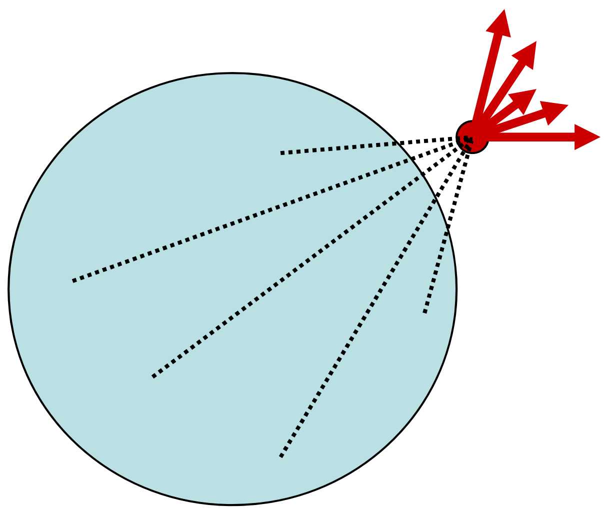
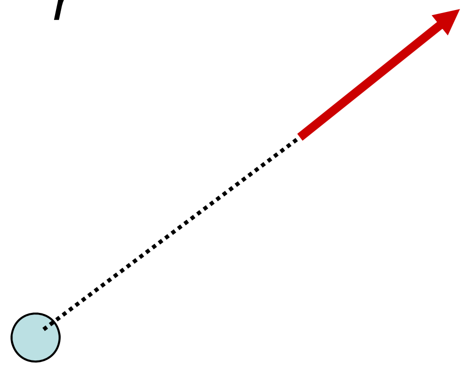
Reading: Chapter 28

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}$$



For *r* > *a*

$\equiv$



Gauss's Law

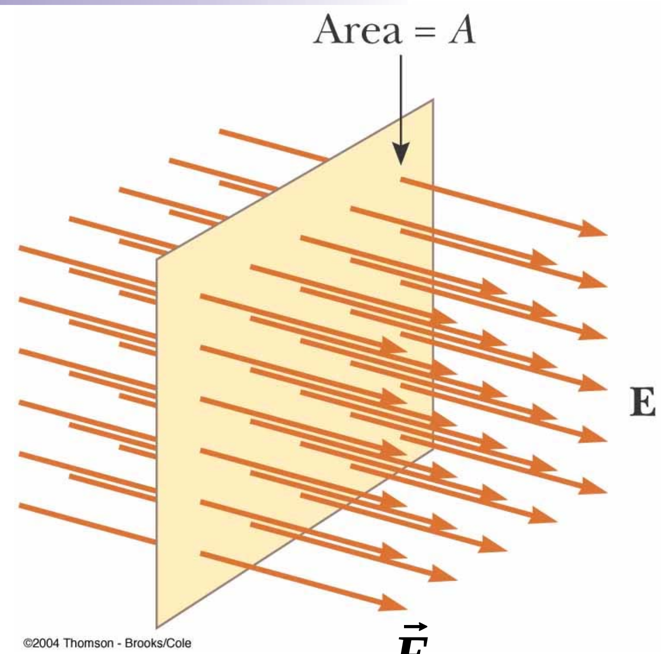
Chapter 28

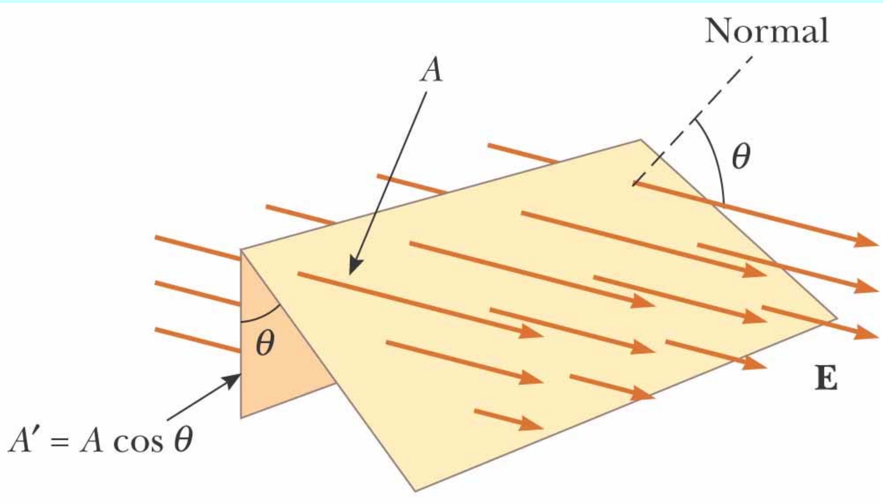
Gauss's Law

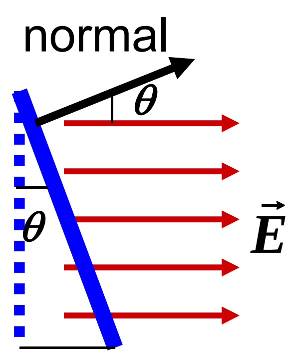
Electric Flux

Definition:

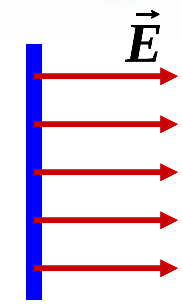
- ***Electric flux*** is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field
- $\Phi_E = EA$
- The field lines may make some angle θ with the perpendicular to the surface
- Then $\Phi_E = EA \cos \theta$







$$\Phi_E = EA\cos\theta$$

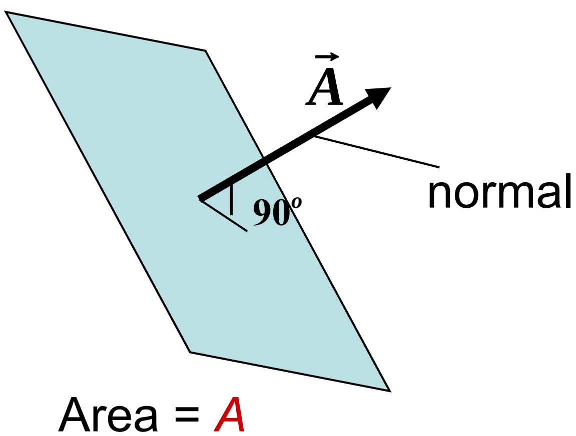


$$\Phi_E = EA$$

Electric Flux: Surface as a Vector

Vector, corresponding to a Flat Surface of Area *A*, is determined by the following rules:

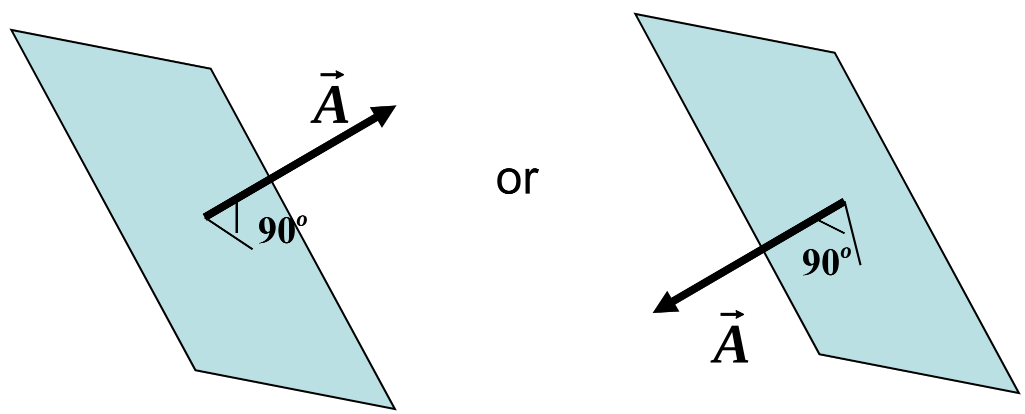
- **the vector is orthogonal to the surface**
- **the magnitude of the vector is equal to the area *A***



The first rule

- **the vector is orthogonal to the surface**

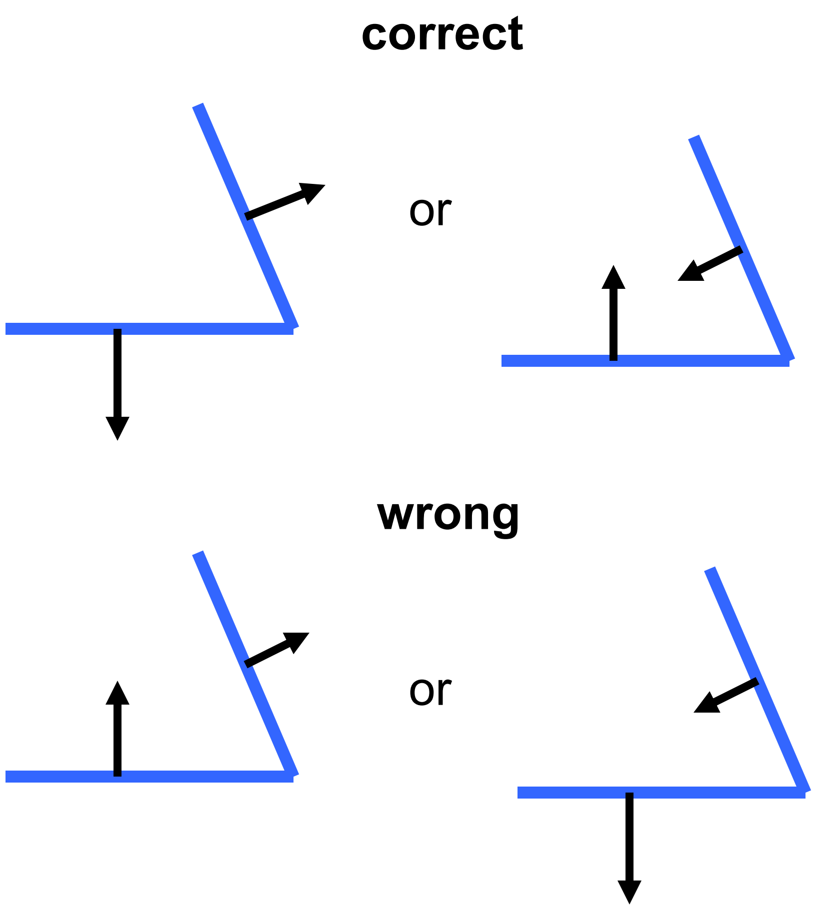
does not determine the direction of $\vec{A}$. There are still two possibilities:

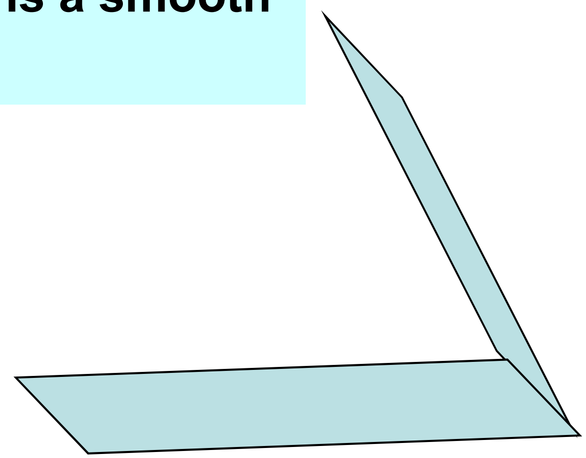


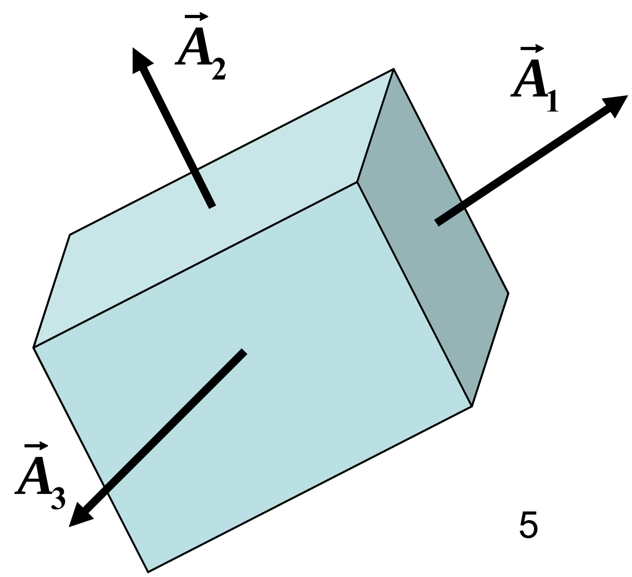
You can choose any of them

Electric Flux: Surface as a Vector

If we consider more complicated surface then the directions of vectors should be adjusted, so the direction of vector is a smooth function of the surface point



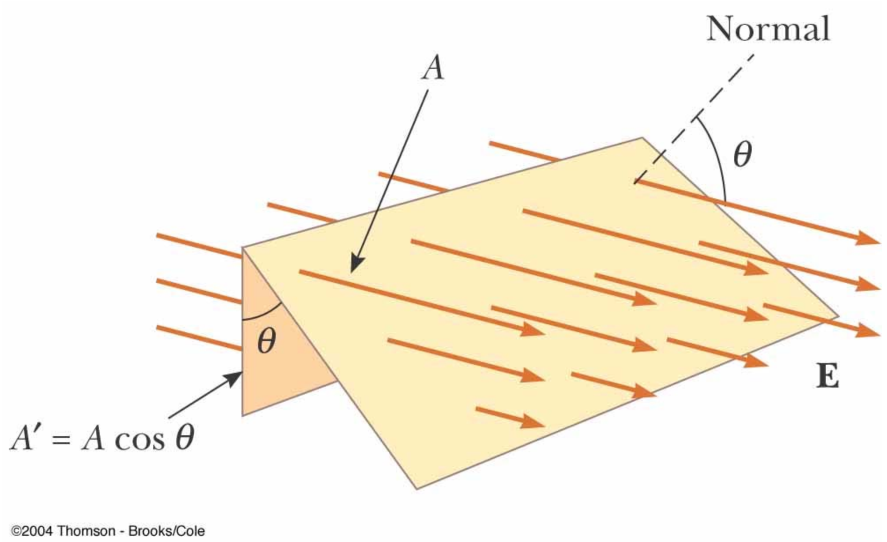


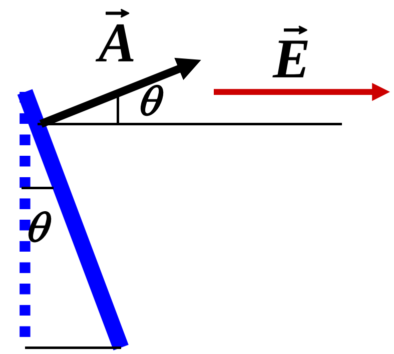


Electric Flux

Definition:

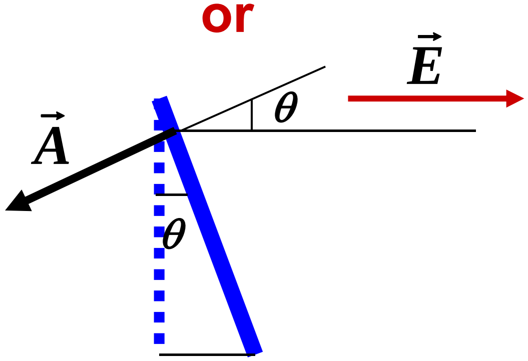
- ***Electric flux*** is the scalar product of electric field and the vector $\vec{A}$
- $\Phi = \vec{E}\vec{A}$





$$\Phi_E = \vec{E}\vec{A} = EA\cos\theta > 0$$

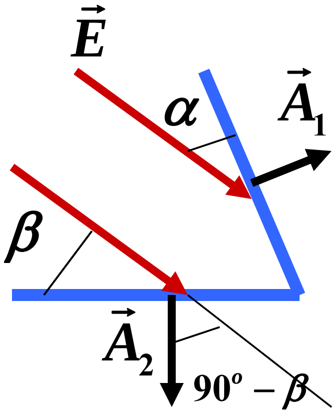
or

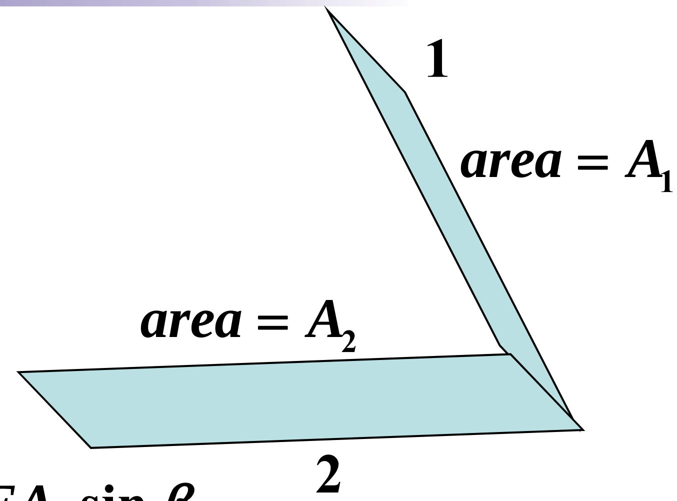


$$\Phi_E = \vec{E}\vec{A} = -EA\cos\theta < 0$$

Electric Flux

$$\Phi_E = \Phi_1 + \Phi_2$$





$$\Phi_2 = \vec{E}\vec{A}_2 = EA_2 \cos(90^o - \beta) = EA_2 \sin\beta$$

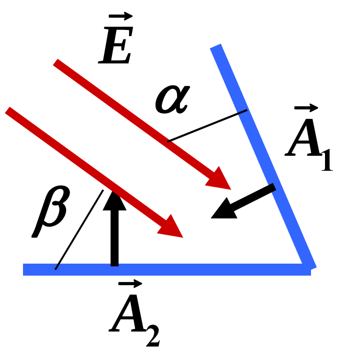
$$\Phi_1 = \vec{E}\vec{A}_1 = EA_1 \sin\alpha$$

