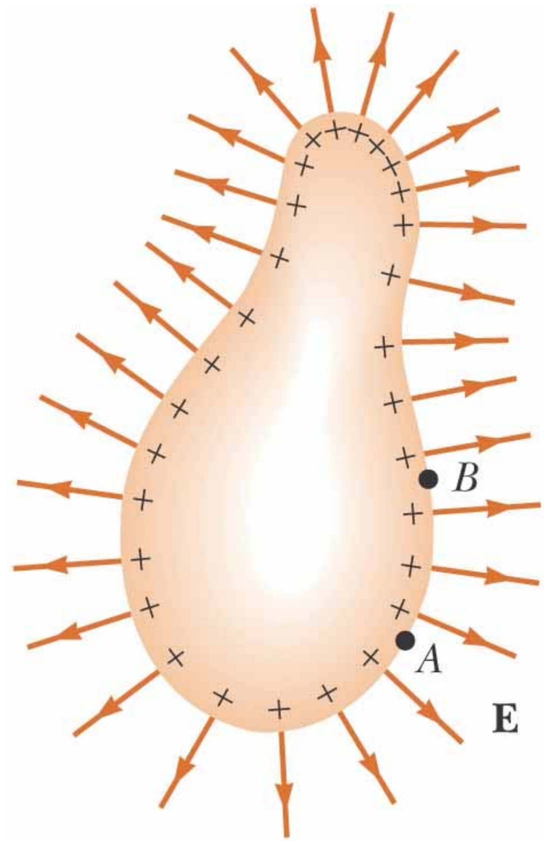
Chapter 29

Electric Potential: Charged Conductor

Electric Potential: Charged Conductor

- Consider two points (***A*** and ***B***) on the surface of the charged conductor
- ***E*** is always perpendicular to the displacement $d\mathbf{s}$
- Therefore, $\mathbf{E} \cdot d\mathbf{s} = 0$
- Therefore, the potential difference between ***A*** and ***B*** is also **zero!!!!**



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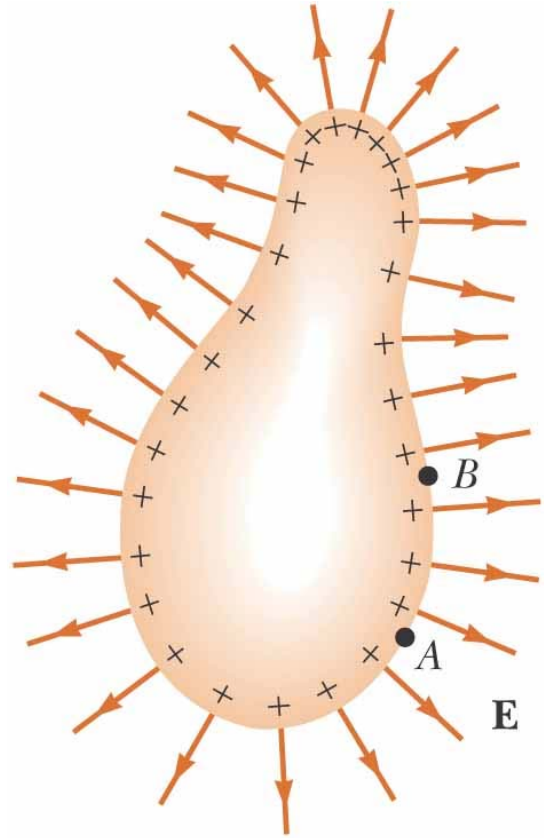
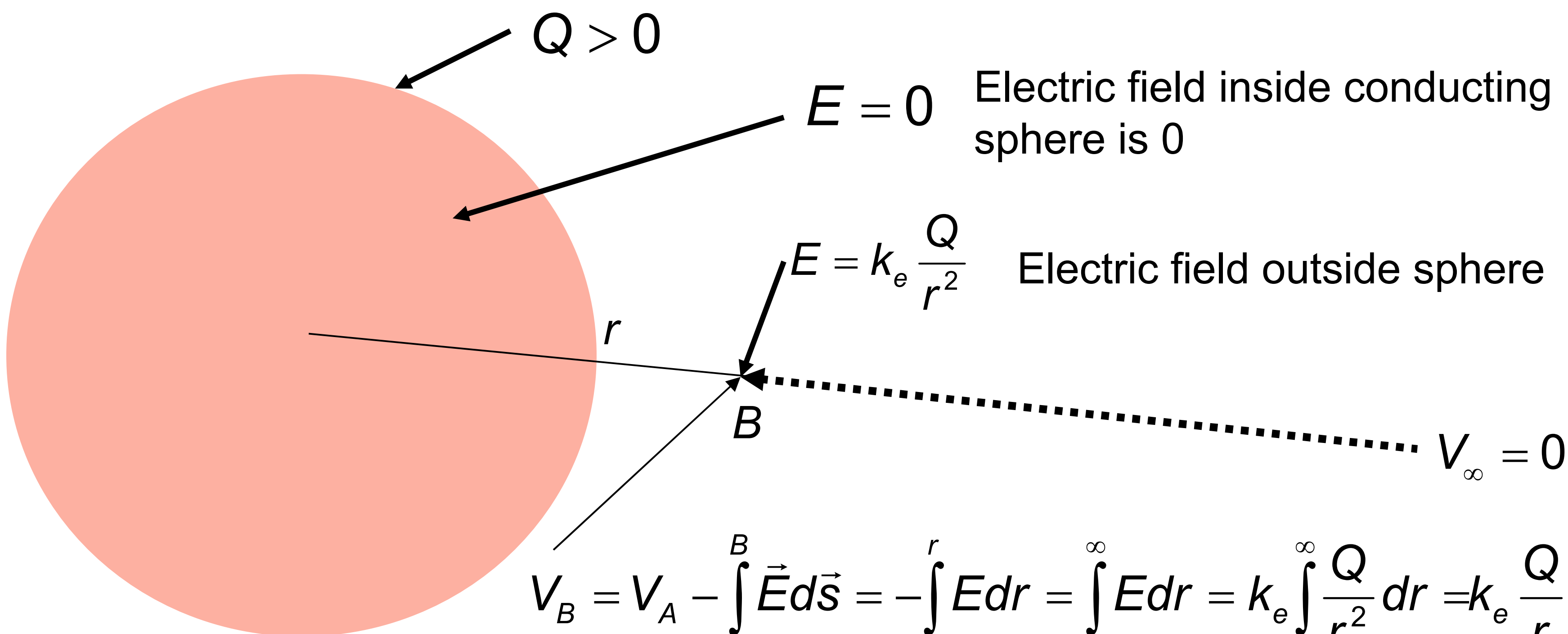
Electric Potential: Charged Conductor

- The potential difference between ***A*** and ***B*** is **zero!!!!**
- Therefore ***V*** is constant everywhere on the surface of a charged conductor in equilibrium
  - **Δ*V* = 0** between any two points on the surface
- The surface of any charged conductor is an equipotential surface
- Because the electric field is zero inside the conductor, the electric potential is **constant everywhere inside the conductor** and equal to the value at the surface



