energy, integrate and

$$U = L\int_0^I I\ dI = L\frac{I^2}{2}$$

Energy of a Magnetic Field

- Given U = ½ $L\,I^2$
- For Solenoid: $L = \mu_0 n^2 A\ell \qquad I = \dfrac{B}{\mu_0 n}$



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$$U = \frac{1}{2}\mu_0 n^2 A\ell\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}A\ell$$

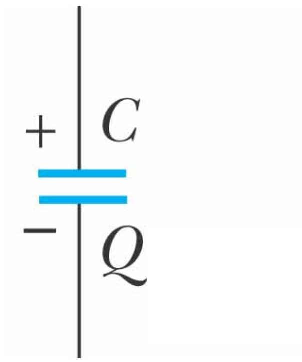
- Since $A\ell$ is the volume of the solenoid, the magnetic energy density, u_B is

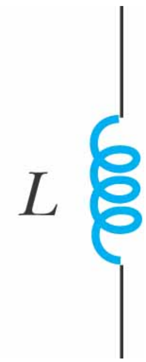
$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0}$$

- **This applies to any region in which a magnetic field exists (not just the solenoid)**

Energy of Magnetic and Electric Fields

$$U_C = C\frac{Q^2}{2}$$



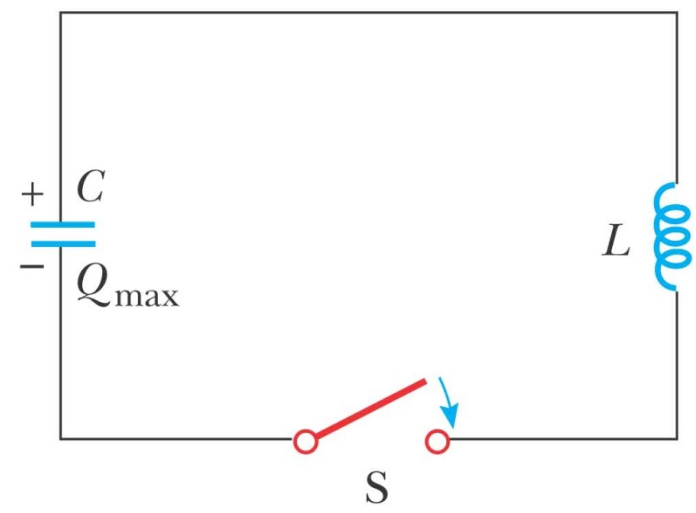


$$U_L = L\frac{I^2}{2}$$

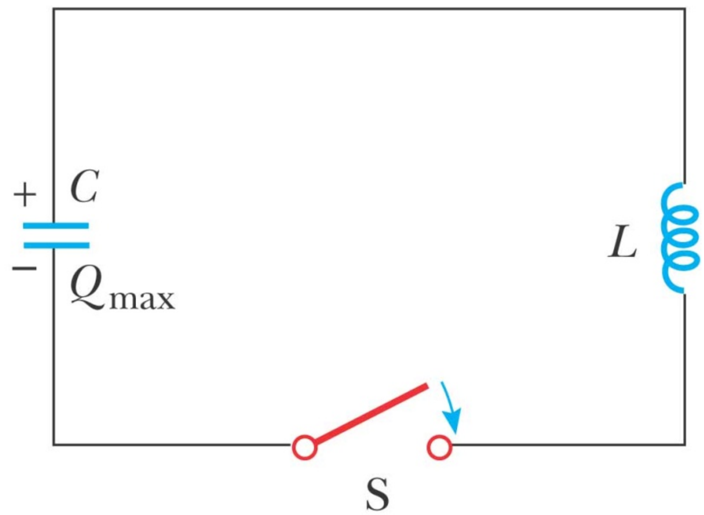
LC Circuit

LC Circuit

- A capacitor is connected to an inductor in an LC circuit
- Assume the capacitor is initially charged and then the switch is closed
- Assume no resistance and no energy losses to radiation

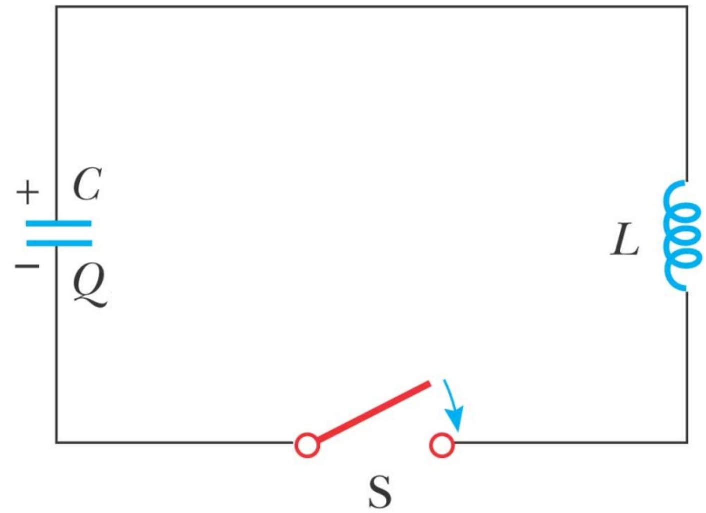


LC Circuit



- **With zero resistance, no energy is transformed into internal energy**
- The capacitor is fully charged
  - The energy *U* in the circuit is stored in the electric field of the capacitor
  - The energy is equal to $Q^2_{max} / 2C$
  - The current in the circuit is **zero**
  - No energy is stored in the inductor
- The switch is closed