$$\Phi_E = EA_1 \sin\alpha + EA_2 \sin\beta$$

flux is positive

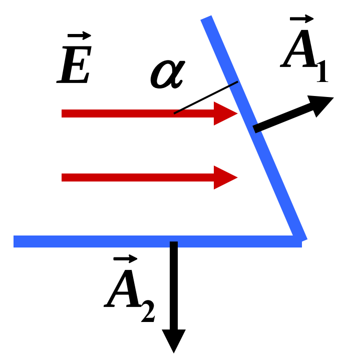
---

if



then $\Phi_E = -EA_1 \cos\alpha - EA_2 \cos\beta$

flux is negative



$$\Phi_E = EA_1 \sin\alpha$$

$$\Phi_2 = 0$$

$\vec{A}_2$ **and** $\vec{E}$ **are** **orthogonal**

Electric Flux

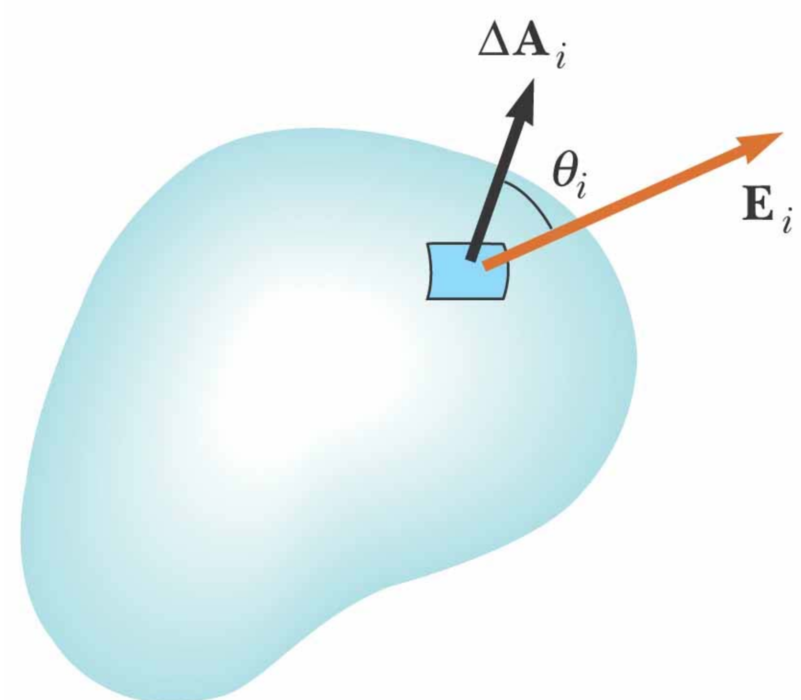
- In the more general case, look at a small flat area element

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$

- In general, this becomes

$$\Phi_E = \lim_{\Delta\vec{A}_i \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

- The surface integral means the integral must be evaluated over the surface in question
- The units of electric flux will be N·m²/C²

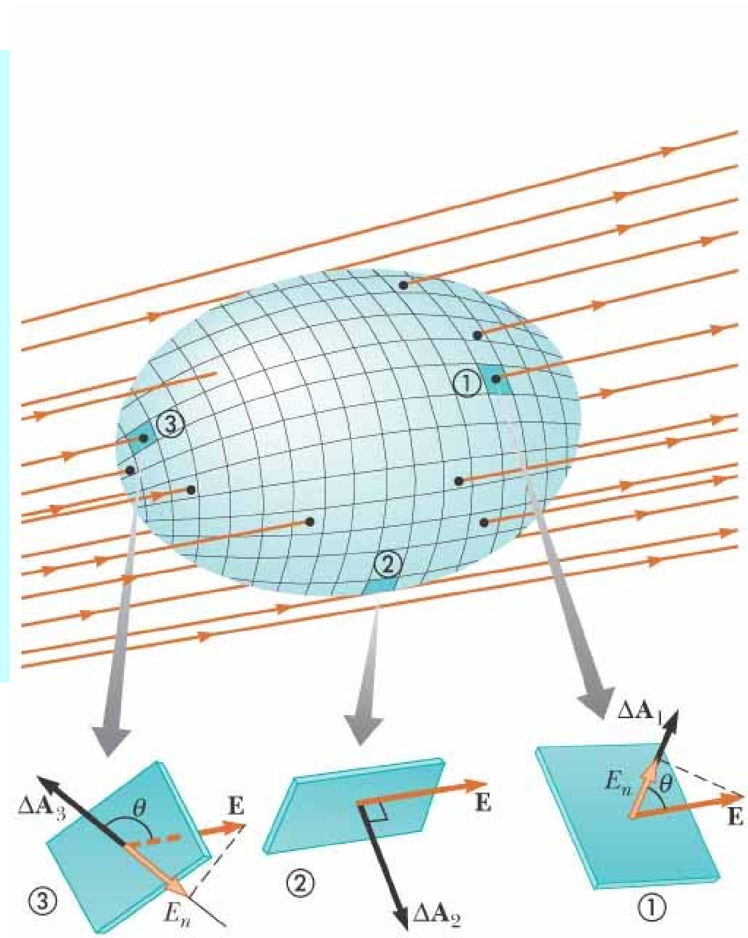


Electric Flux: Closed Surface

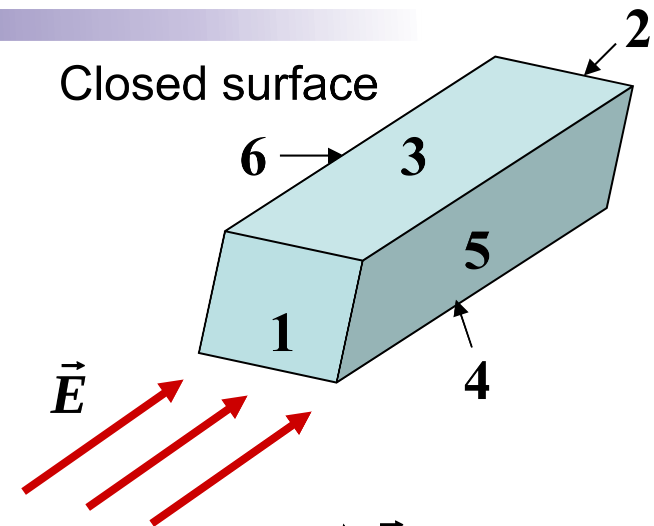
The vectors $\Delta\vec{A}_i$ point in different directions

- At each point, they are perpendicular to the surface
- By convention, they point outward

$$\Phi_E = \lim_{\Delta\vec{A}_i \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



Electric Flux: Closed Surface

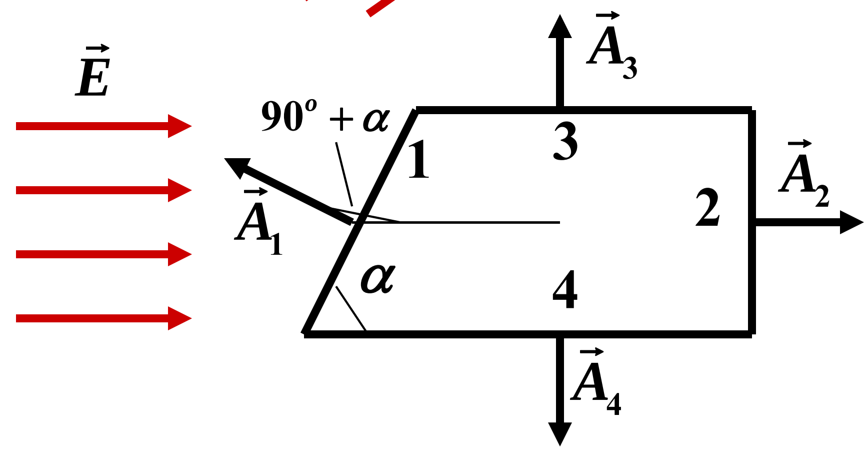


$$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$$

$\vec{E}$ is orthogonal to $\vec{A}_3$, $\vec{A}_4$, $\vec{A}_5$, and $\vec{A}_6$

Then $\Phi_3 = \vec{E}\vec{A}_3 = 0 \quad \Phi_4 = \vec{E}\vec{A}_4 = 0$

$$\Phi_5 = \vec{E}\vec{A}_5 = 0 \quad \Phi_6 = \vec{E}\vec{A}_6 = 0$$



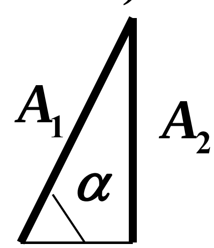
$$\Phi_E = \Phi_1 + \Phi_2$$

$$\Phi_1 = \vec{E}\vec{A}_1 = EA_1 \cos(90^o + \alpha) = -EA_1 \sin\alpha$$

$$\Phi_2 = \vec{E}\vec{A}_2 = EA_2$$

$$\Phi_E = E(A_2 - A_1 \sin\alpha)$$

but $A_2 = A_1 \sin\alpha$



Then $\Phi_E = 0$

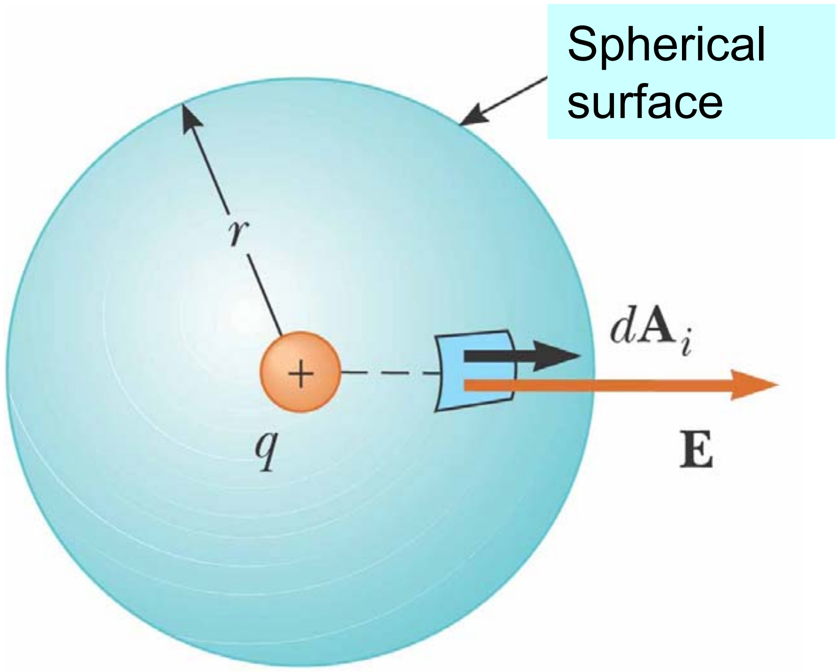
(no charges inside closed surface)

Electric Flux: Closed Surface

- A positive point charge, q, is located at the center of a sphere of radius r
- The magnitude of the electric field everywhere on the surface of the sphere is

  $E = k_e q / r^2$
- Electric field is perpendicular to the surface at every point, so

  $\vec{E}$ has the same direction as $\vec{A}$ at every point.