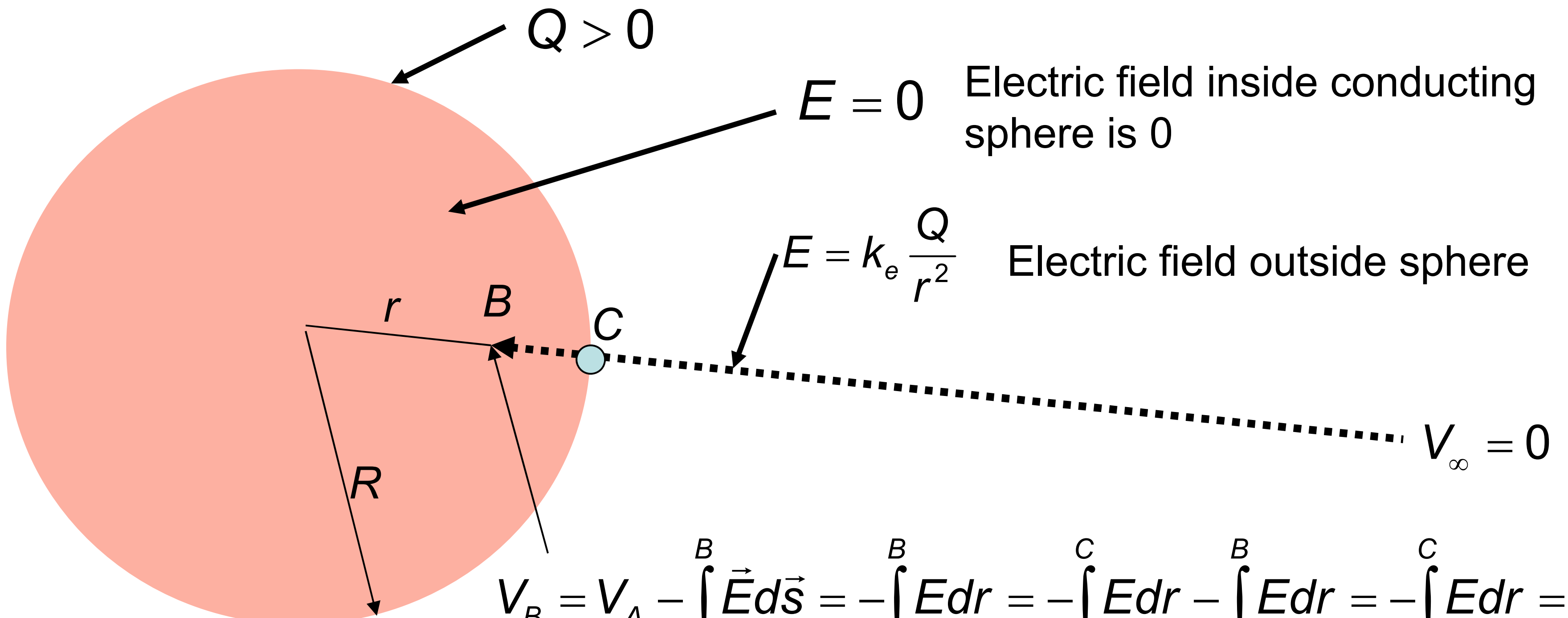
Electric Potential: Conducting Sphere: Example



$$V_B = V_A - \int_A^B \vec{E} d\vec{s} = -\int_\infty^r E dr = \int_r^\infty E dr = k_e \int_r^\infty \frac{Q}{r^2} dr = k_e \frac{Q}{r}$$

The same expression for potential (outside sphere) as for the point charge (at the center of the sphere).

Electric Potential: Conducting Sphere: Example



Electric field inside conducting sphere is 0

$$E = k_e \frac{Q}{r^2}$$

Electric field outside sphere

$$V_B = V_A - \int_A^B \vec{E} d\vec{s} = -\int_A^B E dr = -\int_A^C E dr - \int_C^B E dr = -\int_A^C E dr =$$

$$= k_e \int_R^\infty \frac{Q}{r^2} dr = k_e \frac{Q}{R}$$

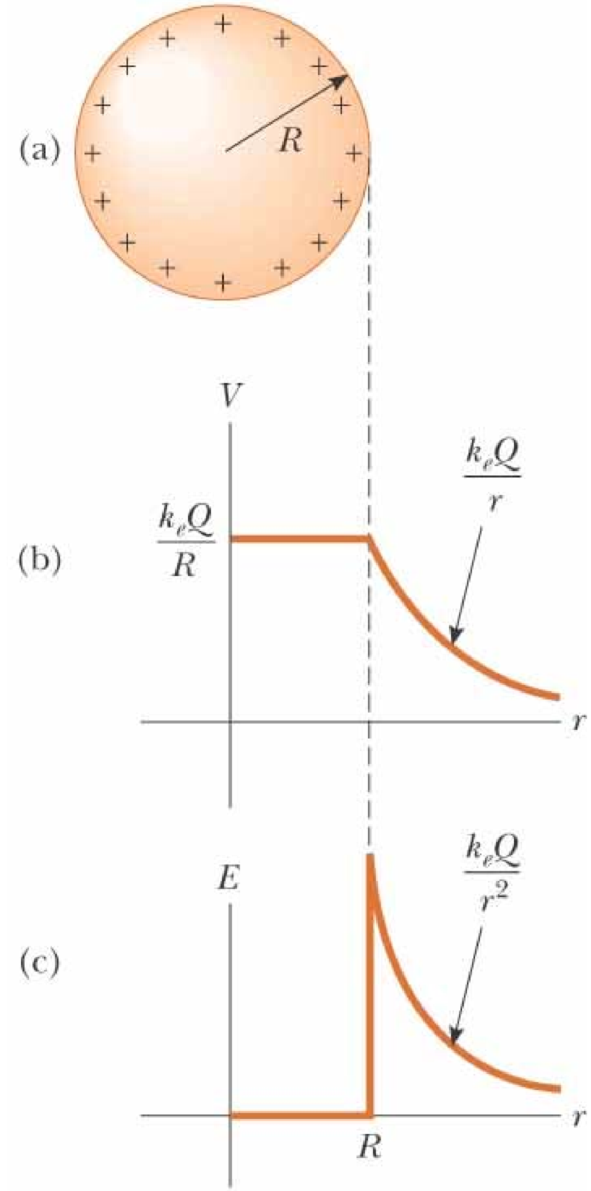
The electric potential is constant everywhere inside the conducting sphere

Electric Potential: Conducting Sphere: Example

$$V_r = k_e \frac{Q}{r} \qquad \text{for } \boldsymbol{r > R}$$

$$V_{sphere} = k_e \frac{Q}{R} \qquad \text{for } \boldsymbol{r < R}$$

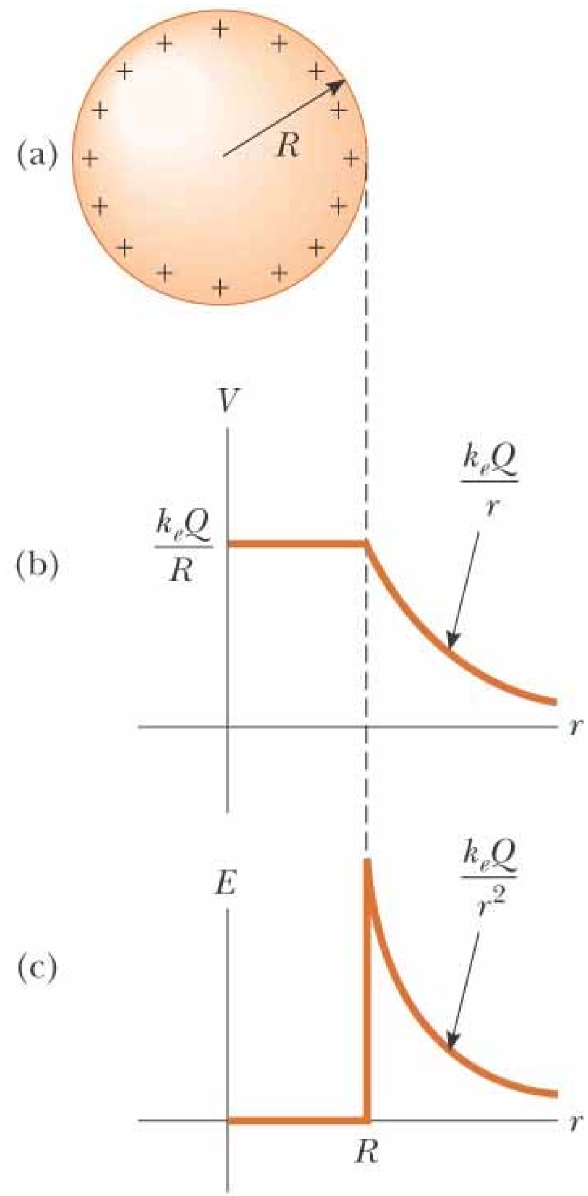
The potential of conducting sphere!!