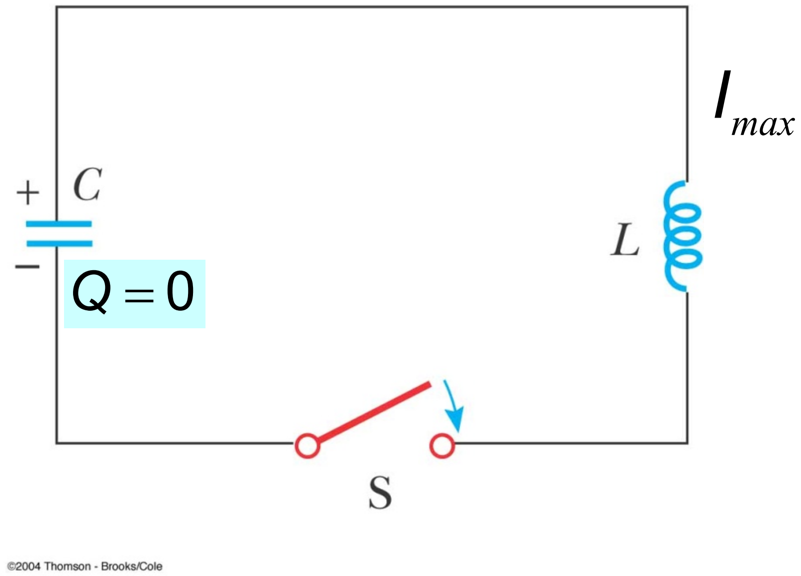
LC Circuit



$$I = \frac{dQ}{dt}$$

- The current is equal to the rate at which the charge changes on the capacitor
  - As the capacitor discharges, the energy stored in the electric field decreases
  - Since there is now a current, some energy is stored in the magnetic field of the inductor
  - **Energy is transferred from the electric field to the magnetic field**

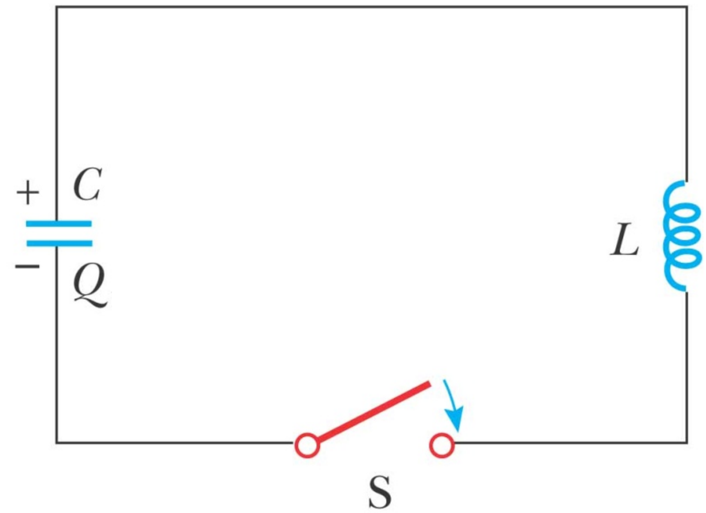
LC circuit



$$I = \frac{dQ}{dt}$$

- The capacitor becomes fully discharged
  - It stores no energy
  - All of the energy is stored in the magnetic field of the inductor
  - The current reaches its maximum value
- The current now decreases in magnitude, recharging the capacitor with its plates having opposite their initial polarity

LC circuit

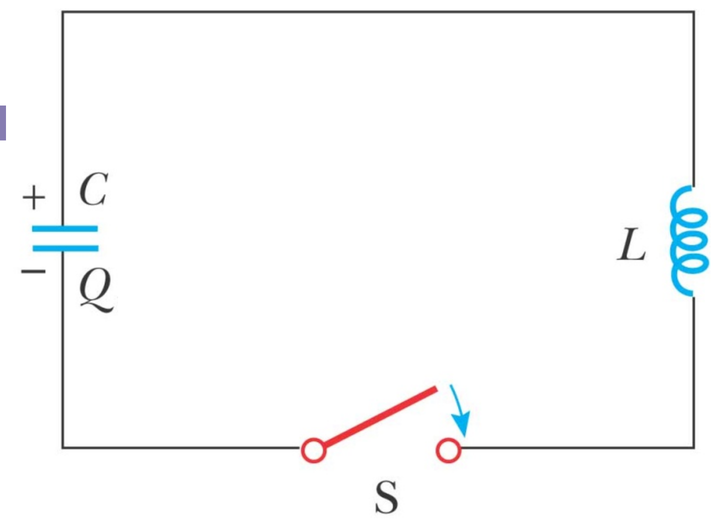


$$I = \frac{dQ}{dt}$$

- Eventually the capacitor becomes fully charged and the cycle repeats
- The energy continues to oscillate between the inductor and the capacitor
- **The total energy stored in the *LC* circuit remains constant in time and equals**

