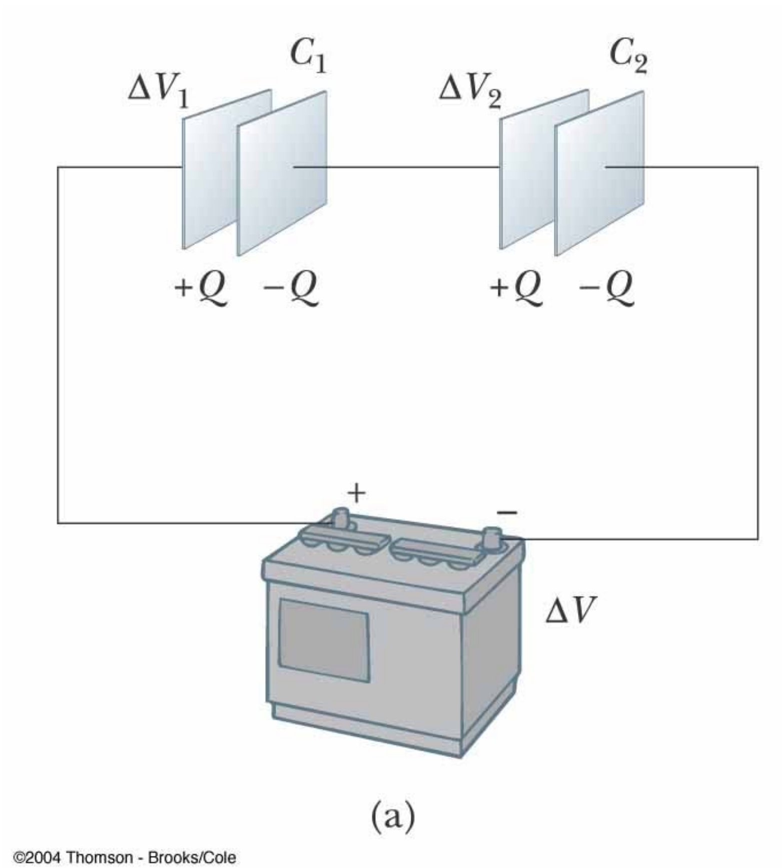
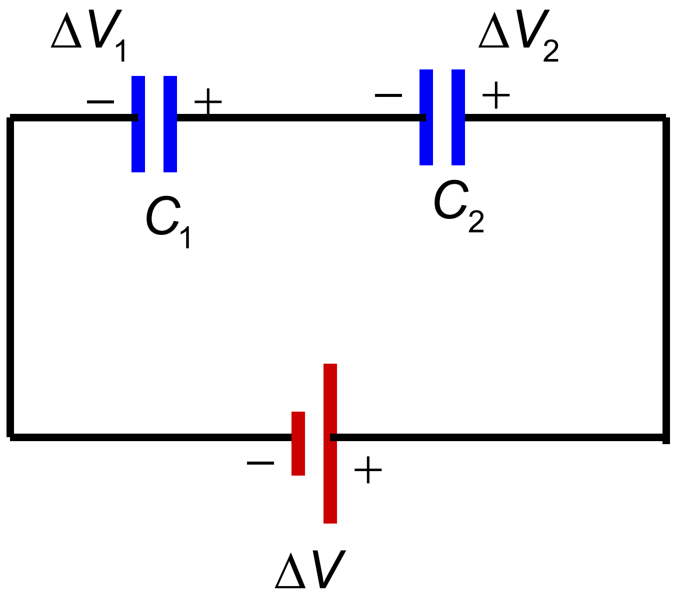
Chapter 31

Fundamentals of Circuits

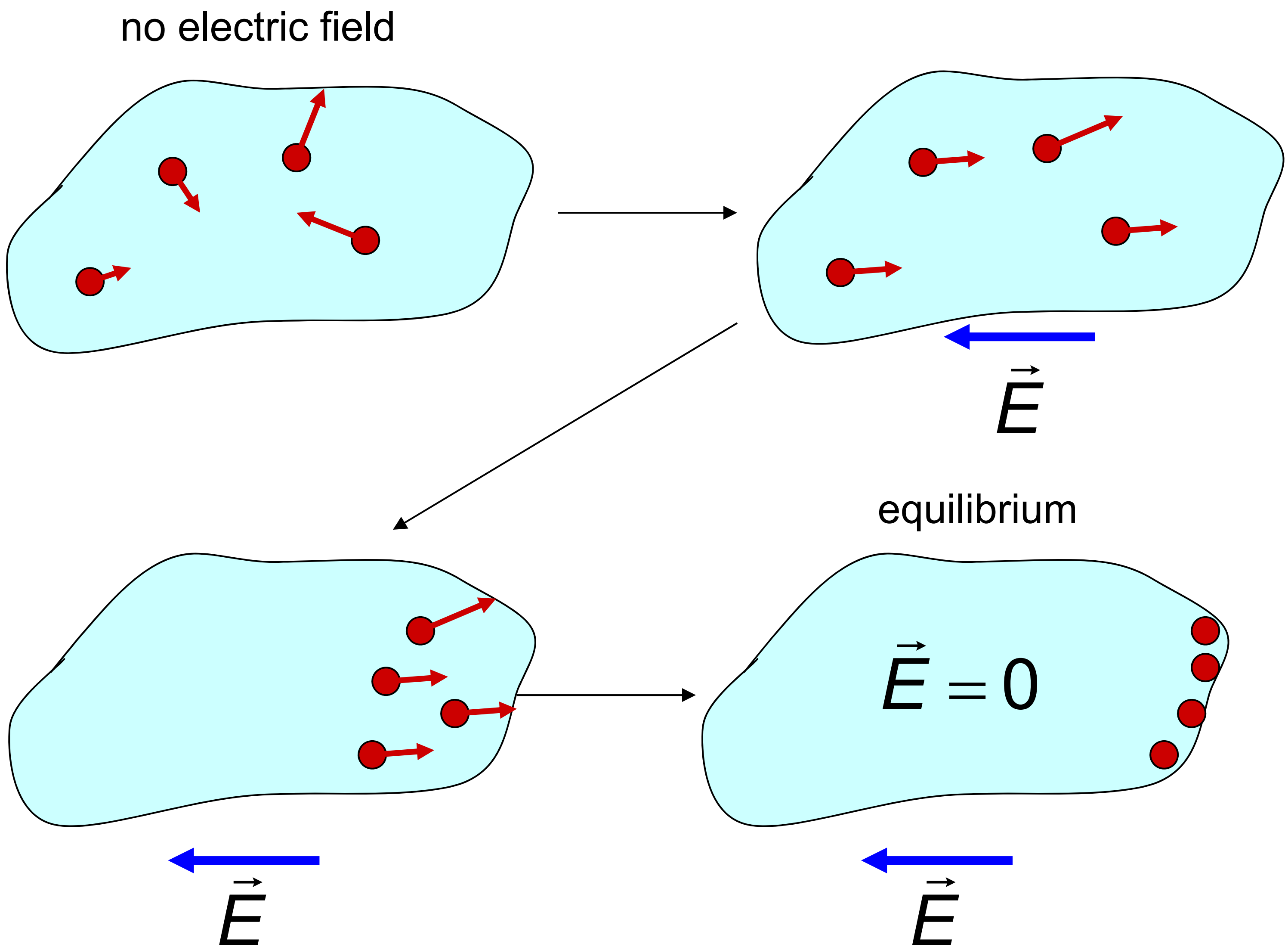
Capacitors in Series



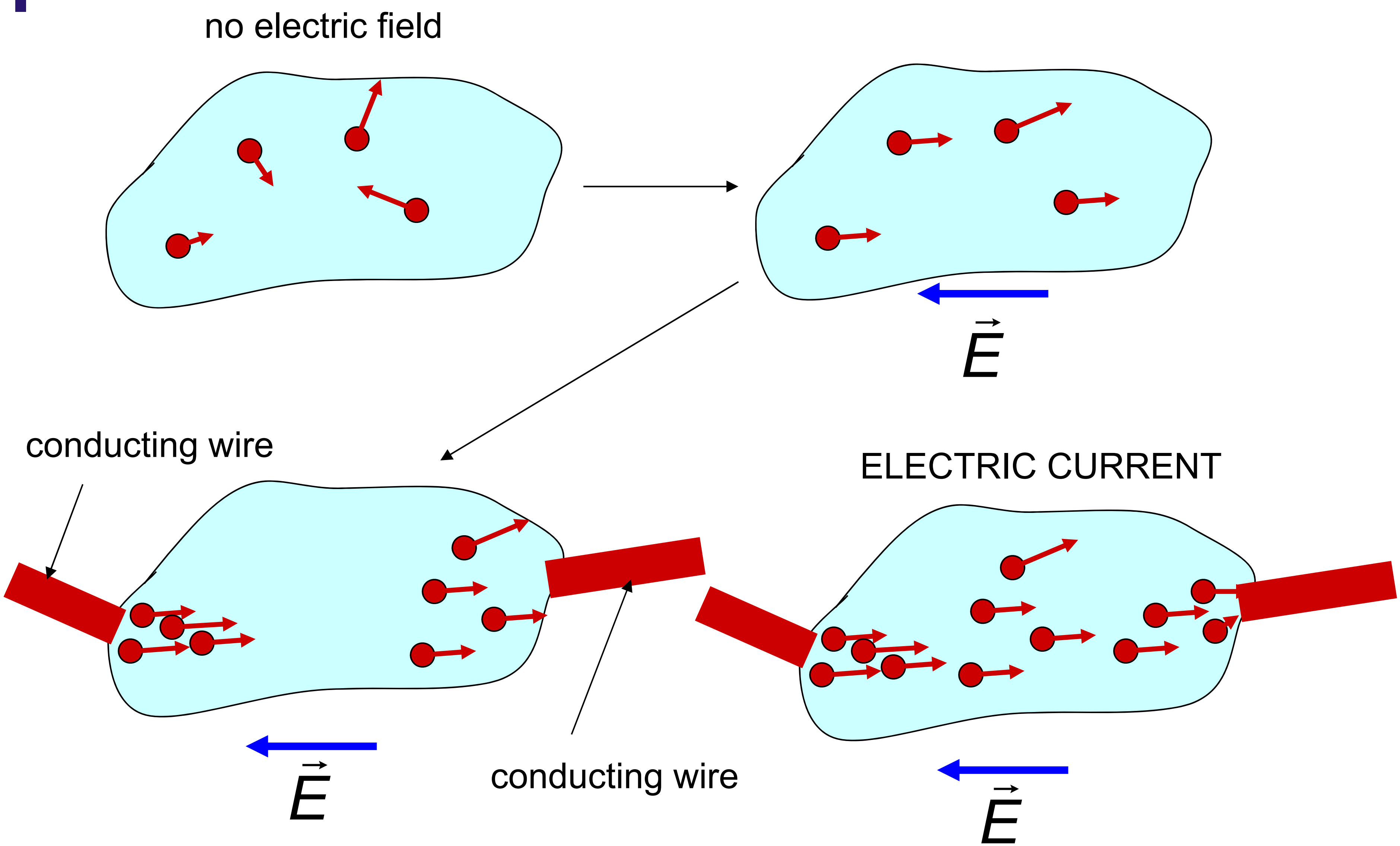


$$\Delta V = \Delta V_1 + \Delta V_2$$

Conductor in Electric Field

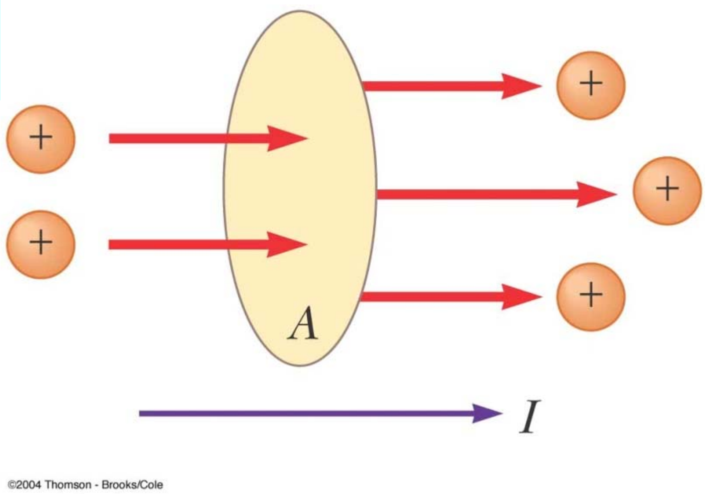


Conductor in Electric Field



Electric Current

- **Electric current** is the rate of flow of charge through some region of space
- The SI unit of current is the **ampere (A)**, **1 A = 1 C / s**



- Assume charges are moving perpendicular to a surface of area ***A***
- If ***ΔQ*** is the amount of charge that passes through ***A*** in time ***Δt***, then the average current is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

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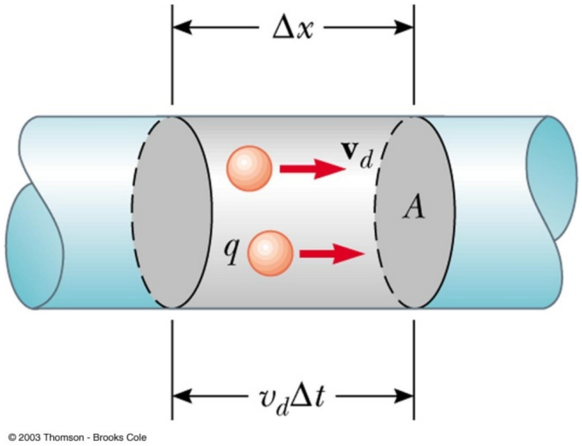
Chapter 31

Ohm's Law

Current Density



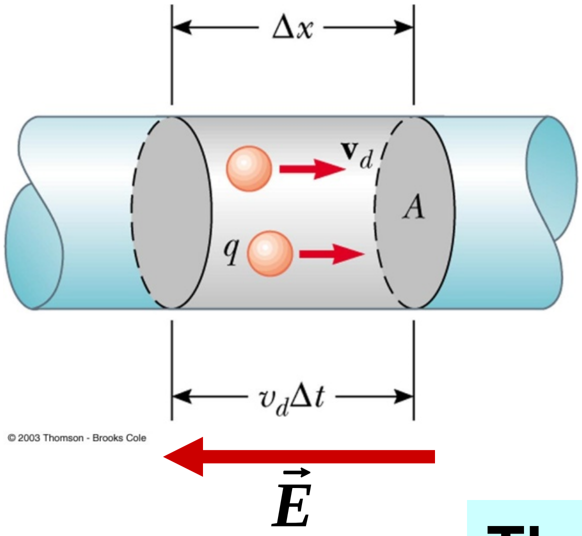
Current density is defined as the current per unit area

$$j = \frac{I}{A}$$

This expression is valid only if the current density is uniform and *A* is perpendicular to the direction of the current

j has **SI** units of **A/m²**

Ohm's Law



Ohm's Law:
Current density is proportional to electric field

$$j = \sigma E$$

The constant of proportionality, σ, is called the conductivity of the conductor.