Conducting Sphere: Example

What is the potential of conducting sphere with radius 0.1 m and charge $1\mu C$?

$$V_{sphere} = k_e \frac{Q}{R} = 9 \cdot 10^9 \frac{10^{-6} C}{0.1 m} = 90000 V$$

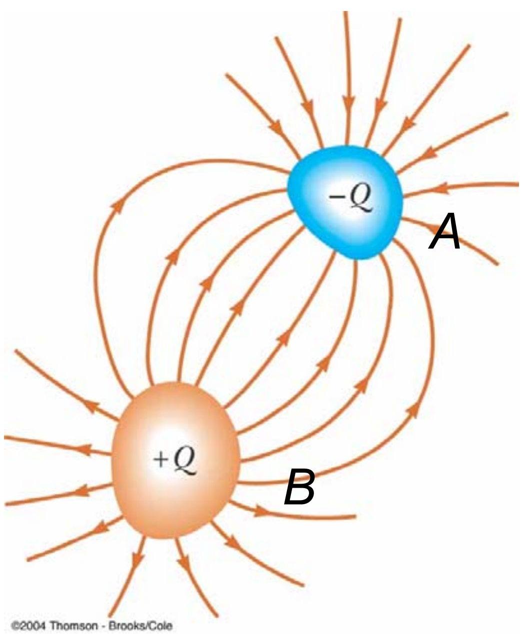


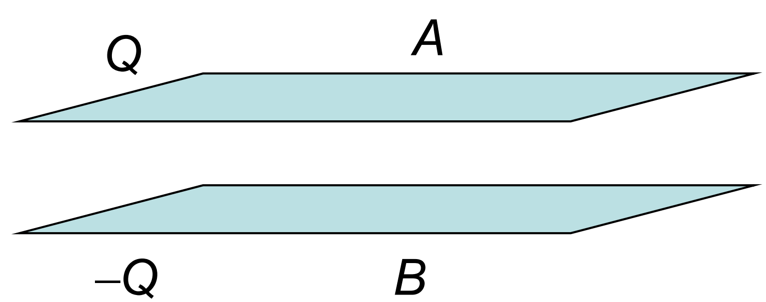
Chapter 30

Capacitance

Capacitors

- **Capacitors are devices that store electric charge**
- A capacitor consists of two conductors
  - These conductors are called ***plates***
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions
- A potential difference exists between the plates due to the charge





Q - the charge of capacitor

$\Delta V = V_A - V_B$ - a potential difference of capacitor

Capacitors

- A capacitor consists of two conductors

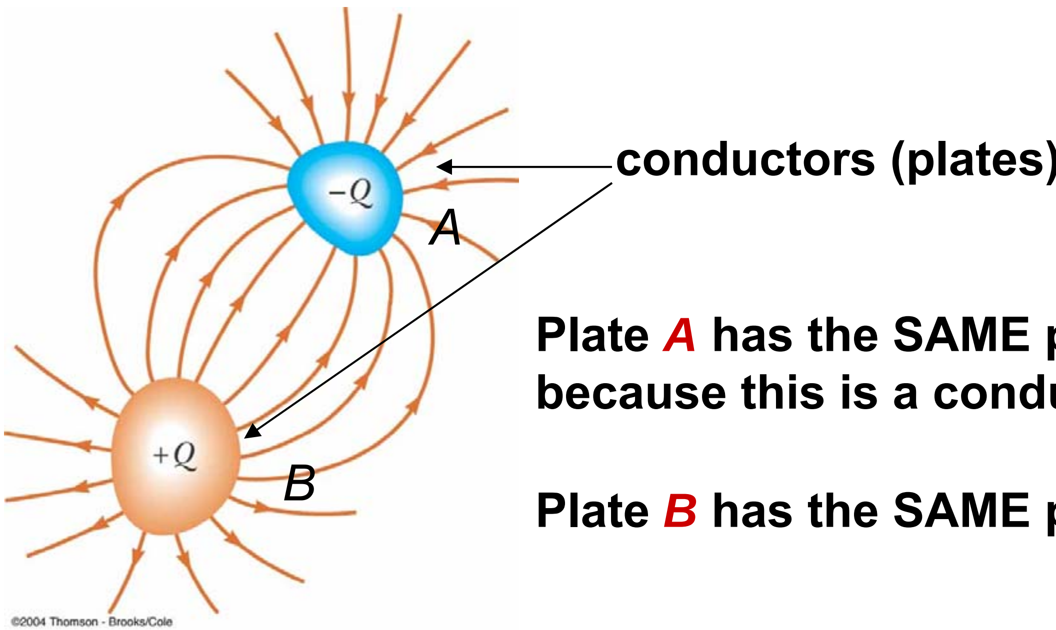


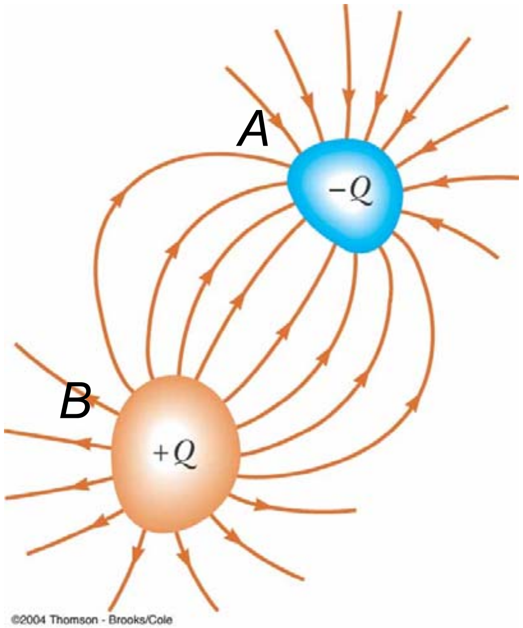
Plate *A* has the SAME potential at all points because this is a conductor .

Plate *B* has the SAME potential at all points.

So we can define the potential difference between the plates:

$$\Delta V = V_A - V_B$$

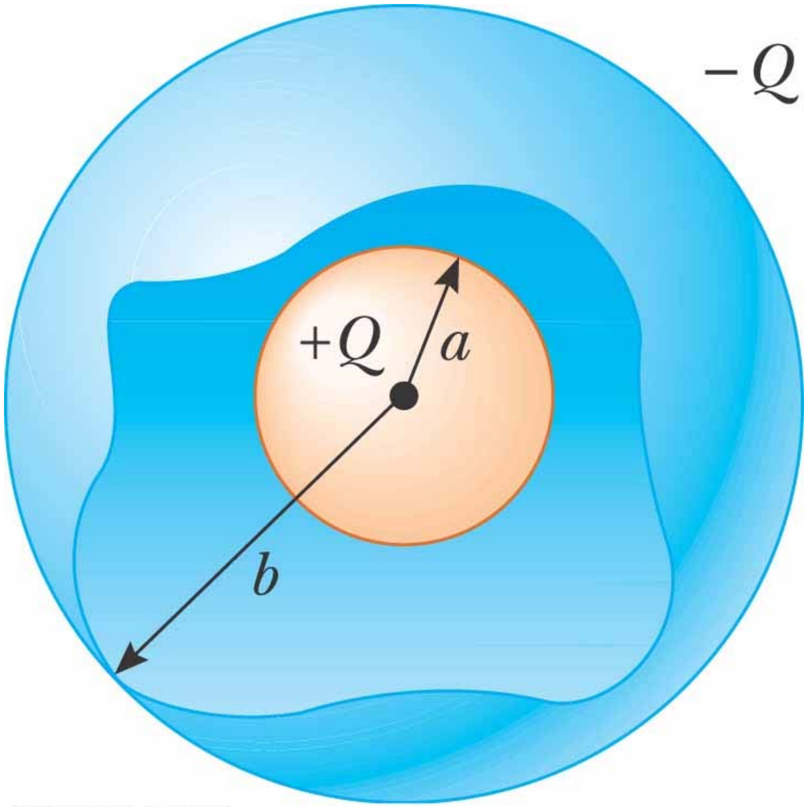
Capacitance of Capacitor



$$C = \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the ***farad*** (F) = C/V.
- Capacitance is always a positive quantity
- The capacitance of a given capacitor is constant and determined only by geometry of capacitor
- The farad is a large unit, typically you will see microfarads (μF) and picofarads (***pF***)

Capacitor: Spherical Capacitor



$$C = \frac{Q}{\Delta V}$$

No electric field outside of the capacitor (because the total charge is 0). The field inside the capacitor is due to small sphere.