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2} L I^2$$

LC circuit



$$\frac{Q}{C} = -L\frac{dI}{dt}$$

$$I = \frac{dQ}{dt}$$

$$\frac{Q}{C} = -L\frac{d^2Q}{dt^2}$$

Solution: $Q = Q_{max}\cos(\omega t + \phi)$

$$\frac{Q_{max}}{C}\cos(\omega t + \phi) = LQ_{max}\omega^2\cos(\omega t + \phi)$$

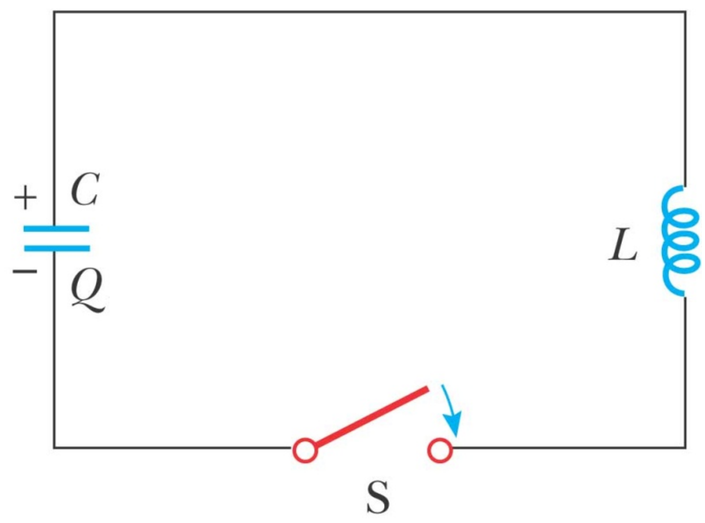
$$\omega^2 = \frac{1}{LC}$$

It is the *natural frequency* of oscillation of the circuit

LC circuit

$$Q = Q_{max} \cos(\omega t + \phi)$$

$$\omega^2 = \frac{1}{LC}$$



- The current can be expressed as a function of time

$$I = \frac{dQ}{dt} = -\omega Q_{max} \sin(\omega t + \phi)$$

- The total energy can be expressed as a function of time

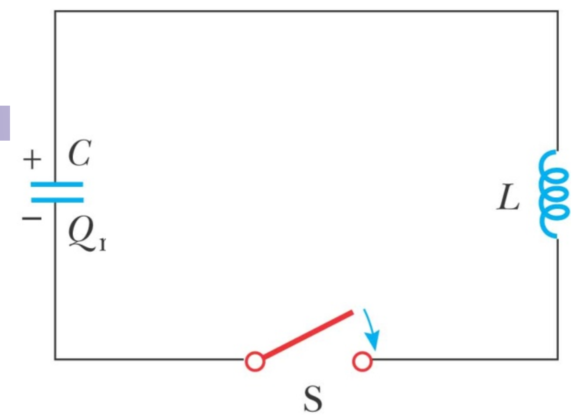
$$U = U_C + U_L = \frac{Q_{max}^2}{2c} \cos^2 \omega t + \frac{1}{2} L I_{max}^2 \sin^2 \omega t = \frac{Q_{max}^2}{2c}$$