The conductivity depends only on the material of conductor.

Simplified model of electron motion in conductor gives

$$\sigma = \frac{n\tau q^2}{m}$$

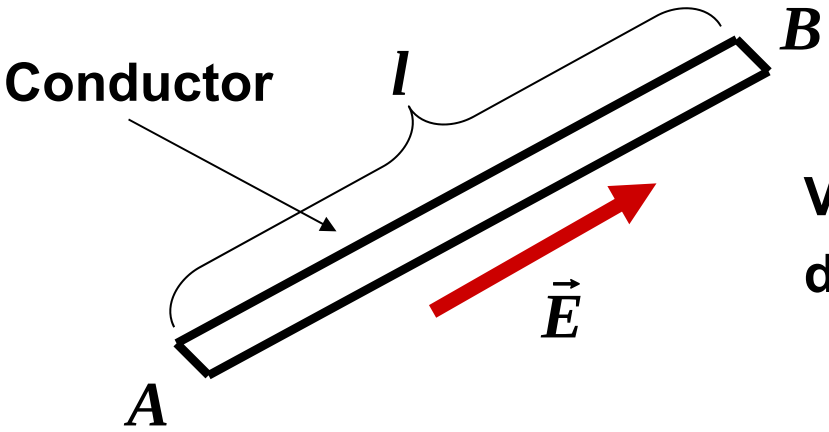
τ **- is the material dependent characteristic of conductor.**

Ohm's Law

$$j = \sigma E$$

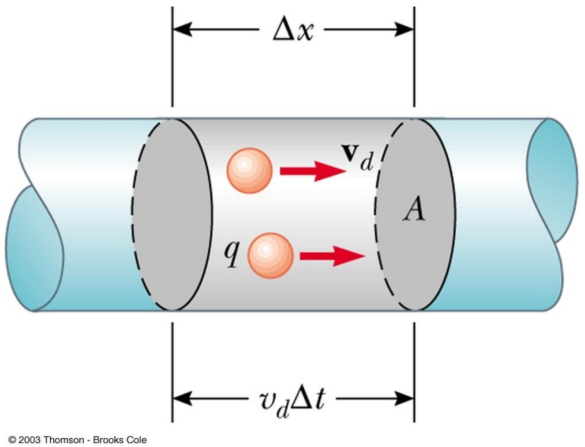
- **Ohm's law** states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current
  - Most metals, but not all, obey Ohm's law
  - Materials that obey Ohm's law are said to be ***ohmic***
  - Materials that do not obey Ohm's law are said to be ***nonohmic***
- Ohm's law is not a fundamental law of nature
- Ohm's law is an empirical relationship valid only for certain materials

Ohm's Law



Voltage across the conductor (potential difference between points A and B)

$$\Delta V = V_B - V_A = El$$



where electric field is the same along the conductor. Then

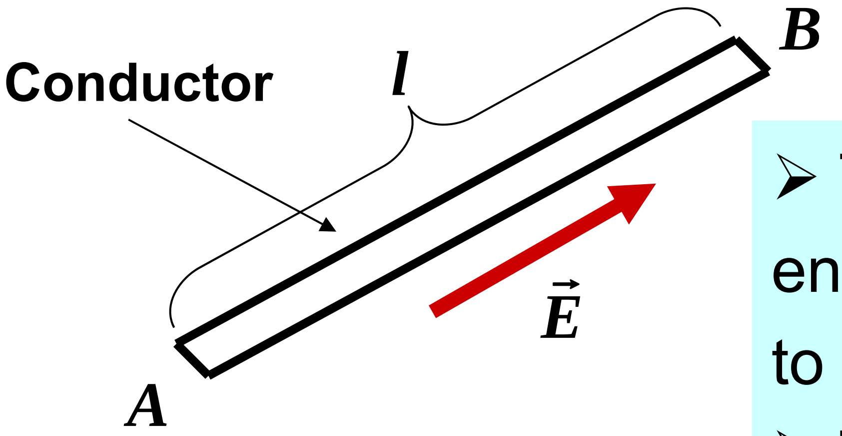
$$E = \frac{\Delta V}{l} \qquad j = \frac{I}{A}$$

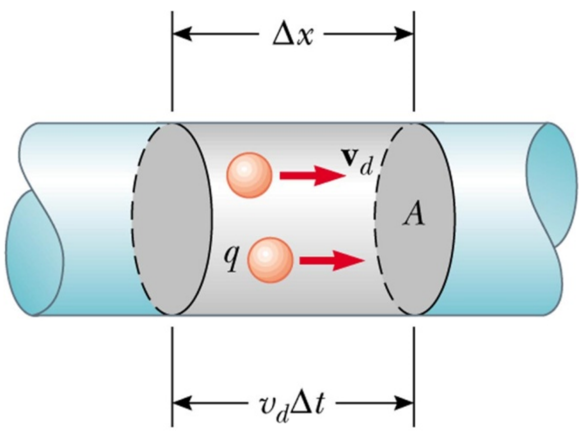
$$\boxed{j = \sigma E}$$

$$E = \frac{\Delta V}{l} = \frac{1}{\sigma} j = \frac{I}{\sigma A}$$

Another form of the Ohm's Law $\longrightarrow$ $\boxed{\Delta V = \frac{l}{\sigma A} I = RI}$

Ohm's Law: Resistance





- The voltage applied across the ends of the conductor is proportional to the current through the conductor
- The constant of proportionality is called the **resistance** of the conductor

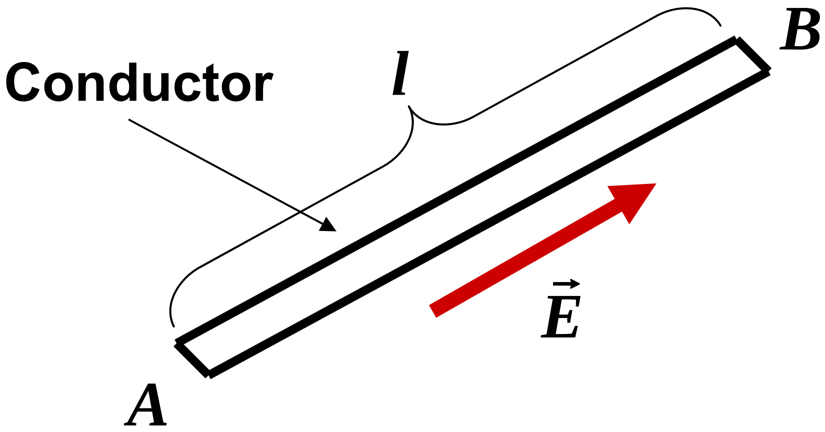
$$\Delta V = RI$$

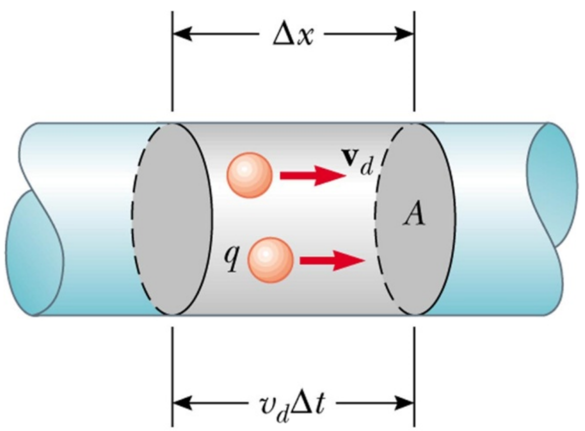
resistance

SI units of resistance are ***ohms*** (Ω)

1 Ω = 1 V / A

Ohm's Law: Resistance





$$\Delta V = RI$$

resistance

$$R = \frac{l}{\sigma A}$$

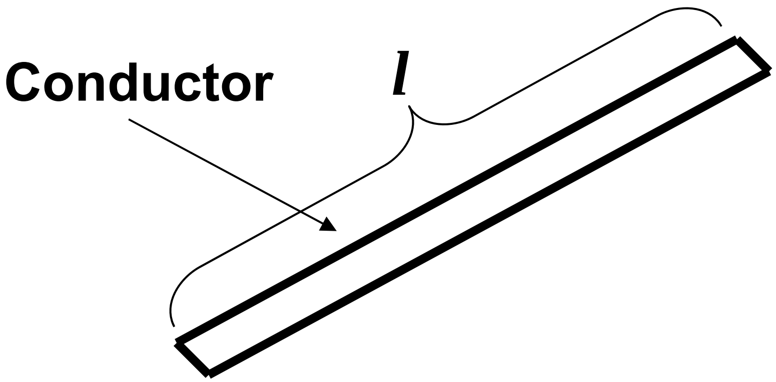
Or

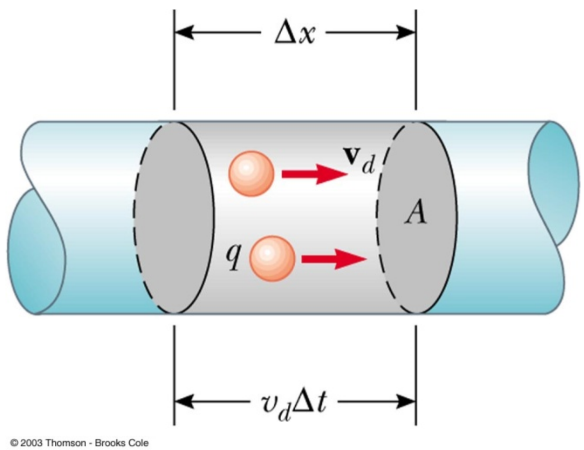
$$R = \frac{\rho l}{A}$$

where $\rho = 1/\sigma$ is the resistivity – the inverse of the conductivity

Resistivity has SI units of ohm-meters (Ω m)

Resistance: Example





$$R = \rho \frac{l}{A}$$

The same amount of material has been used to fabricate the wire with uniform cross-section and length ***l/3***. What is the resistance of the wire?

$$R_1 = \rho \frac{l_1}{A_1}$$

$$l_1 A_1 = lA \qquad\qquad l_1 = l/3$$

$$A_1 = \frac{lA}{l_1} = 3A$$

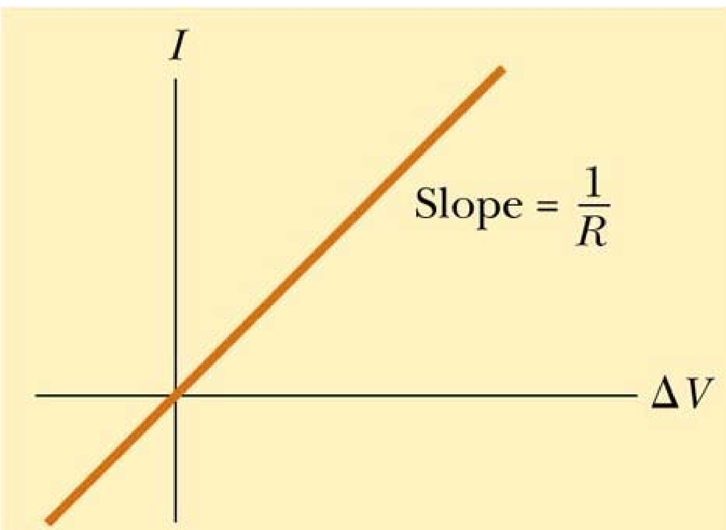
$$R_1 = \rho \frac{l_1}{A_1} = \rho \frac{l/3}{3A} = \rho \frac{l}{9A} = \frac{R}{9}$$