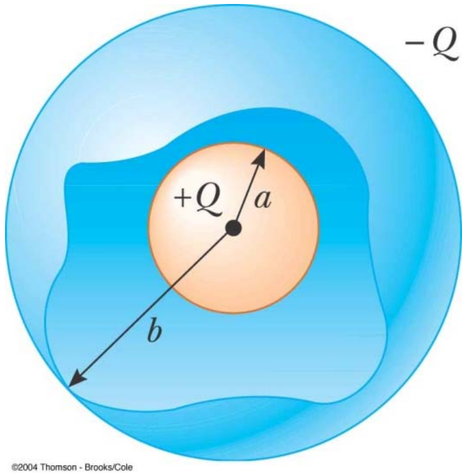
The potential difference is only due to a small sphere:

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

The capacitance:

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b-a\right)}$$

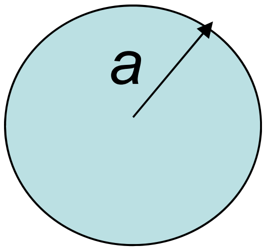
Capacitor: Isolated Sphere



$$C = \frac{Q}{\Delta V}$$

$$b >> a$$

$$\longrightarrow$$



The capacitance:

$$b >> a$$

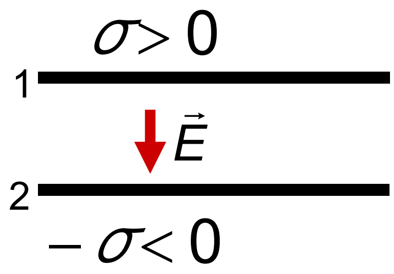
$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b-a\right)} \approx \frac{ab}{k_e b} = \frac{a}{k_e}$$

Capacitor: Parallel Plates

$$E = 0$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = 0$$

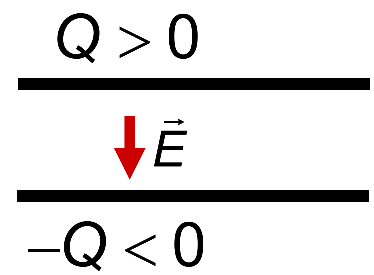


$$V = 0$$

$$V = -\frac{\sigma}{\varepsilon_0} x$$

$$V = -\frac{\sigma}{\varepsilon_0} h$$

$$\sigma = \frac{Q}{S}$$



The potential difference:

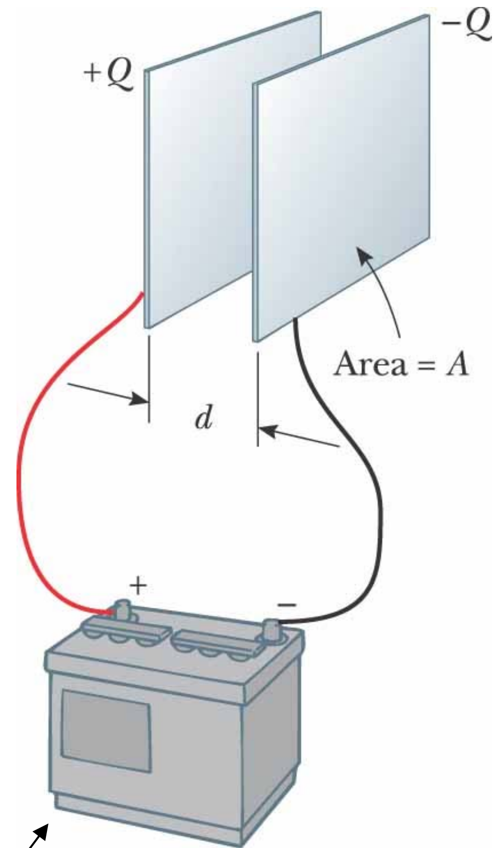
$$\Delta V = \frac{\sigma}{\varepsilon_0} h = \frac{Q}{\varepsilon_0 S} h$$

The capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 S}{h}$$

Capacitor: Charging

- Each plate is connected to a terminal of the battery
- The battery establishes an electric field in the connecting wires
- This field applies a force on electrons in the wire just outside of the plates
- The force causes the electrons to move onto the negative plate
- This continues until equilibrium is achieved
  - The plate, the wire and the terminal are all at the same potential
- At this point, there is no field present in the wire and there is no motion of electrons

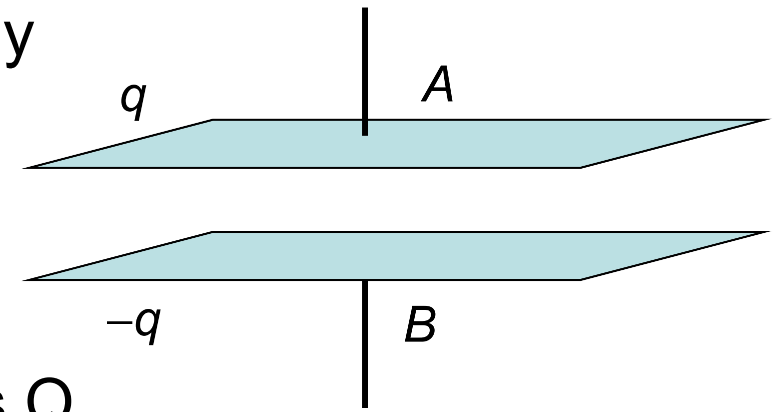


Battery- produce the fixed voltage – the fixed potential difference

Energy Stored in a Capacitor

➢ Assume the capacitor is being charged and, at some point, has a charge ***q*** on it

➢ The work needed to transfer a small charge Δq from one plate to the other is equal to the change of potential energy



$$dW = \Delta V dq = \frac{q}{C} dq$$

➢ If the final charge of the capacitor is Q, then the total work required is

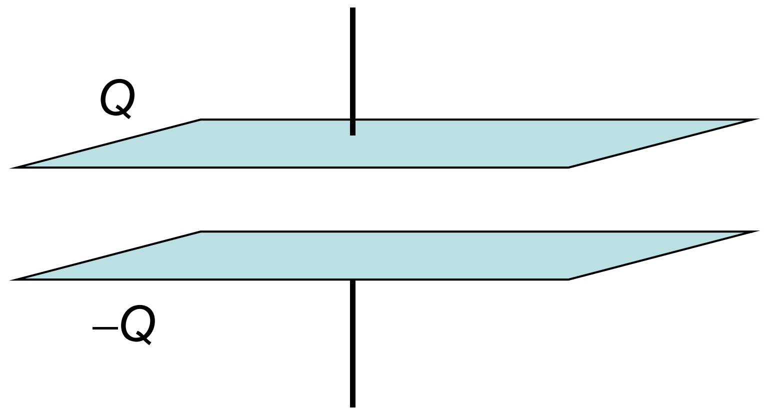
$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy Stored in a Capacitor

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}$$

➢ The work done in charging the capacitor is equal to the electric potential energy *U* of a capacitor

$$Q = C\Delta V$$



$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

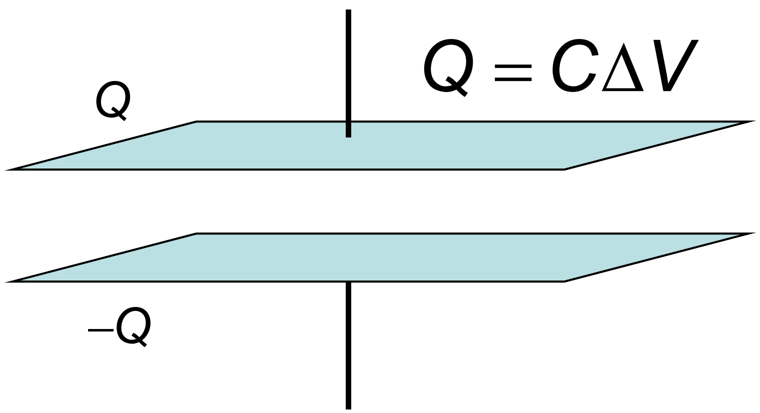
This applies to a capacitor of **any geometry**

Energy Stored in a Capacitor: Application

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

One of the main application of capacitor:

➢ capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse

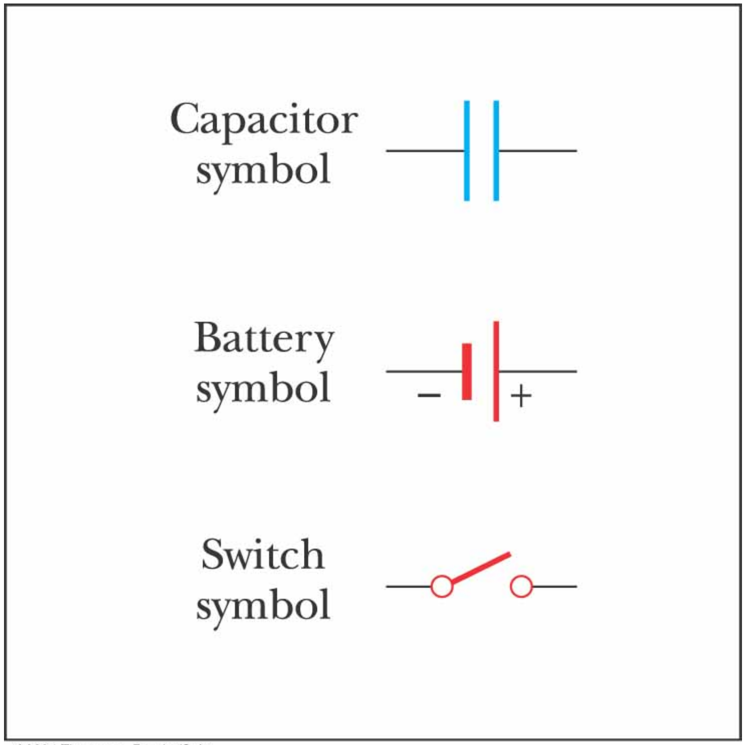


Chapter 30

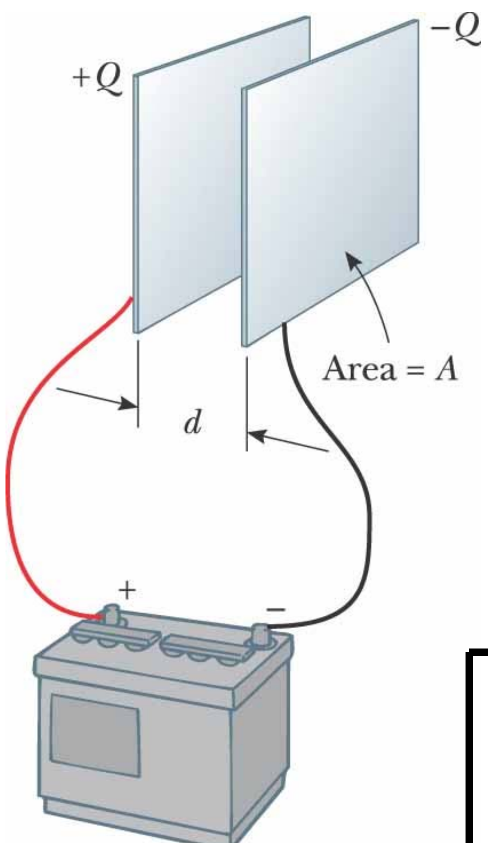
Capacitance and Electrical Circuit

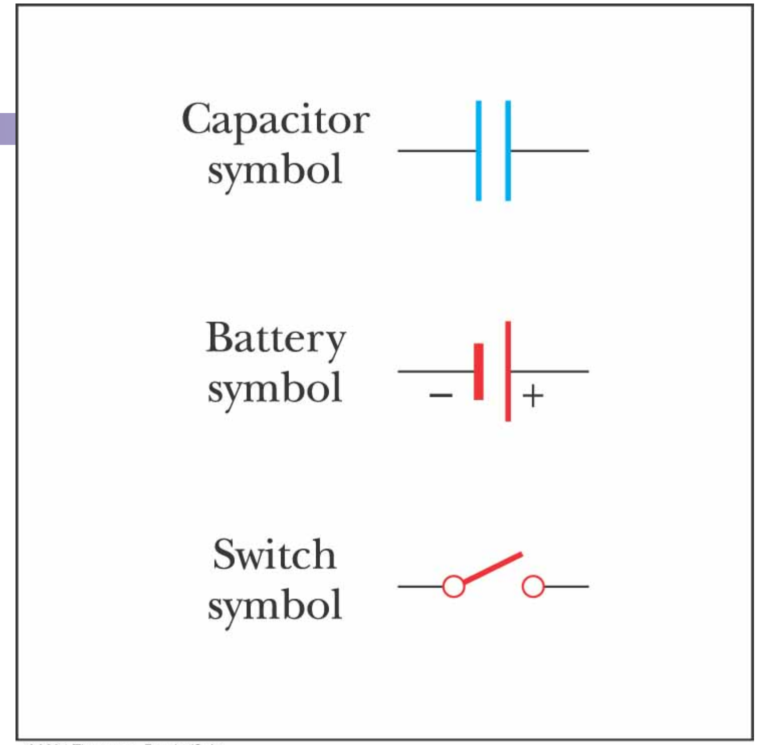
Electrical Circuit

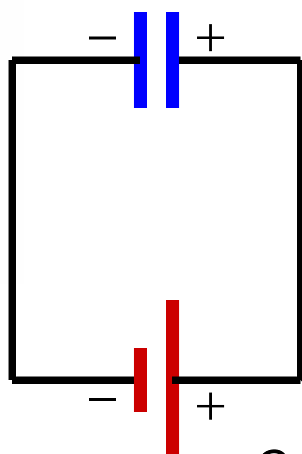
- A circuit diagram is a simplified representation of an actual circuit
- Circuit symbols are used to represent the various elements
- Lines are used to represent wires
- The battery's positive terminal is indicated by the longer line

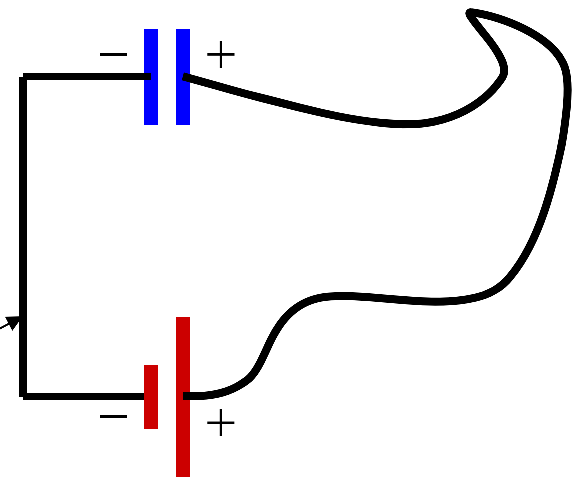


Electrical Circuit



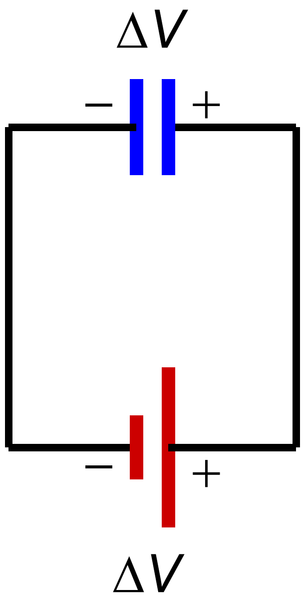






Conducting wires.
In equilibrium all the points of the wires have the same potential

Electrical Circuit

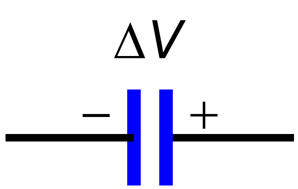


The battery is characterized by the voltage – the potential difference between the contacts of the battery

In equilibrium this potential difference is equal to the potential difference between the plates of the capacitor.

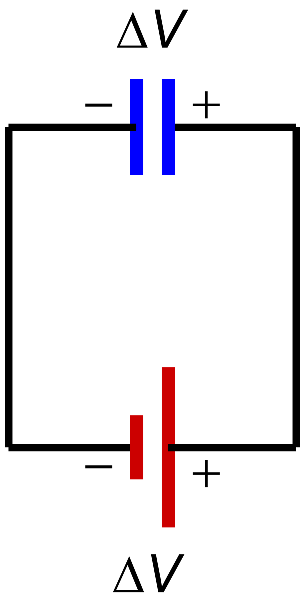
Then the charge of the capacitor is

$$Q = C\Delta V$$

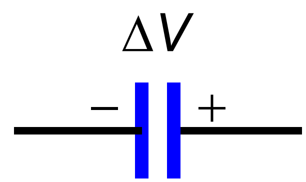


If we disconnect the capacitor from the battery the capacitor will still have the charge ***Q*** and potential difference ΔV

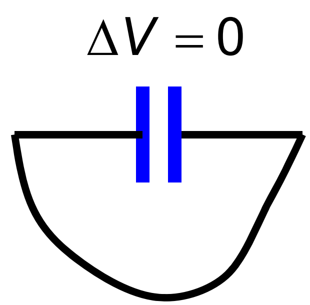
Electrical Circuit



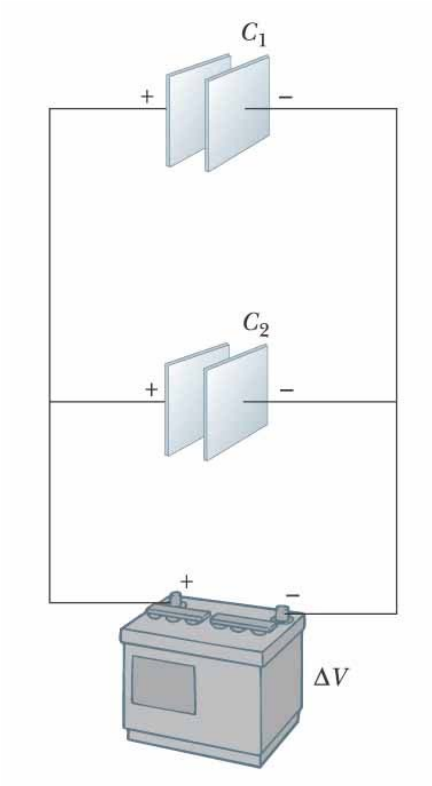
$$Q = C\Delta V$$



If we connect the wires the charge will disappear and there will be no potential difference



