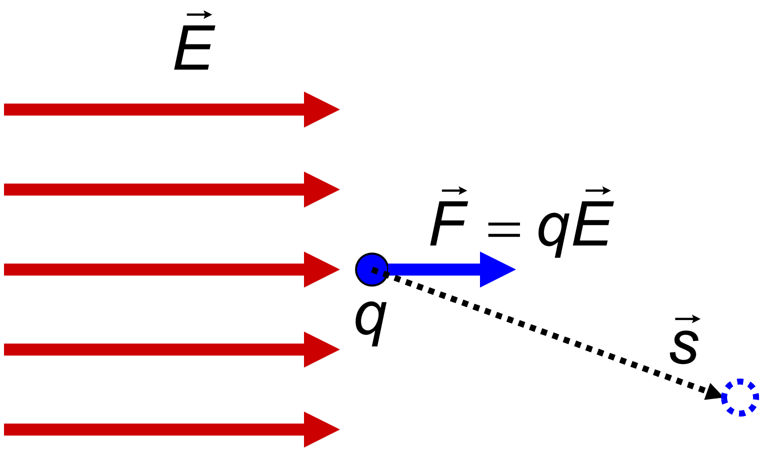
Chapter 29

Electric Potential

Reading: Chapter 29

Electrical Potential Energy

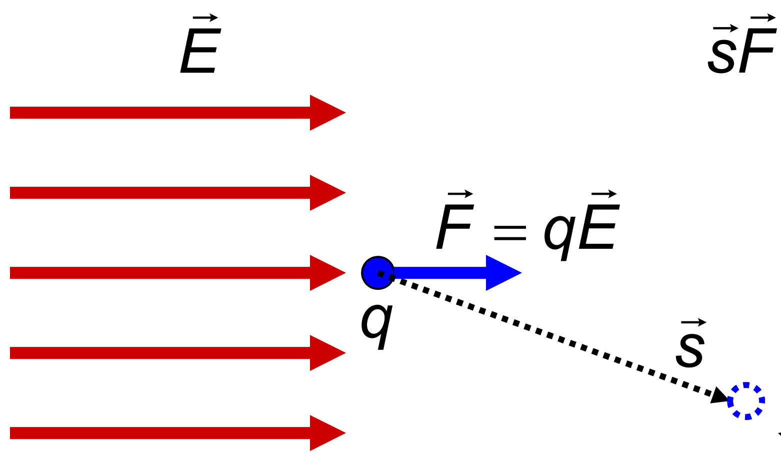
When a test charge is placed in an electric field,
it experiences a force



If $\vec{s}$ is an infinitesimal displacement of test charge, then the work done by electric force during the motion of the charge is given by

$$\vec{s}\vec{F} = q\vec{s}\vec{E}$$

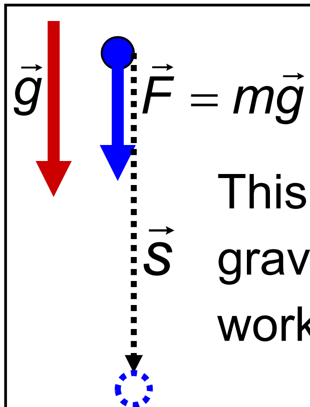
Electrical Potential Energy



$$\vec{s}\vec{F} = q\vec{s}\vec{E}$$

This is the work **done by electric field.**

In this case work is positive.



This is very similar to gravitational force: the work done by force is

$$\vec{s}\vec{F} = m\vec{g}\vec{s}$$

The change of potential energy is

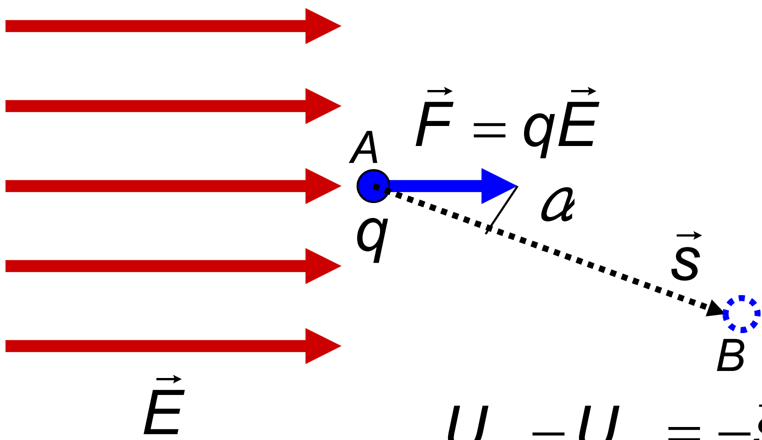
$$\Delta U = -\vec{s}\vec{F} = -m\vec{g}\vec{s}$$

Because the positive work is done, the potential energy of charge-field system should decrease. So the change of potential energy is

$$\Delta U = -\vec{s}\vec{F} = -q\vec{s}\vec{E}$$

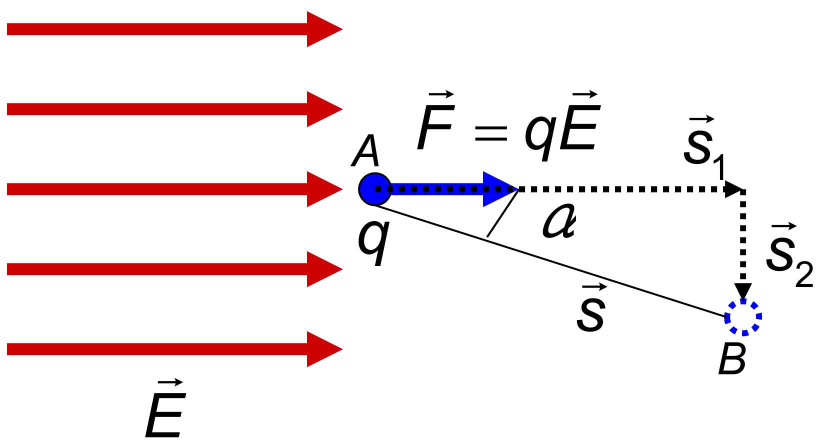
sign minus

Electrical Potential Energy: Example



$$U_B - U_A = -\vec{s}\vec{F} = -q\vec{s}\vec{E} = -qsE\cos\alpha$$

$$U_B = U_A - qsE\cos\alpha$$



$$U_B - U_A = -\vec{s}_1\vec{F} - \vec{s}_2\vec{F} = -qs_1E$$

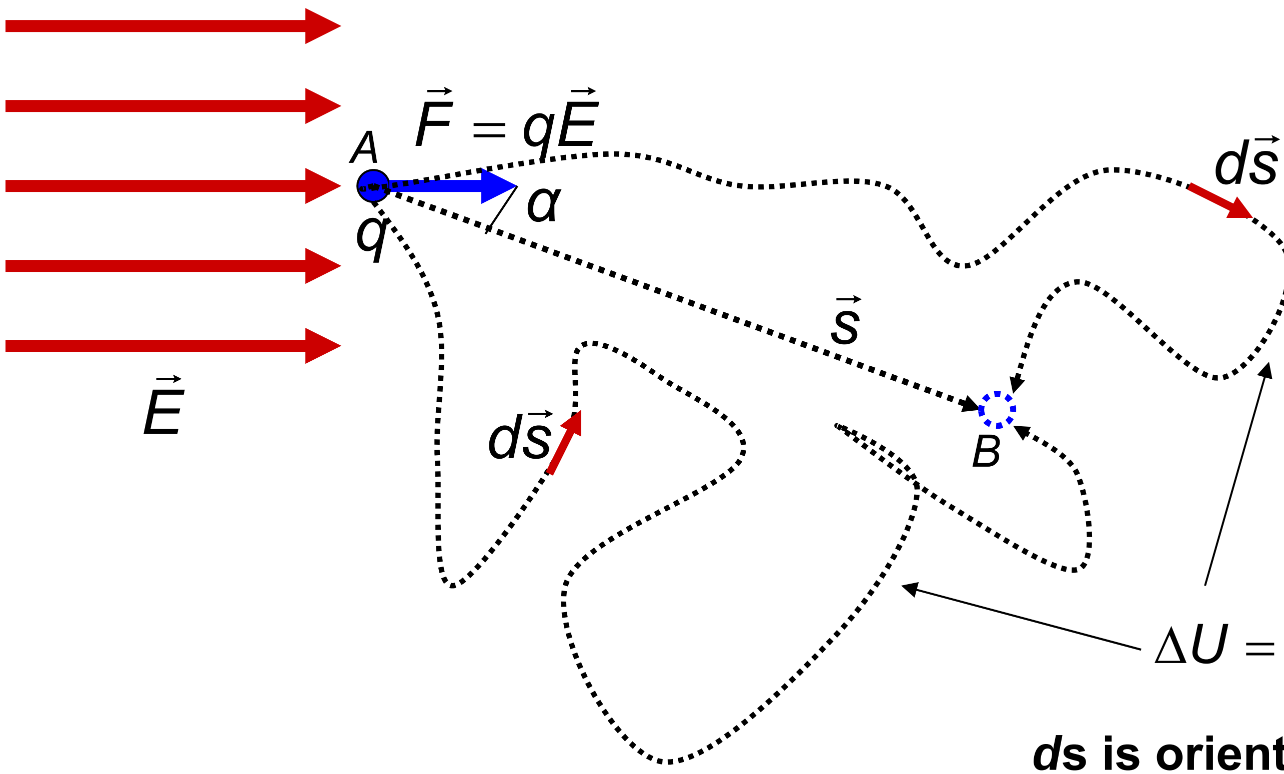
$$\vec{s}_2\vec{E} = 0 \qquad s_1 = s\cos\alpha$$

$$U_B = U_A - qsE\cos\alpha$$

The change of potential energy does not depend on the path:

The electric force is conservative

Electrical Potential Energy



$$\Delta U = U_B - U_A = -q\int_A^B \vec{E} \cdot d\vec{s}$$

ds **is oriented tangent to a path through space**

For all paths: $U_B = U_A - qsE\cos\alpha$

The electric force is conservative

Electric Potential

Electric potential is the potential energy per unit charge,

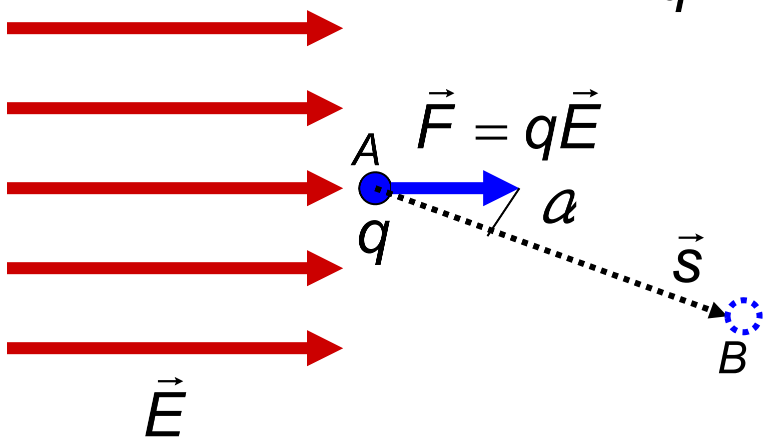
$$V = \frac{U}{q}$$

The potential is independent of the value of ***q***.