Ohm's Law

$$\vec{j} = \sigma \vec{E} \qquad \Delta V = RI$$

– Materials that obey Ohm's law are said to be ***ohmic***

– Materials that do not obey Ohm's law are said to be ***nonohmic***



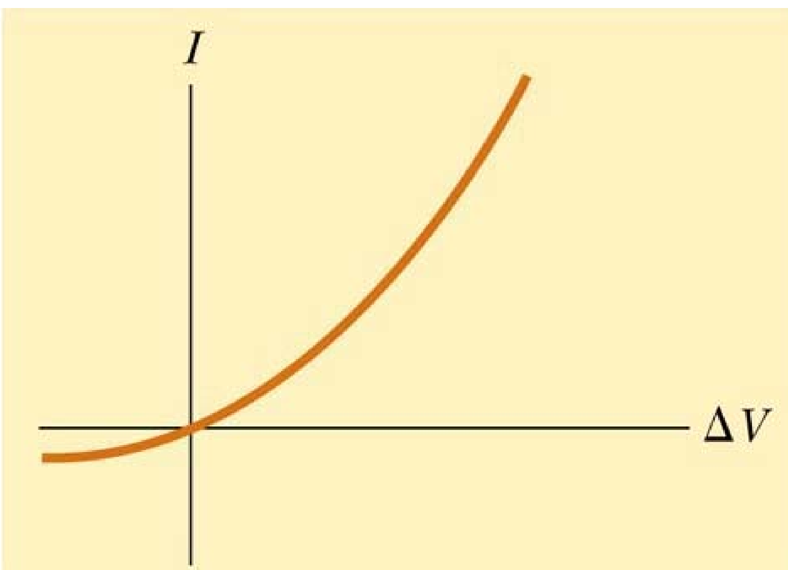
An ohmic device

- The resistance is constant over a wide range of voltages
- The relationship between current and voltage is linear
- The slope is related to the resistance

Ohm's Law

$$\vec{j} = \sigma \vec{E} \qquad \Delta V = RI$$

- Materials that obey Ohm's law are said to be ***ohmic***
- Materials that do not obey Ohm's law are said to be ***nonohmic***



Nonohmic materials

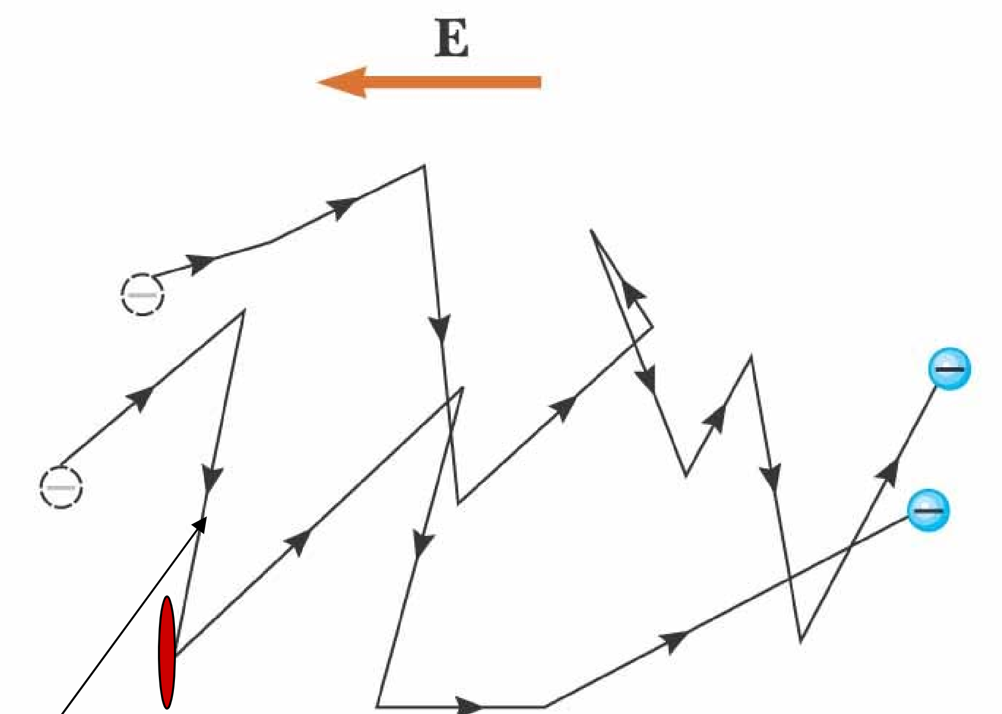
➢ The current-voltage relationship is nonlinear

Chapter 31

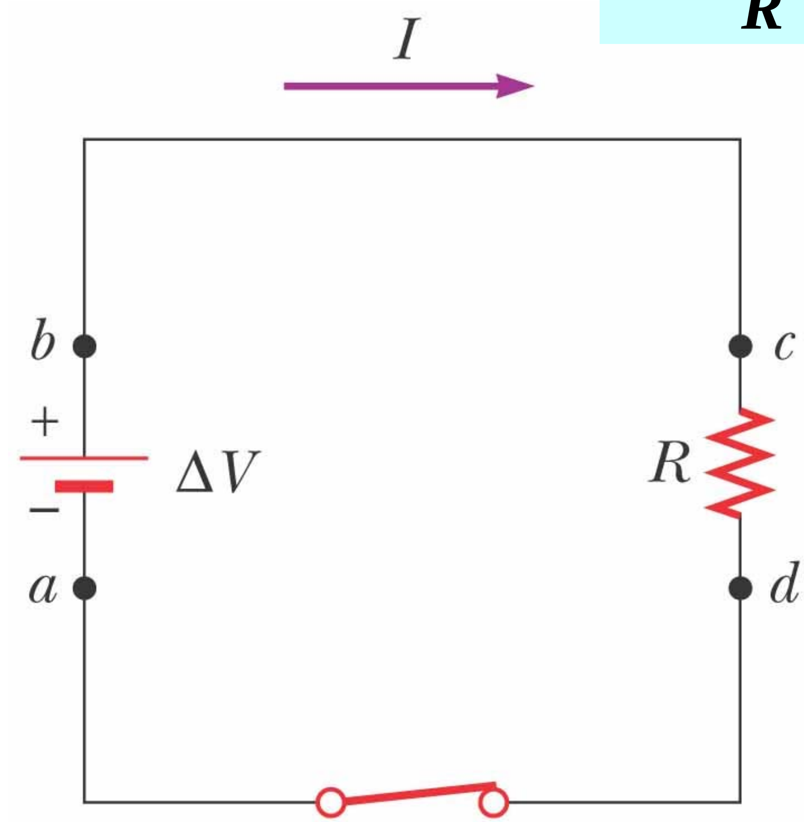
Electric Power

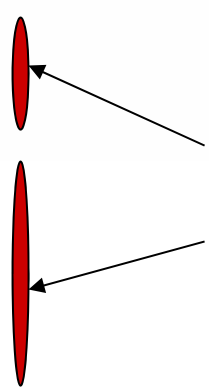
Electrical Power

$$I = \frac{\Delta V}{R}$$



$$\vec{v}_f = \vec{v}_i + \frac{q\vec{E}}{m}t$$





Before the collision

After the collision

Electrical Power

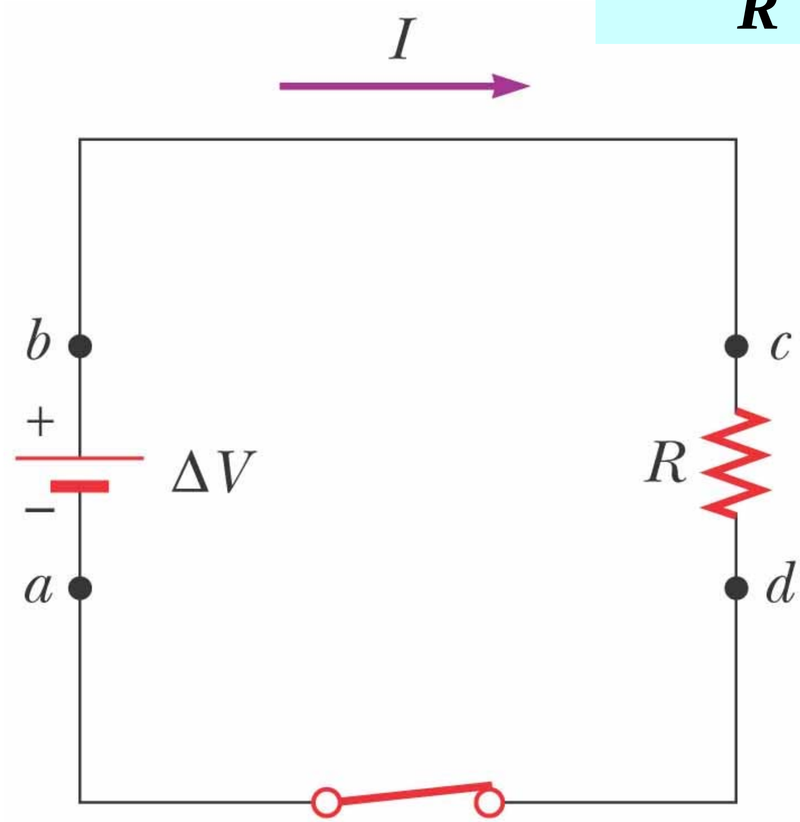
$$I = \frac{\Delta V}{R}$$

➢ As a charge moves from *a* to *b*, the electric potential energy of the system increases by $Q\Delta V$

➢ The chemical energy in the battery must decrease by the same amount

➢ As the charge moves through the resistor (*c* to *d*), the system loses this electric potential energy during collisions of the electrons with the atoms of the resistor

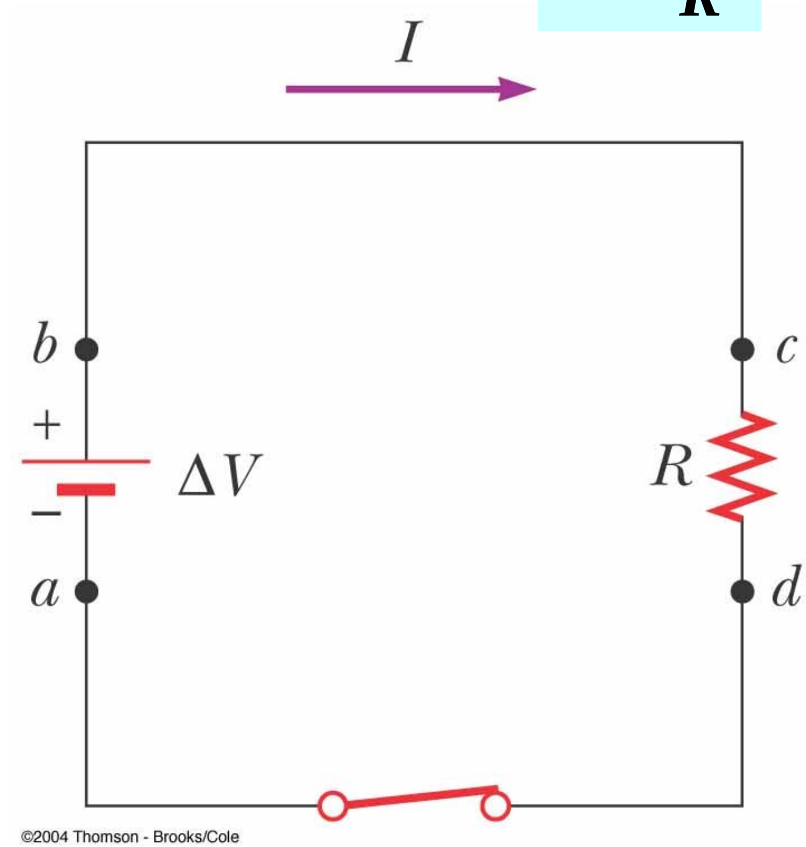
➢ This energy is transformed into internal energy in the resistor



Electrical Power

$$I = \frac{\Delta V}{R}$$

➢ The **power** is the rate at which the energy is delivered to the resistor



$\Delta U = Q\Delta V$ - the energy delivered to the resistor when charge **Q** moves from **a** to **b** (or from **c** to **d**)

The power:

$$P = \frac{\Delta U}{\Delta t} = \frac{Q}{\Delta t}\Delta V = I\Delta V$$

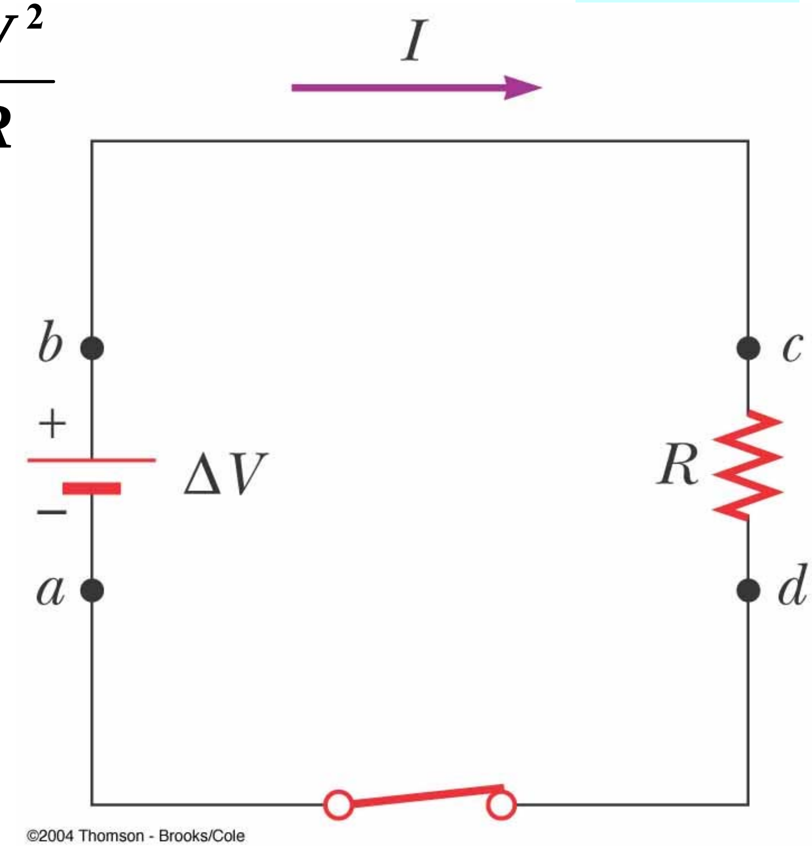
$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