Electric Flux: Closed Surface

$\vec{E}$ has the same direction as $\vec{A}$ at every point.

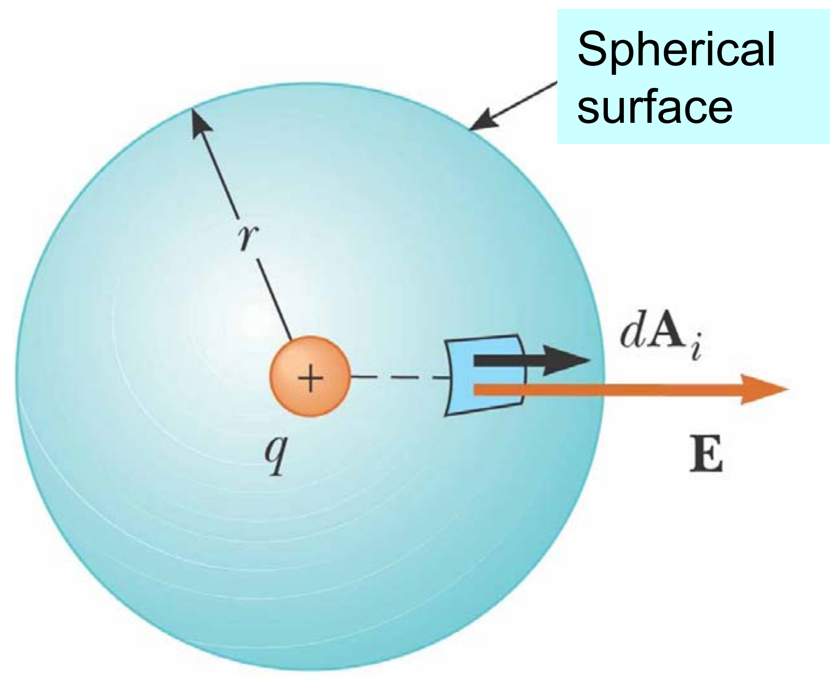
$$E = k_e \frac{q}{r^2}$$

Then

$$\Phi = \sum_i \vec{E}_i d\vec{A}_i = E \sum_i dA_i =$$

$$= EA_0 = E\,4\pi r^2 = 4\pi r^2 k_e \frac{q}{r^2} =$$

$$= 4\pi k_e q = \frac{q}{\varepsilon_0}$$ **Gauss's Law**



Φ does not depend on ***r*** ⟵ ONLY BECAUSE $E \propto \frac{1}{r^2}$

Electric Flux: Closed Surface

$\vec{E}$ and $\vec{A}$ have opposite directions at every point.

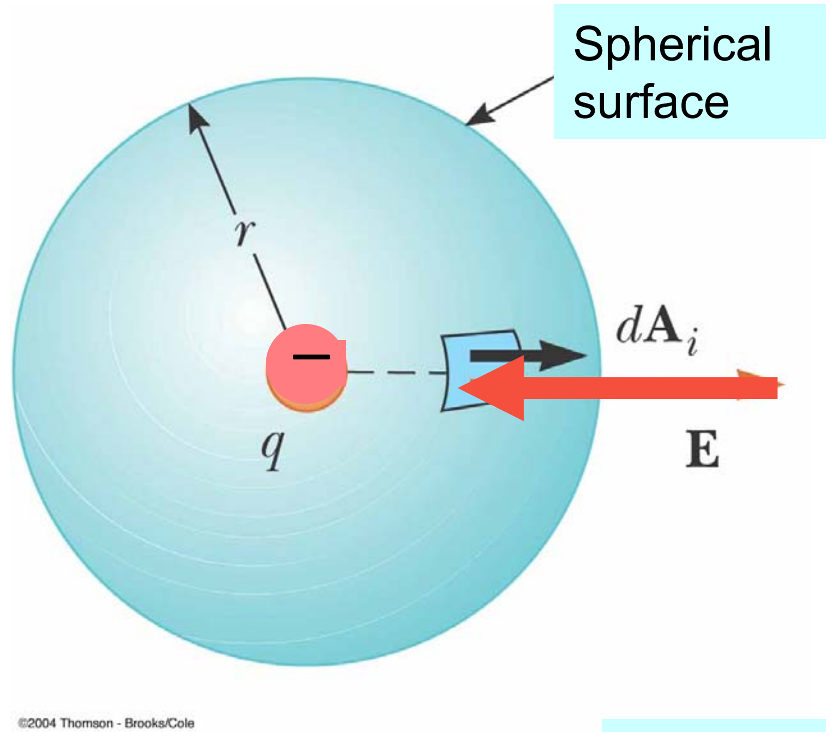
$$E = k_e \frac{|q|}{r^2}$$

Then

$$\Phi = \sum_i \vec{E}_i d\vec{A}_i = -E \sum_i dA_i =$$

$$= -EA_0 = -E\,4\pi r^2 = -4\pi r^2 k_e \frac{|q|}{r^2} =$$

$$= -4\pi k_e |q| = \frac{q}{\varepsilon_0}$$ **Gauss's Law**

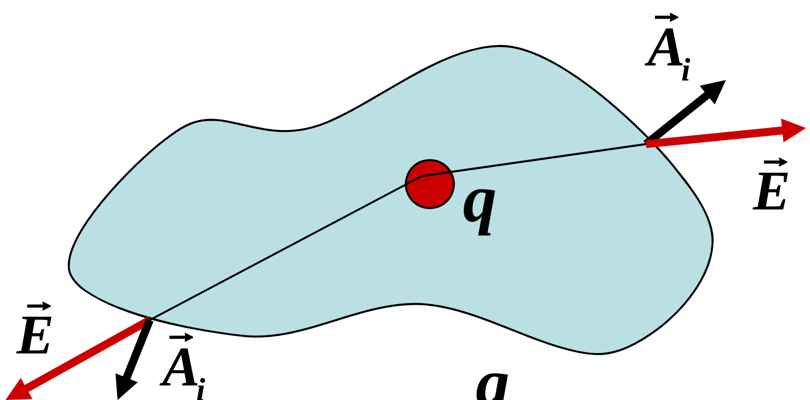


Φ does not depend on ***r*** ⟵ ONLY BECAUSE $E \propto \frac{1}{r^2}$

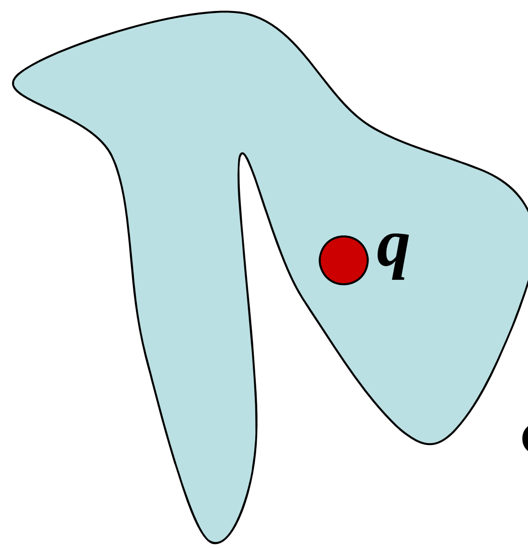
Gauss's Law

- The net flux through any closed surface surrounding a point charge, q, is given by $q/\varepsilon o$ and is **independent of the shape of that surface**