The potential has a value at every point in an electric field

Only the *difference* in potential is the meaningful quantity.

$$V_B - V_A = \frac{U_B}{q} - \frac{U_A}{q} = -\frac{\vec{s}\vec{F}}{q} = -\vec{s}\vec{E} = -sE\cos\alpha$$

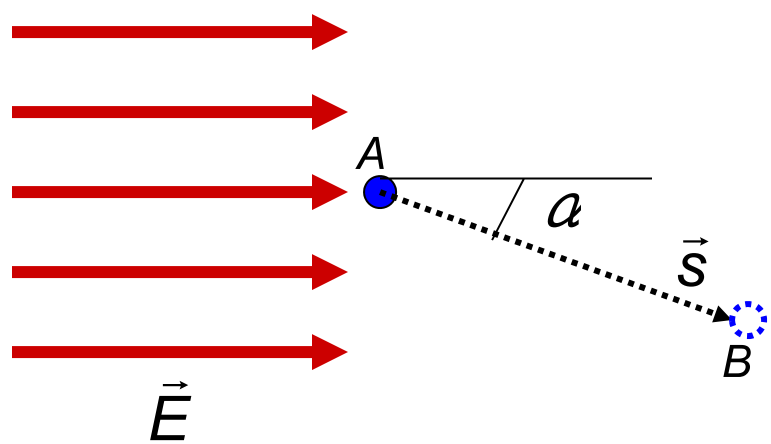


Electric Potential

To find the potential at every point

- 1. we assume that the potential is equal to **0** at some point, for example at point **A**, $V_A = 0$
- 2. we find the potential at any point B from the expression

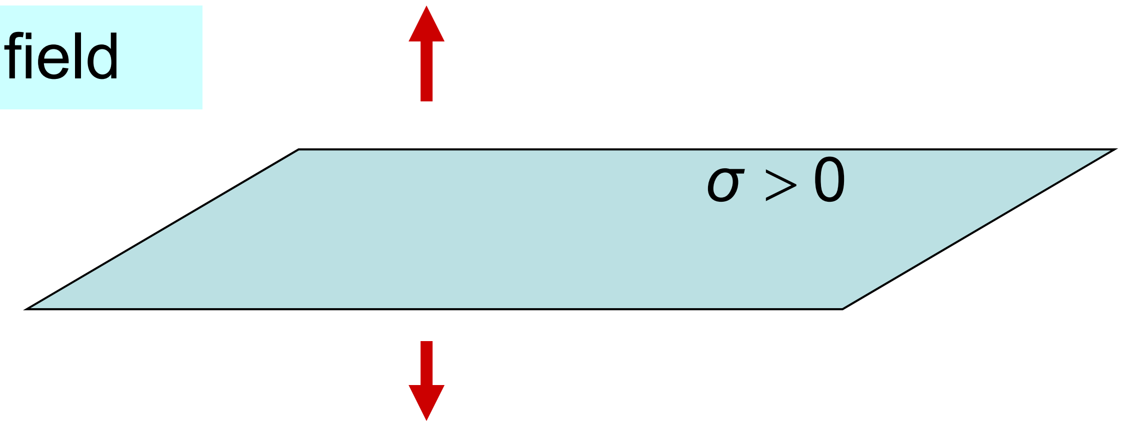
$$V_B = V_A - \int_A^B \vec{E} d\vec{s} = -\int_A^B \vec{E} d\vec{s}$$



$$V_B = -sE \cos a$$

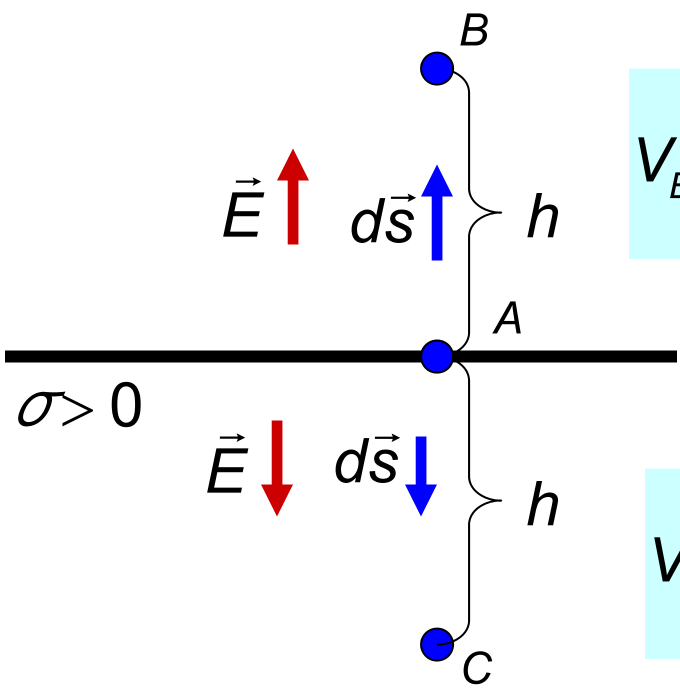
Electric Potential: Example

Plane: Uniform electric field



$$V_A = 0$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

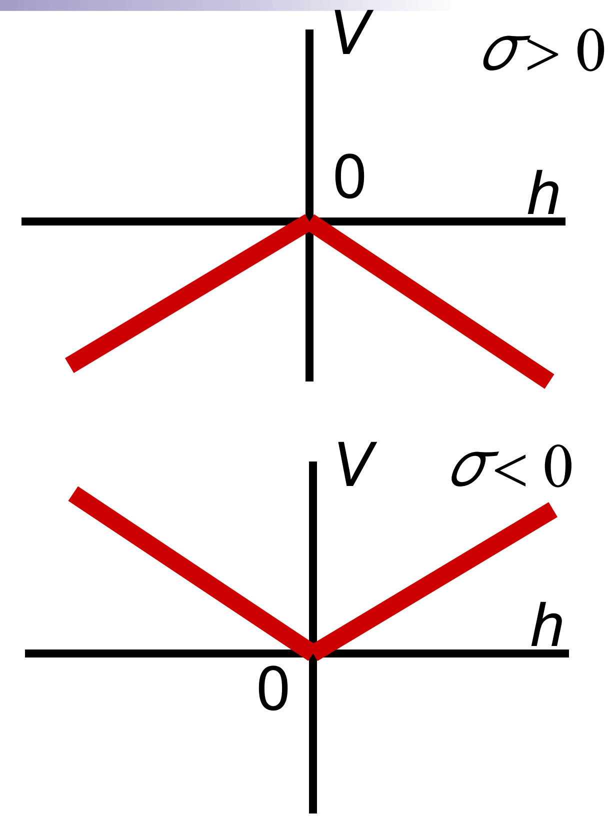


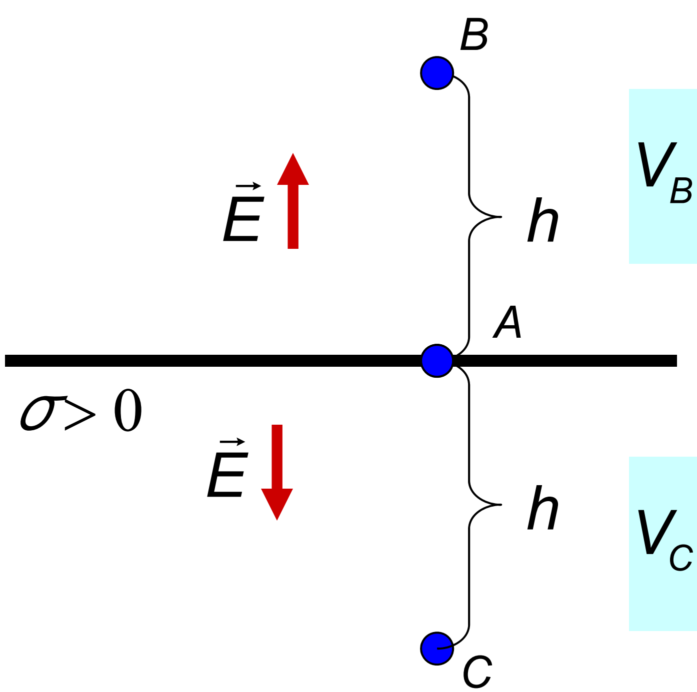
$$V_B = V_A - \int_A^B \vec{E}d\vec{s} = -E\int_A^B ds = -hE = -\frac{\sigma h}{2\varepsilon_0}$$

$$V_C = V_A - \int_A^C \vec{E}d\vec{s} = -E\int_A^C ds = -hE = -\frac{\sigma h}{2\varepsilon_0}$$