$$\frac{Q_{max}^2}{2c} = \frac{1}{2} L I_{max}^2$$

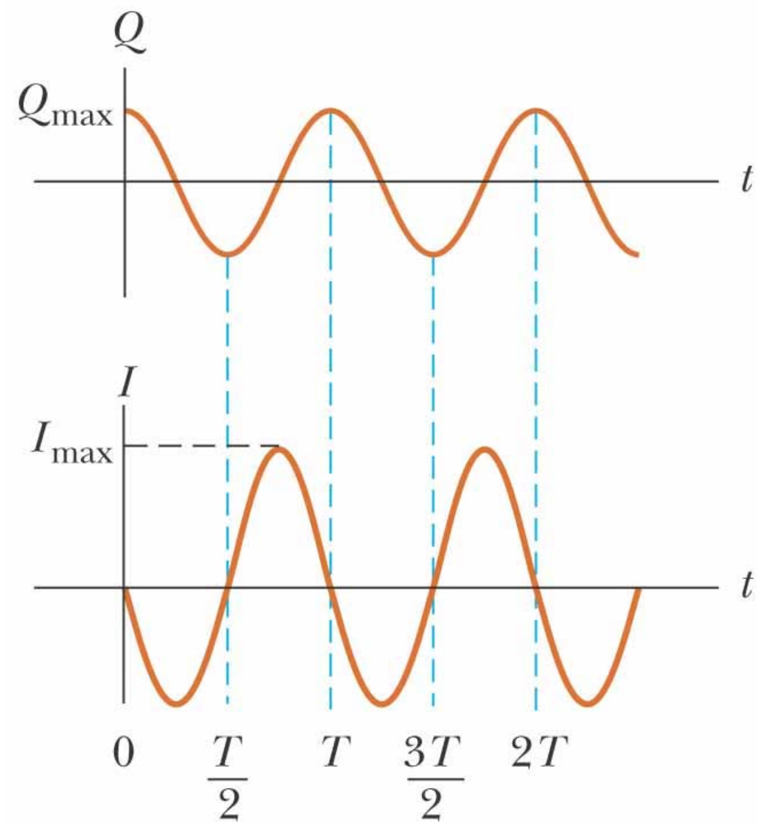
LC circuit

$$Q = Q_{max} \cos(\omega t + \phi)$$

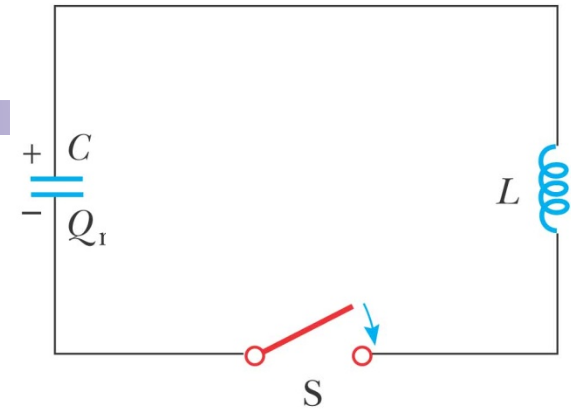
$$I = -\omega Q_{max} \sin(\omega t + \phi)$$



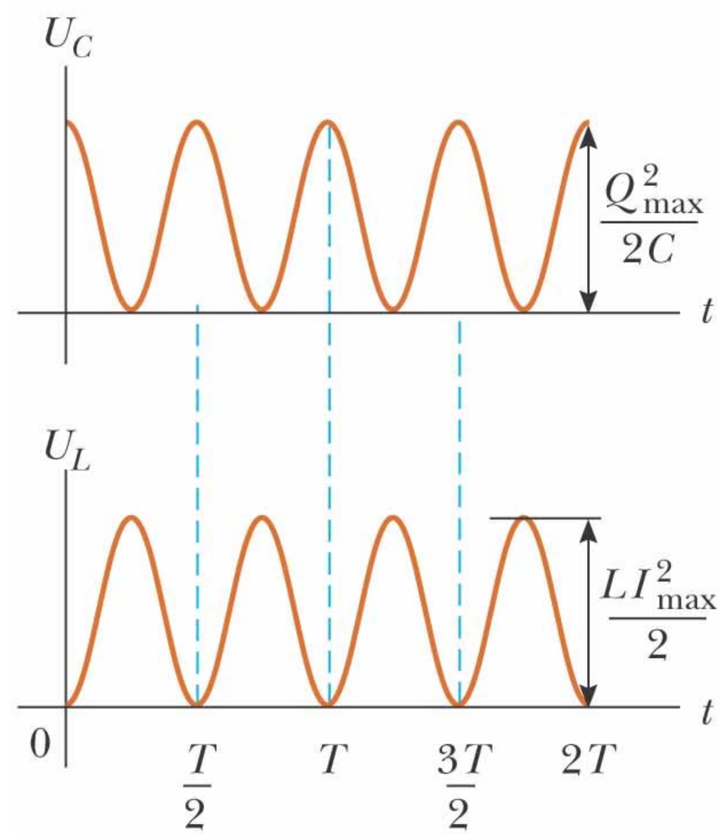
- The charge on the capacitor oscillates between Q_{max} and $-Q_{max}$
- The current in the inductor oscillates between I_{max} and $-I_{max}$
- Q and I are 90^o out of phase with each other
  - So when Q is a maximum, I is zero, etc.



LC circuit

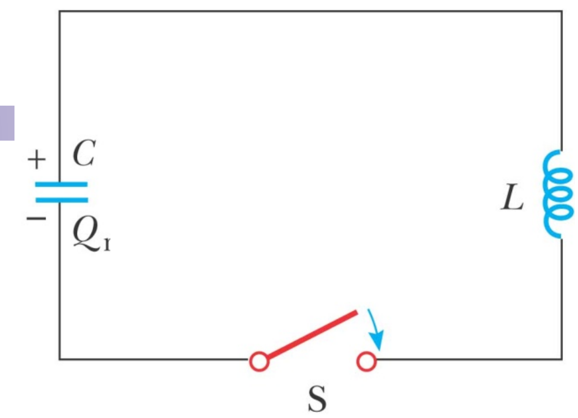


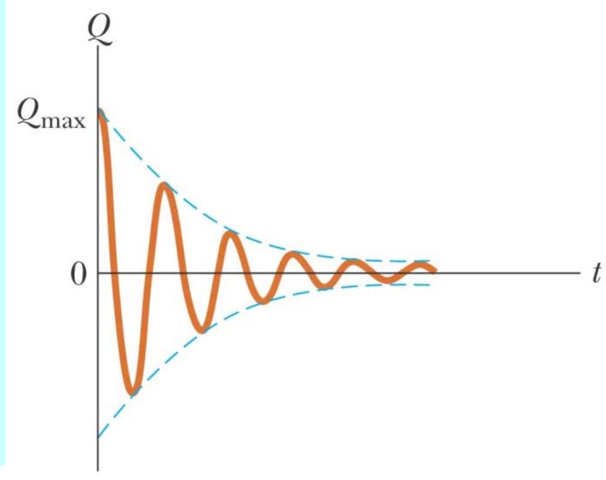
- The energy continually oscillates between the energy stored in the electric and magnetic fields
- When the total energy is stored in one field, the energy stored in the other field is zero