Capacitors in Parallel



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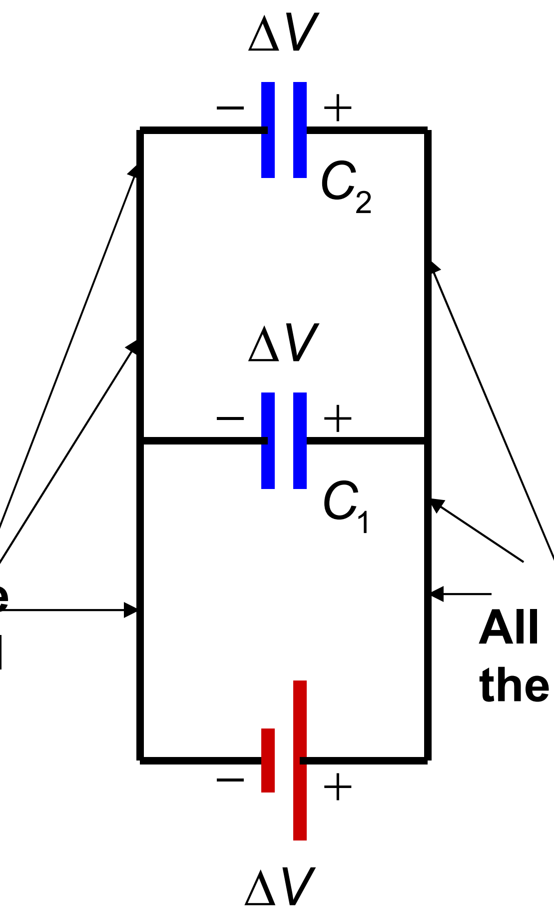


All the points have the same potential

All the points have the same potential

The capacitors 1 and 2 have the same potential difference ΔV

Then the charge of capacitor 1 is $Q_1 = C_1 \Delta V$

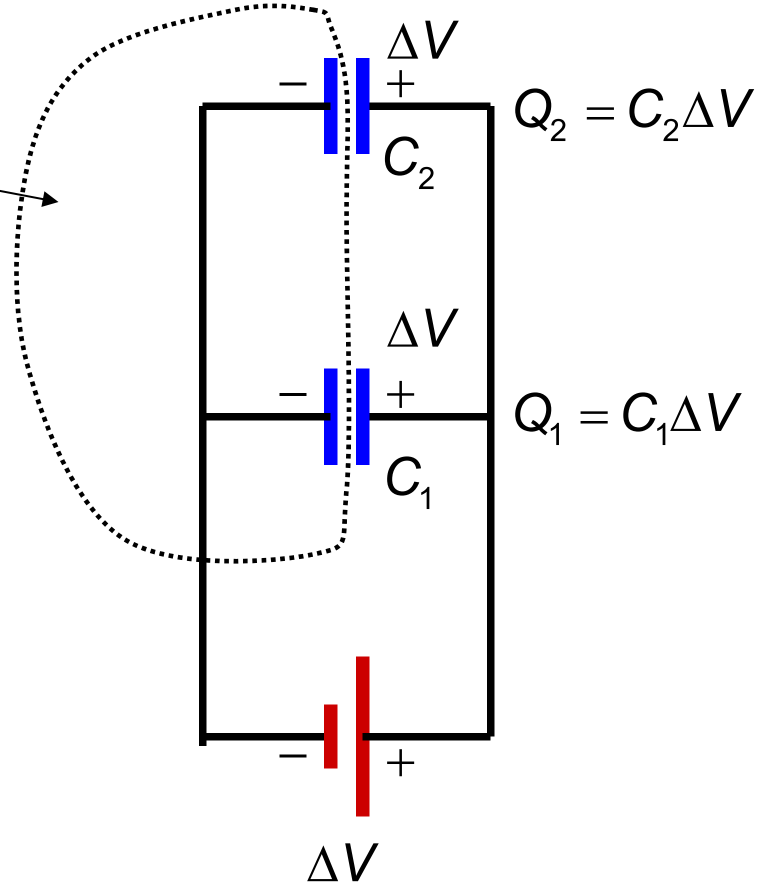
The charge of capacitor 2 is $Q_2 = C_2 \Delta V$

Capacitors in Parallel

The total charge is

$$Q = Q_1 + Q_2$$

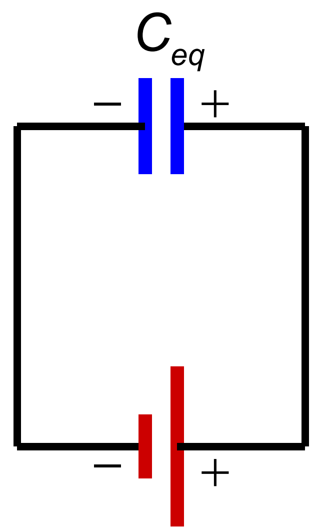
$$Q = C_1\Delta V + C_2\Delta V = (C_1 + C_2)\Delta V$$



This relation is equivalent to the following one

$$Q = C_{eq}\Delta V$$

$$C_{eq} = C_1 + C_2$$

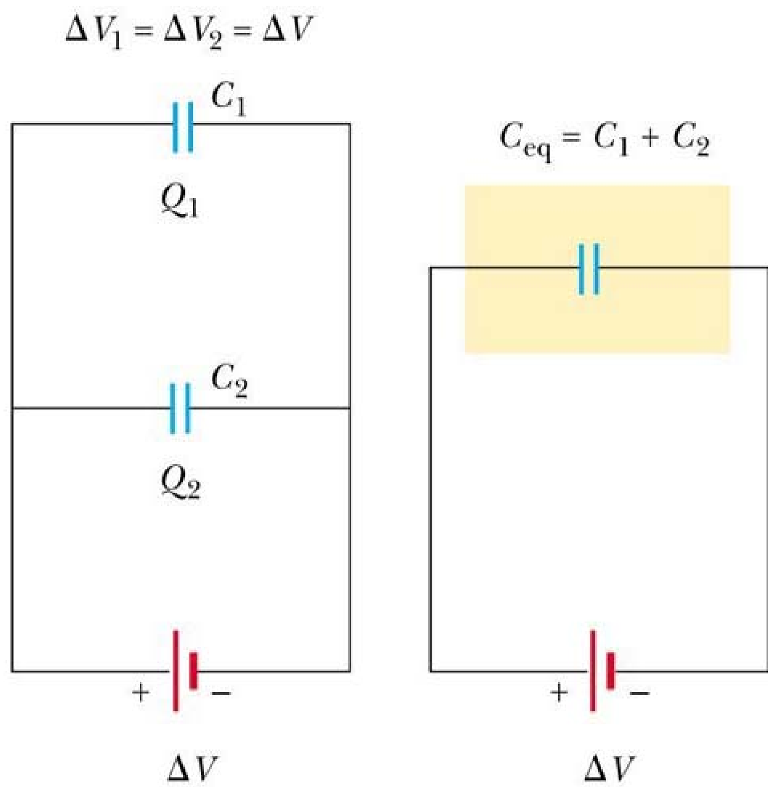


Capacitors in Parallel

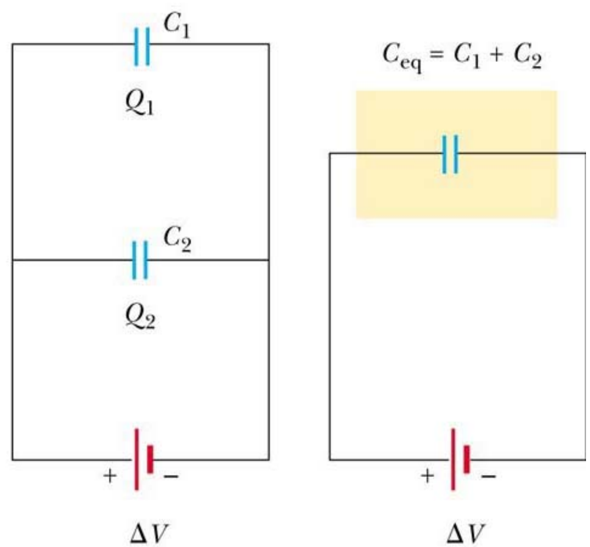
- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}