Electric Potential: Example

Plane: Uniform electric field





$$V_B = -\frac{\sigma h}{2\varepsilon_0}$$

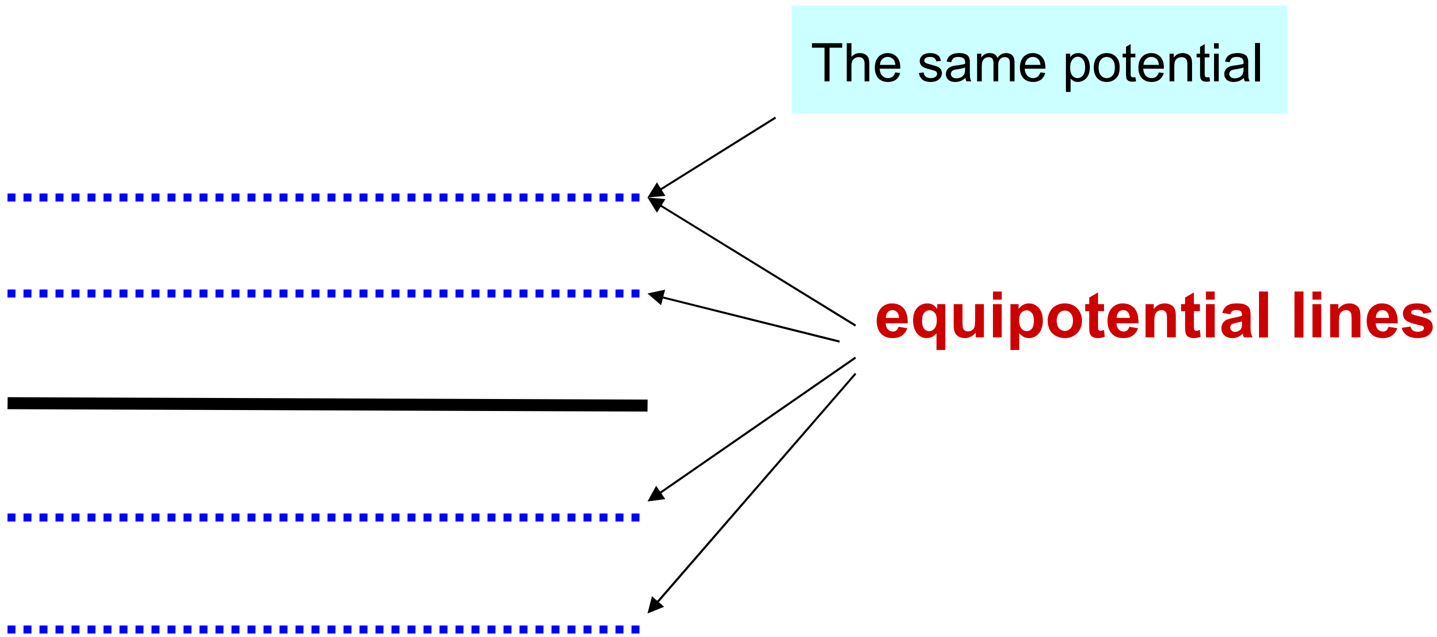
$$V_C = -\frac{\sigma h}{2\varepsilon_0}$$

All points with the same *h* have the same potential

Electric Potential: Example

Plane: Uniform electric field

$$V = -\frac{\sigma h}{2\varepsilon_0}$$

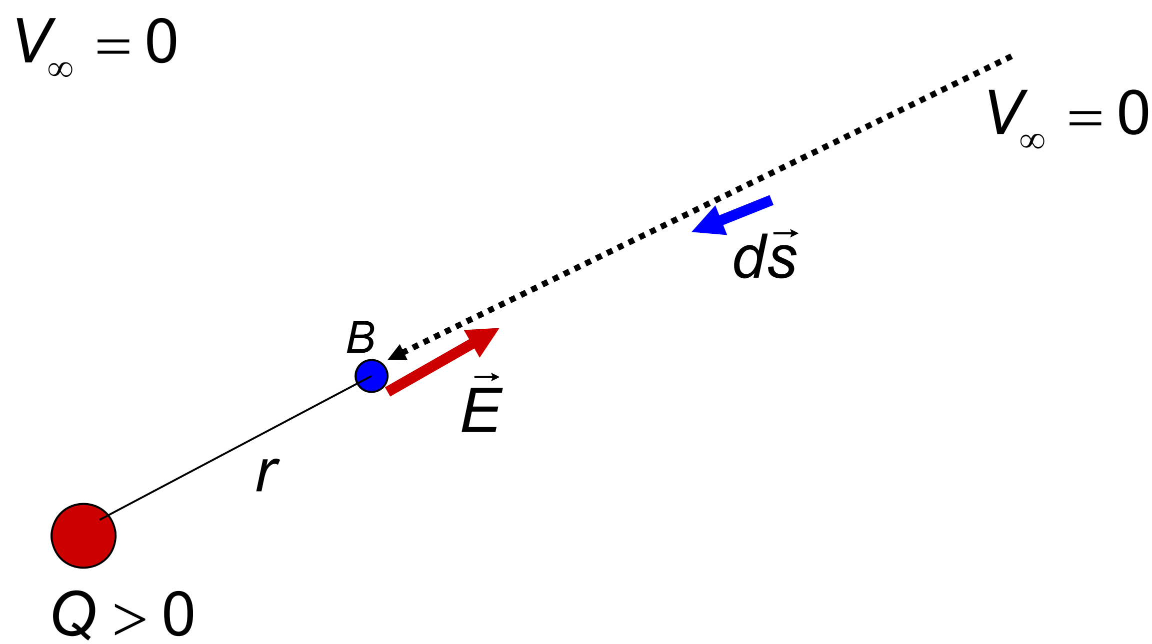


$$V = -\frac{\sigma h}{2\varepsilon_0}$$

Electric Potential: Example

Point Charge

$$E_r = k_e \frac{Q}{r^2}$$



$$V_B = V_A - \int_\infty^B \vec{E} d\vec{s} = -\int_\infty^r E_r dr = -k_e Q \int_\infty^r \frac{dr}{r^2} = k_e Q \frac{1}{r}\Bigg|_\infty^r = k_e \frac{Q}{r}$$

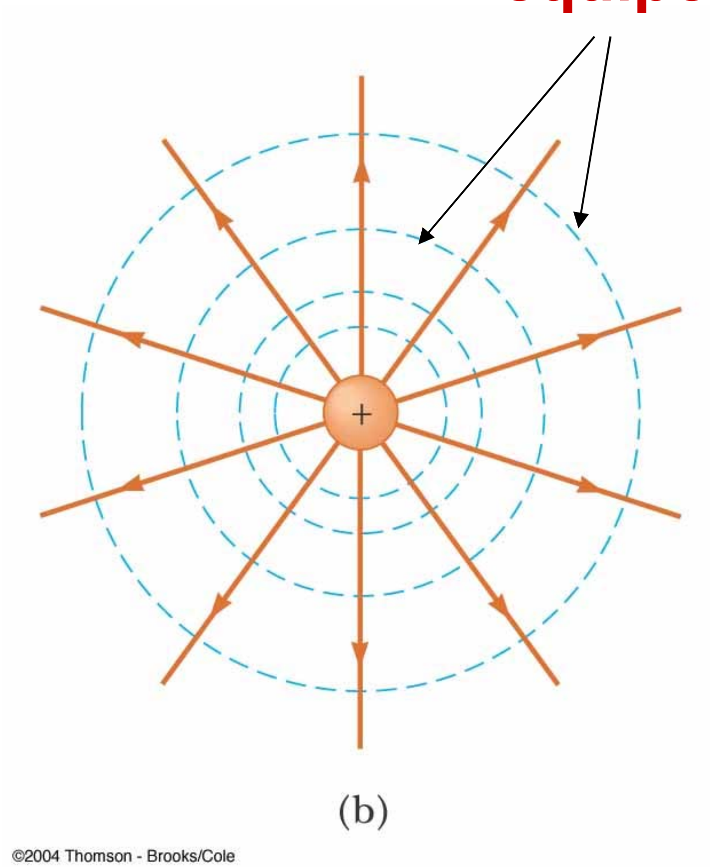
Electric Potential: Example

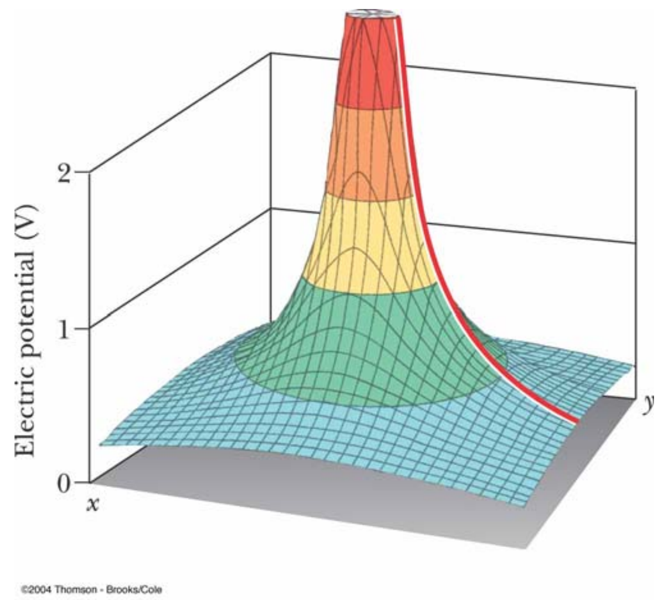
Point Charge

$$V_\infty = 0$$

$$V_r = k_e \frac{Q}{r}$$

equipotential lines





Electric Potential: Example

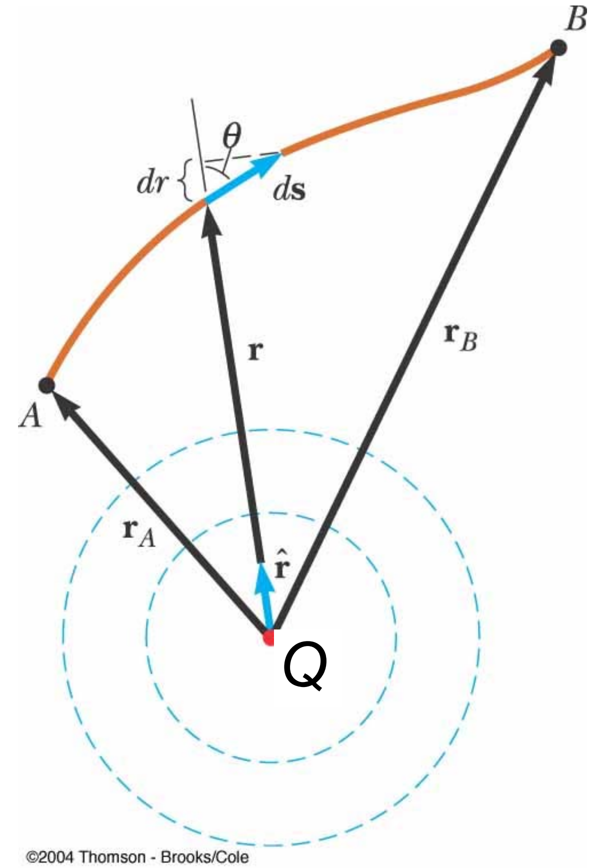
Point Charge

$$V_r = k_e \frac{Q}{r}$$

$$V_\infty = 0$$

- The potential difference between points *A* and *B* will be

$$V_B - V_A = k_e Q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = -\int_A^B \vec{E} d\vec{s}$$