LC circuit

- In actual circuits, there is always some resistance
- Therefore, there is some energy transformed to internal energy
- Radiation is also inevitable in this type of circuit
- The total energy in the circuit continuously decreases as a result of these processes

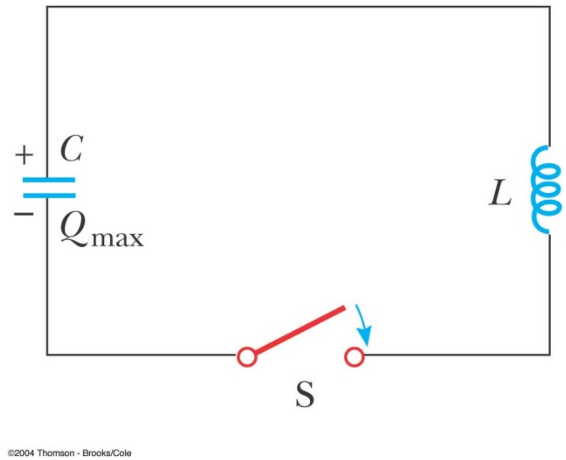




(a)

Problem 2

A capacitor in a series ***LC*** circuit has an initial charge Q_{max} and is being discharged. Find, in terms of ***L*** and *C*, the flux through each of the N turns in the coil, when the charge on the capacitor is $Q_{max}/2$.



The total energy is conserved:

$$\frac{Q_{max}^2}{2C} = \frac{Q^2}{2C} + \frac{1}{2} L I^2 \qquad Q = \frac{Q_{max}}{2}$$

$$\frac{1}{2} L I^2 = \frac{Q_{max}^2}{2C} - \frac{Q^2}{2C} = \frac{Q_{max}^2}{2C} - \frac{1}{4}\frac{Q_{max}^2}{2C} = \frac{3Q_{max}^2}{8C}$$

$$I = \frac{\sqrt{3}}{2\sqrt{CL}} Q_{max} \qquad \Phi = LI = \sqrt{\frac{3L}{C}}\frac{Q_{max}}{2} \qquad \Phi_1 = \frac{\Phi}{N} = \sqrt{\frac{3L}{C}}\frac{Q_{max}}{2N}$$

Chapter 31

Maxwell's Equations

Maxwell's Equations

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \quad \text{Gauss's law (electric)}$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = 0 \quad \text{Gauss's law in magnetism}$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \quad \text{Faraday's law}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \text{Ampere-Maxwell law}$$

Chapter 34

Electromagnetic Waves

Maxwell Equations – Electromagnetic Waves

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \qquad \oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad \oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Electromagnetic waves – solutions of Maxwell equations

- Empty space: ***q* = 0, *I* = 0**

$$\oint \mathbf{E}\cdot d\mathbf{A} = 0 \qquad \oint \mathbf{B}\cdot d\mathbf{A} = 0$$

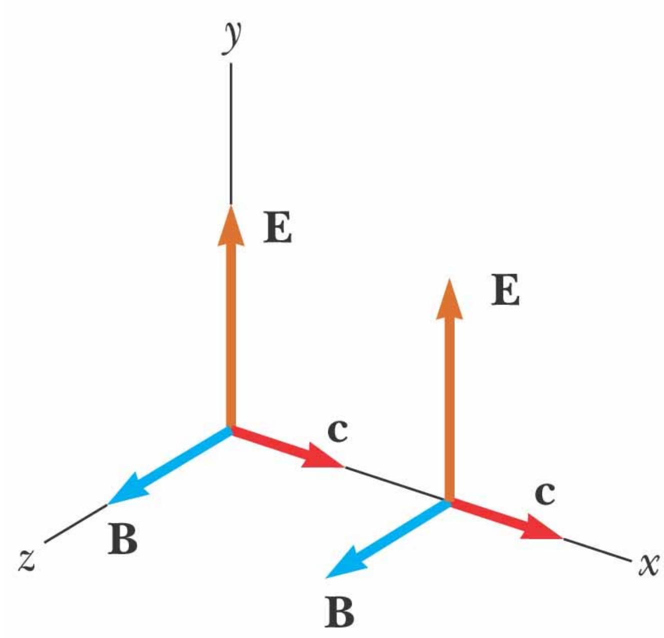
$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad \oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Solution – Electromagnetic Wave

Plane Electromagnetic Waves

- Assume EM wave that travel in x-direction
- Then Electric and Magnetic Fields are orthogonal to x
- This follows from the first two Maxwell equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0 \qquad \oint \mathbf{B} \cdot d\mathbf{A} = 0$$