<u>Units:</u> ***I*** is in A, ***R*** is in Ω, ***V*** is in V, and ***P*** is in W

Electrical Power

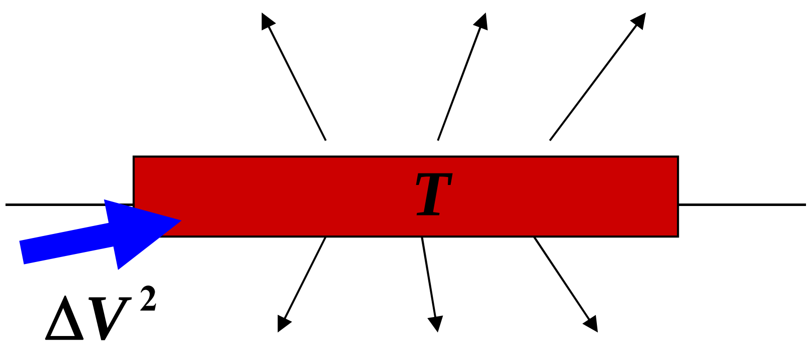
$$I = \frac{\Delta V}{R}$$

The power: $$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$



$$P = \frac{\Delta V^2}{R(T)}$$ **Will increase the temperature of conductor**

Electromagnetic waves (light), $P_{EMW}(T)$



$$P = \frac{\Delta V^2}{R(T)}$$

Heat transfer to air $P_{air}(T) = \alpha(T - T_0)$

$$P = \frac{\Delta V^2}{R(T)} = P_{EMW}(T) + \alpha(T - T_0)$$

Power: Example

A 1000-W heating coil designed to operate from 110 V is made of Nichrome wire 0.5 mm in diameter. Assuming that the resistivity of the Nichrome remains constant at its 20 C value, find the length of wire used.

$$\rho_N = 1.5 \cdot 10^{-6} \Omega \cdot m \qquad R = \rho_N \frac{l}{A} \qquad A = \pi \frac{d^2}{4}$$

$$P = I \Delta V = I^2 R = \frac{U^2}{R}$$

$$R = \frac{U^2}{P}$$

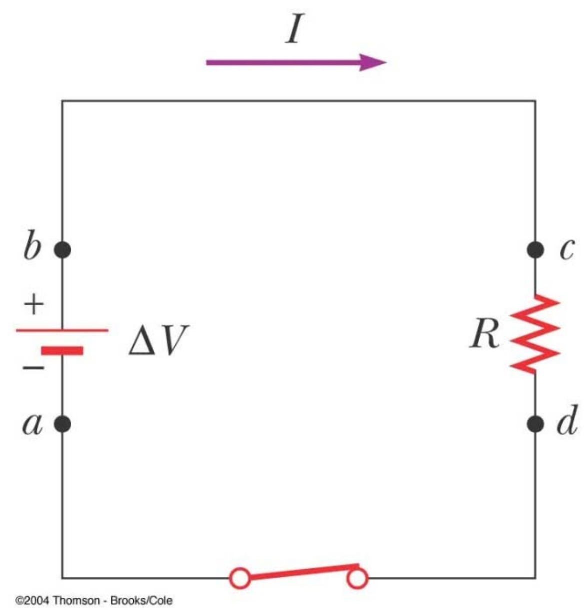
$$l = A \frac{R}{\rho_N} = A \frac{U^2}{\rho_N P} = \pi \frac{d^2}{4} \frac{U^2}{\rho_N P} = \frac{3.14 \cdot 0.5^2 \cdot 10^{-6} \cdot 110^2}{4 \cdot 1.5 \cdot 10^{-6} \cdot 1000} m = 1.58 m$$

Chapter 31

Direct Current

Direct Current

- When the current in a circuit has a constant magnitude and direction, the current is called ***direct current***
- Because the potential difference between the terminals of a battery is constant, the battery produces direct current
- The battery is known as a source of **emf** (electromotive force)



Resistors in Series

For a series combination of resistors, the currents are the same in all the resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval

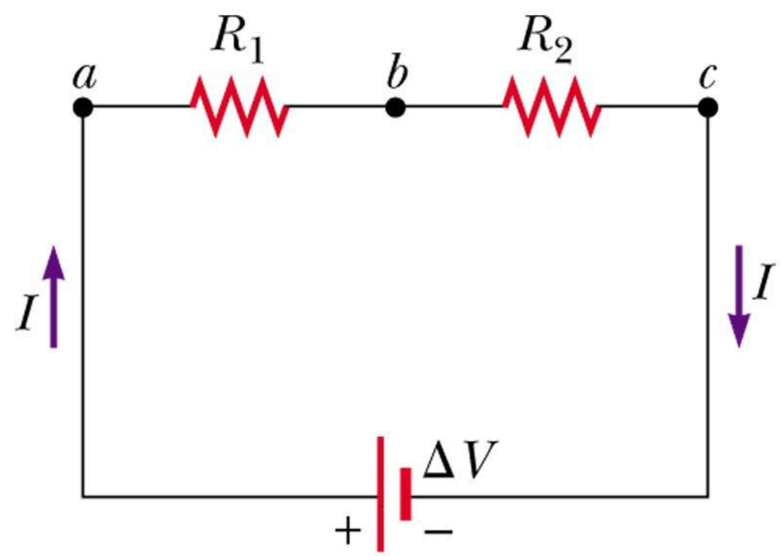
Ohm's law: $V_c - V_b = IR_2$

$$V_b - V_a = IR_1$$

$$V_c - V_a = V_c - V_b + V_b - V_a =$$

$$= IR_2 + IR_1 = I\left(R_1 + R_2\right)$$

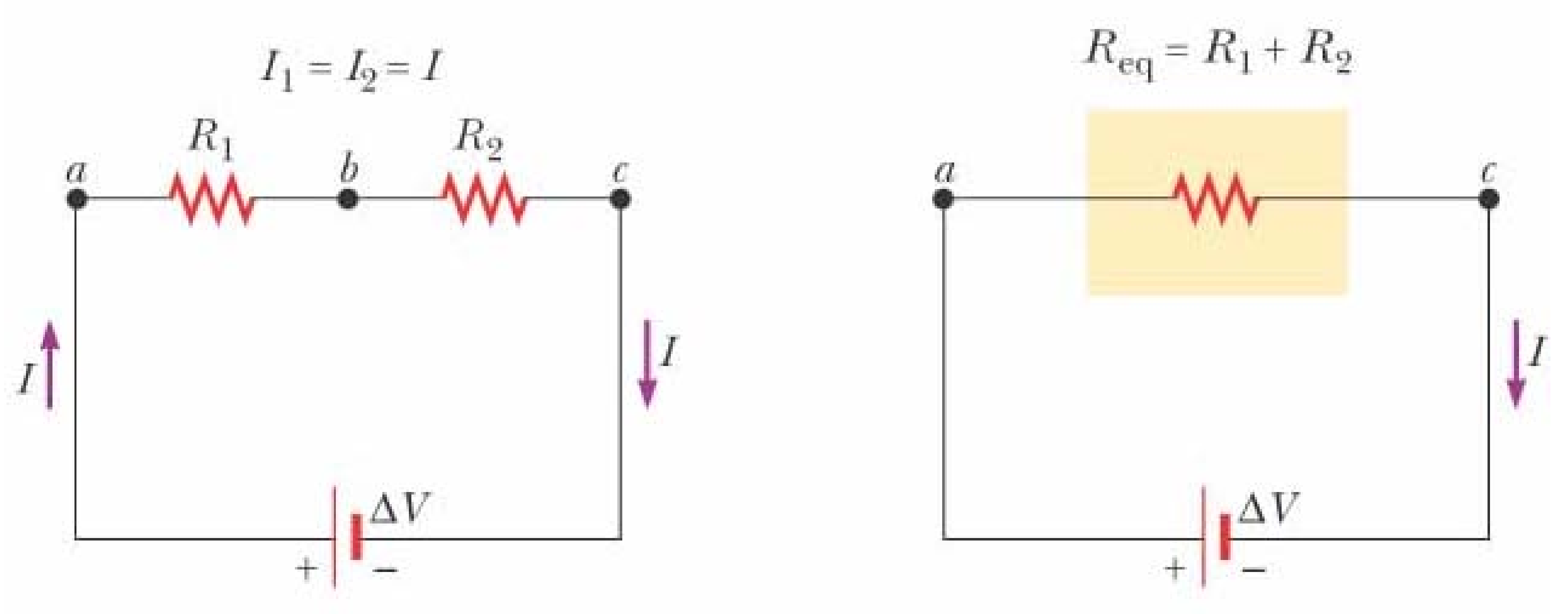
$$\boxed{R_{eq} = R_1 + R_2}$$



(b)

The equivalent resistance has the same effect on the circuit as the original combination of resistors

Resistors in Series



- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance

Resistors in Parallel

- The potential difference across each resistor is the same because each is connected directly across the battery terminals
- The current, *I*, that enters a point must be equal to the total current leaving that point

$$I = I_1 + I_2$$

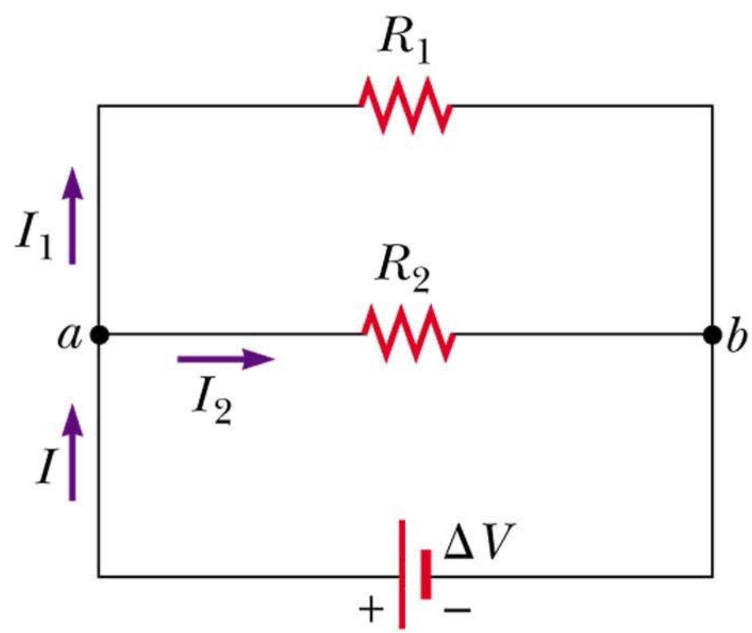
- Consequence of Conservation of Charge

Ohm's law: $V_b - V_a = \Delta V = I_1 R_1$

$$V_b - V_a = \Delta V = I_2 R_2$$

Conservation of Charge: $I = I_1 + I_2$

$$I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$



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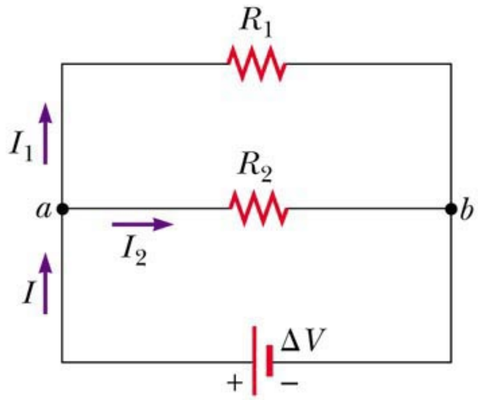
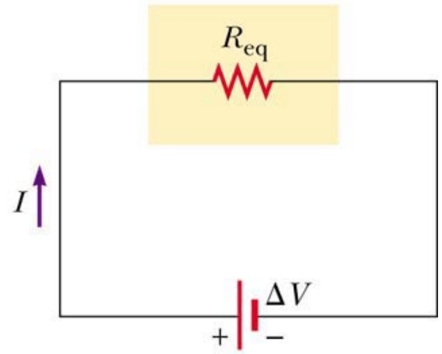
Resistors in Parallel