Units

- Units of potential: ***1 V = 1 J/C***
  - ***V*** is a volt
  - It takes one joule (***J***) of work to move a 1-coulomb (***C***) charge through a potential difference of 1 volt (***V***)

❑ Another unit of energy that is commonly used in atomic and nuclear physics is the ***electron-volt***

❑ One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of ***1 volt***

$$1\ eV = 1.60 \times 10^{-19}\ J$$

Potential and Potential Energy

- If we know the electric potential then the potential energy of a point charge ***q*** is

$$U = qV$$

(this is similar to the relation between an electric force and an electric field)

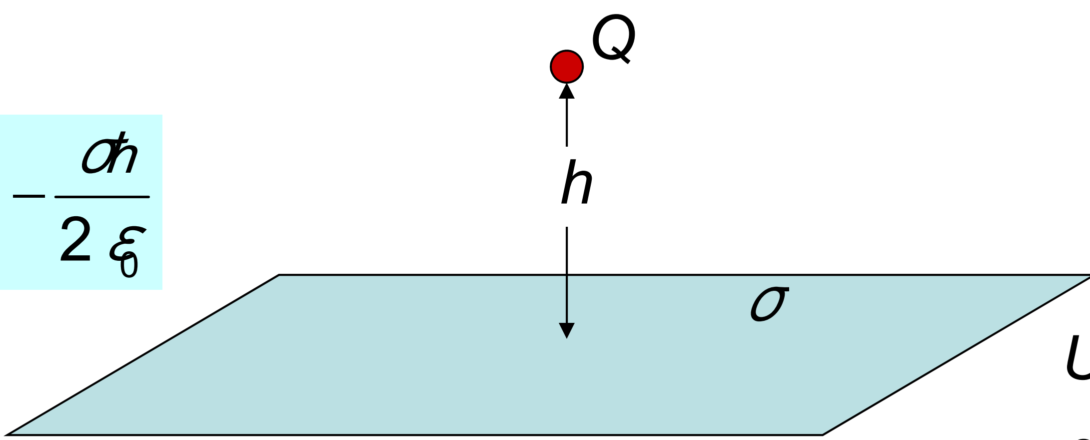
$$\vec{F} = q\vec{E}$$

Potential Energy: Example

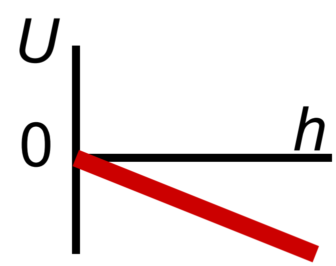
What is the potential energy of a point charge ***q*** in the field of uniformly charged plane?

$$U = QV$$

$$V = -\frac{\sigma h}{2\varepsilon_0}$$

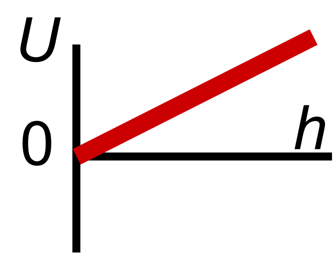


$$U = qV = -\frac{\sigma Q}{2\varepsilon_0}h$$



$\sigma Q > 0$

repulsion



$\sigma Q > 0$

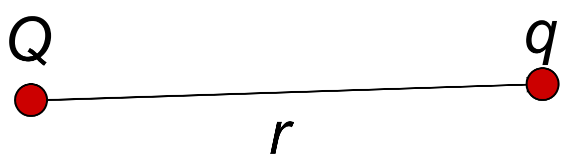
attraction

Potential Energy: Example

What is the potential energy of two point charges ***q*** and ***Q***?

This can be calculated by two methods:

A



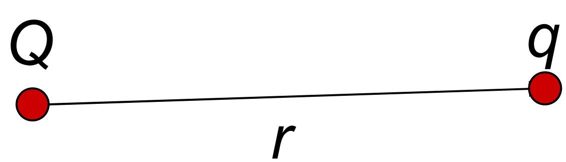
$$U_q = qV_Q$$

$$V_Q = k_e \frac{Q}{r}$$

The potential energy of point charge *q* in the field of point charge *Q*

$$U_q = qV_Q = k_e \frac{qQ}{r}$$

B



$$U_Q = QV_q$$

$$V_q = k_e \frac{q}{r}$$

The potential energy of point charge *Q* in the field of point charge *q*

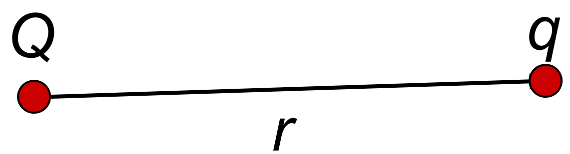
$$U_Q = QV_q = k_e \frac{qQ}{r}$$

In both cases we have the same expression for the energy. This expression gives us the energy of two point charges.

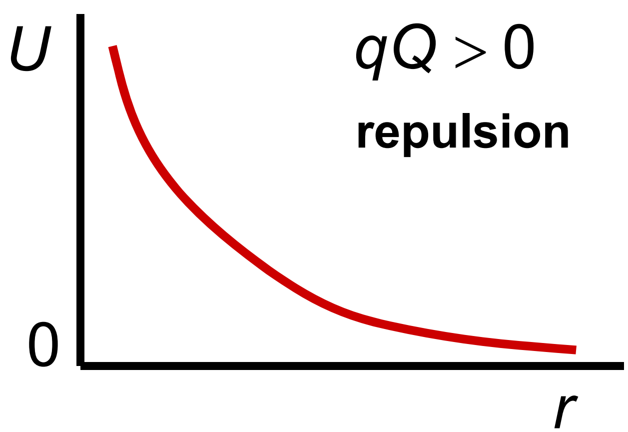
$$U = k_e \frac{qQ}{r}$$

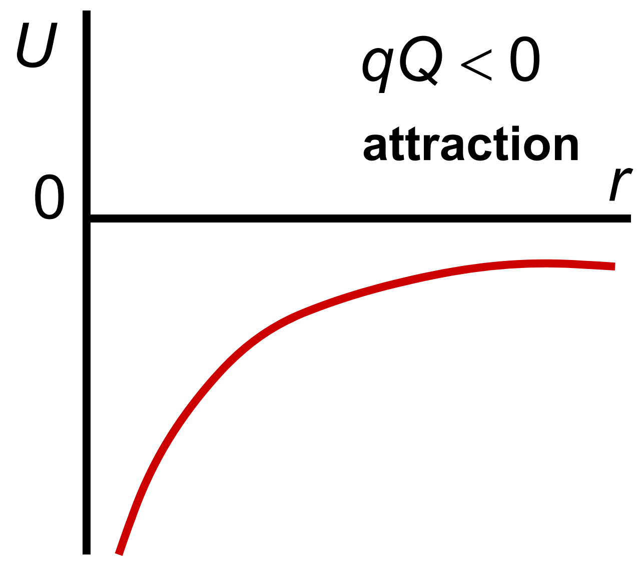
Potential Energy: Example

Potential energy of two point charges:



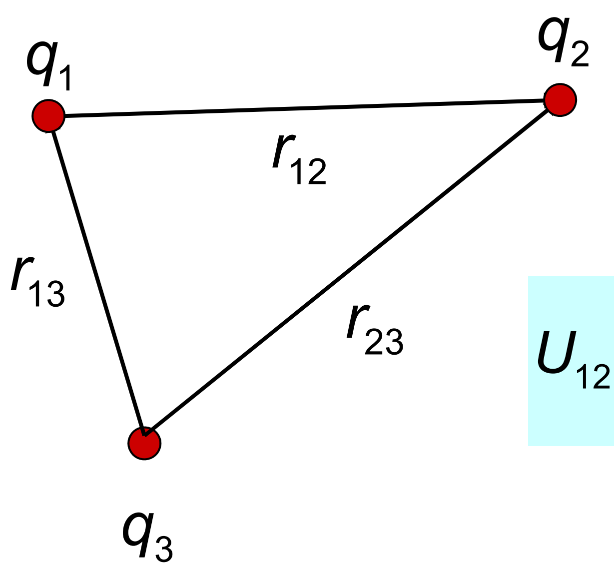
$$U = k_e \frac{qQ}{r}$$





Potential Energy: Example

Find potential energy of three point charges:



$$U = U_{12} + U_{13} + U_{23}$$

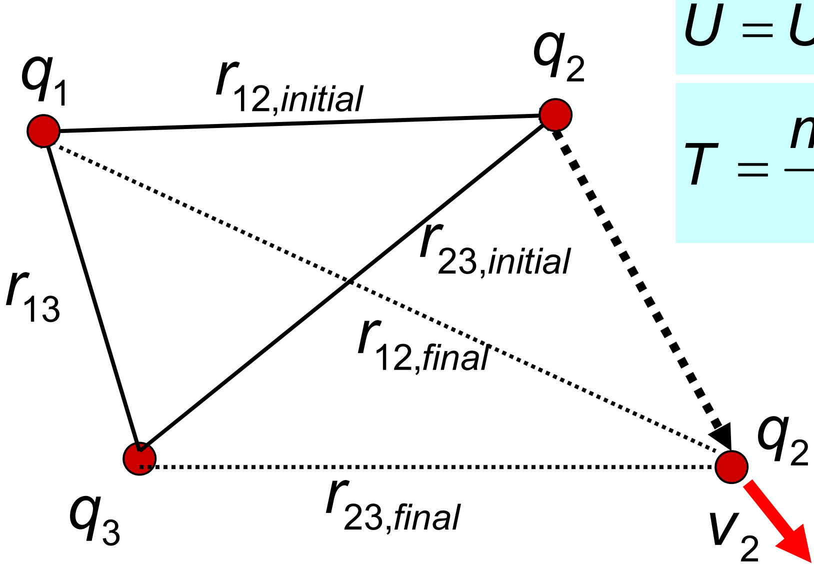
$$U_{12} = k_e \frac{q_1 q_2}{r_{12}} \qquad U_{13} = k_e \frac{q_1 q_3}{r_{13}} \qquad U_{23} = k_e \frac{q_2 q_3}{r_{23}}$$

$$U = U_{12} + U_{13} + U_{23} = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}}$$

Potential Energy: Applications: Energy Conservation

For a closed system: Energy Conservation:

The sum of potential energy and kinetic energy is constant



$$U = U_{12} + U_{13} + U_{23}$$ ***- Potential energy***

$$T = \frac{mv^2}{2}$$ ***- Kinetic energy***

Example: Particle 2 is released from the rest. Find the speed of the particle when it will reach point P.