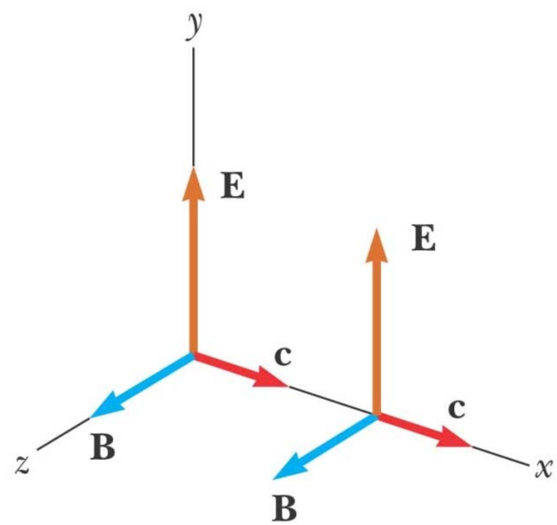
Plane Electromagnetic Waves

If Electric Field and Magnetic Field depend only on ***x*** and ***t*** then the third and the forth Maxwell equations can be rewritten as

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

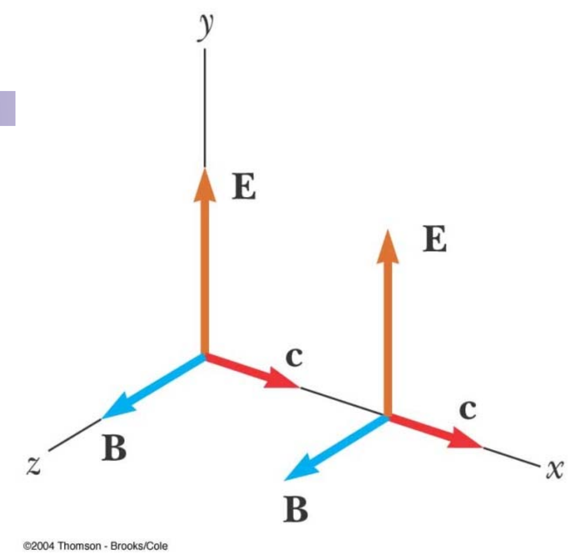
$$\Downarrow$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad and \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$



Plane Electromagnetic Waves

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad and \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$



Solution:

$$E = E_{max} \cos(kx - \omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = -E_{max} k^2 \cos(kx - \omega t) \qquad \frac{\partial^2 E}{\partial t^2} = -E_{max} \omega^2 \cos(kx - \omega t)$$

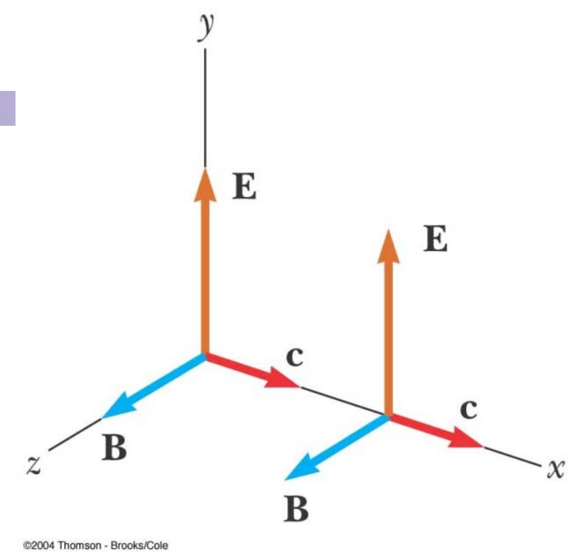
$$E_{max} k^2 \cos(kx - \omega t) = \mu_0 \varepsilon_0 E_{max} \omega^2 \cos(kx - \omega t)$$

$$k = \omega \sqrt{\mu_0 \varepsilon_0}$$

Plane Electromagnetic Waves

$$E = E_{max}\cos(kx - \omega t)$$

$$k = \omega\sqrt{\mu_0 \varepsilon_0}$$



The angular wave number is ***k* = 2π/λ**

- ***λ*** is the wavelength

The angular frequency is ***ω* = 2π*f***

- ***f*** is the wave frequency

$$\frac{2\pi}{\lambda} = 2\pi f\sqrt{\mu_0 \varepsilon_0} \qquad \lambda = \frac{1}{f\sqrt{\mu_0 \varepsilon_0}} = \frac{c}{f}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792 \times 10^8 m/s$$