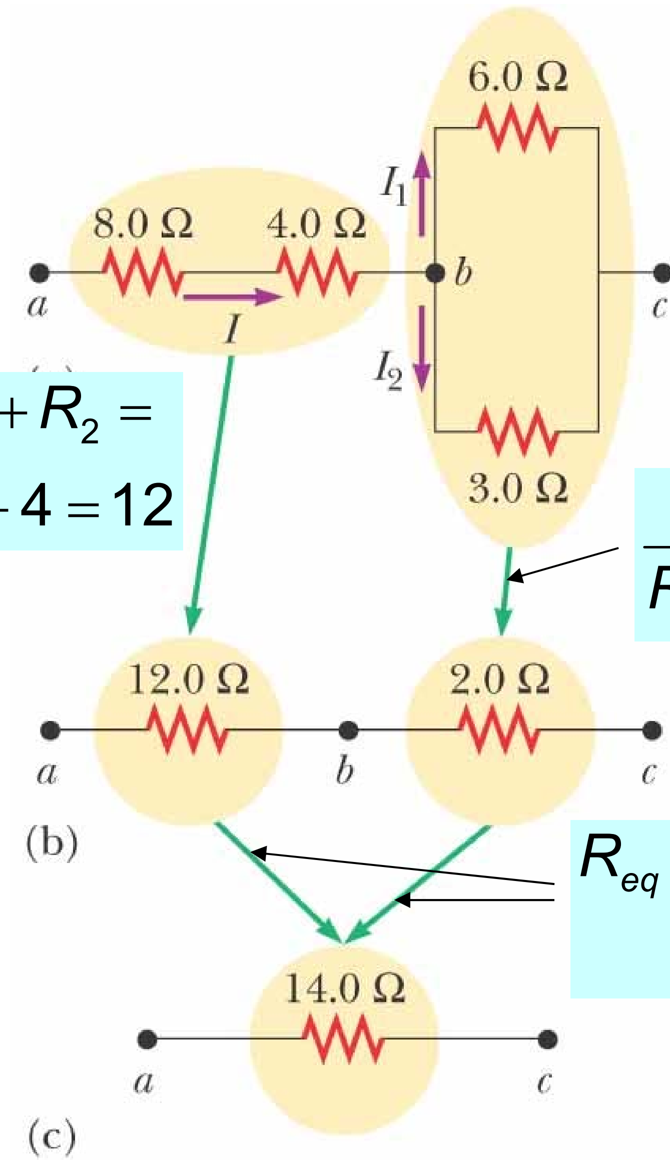




$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- Equivalent Resistance $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$
  - The equivalent is always less than the smallest resistor in the group
- In parallel, each device operates independently of the others so that if one is switched off, the others remain on
- In parallel, all of the devices operate on the same voltage
- The current takes all the paths
  - The lower resistance will have higher currents
  - Even very high resistances will have some currents

Example



$$R_{eq} = R_1 + R_2 = \\ = 8 + 4 = 12$$

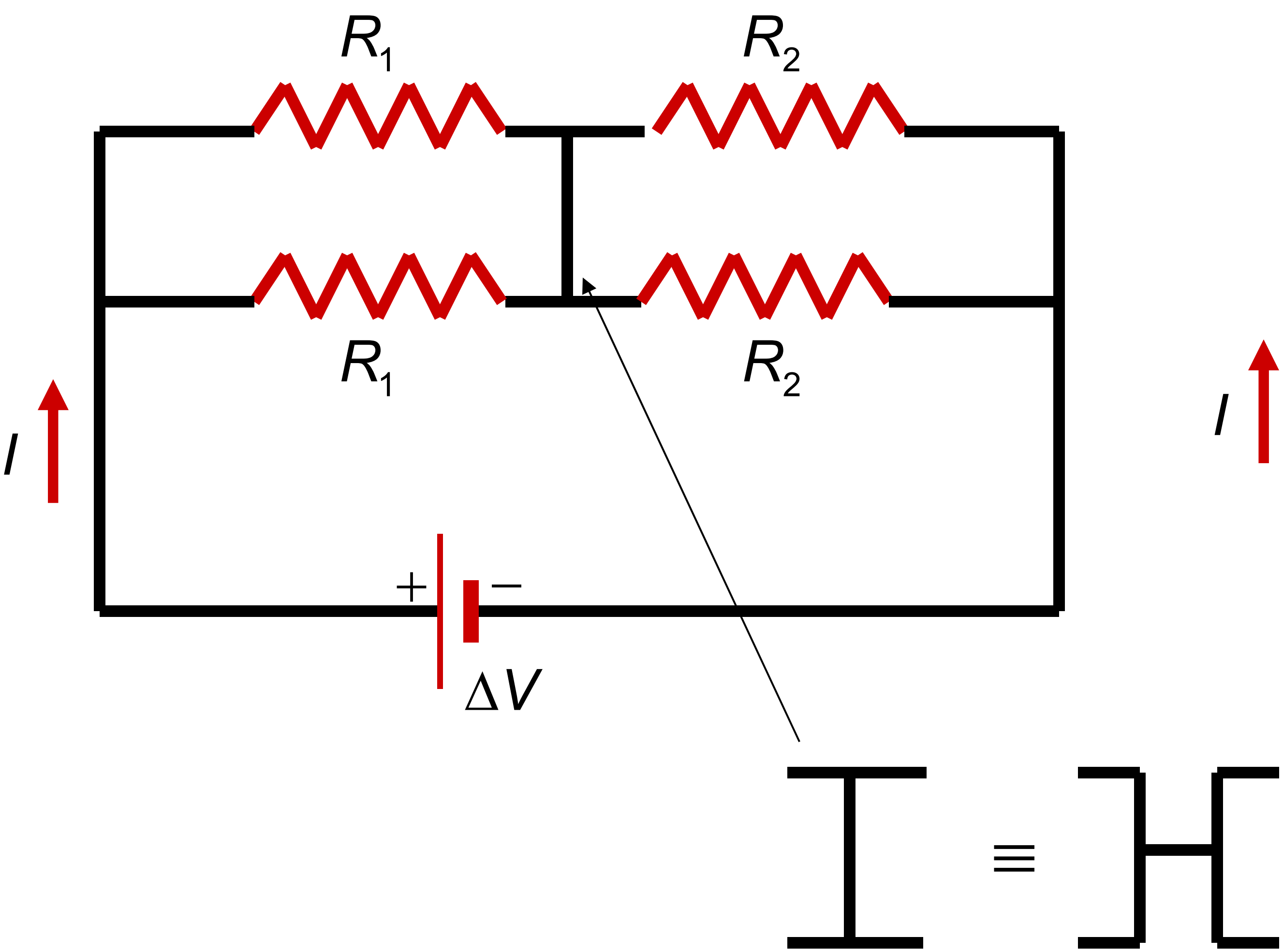
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

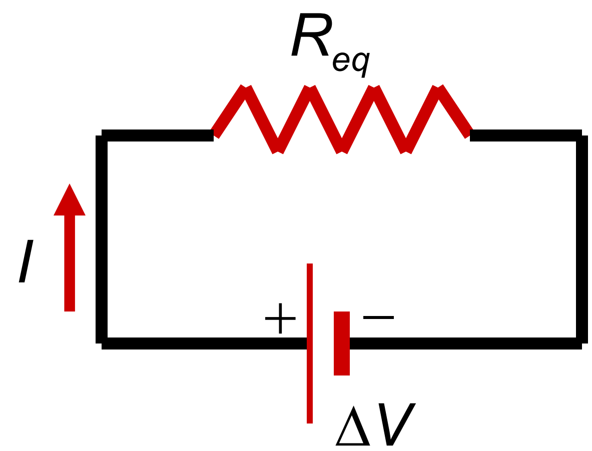
$$R_{eq} = R_1 + R_2 = \\ = 12 + 2 = 14$$

Example

Main question: $R_{eq} = ?$ **or** $I = ?$

$$I = \frac{\Delta V}{R_{eq}}$$



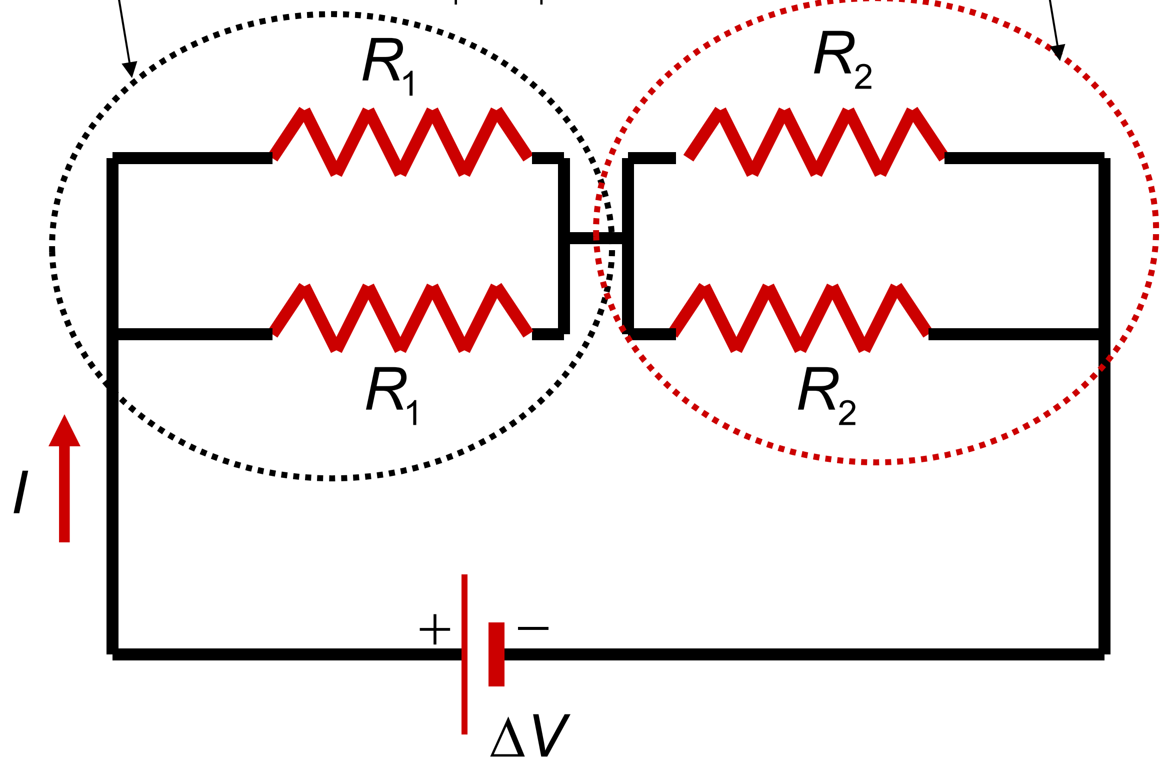


Example

Main question: $R_{eq} = ?$ **or** $I = ?$

in parallel $R_{eq,1} = \frac{R_1 R_1}{R_1 + R_1} = \frac{R_1}{2}$

in parallel $R_{eq,2} = \frac{R_2 R_2}{R_2 + R_2} = \frac{R_2}{2}$

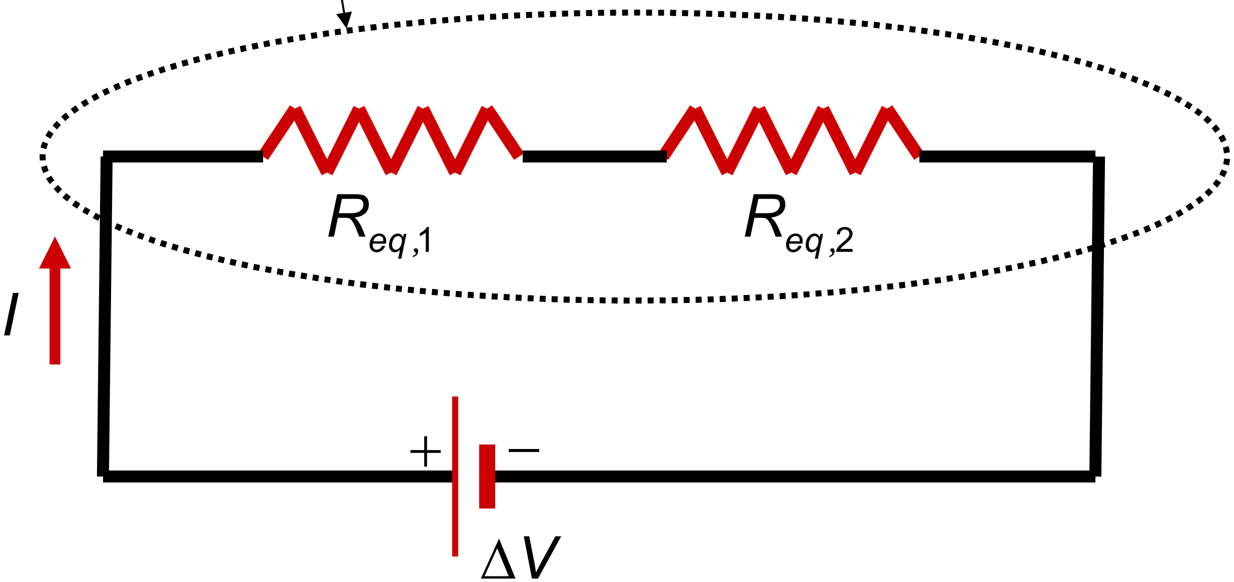


Example

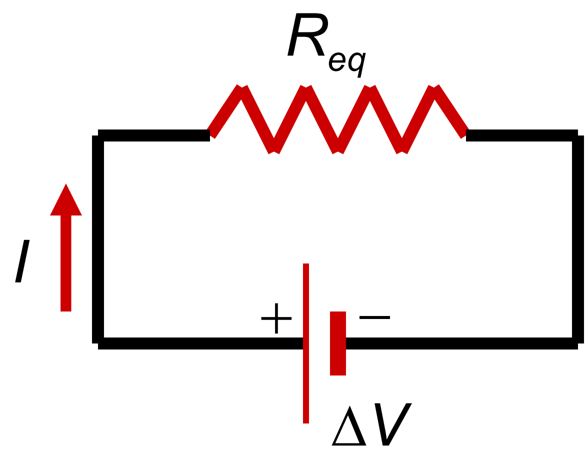
Main question: $R_{eq} = ?$ **or** $I = ?$