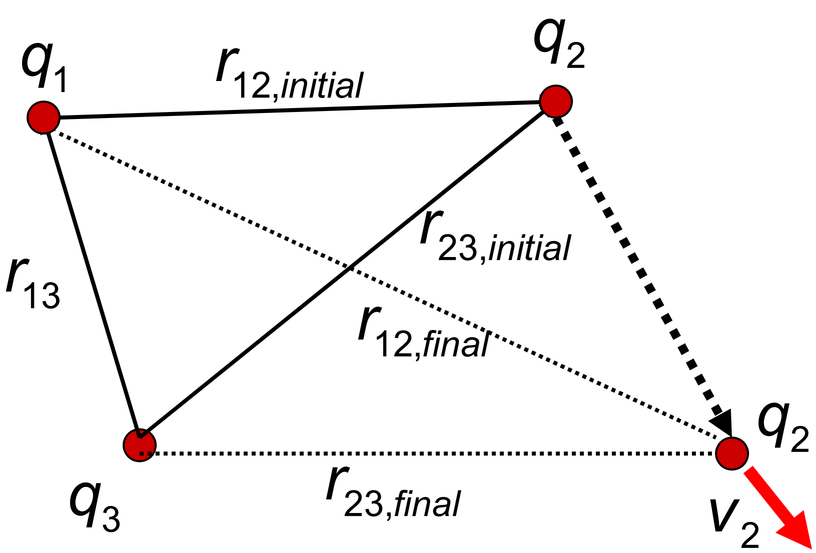
Initial Energy is the sum of kinetic energy and potential energy (velocity is zero – kinetic energy is zero)

$$E_{initial} = T + U = T + U_{12} + U_{13} + U_{23} = k_e \frac{q_1 q_2}{r_{12,initial}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,initial}}$$

Potential Energy: Applications: Energy Conservation

For a closed system: Energy Conservation:

The sum of potential energy and kinetic energy is constant



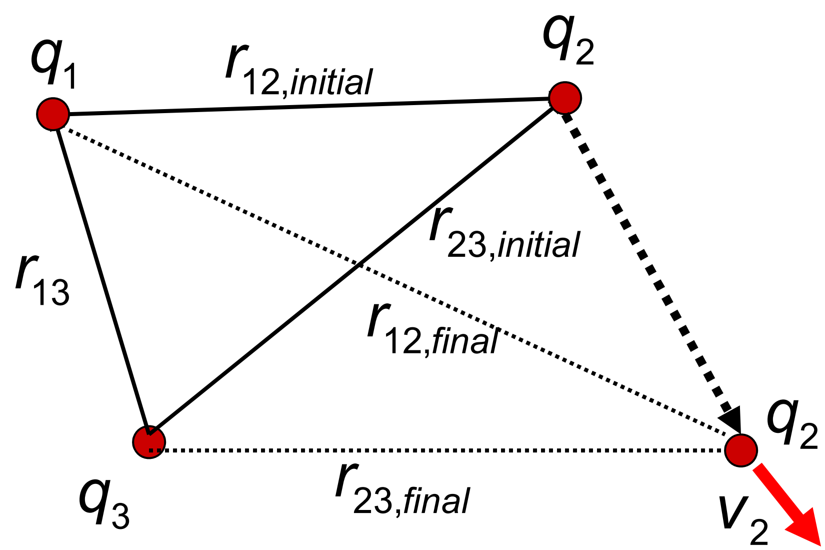
Final Energy is the sum of kinetic energy and potential energy (velocity of particle 2 is nonzero – kinetic energy)

$$E_{final} = T + U = T + U_{12} + U_{13} + U_{23} = \frac{m_2 v_2^2}{2} + k_e \frac{q_1 q_2}{r_{12,final}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,final}}$$

Potential Energy: Applications: Energy Conservation

For a closed system: Energy Conservation:

The sum of potential energy and kinetic energy is constant



Final Energy = Initial Energy

$$k_e \frac{q_1 q_2}{r_{12,initial}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,initial}} = \frac{m_2 v_2^2}{2} + k_e \frac{q_1 q_2}{r_{12,final}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,final}}$$

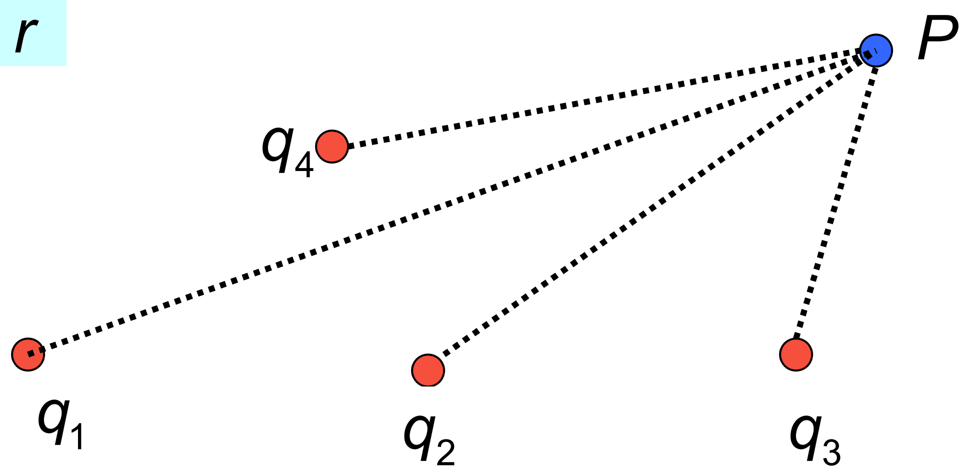
$$\frac{m_2 v_2^2}{2} = k_e \frac{q_1 q_2}{r_{12,initial}} - k_e \frac{q_1 q_2}{r_{12,final}} + k_e \frac{q_2 q_3}{r_{23,initial}} - k_e \frac{q_2 q_3}{r_{23,final}}$$

$$v_2 = \sqrt{\frac{2}{m_2}\left[k_e q_1 q_2 \left(\frac{1}{r_{12,initial}} - \frac{1}{r_{12,final}}\right) + k_e q_2 q_3 \left(\frac{1}{r_{23,initial}} - \frac{1}{r_{23,final}}\right)\right]}$$

Electric Potential: Continuous Charge Distribution

Electric Potential of Multiple Point Charge

$$V_r = k_e \frac{Q}{r}$$



$$V_r = V_1 + V_2 + V_3 + V_4 = k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2} + k_e \frac{q_3}{r_3} + k_e \frac{q_4}{r_4}$$

The potential is a **scalar sum.**

The electric field is a **vector sum.**

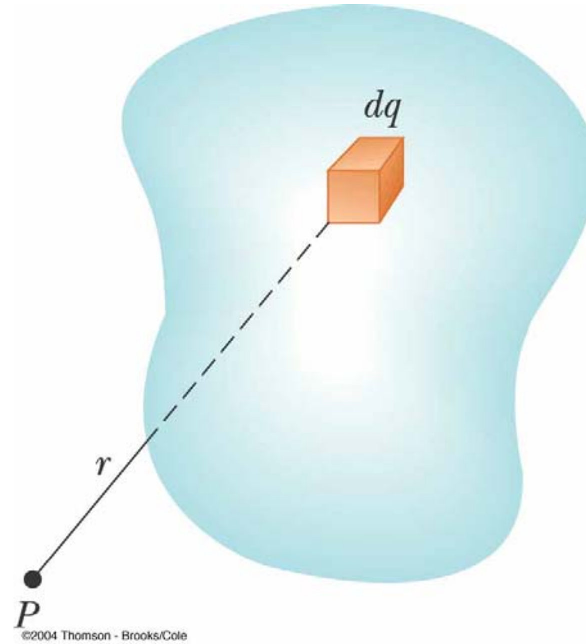
Electric Potential of Continuous Charge Distribution

- Consider a small charge element *dq*
  - Treat it as a point charge
- The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$

- To find the total potential, you need to integrate to include the contributions from all the elements

$$V = k_e \int \frac{dq}{r}$$



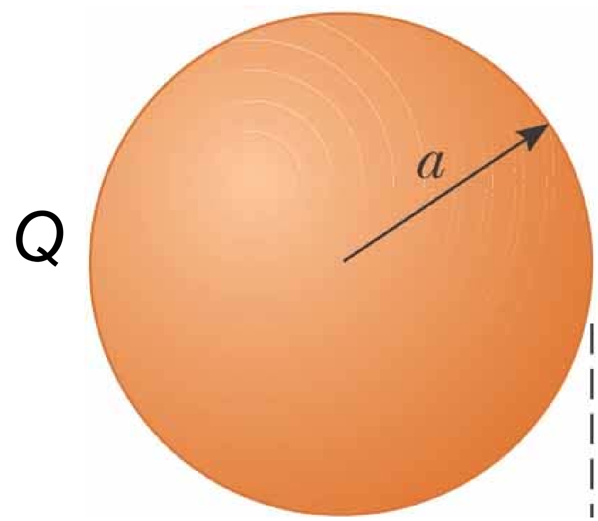
$$V_\infty = 0$$

The potential is a **scalar sum.**

The electric field is a **vector sum.**

Spherically Symmetric Charge Distribution

Uniformly distributed charge *Q*



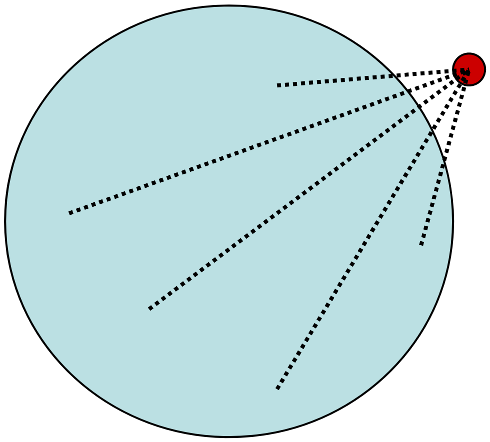
$$V = ?$$

Spherically Symmetric Charge Distribution

Two approaches:

“Complicated” Approach A:

$$V = k_e \int \frac{dq}{r}$$

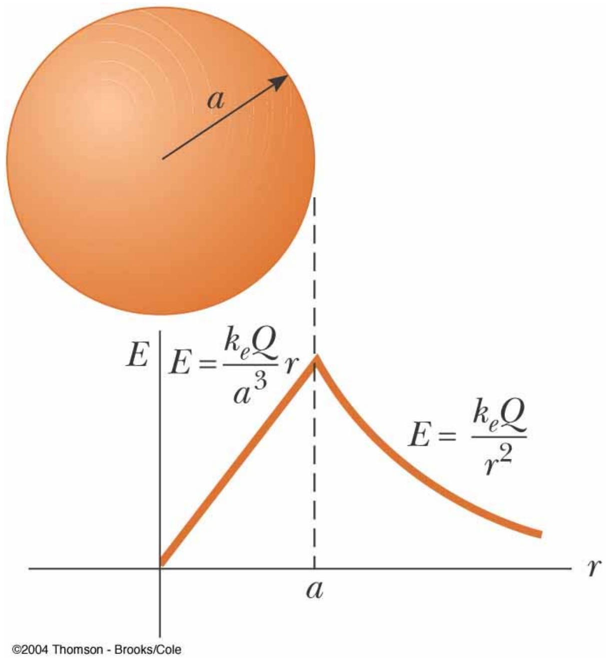


“Simple” Approach B:

$$V_B = V_A - \int_{\infty}^{B} \vec{E} d\vec{s}$$

(simple - only because we know ***E(r)***)

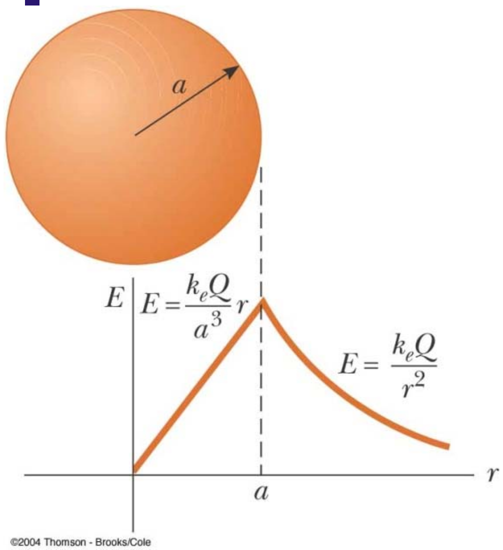
Spherically Symmetric Charge Distribution



$$E_r = k_e \frac{Q}{a^3} r \qquad r < a$$

$$E_r = k_e \frac{Q}{r^2} \qquad r > a$$

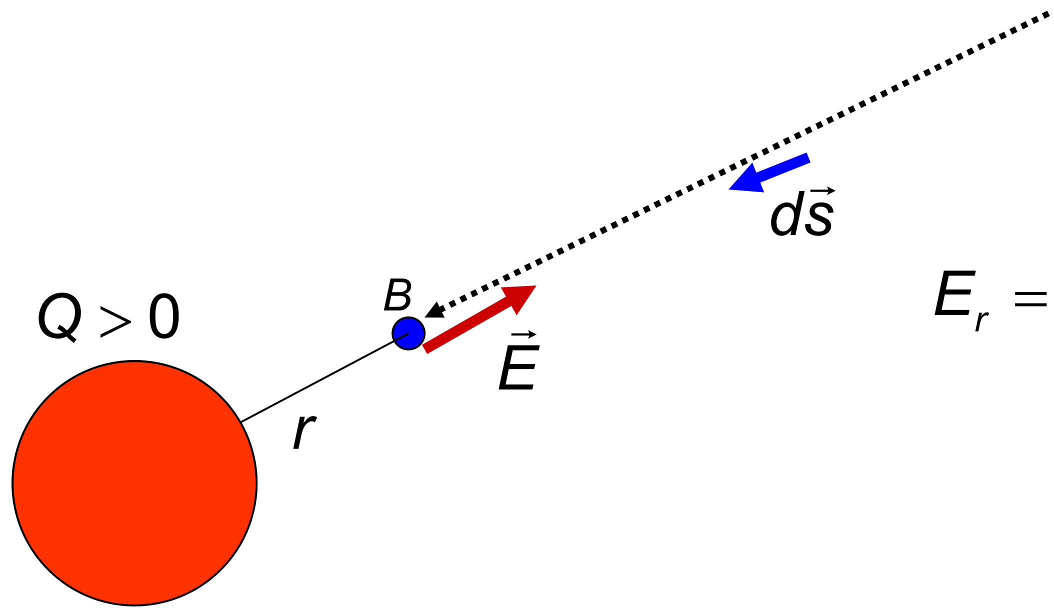
Spherically Symmetric Charge Distribution



$$V_\infty = 0$$

$$r > a$$

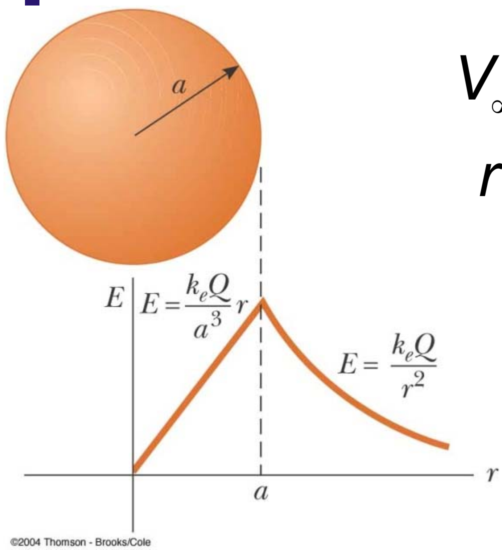
$$V_\infty = 0$$



$$E_r = k_e \frac{Q}{r^2}$$

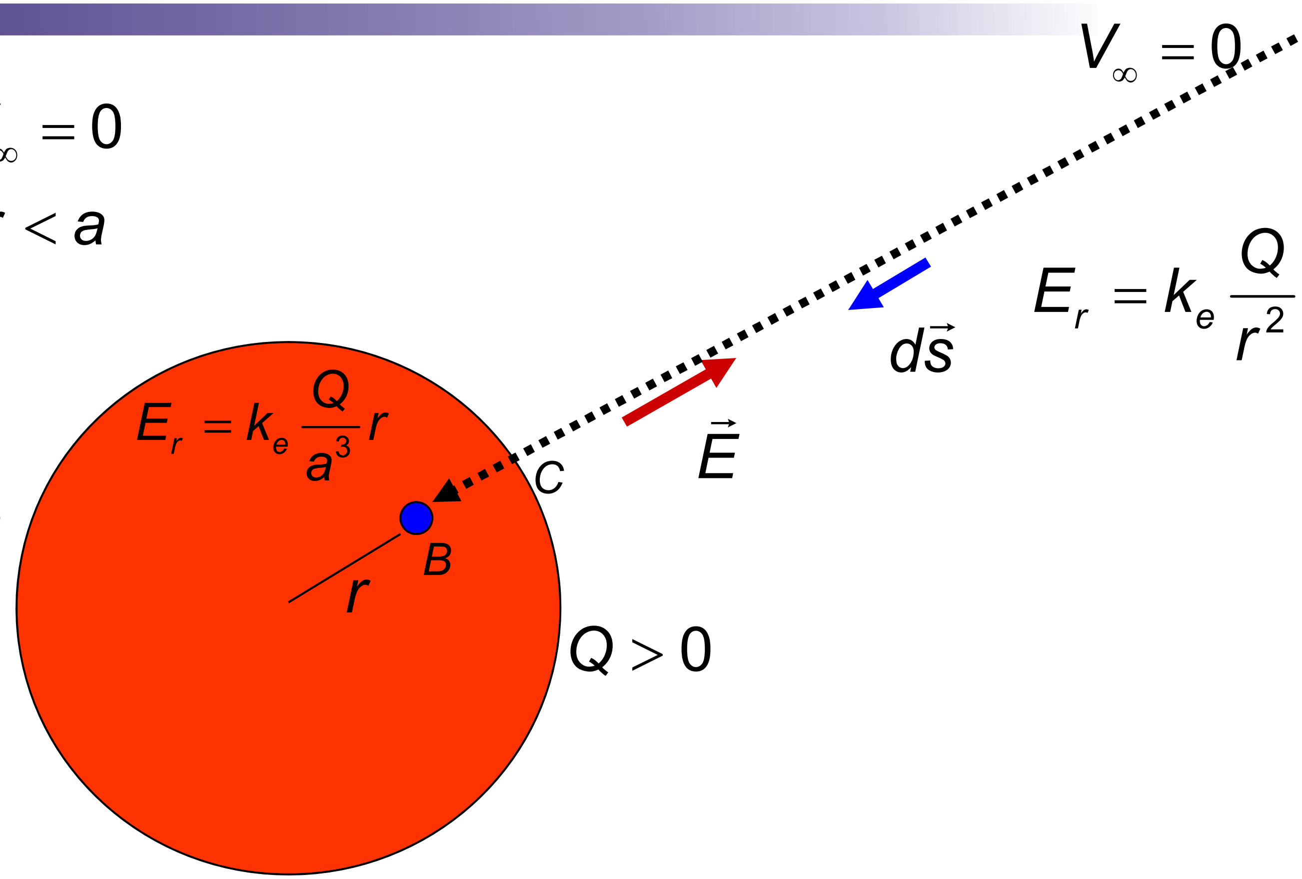
$$V_B = V_A - \int_\infty^B \vec{E} d\vec{s} = -\int_\infty^r E_r dr = -k_e Q \int_\infty^r \frac{dr}{r^2} = k_e Q \frac{1}{r}\Big|_\infty^r = k_e \frac{Q}{r}$$