- The net electric flux through a closed surface that surrounds **no charge** is **zero**



$$\Phi = \frac{q}{\varepsilon_0}$$



$$\Phi = \frac{q}{\varepsilon_0}$$

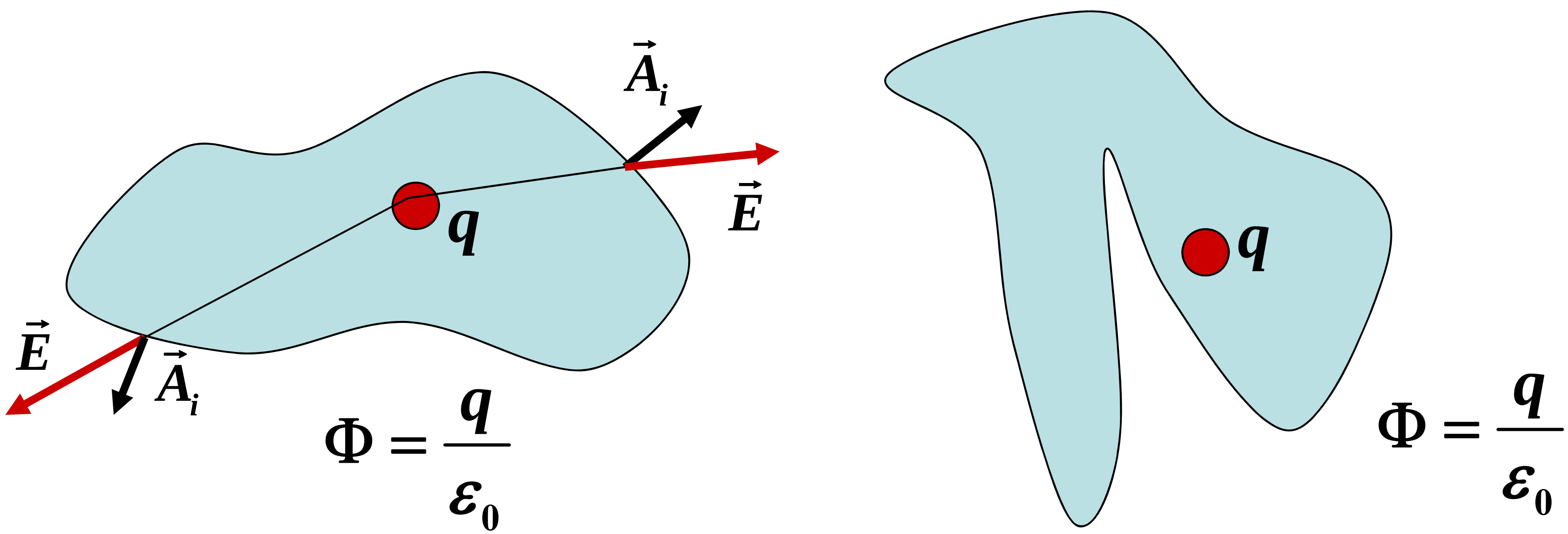
Gauss's Law

- Gauss's law states

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_o}$$

- q_{in} is the net charge inside the surface

- **E** is the *total electric field* and may have contributions from charges both inside and outside of the surface



Gauss's Law

- Gauss's law states

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_o}$$

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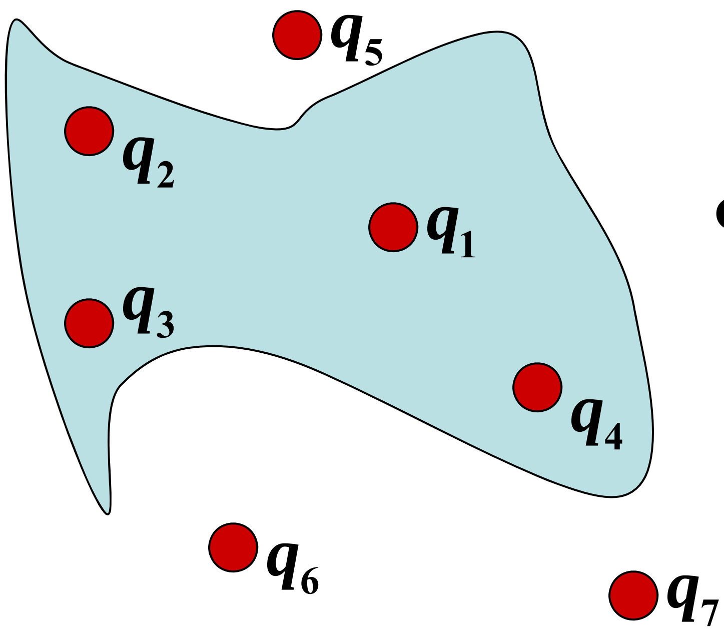


$$\Phi = \frac{q_1 + q_2 + q_3 + q_4}{\varepsilon_0}$$

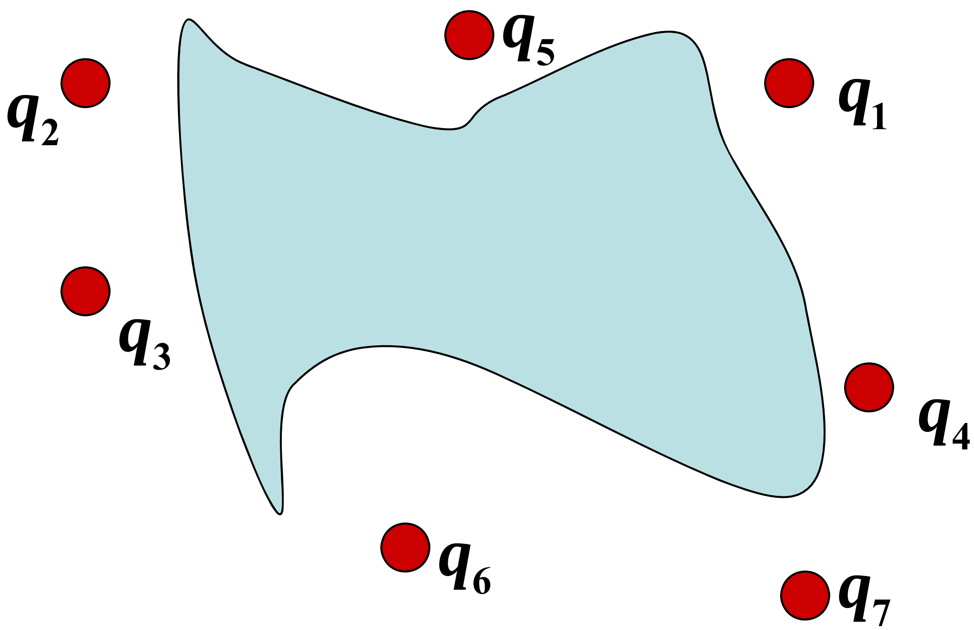
Gauss's Law

- **Gauss's law states**

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

- **q_{in} is the net charge inside the surface**

- **E is the *total electric field* and may have contributions from charges both inside and outside of the surface**



$$\Phi = 0$$

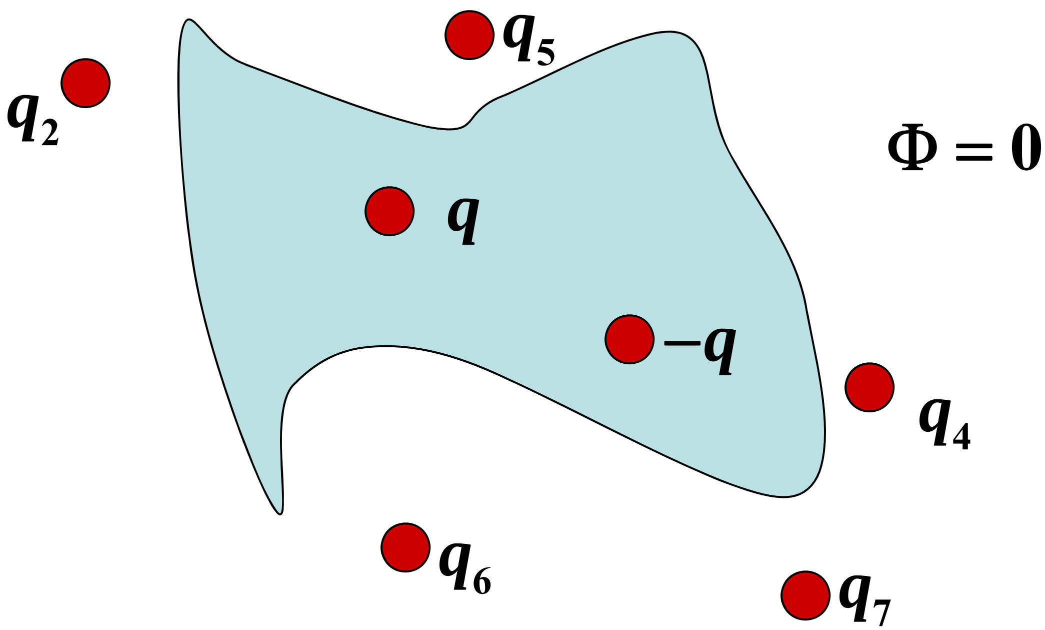
Gauss's Law

- Gauss's law states

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

- q_{in} is the net charge inside the surface

- E is the *total electric field* and may have contributions from charges both inside and outside of the surface