$$R_{eq,1} = \frac{R_1}{2} \qquad R_{eq,2} = \frac{R_2}{2}$$

in series $\quad R_{eq} = R_{eq,1} + R_{eq,2} = \dfrac{R_1 + R_2}{2}$



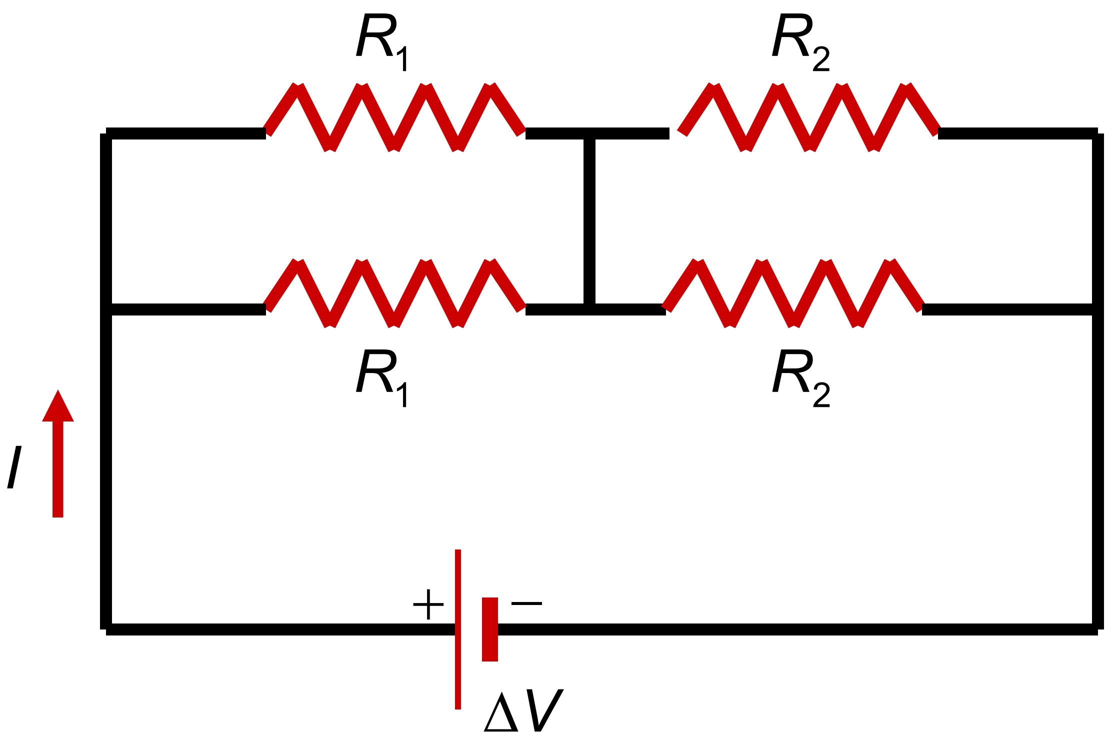
$$I = \frac{\Delta V}{R_{eq}} = \frac{2\Delta V}{R_1 + R_2}$$

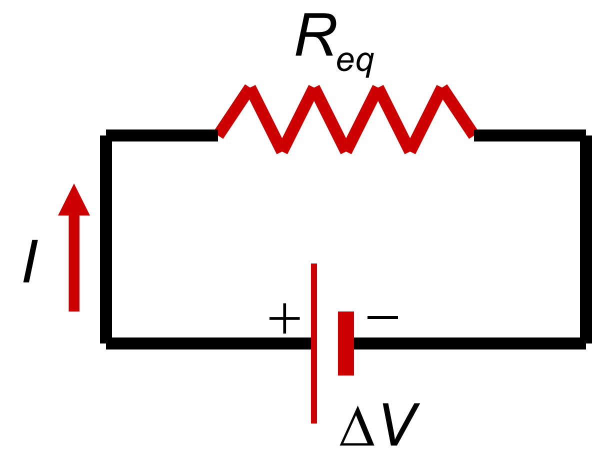


Example

Main question: $R_{eq} = ?$ **or** $I = ?$

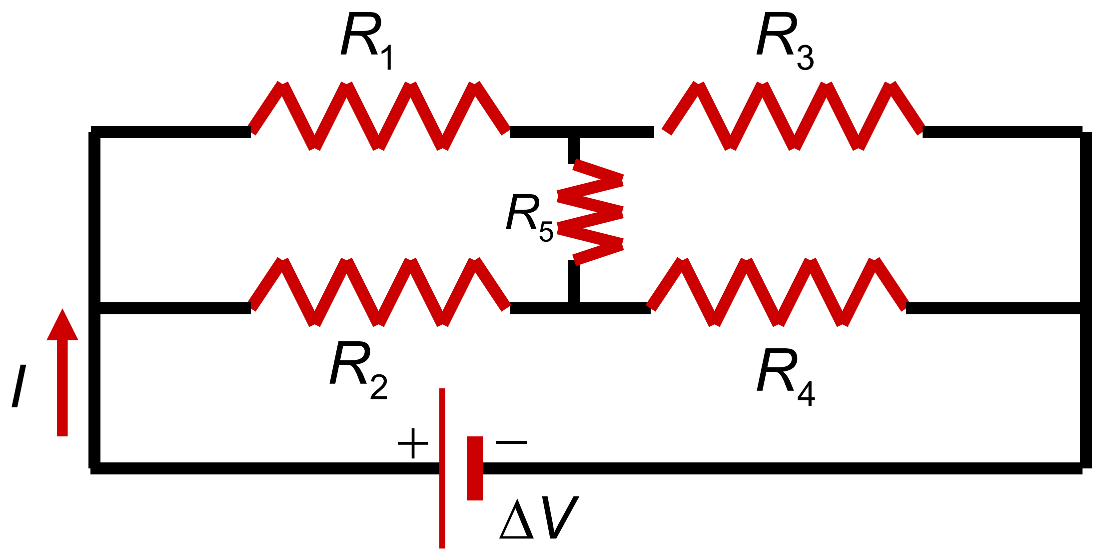
$$I = \frac{\Delta V}{R_{eq}}$$

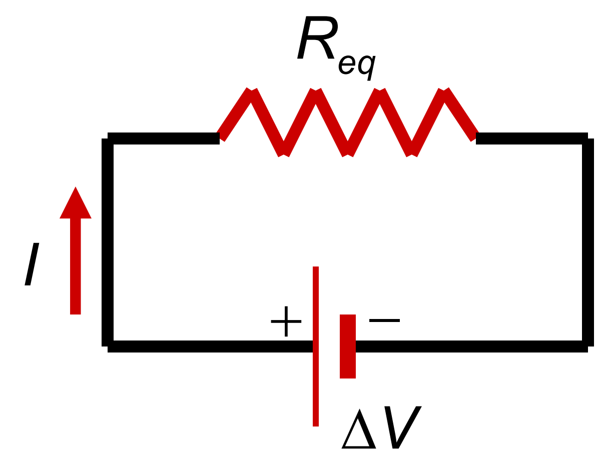




Example

Main question: $R_{eq} = ?$ **or** $I = ?$





To find R_{eq} **you need to use Kirchhoff's rules.**

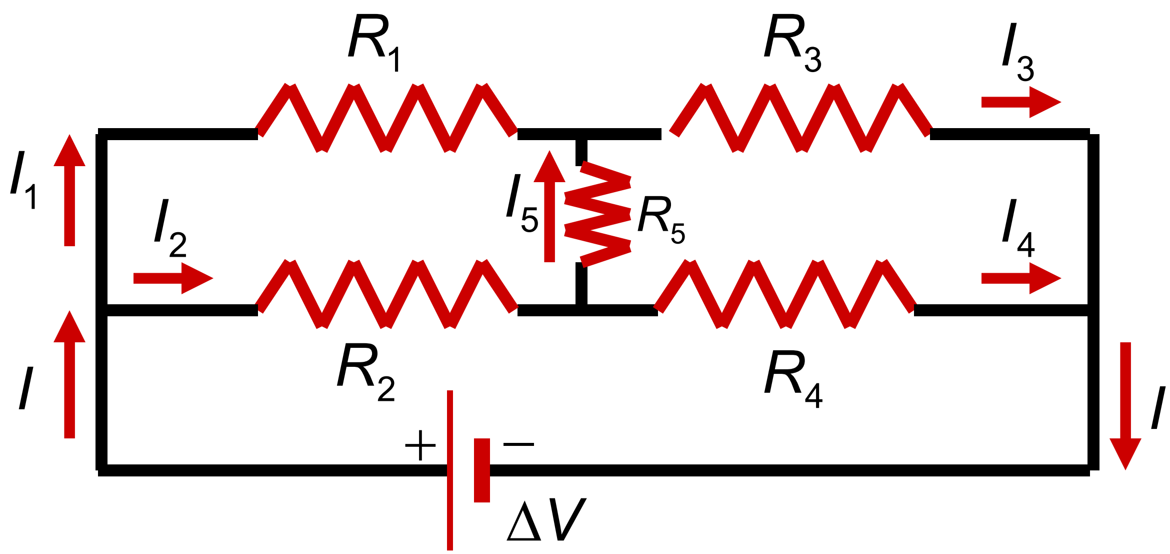
Chapter 31

Kirchhoff's rules

Kirchhoff's rules

- There are two Kirchhoff's rules

- To formulate the rules you need, at first, to choose the directions of current through all resistors. If you choose the wrong direction, then after calculation the corresponding current will be negative.



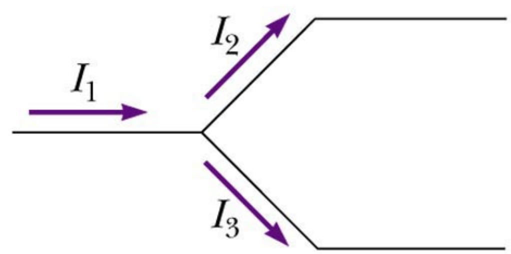
Junction Rule

- The first Kirchhoff's rule – Junction Rule:
- The sum of the currents entering any junction must equal the sum of the currents leaving that junction

  \- A statement of Conservation of Charge

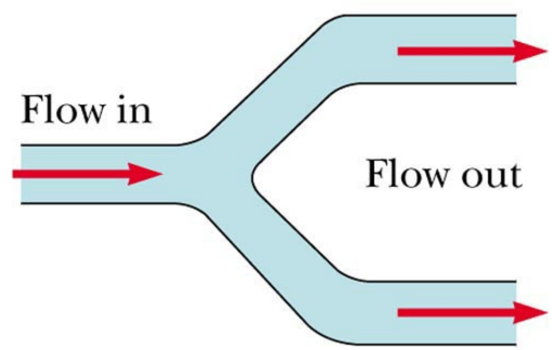
$$\sum I_{in} = \sum I_{out}$$

$$I_1 = I_2 + I_3$$



(a)

In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit

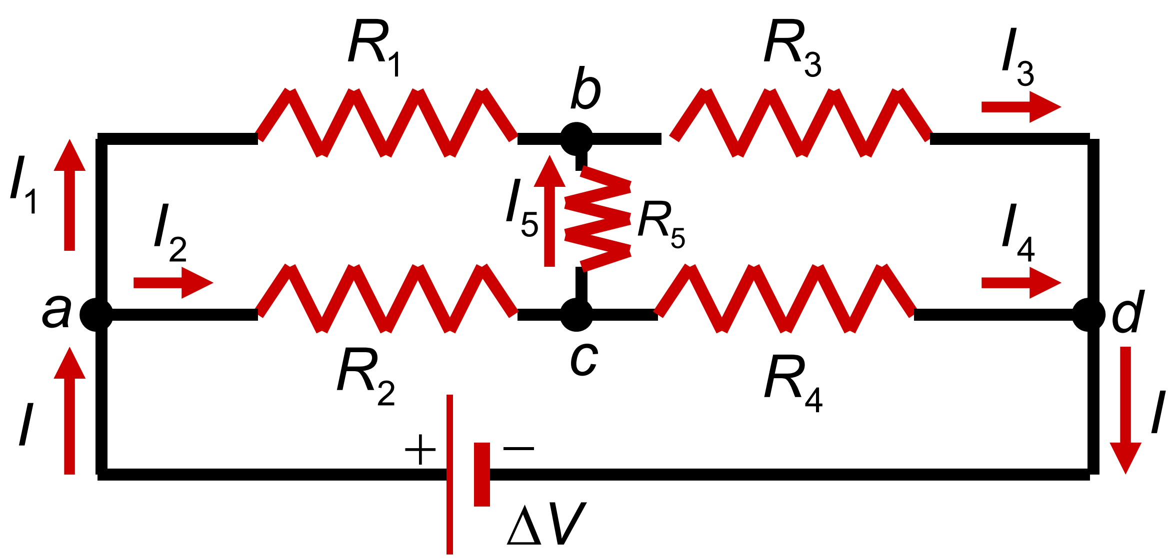


(b)

Junction Rule

- The first Kirchhoff's rule – Junction Rule: $\sum I_{in} = \sum I_{out}$
- In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit

- There are 4 junctions: a, b, c, d.
- We can write the Junction Rule for any three of them