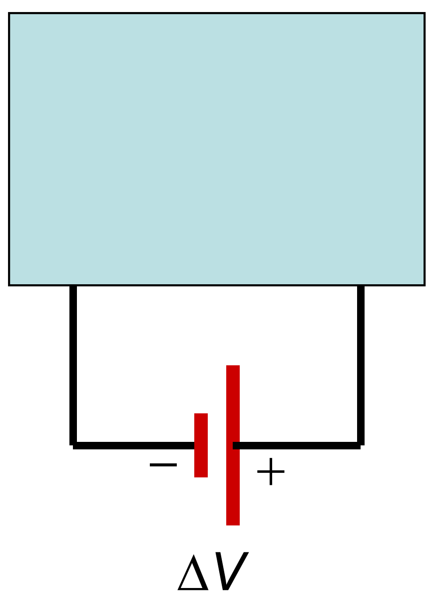
- The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors

$$Q = C_{eq}\Delta V$$



Capacitors

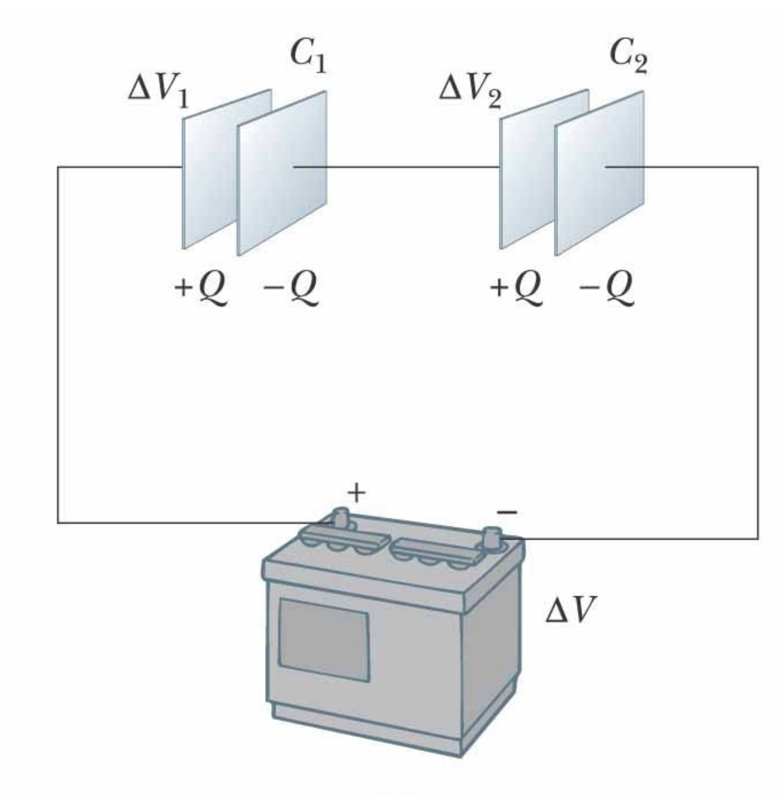




The equivalence means that

$$Q = C_{eq} \Delta V$$

Capacitors in Series



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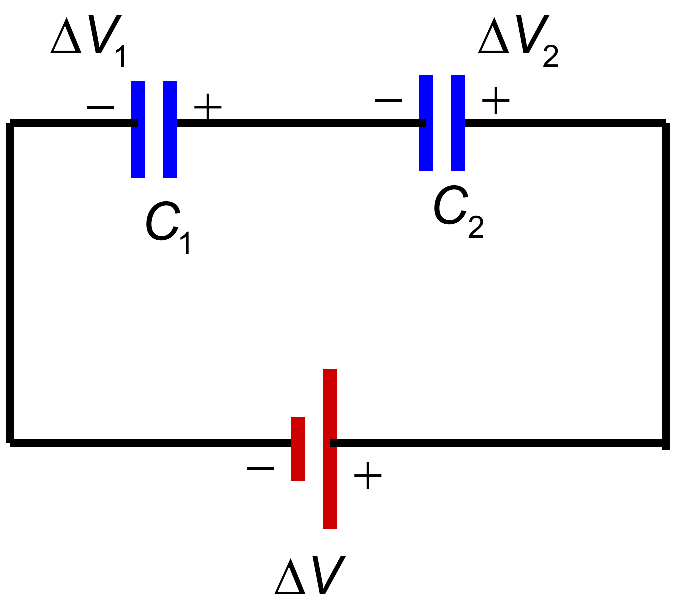


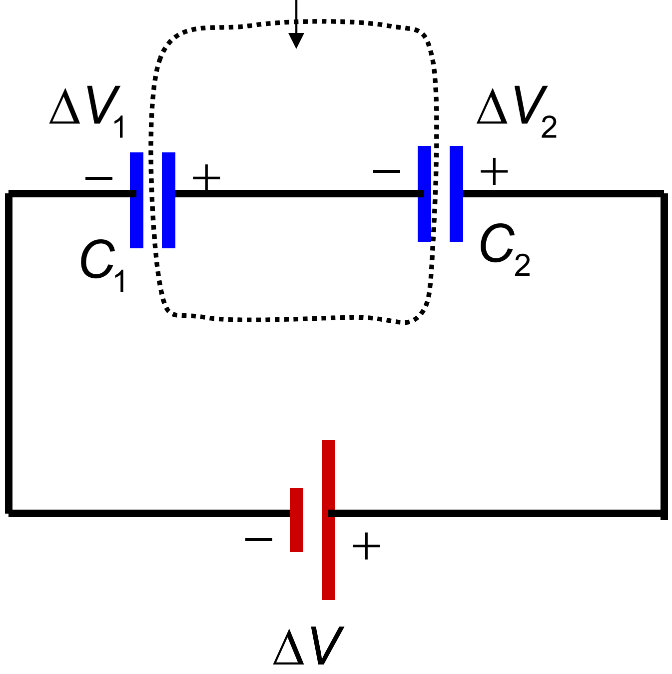
$$\Delta V = \Delta V_1 + \Delta V_2$$

Capacitors in Series

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

The total charge
is equal to 0

$$Q_1 = Q_2 = Q$$



$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

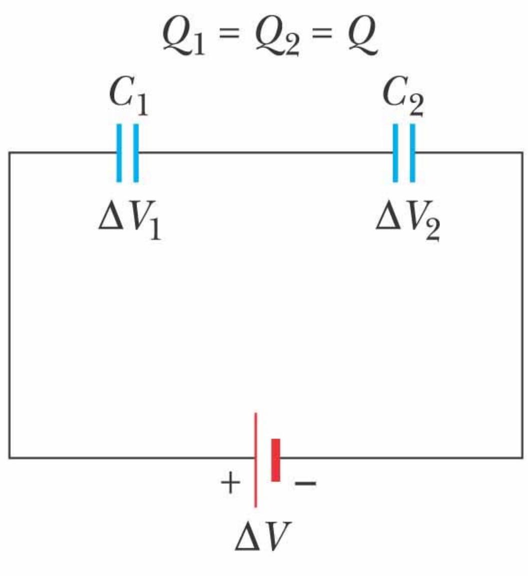
$$\Delta V = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

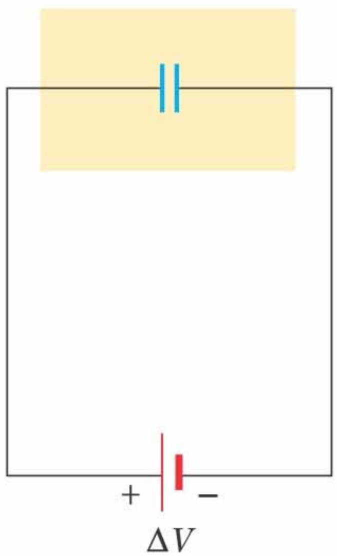
Capacitors in Series

- An equivalent capacitor can be found that performs the same function as the series combination
- The potential differences add up to the battery voltage