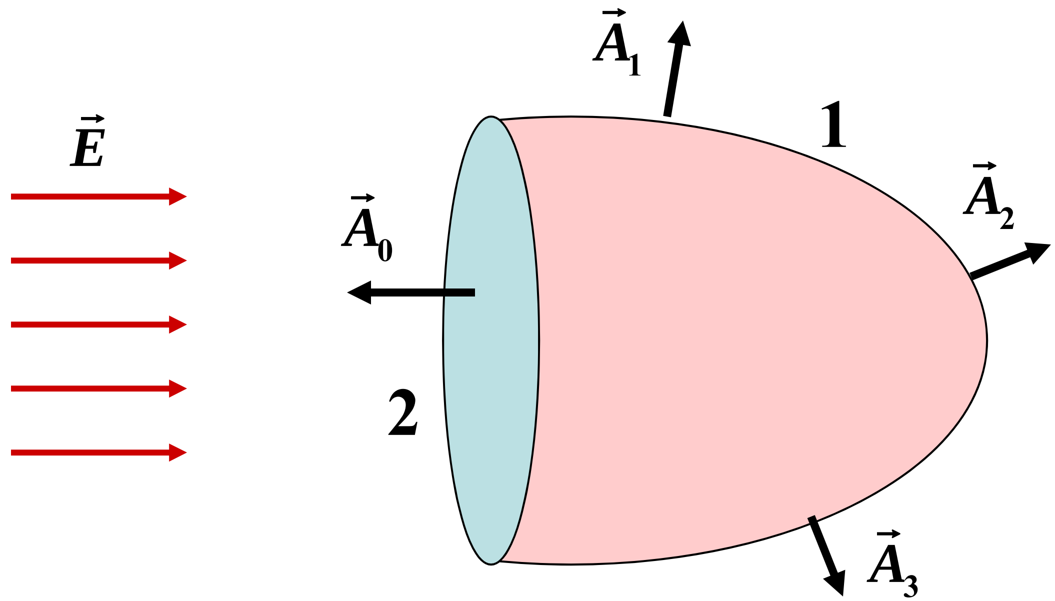
Gauss's Law: Problem

What is the flux through surface 1

$$\Phi_1 + \Phi_2 = 0$$

$$\Phi_2 = \vec{E}\vec{A}_0 = -EA_0$$

$$\Phi_1 = -\Phi_2 = EA_0$$



Chapter 28

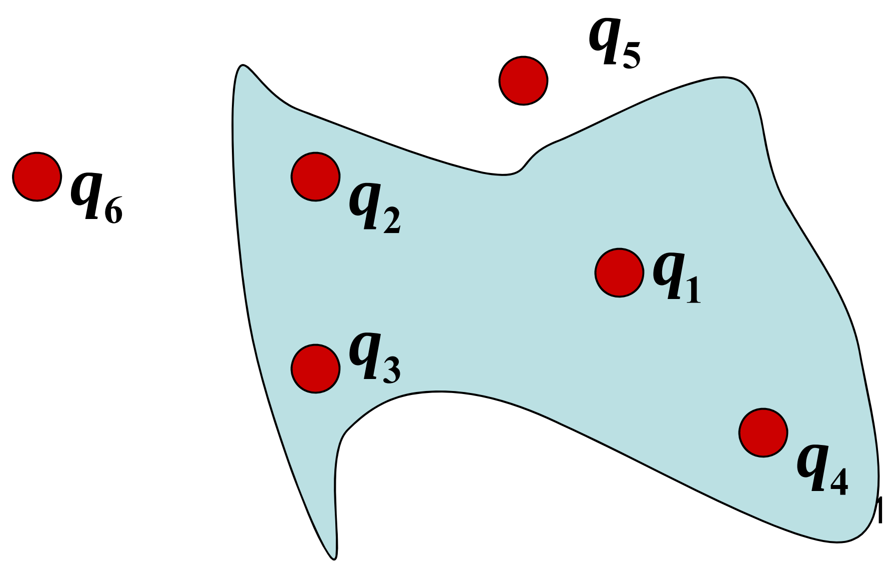
Gauss's Law: Applications

Gauss's Law: Applications

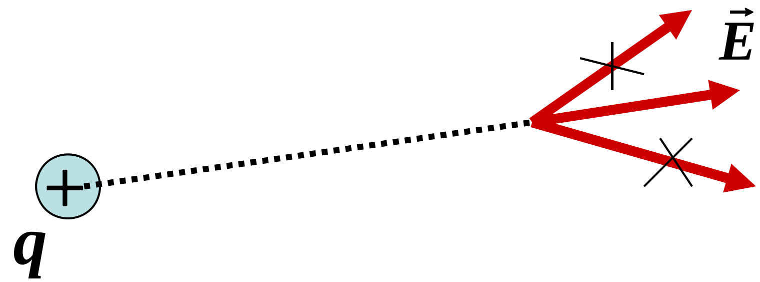
- Although Gauss's law can, in theory, be solved to find E for any charge configuration, in practice it is limited to symmetric situations
- To use Gauss's law, you want to choose a Gaussian surface over which the surface integral can be simplified and the electric field determined
- Take advantage of symmetry
- Remember, the gaussian surface is a surface you choose, it does not have to coincide with a real surface

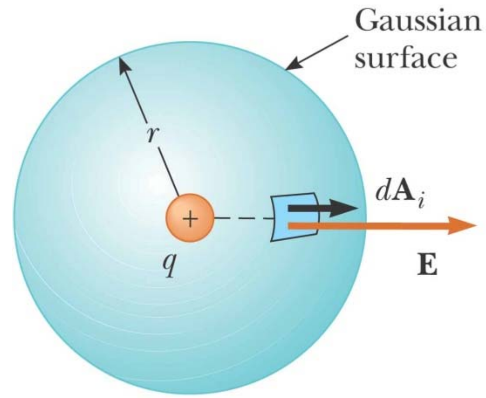
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_o}$$

$$\Phi = \frac{q_1 + q_2 + q_3 + q_4}{\varepsilon_0}$$



Gauss's Law: Point Charge





SYMMETRY:

$\vec{E}$ **- direction - along the radius**

$\vec{E}$ **- depends only on radius, *r***

Gaussian Surface – Sphere

Only in this case the magnitude of electric field is constant on the Gaussian surface and the flux can be easily evaluated

$$\Phi = \frac{q}{\varepsilon_0}$$ - Gauss's Law

$$\Phi = \sum_i \vec{E}_i d\vec{A}_i = E \sum_i dA_i = EA_0 = E\,4\pi r^2$$ **- definition of the Flux**

Then $$\frac{q}{\varepsilon_0} = 4\pi r^2 E$$

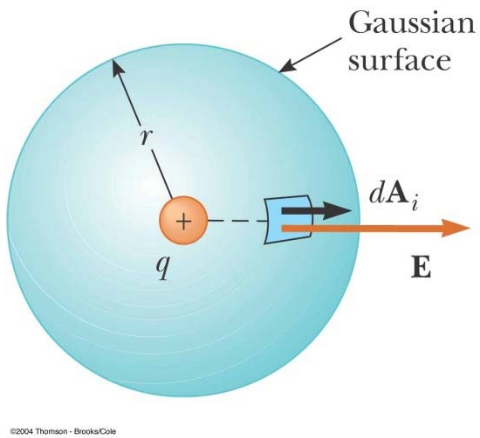
$$E = k_e \frac{q}{r^2}$$

Gauss's Law: Applications

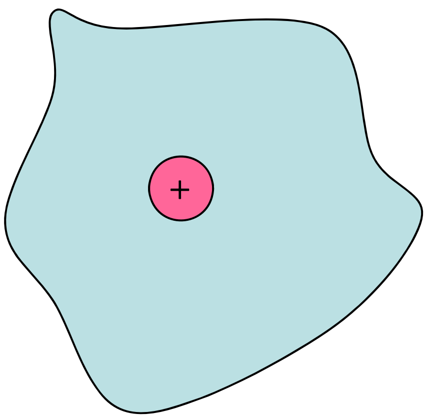
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_o}$$

- Try to choose a surface that satisfies one or more of these conditions:
  - The value of the electric field can be argued from symmetry to be **constant over the surface**
  - The dot product of **E**·*d***A** can be expressed as a simple algebraic product *EdA* because **E** and *d***A** **are parallel**
  - The dot product is 0 because **E** and *d***A** **are perpendicular**
  - The field can be argued to be zero over the surface

correct Gaussian surface



wrong Gaussian surface



Gauss's Law: Applications

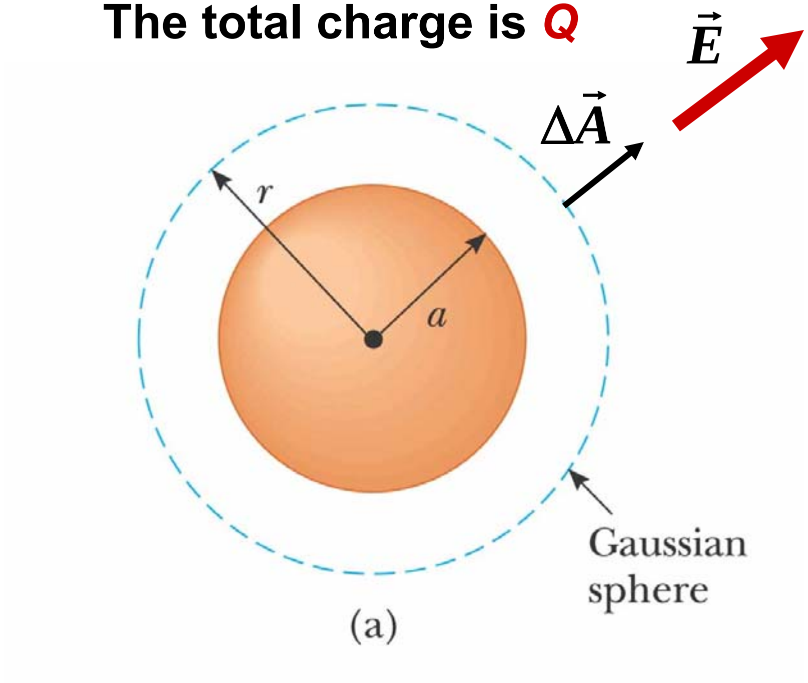
Spherically Symmetric Charge Distribution

SYMMETRY:

$\vec{E}$ **- direction - along the radius**

$\vec{E}$ **- depends only on radius, *r***

The total charge is *Q*



- Select a sphere as the gaussian surface
- For $r > a$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA = 4\pi r^2 E = \frac{q_{\text{in}}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}$$

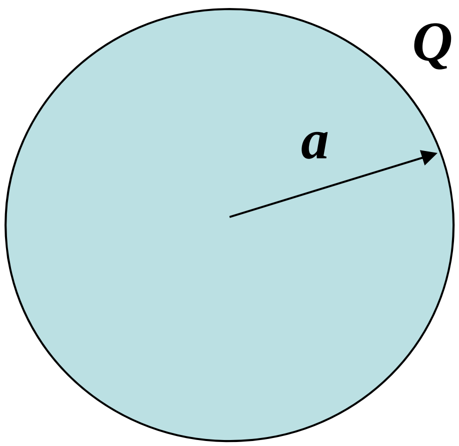
The electric field is the same as for the point charge *Q*

Gauss's Law: Applications

Spherically Symmetric Charge Distribution

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}$$

The electric field is the same as for the point charge *Q* !!!!!