(a) $I = I_1 + I_2$

(b) $I_1 + I_5 = I_3$

(c) $I_2 = I_4 + I_5$

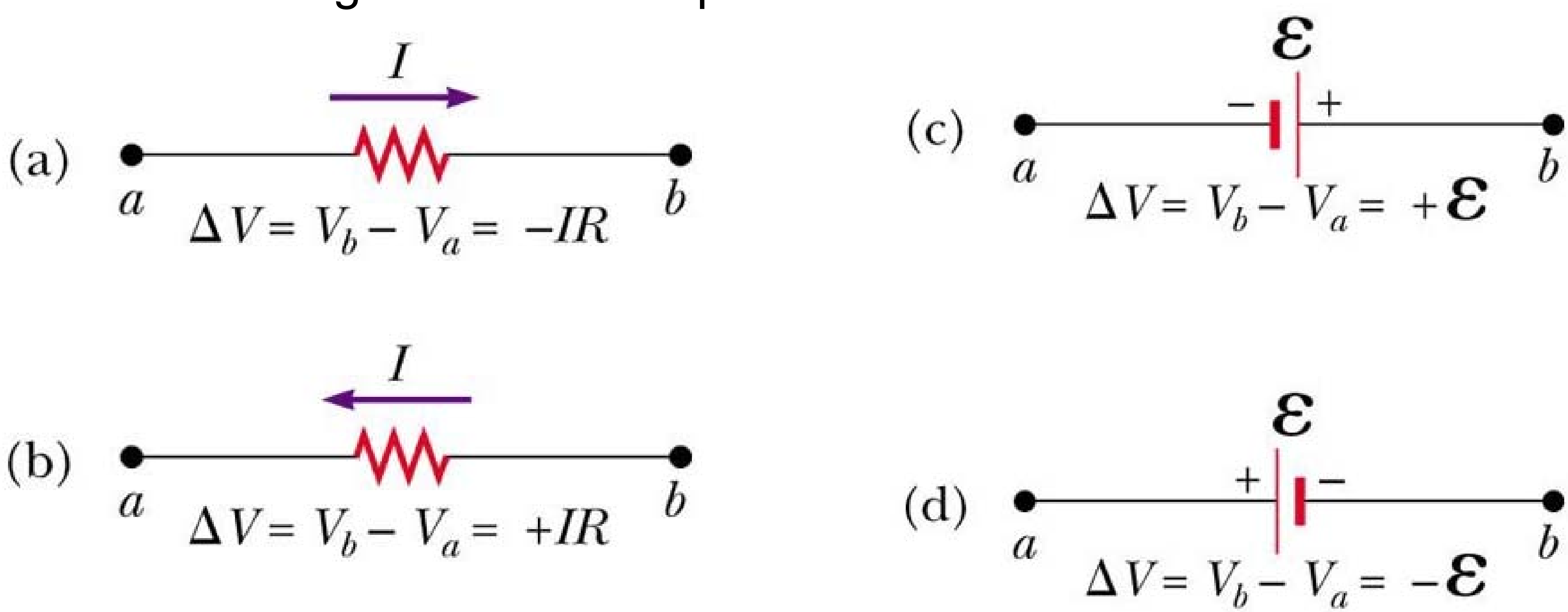
Loop Rule

- The second Kirchhoff's rule – Loop Rule:
  - The sum of the potential differences across all the elements around any closed circuit loop must be zero

    - A statement of Conservation of Energy

$$\sum_{closed\ loop} \Delta V = 0$$

Traveling around the loop from ***a*** to ***b***



Loop Rule

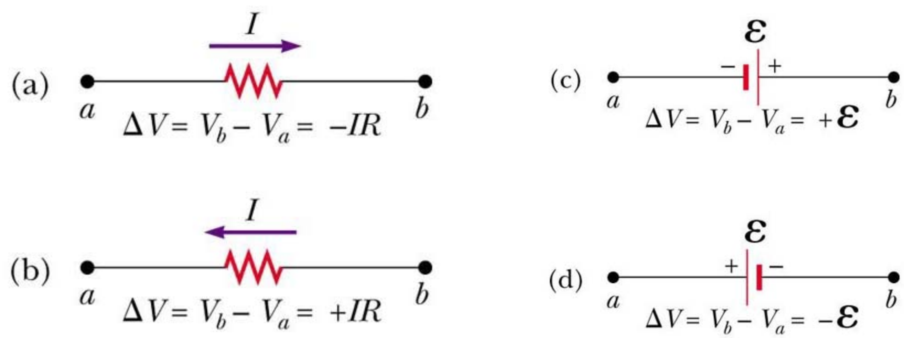
- The second Kirchhoff's rule – Loop Rule:

In general, the number of times the Loop Rule can be used is one fewer than the number of possible loops in the circuit

$$\sum_{closed\ loop} \Delta V = 0$$

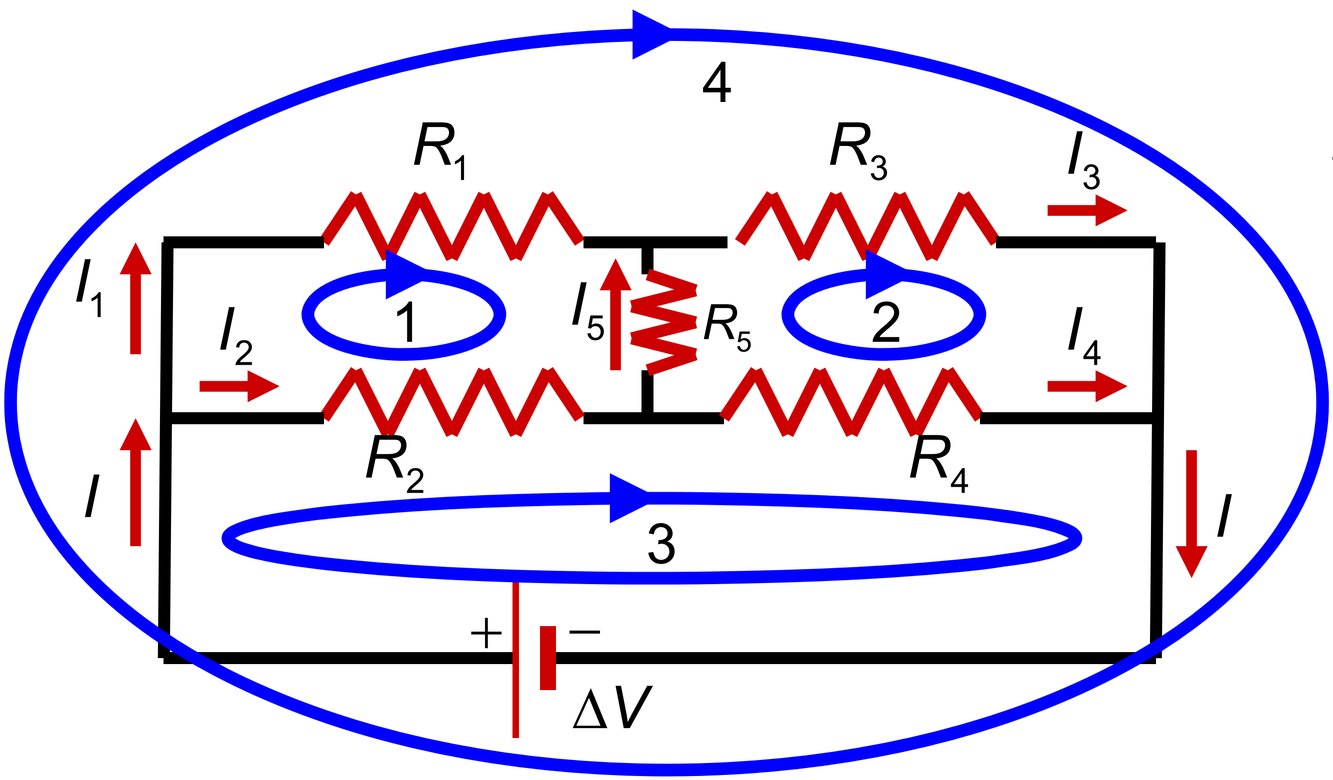
Loop Rule

➢ The second Kirchhoff's rule – Loop Rule: $\sum_{\text{closed loop}} \Delta V = 0$



There are 4 loops.

We need to write the Loop Rule for 3 loops



Loop 1:

$$-I_1R_1 + I_5R_5 + I_2R_2 = 0$$

Loop 2:

$$-I_3R_3 - I_5R_5 + I_4R_4 = 0$$

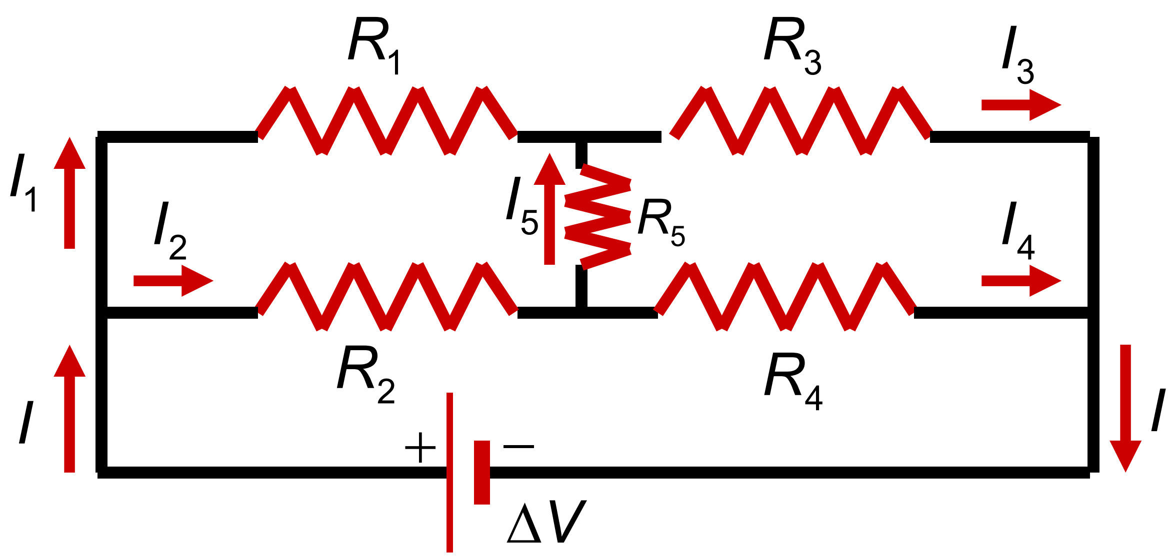
Loop 3:

$$\Delta V - I_2R_2 - I_4R_4 = 0$$

Kirchhoff's Rules

Junction Rule $\sum I_{in} = \sum I_{out}$

➢ Loop Rule $\sum_{closed\ loop} \Delta V = 0$



$$I = I_1 + I_2$$

$$I_1 + I_5 = I_3$$

$$I_2 = I_4 + I_5$$

$$-I_1R_1 + I_5R_5 + I_2R_2 = 0$$

$$-I_3R_3 - I_5R_5 + I_4R_4 = 0$$

$$\Delta V - I_2R_2 - I_4R_4 = 0$$

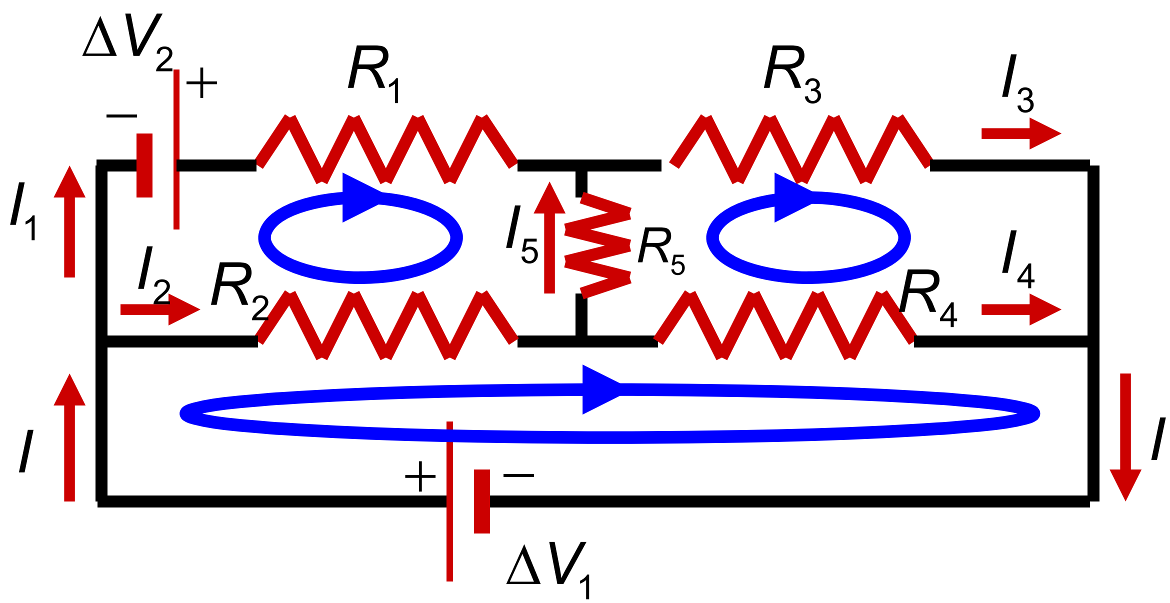
We have 6 equations and 6 unknown currents.