Spherically Symmetric Charge Distribution



$V_\infty = 0$

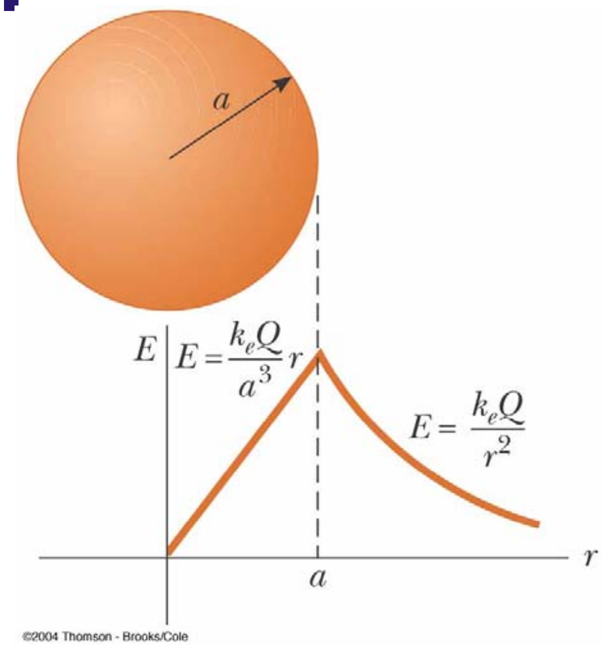
$r < a$

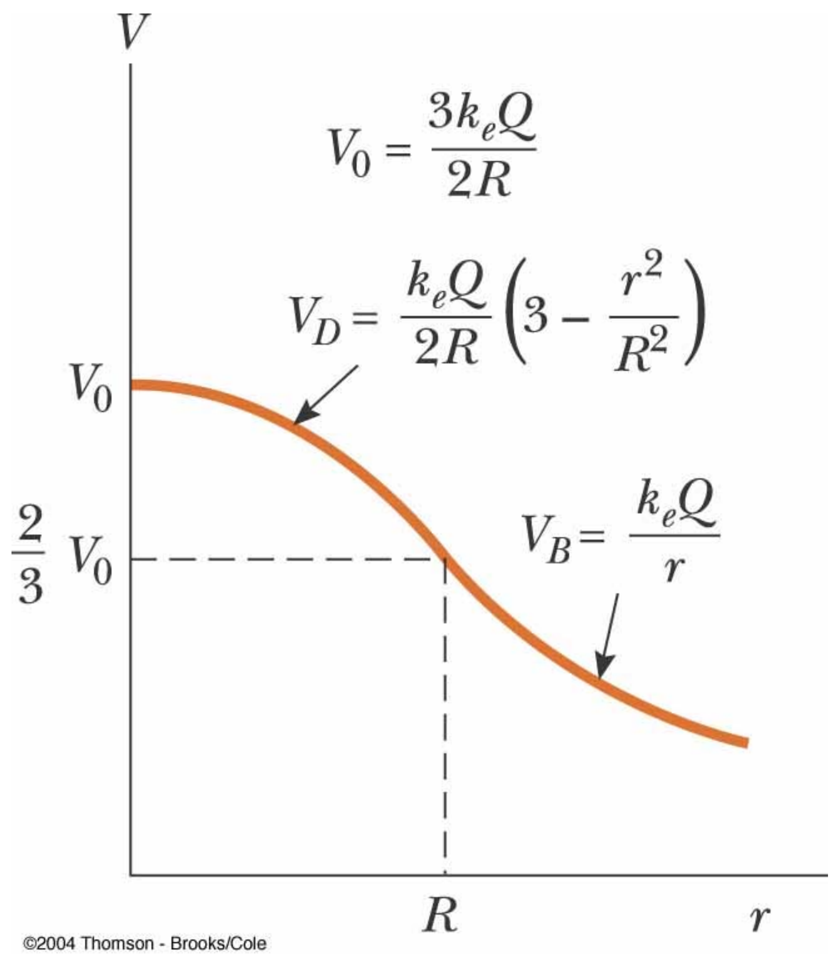


$$V_B = V_A - \int_\infty^B \vec{E} d\vec{s} = \int_r^\infty E_r dr = \int_r^a E_r dr + \int_a^\infty E_r dr =$$

$$= k_e \frac{Q}{a^3} \int_r^a r dr + k_e Q \int_r^\infty \frac{dr}{r^2} = k_e \frac{Q}{2a^3}(a^2 - r^2) + k_e \frac{Q}{a} = k_e \frac{Q}{2a}\left(3 - \frac{r^2}{a^2}\right)$$