For $r > a$

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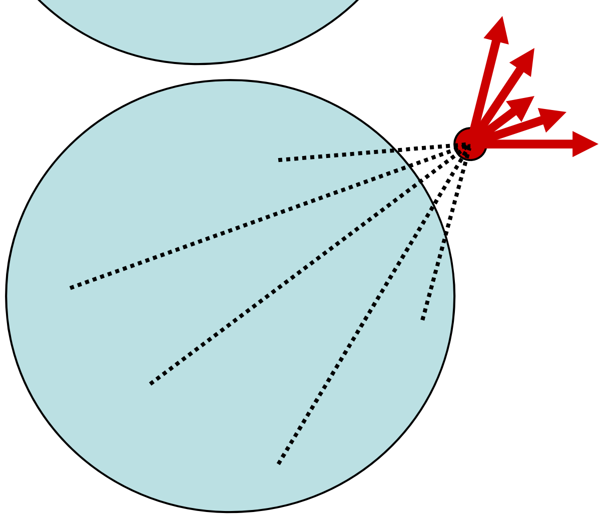
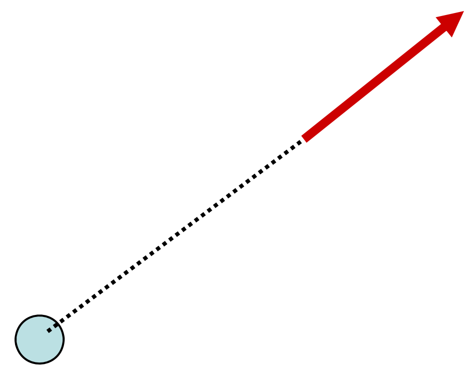


For $r > a$

$\equiv$



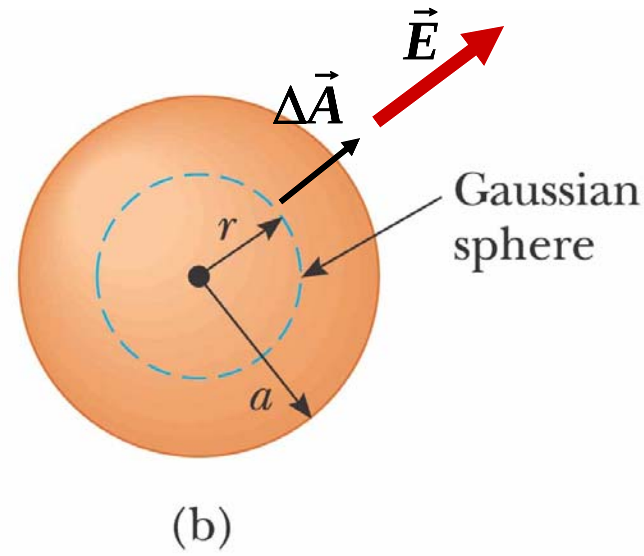
Gauss's Law: Applications

Spherically Symmetric Charge Distribution

SYMMETRY:

$\vec{E}$ - direction - along the radius

$\vec{E}$ - depends only on radius, *r*



- Select a sphere as the gaussian surface, $r < a$

$$q_{in} = \frac{Q}{\frac{4}{3}\pi a^3}\frac{4}{3}\pi r^3 = Q\frac{r^3}{a^3} < Q$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E dA = 4\pi r^2 E = \frac{q_{\text{in}}}{\varepsilon_o}$$

$$E = \frac{q_{\text{in}}}{4\pi\varepsilon_o r^2} = k_e \frac{Qr^3}{a^3}\frac{1}{r^2} = k_e \frac{Q}{a^3} r$$

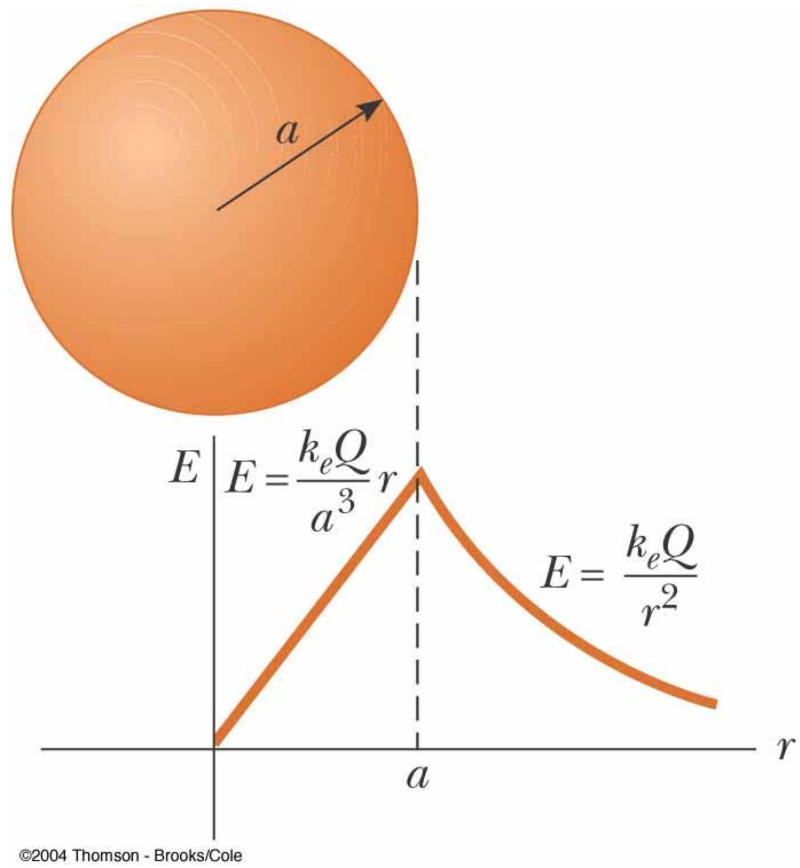
Gauss's Law: Applications

Spherically Symmetric Charge Distribution

- Inside the sphere, ***E*** varies linearly with *r*

  ***E* → 0** as ***r* → 0**

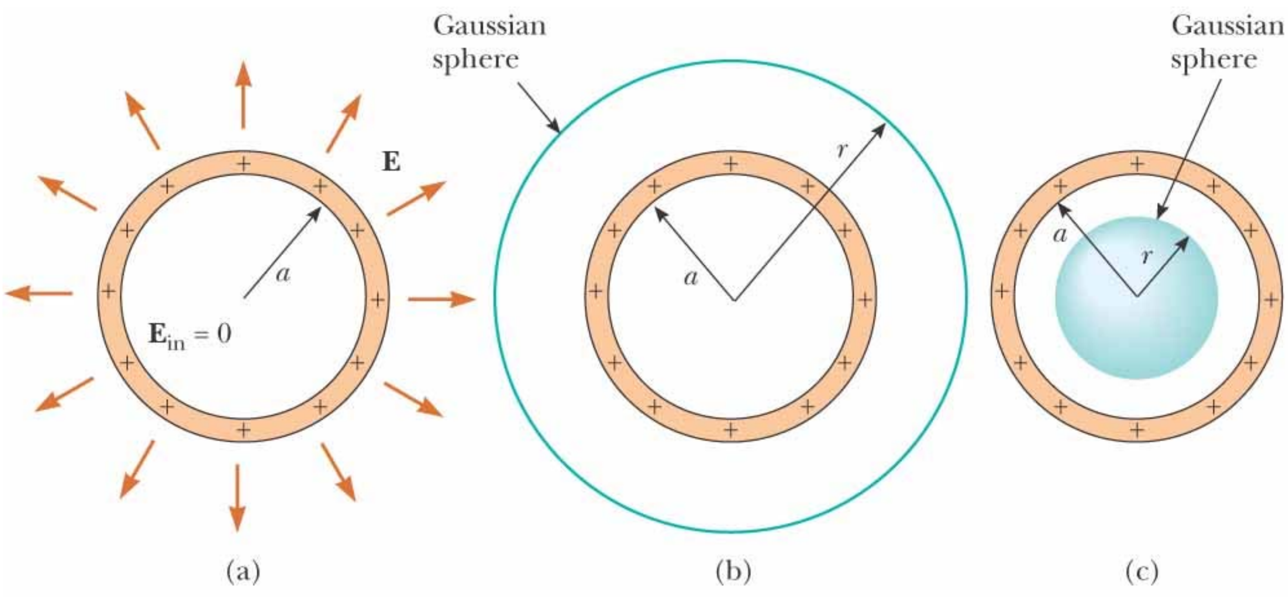
- The field outside the sphere is equivalent to that of a point charge located at the center of the sphere