(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

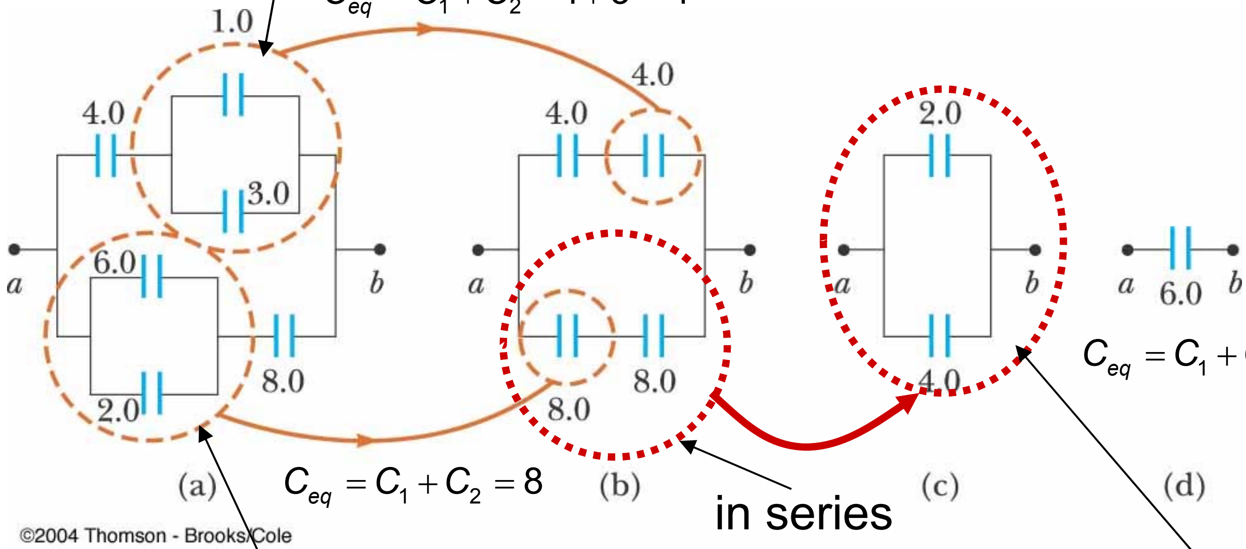


(c)

Example

in parallel

$$C_{eq} = C_1 + C_2 = 1 + 3 = 4$$



$$C_{eq} = C_1 + C_2 = 8$$

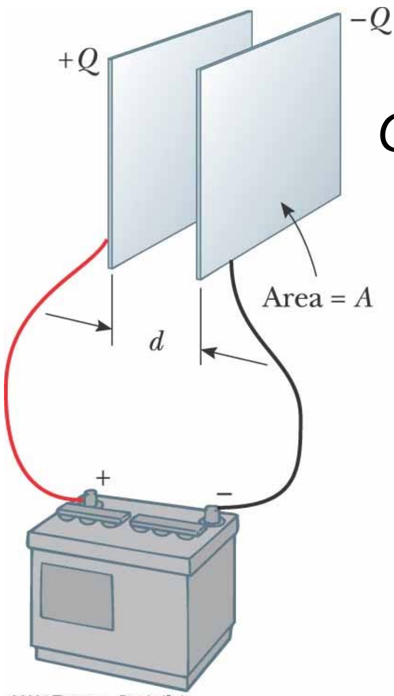
in series

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \cdot 8}{8 + 8} = 4$$

$$C_{eq} = C_1 + C_2 = 6$$

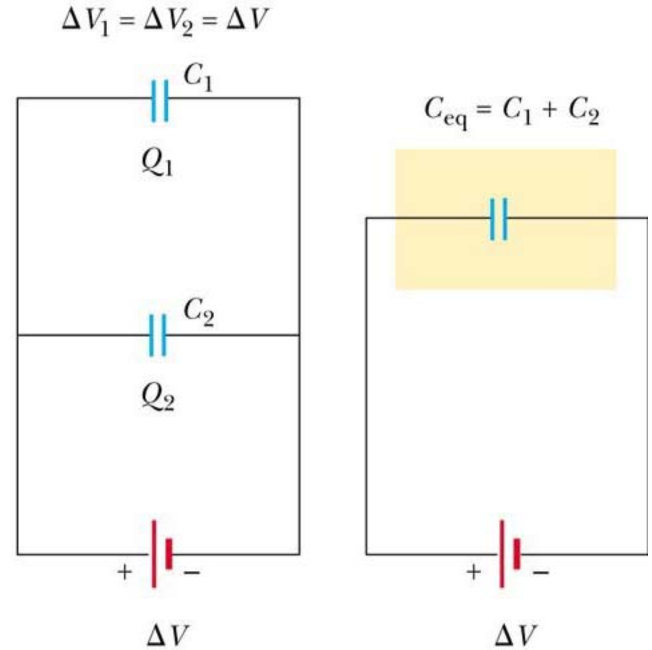
in parallel

in parallel

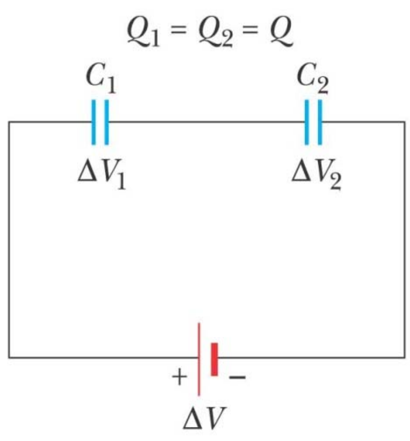


+Q
−Q
Area = A
d
+
−

$$Q = C\Delta V$$

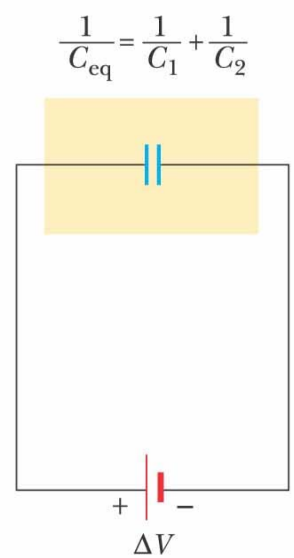


$\Delta V_1 = \Delta V_2 = \Delta V$
C_1
Q_1
C_2
Q_2
+
−
ΔV
$C_{eq} = C_1 + C_2$
+
−
ΔV



$Q_1 = Q_2 = Q$
C_1
C_2
ΔV_1
ΔV_2
+
−
ΔV

(b)

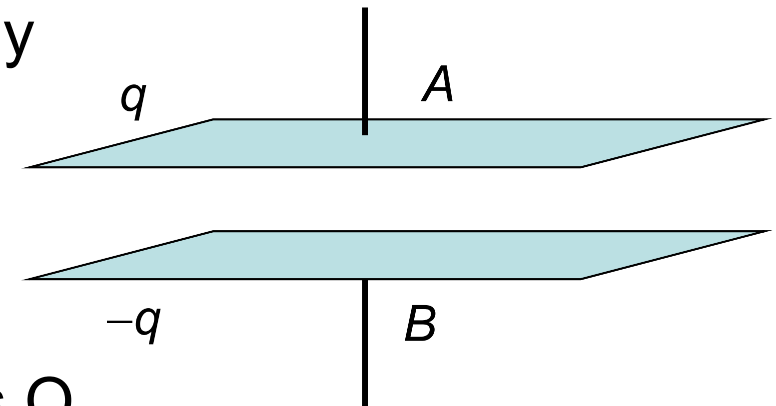


$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
+
−
ΔV

(c)

Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge ***q*** on it
- The work needed to transfer a small charge Δq from one plate to the other is equal to the change of potential energy



$$dW = \Delta V dq = \frac{q}{C} dq$$

- If the final charge of the capacitor is Q, then the total work required is

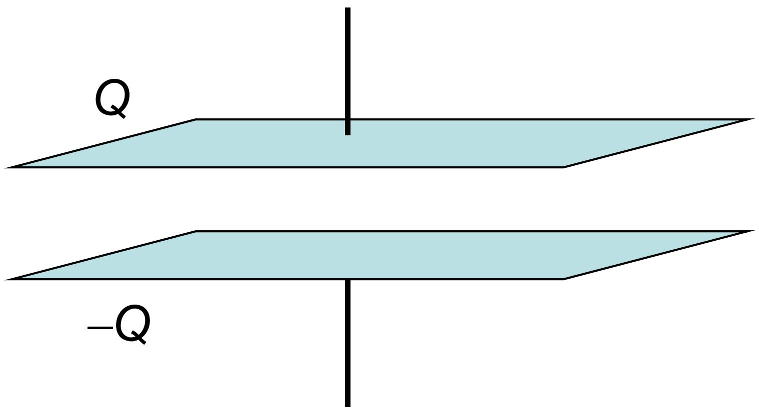
$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy Stored in a Capacitor

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

➢ The work done in charging the capacitor is equal to the electric potential energy *U* of a capacitor

$$Q = C\Delta V$$



$$U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

This applies to a capacitor of **any geometry**

Energy Stored in a Capacitor: Application

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

One of the main application of capacitor:

- capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse

$$Q = C\Delta V$$