$$R_{eq} = \frac{\Delta V}{I}$$

Kirchhoff's Rules

Junction Rule $\sum I_{in} = \sum I_{out}$

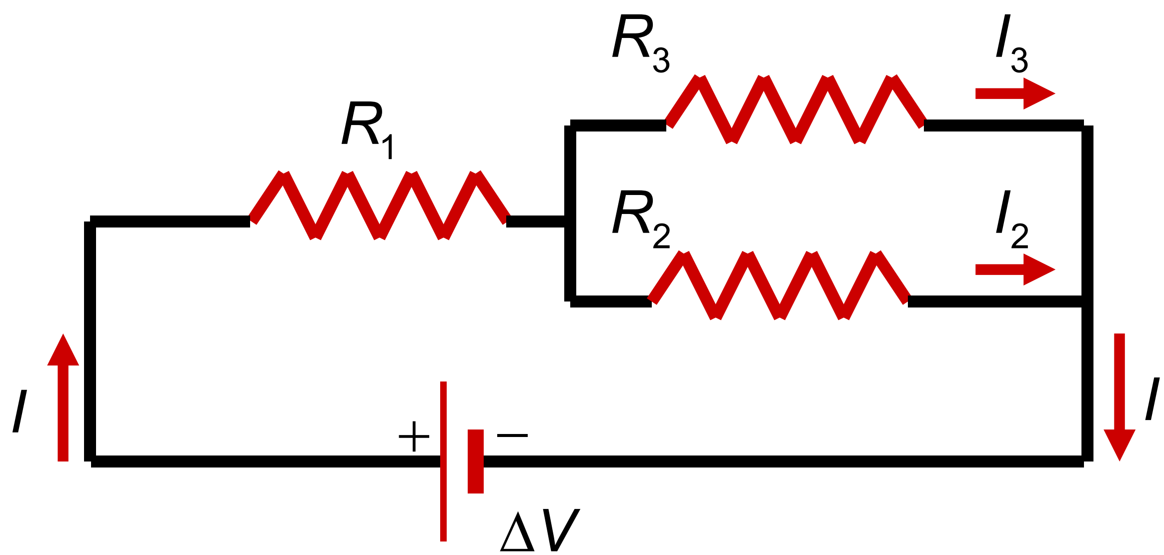
- Loop Rule $\sum_{closed\ loop} \Delta V = 0$

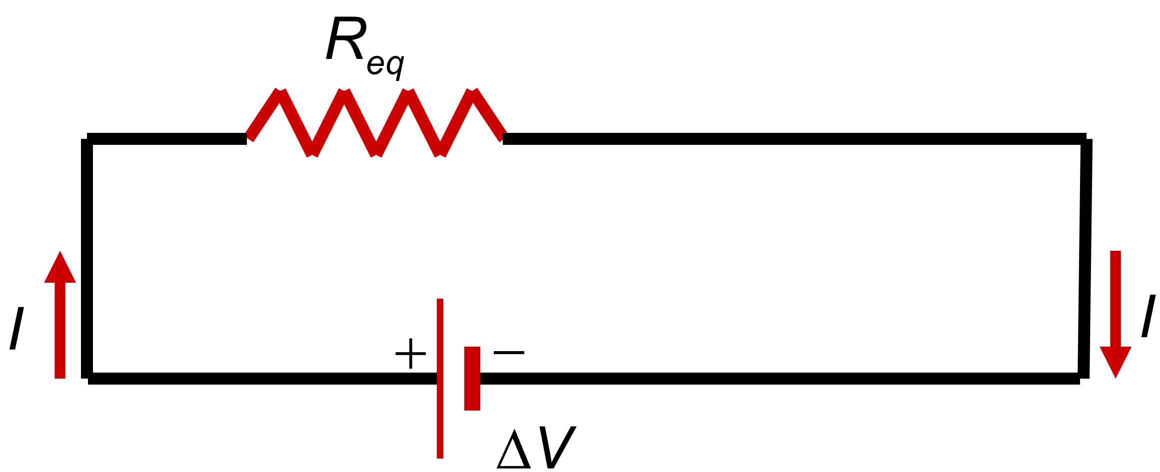


$$I = I_1 + I_2$$
$$I_1 + I_5 = I_3$$
$$I_2 = I_4 + I_5$$
$$-I_1R_1 + I_5R_5 + I_2R_2 + \Delta V_2 = 0$$
$$-I_3R_3 - I_5R_5 + I_4R_4 = 0$$
$$\Delta V_1 - I_2R_2 - I_4R_4 = 0$$

We have 6 equations and 6 unknown currents.

Example 1

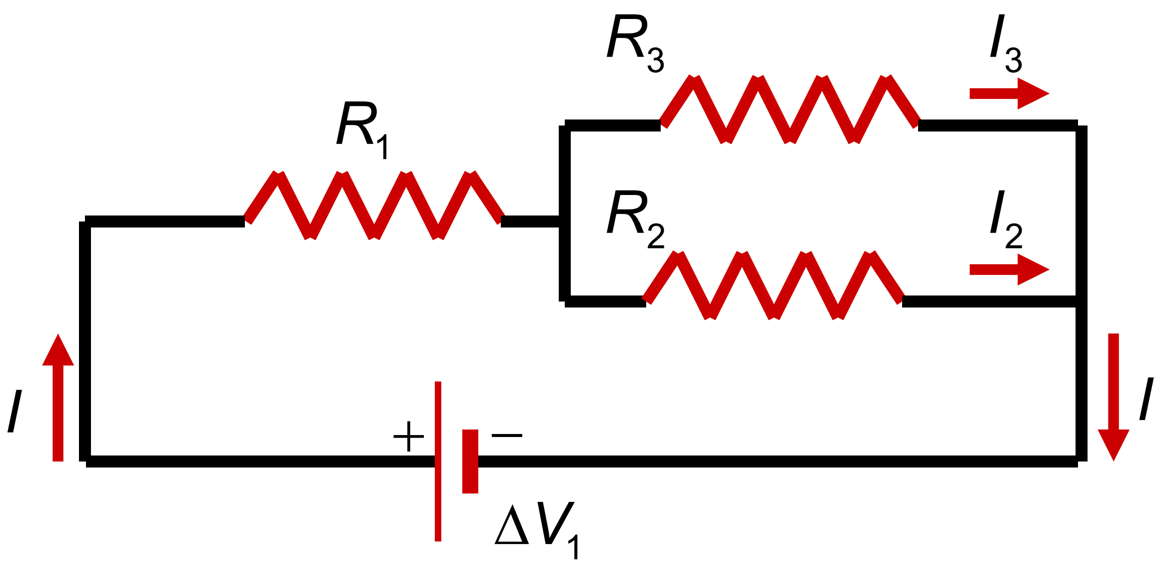




$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$I = \frac{\Delta V}{R_{eq}}$$

Example 1



$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

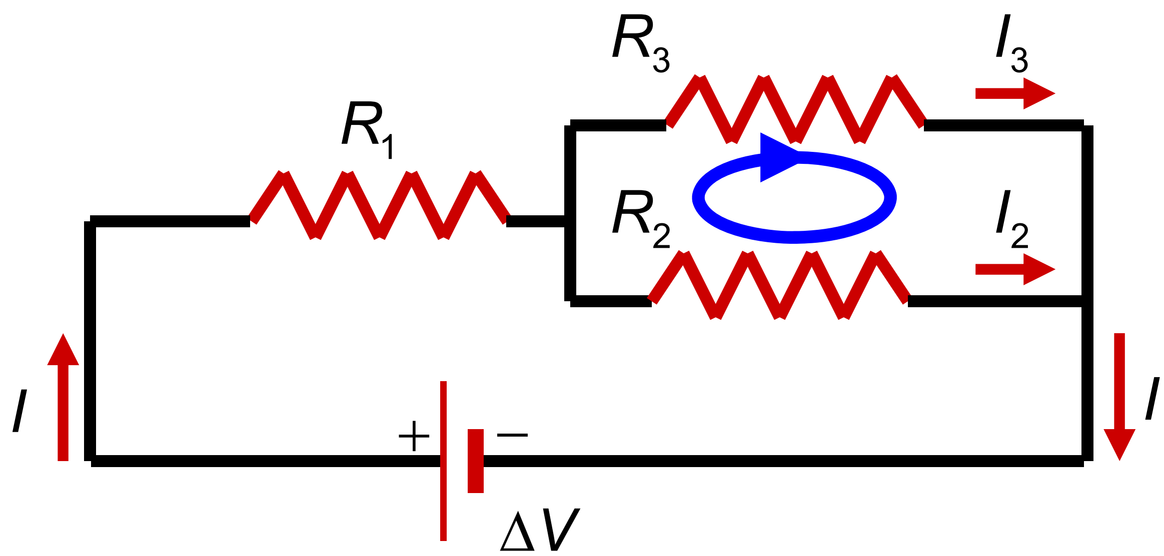
$$I = \frac{\Delta V}{R_{eq}}$$

$$I = I_2 + I_3$$

$$I_2 R_2 = I_3 R_3 \qquad I_2 = I_3 \frac{R_3}{R_2}$$

$$I = I_3 \left(1 + \frac{R_3}{R_2}\right) \qquad I_3 = \frac{I R_2}{R_2 + R_3} \qquad I_2 = \frac{I R_3}{R_2 + R_3}$$

Example 1: solution based on Kirchhoff's Rules



$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

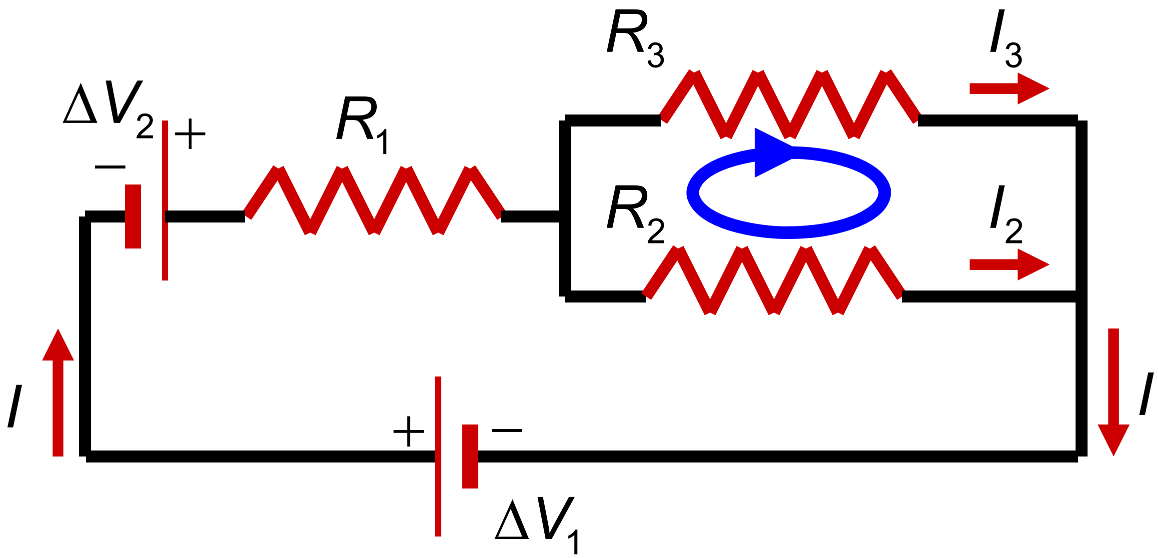
$$\Delta V - I_2 R_2 - I R_1 = 0$$

$$I_2 = I_3 \frac{R_3}{R_2} \qquad I_3 = \frac{I R_2}{R_2 + R_3} \qquad I_2 = \frac{I R_3}{R_2 + R_3}$$

$$\Delta V - \frac{I R_3}{R_2 + R_3} R_2 - I R_1 = 0$$

$$I = \frac{\Delta V}{\dfrac{R_3 R_2}{R_2 + R_3} + R_1} = \frac{\Delta V}{R_{eq}}$$

Example 2

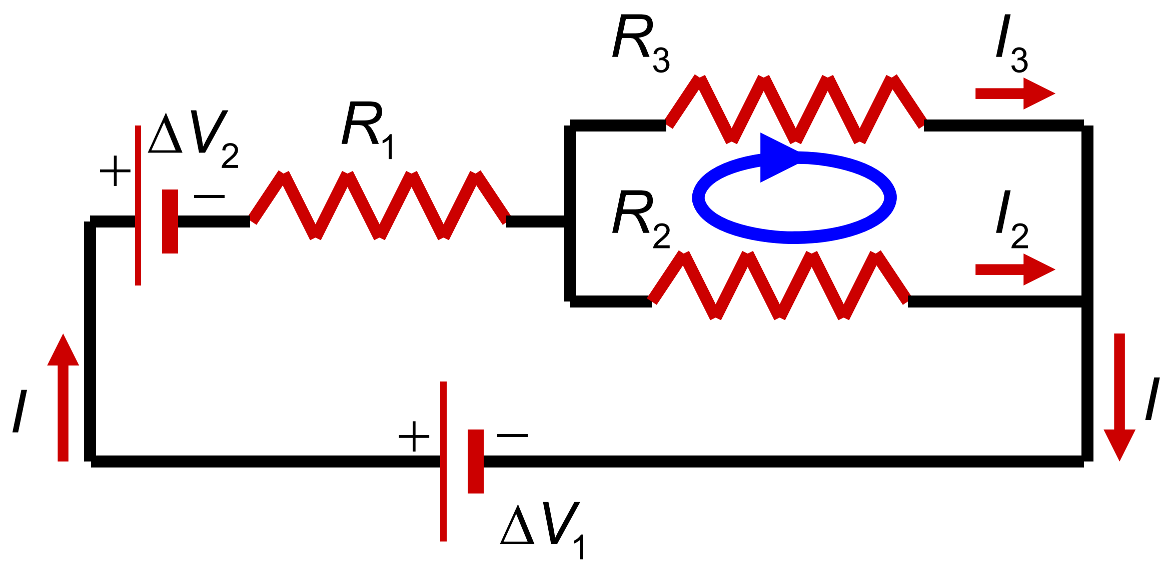


$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

$$\Delta V_1 + \Delta V_2 - I_2 R_2 - I R_1 = 0$$

Example 3



$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

$$\Delta V_1 - \Delta V_2 - I_2 R_2 - I R_1 = 0$$