- speed of light

Plane Electromagnetic Waves

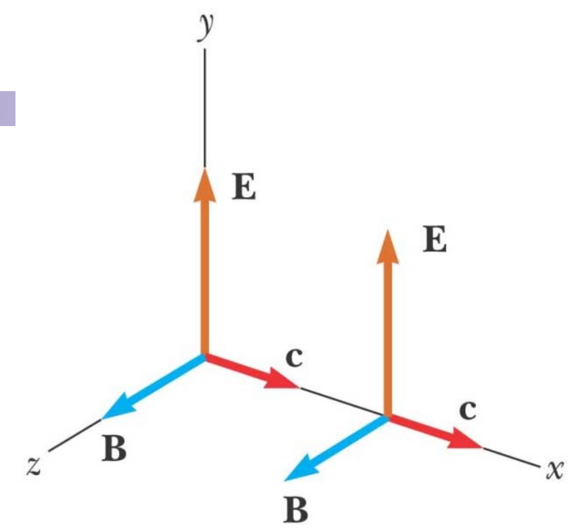
$$E = E_{max}\cos(kx - \omega t)$$

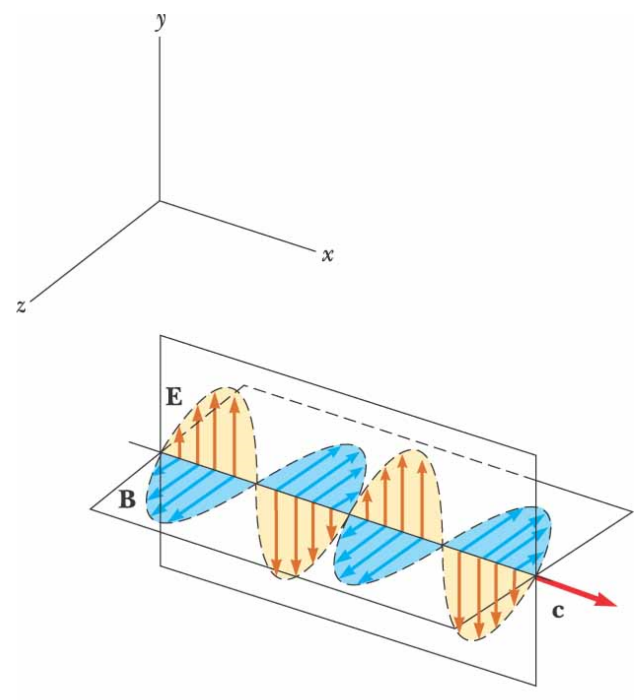
$$H = H_{max}\cos(kx - \omega t)$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \omega = ck \qquad \lambda = \frac{c}{f}$$

$$\frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = \frac{E}{B} = c$$

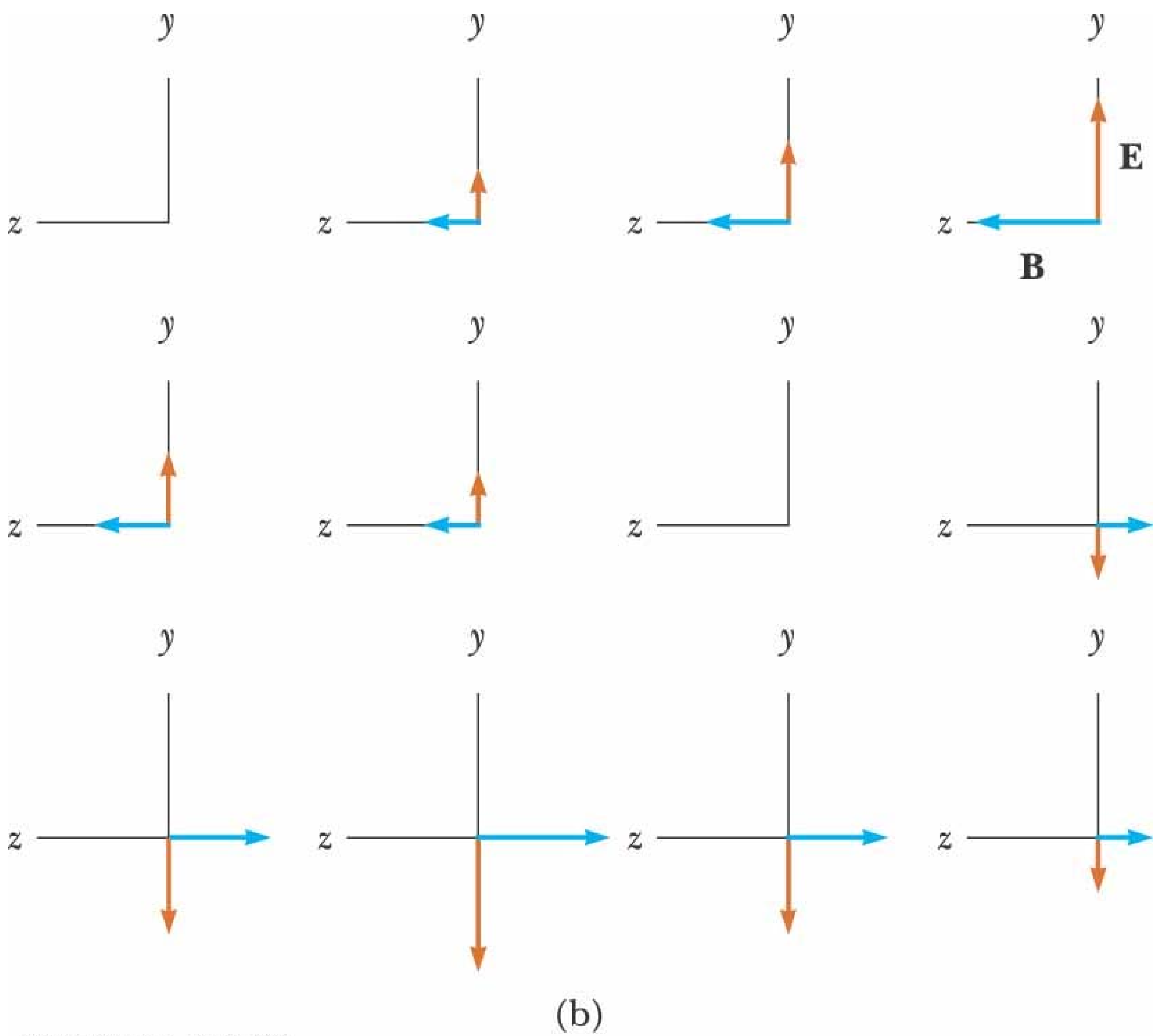
E and ***B*** vary sinusoidally with ***x***