Spherically Symmetric Charge Distribution

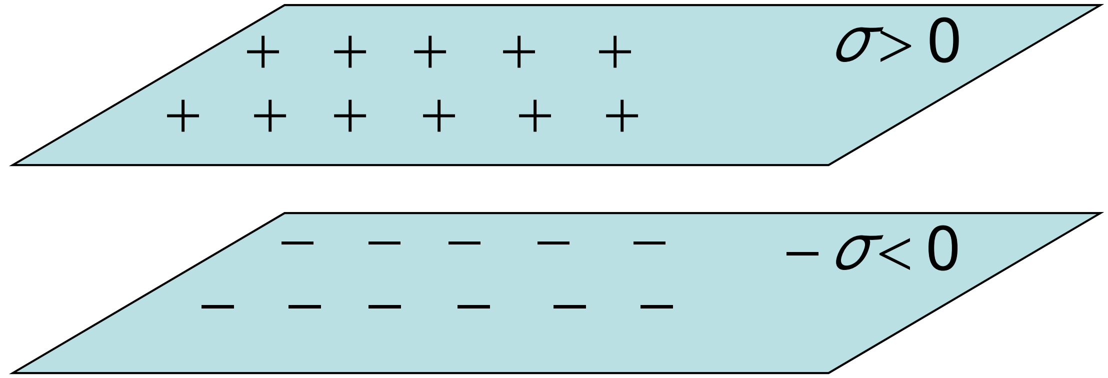


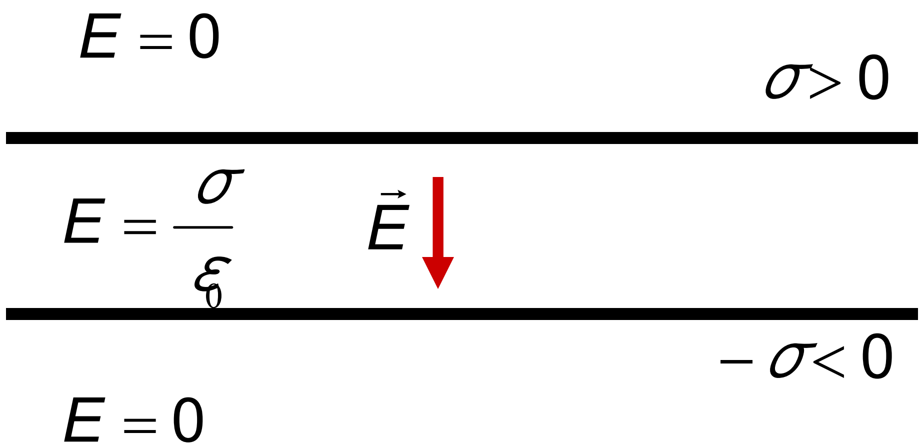


$$V_r = k_e \frac{Q}{r} \qquad r > a$$

$$V_B = k_e \frac{Q}{2a}\left(3 - \frac{r^2}{a^2}\right) \qquad r < a$$

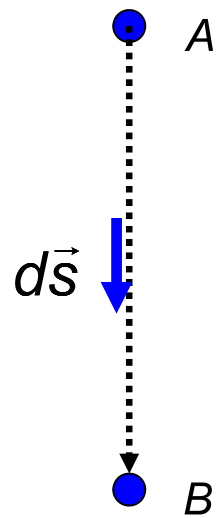
Important Example





$$V = ?$$

Important Example

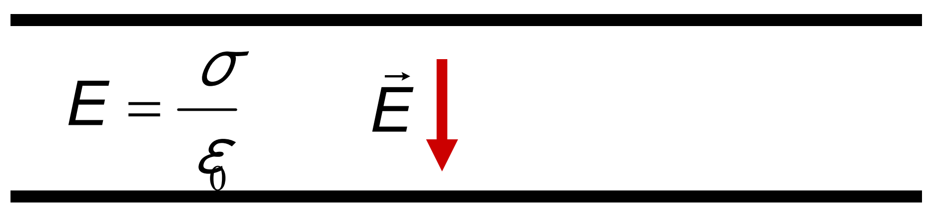


$$V_A = 0$$

$$V_B = -\int_A^B \vec{E}\,d\vec{s} = 0$$

$$E = 0$$

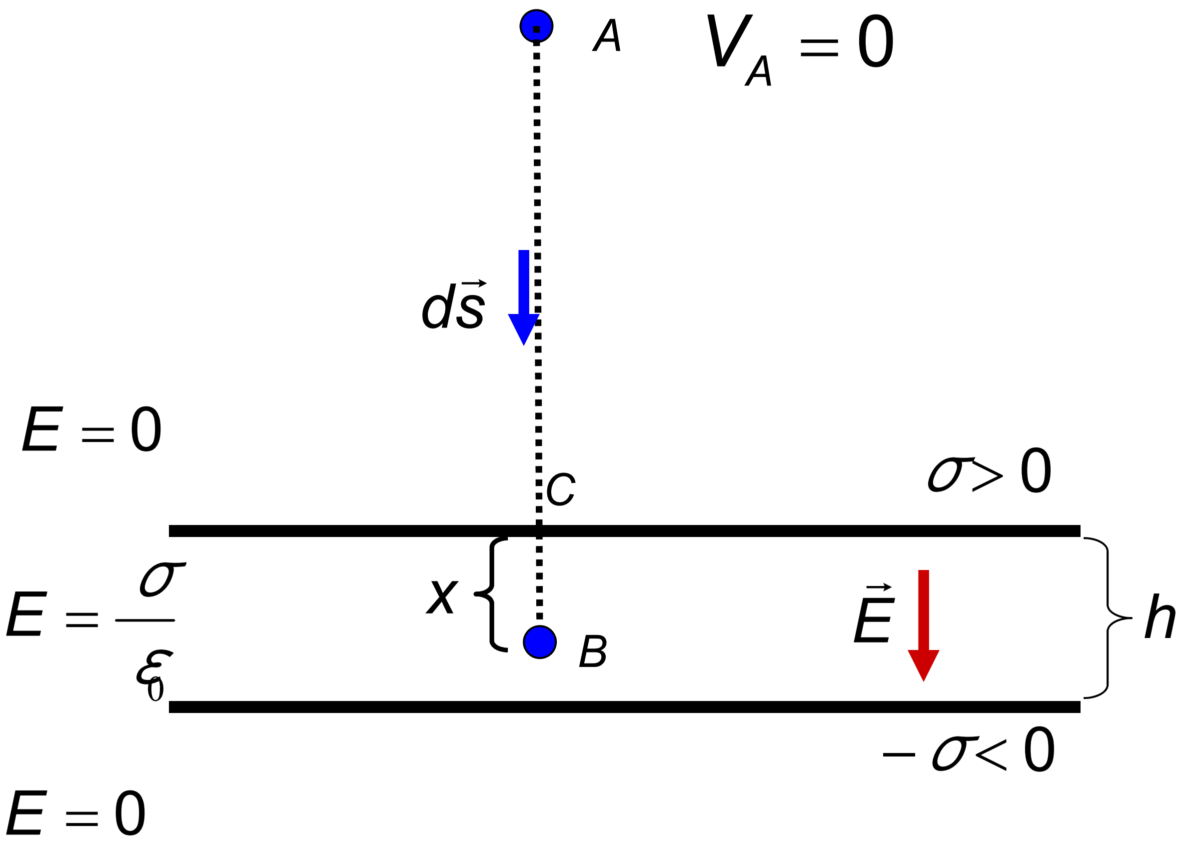
$$\sigma > 0$$



$$-\sigma < 0$$

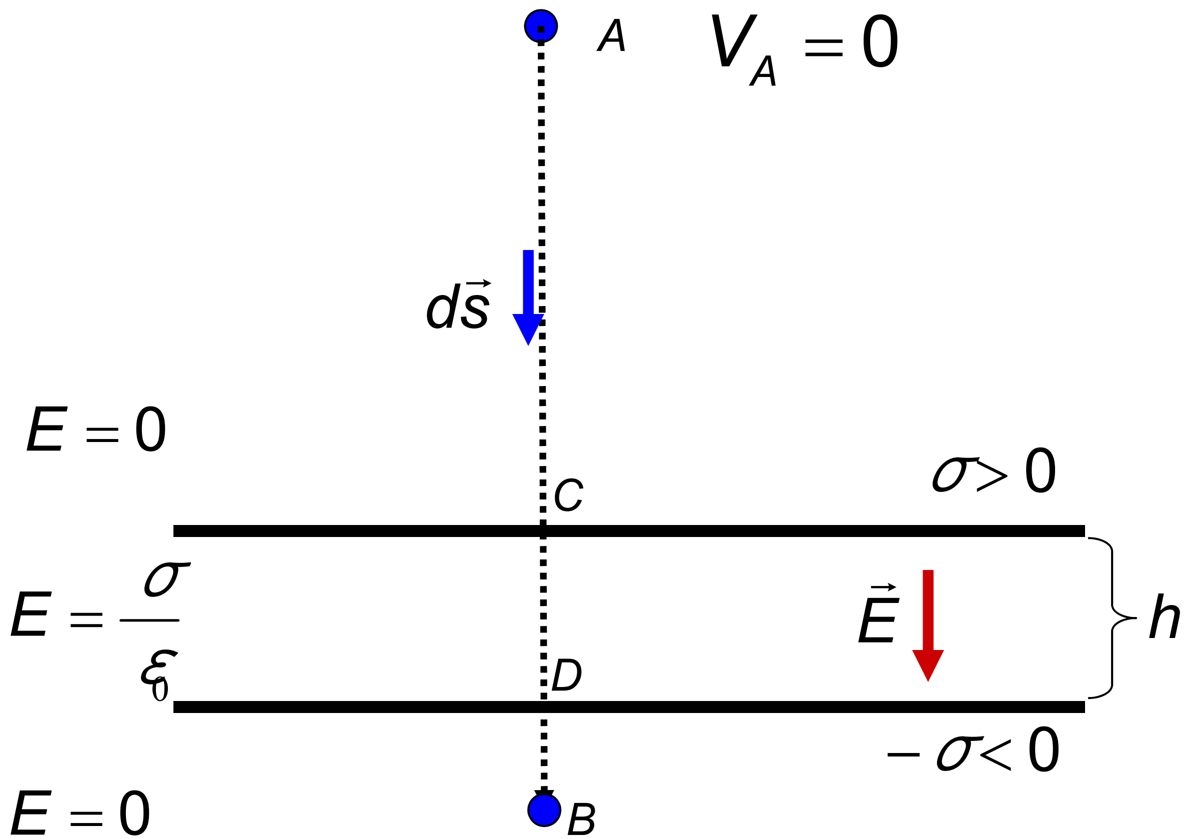
$$E = 0$$

Important Example



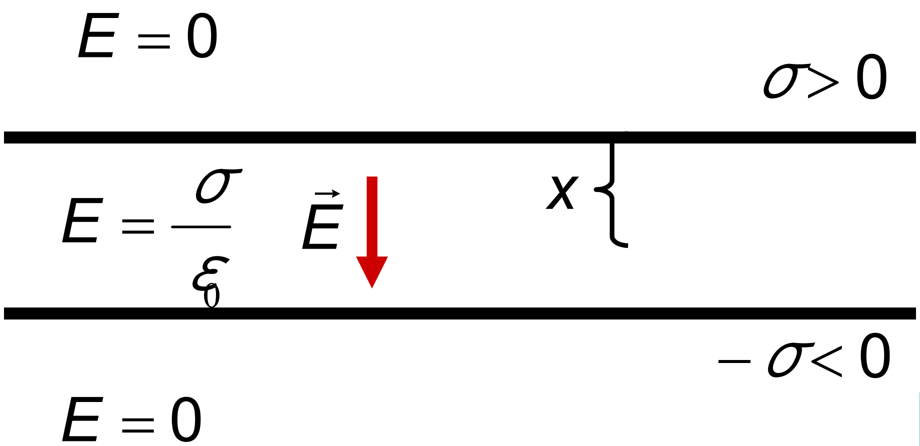
$$V_B = -\int_A^B \vec{E}d\vec{s} = -\int_A^C \vec{E}d\vec{s} - \int_C^B \vec{E}d\vec{s} = -\int_C^B Eds = -Ex$$

Important Example



$$V_B = -\int_A^B \vec{E} d\vec{s} = -\int_A^C \vec{E} d\vec{s} - \int_C^D \vec{E} d\vec{s} - \int_D^B \vec{E} d\vec{s} = -\int_C^D E ds = -Eh$$

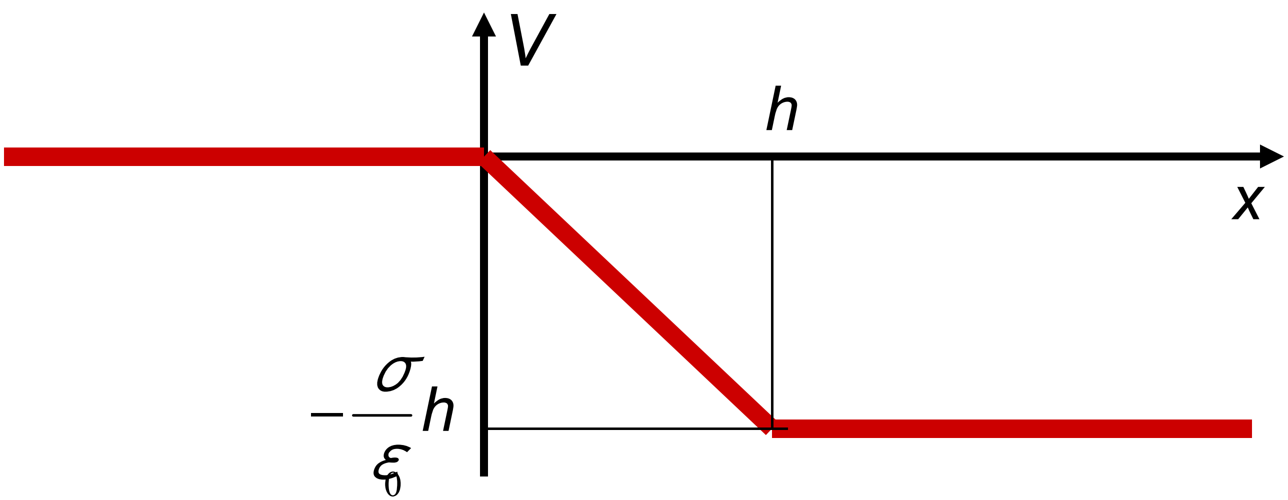
Important Example

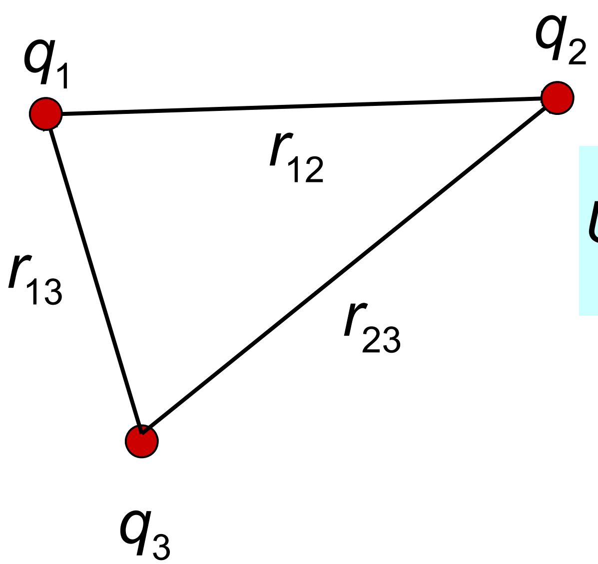


$$V = 0$$

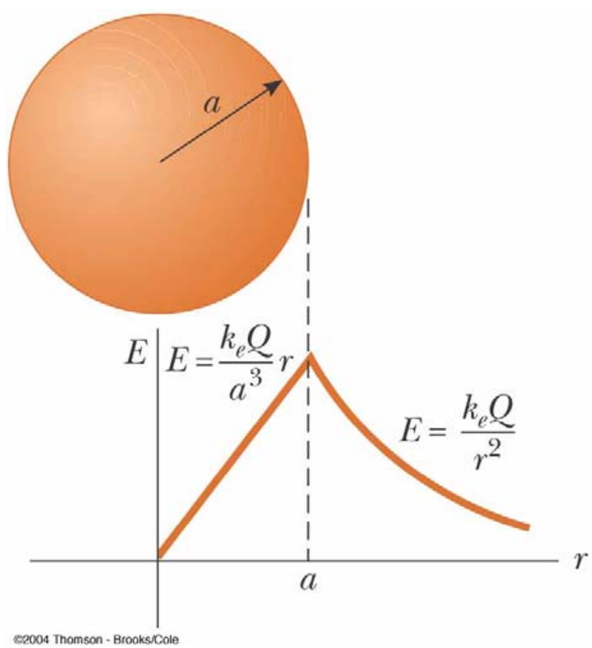
$$V = -\frac{\sigma}{\varepsilon_0} x$$

$$V = -Eh = -\frac{\sigma}{\varepsilon_0} h$$





$$U = U_{12} + U_{13} + U_{23} = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}}$$



$r > a$

$$V_r = k_e \frac{Q}{r}$$

$r < a$

$$V_B = k_e \frac{Q}{2a}\left(3 - \frac{r^2}{a^2}\right)$$