(a)

Time Sequence of Electromagnetic Wave



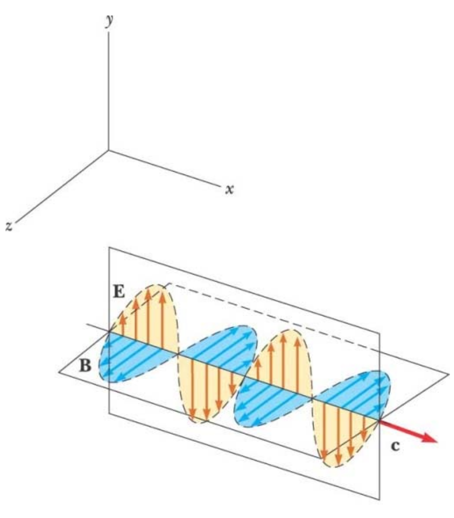
Poynting Vector

- Electromagnetic waves carry energy
- As they propagate through space, they can transfer that energy to objects in their path
- The rate of flow of energy in an em wave is described by a vector, ***S***, called the **Poynting vector**
- The Poynting vector is defined as

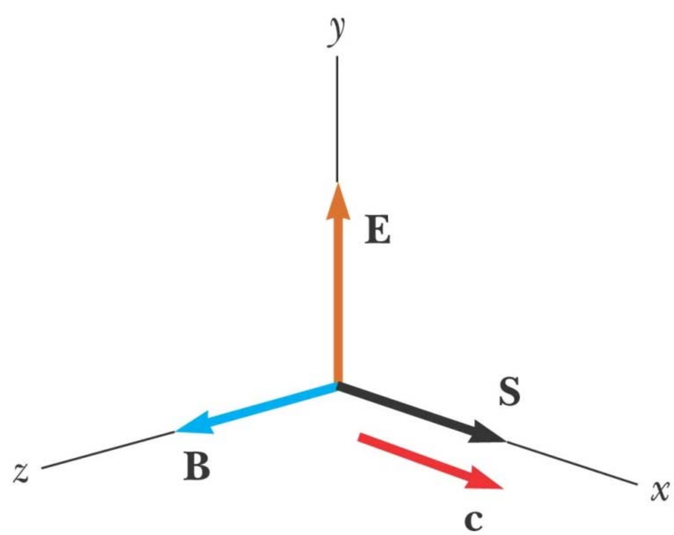
$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$

Poynting Vector

- The direction of Poynting vector is the direction of propagation
- Its magnitude varies in time
- Its magnitude reaches a maximum at the same instant as **E** and **B**



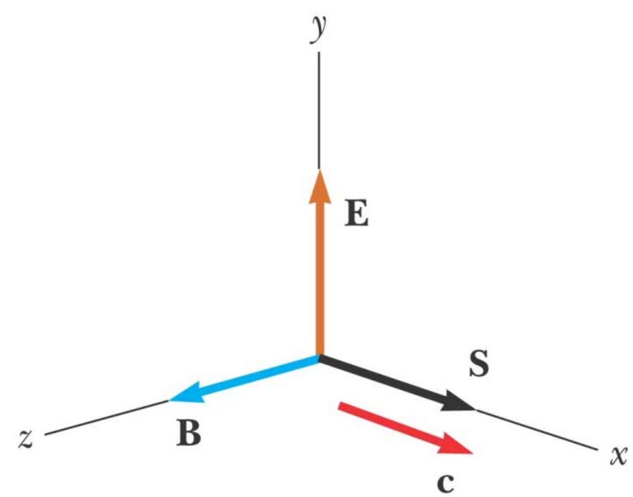
$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$



Poynting Vector

- The magnitude ***S*** represents the rate at which energy flows through a unit surface area perpendicular to the direction of the wave propagation
  - This is the ***power per unit area***
- The SI units of the Poynting vector are **J/s·m² = W/m²**

$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$



The EM spectrum

- Note the overlap between different types of waves
- Visible light is a small portion of the spectrum
- Types are distinguished by frequency or wavelength