Gauss's Law: Applications

Field due to a thin spherical shell

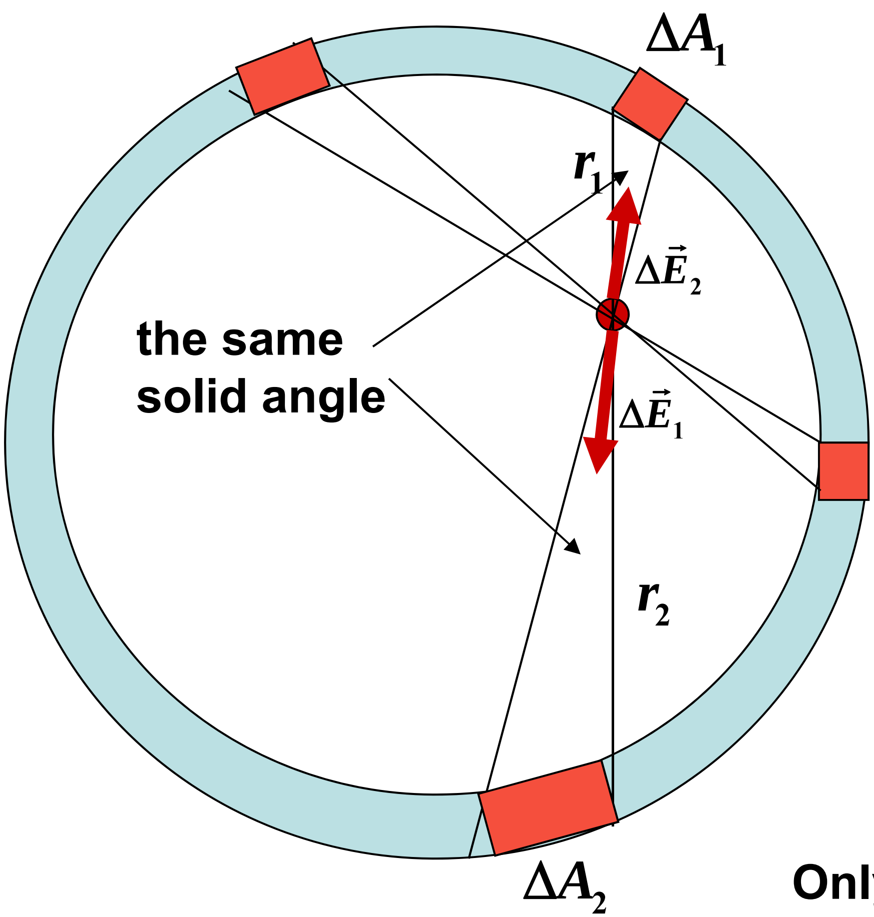
- Use spheres as the gaussian surfaces
- When $r > a$, the charge inside the surface is Q and $E = k_e Q / r^2$
- When $r < a$, the charge inside the surface is **0** and $E = 0$



Gauss's Law: Applications

Field due to a thin spherical shell

- When ***r* < *a***, the charge inside the surface is **0** and ***E* = 0**



$$\Delta q_1 = \sigma \Delta A_1 \qquad \Delta q_2 = \sigma \Delta A_2$$

$$\Delta A_1 = r_1^2 \Delta\Omega \qquad \Delta A_2 = r_2^2 \Delta\Omega$$

$$\Delta E_1 = k_e \frac{\Delta q_1}{r_1^2} = k_e \frac{\sigma \Delta A_1}{r_1^2} = k_e \frac{\sigma r_1^2 \Delta\Omega}{r_1^2} = k_e \sigma \Delta\Omega$$

$$\Delta E_2 = k_e \frac{\Delta q_2}{r_2^2} = k_e \frac{\sigma \Delta A_2}{r_2^2} = k_e \frac{\sigma r_2^2 \Delta\Omega}{r_2^2} = k_e \sigma \Delta\Omega$$

$$\Delta E_1 = \Delta E_2$$

$$\Delta\vec{E}_1 + \Delta\vec{E}_2 = 0$$

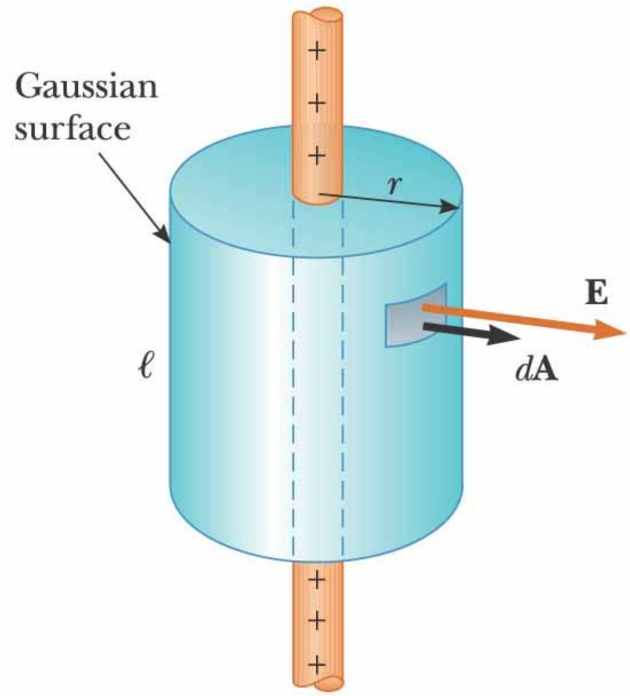
Only because in Coulomb law → $E \propto \frac{1}{r^2}$

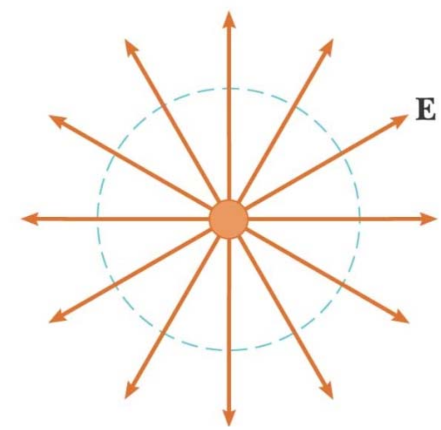
Gauss's Law: Applications

Field from a line of charge

- Select a cylindrical Gaussian surface
  - The cylinder has a radius of ***r*** and a length of ***ℓ***
- **Symmetry:**

  E is constant in magnitude (depends only on radius ***r***) and perpendicular to the surface at every point on the curved part of the surface

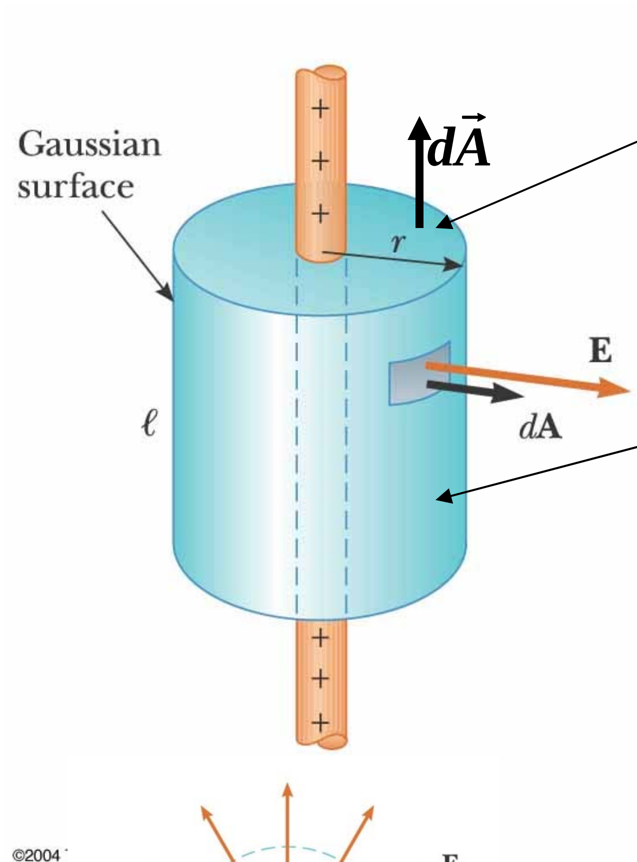




The end view

Gauss's Law: Applications

Field from a line of charge



The flux through this surface is **0**

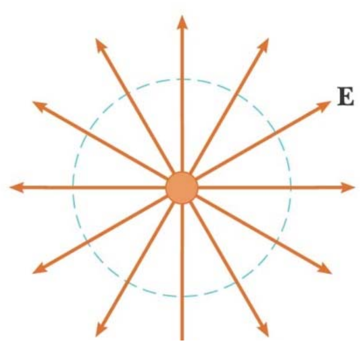
The flux through this surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E\left(2\pi r \ell\right) = \frac{q_{in}}{\varepsilon_o}$$

$$q_{in} = \lambda\ell$$

$$E\left(2\pi r \ell\right) = \frac{\lambda\ell}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o r} = 2k_e\frac{\lambda}{r}$$



The end view

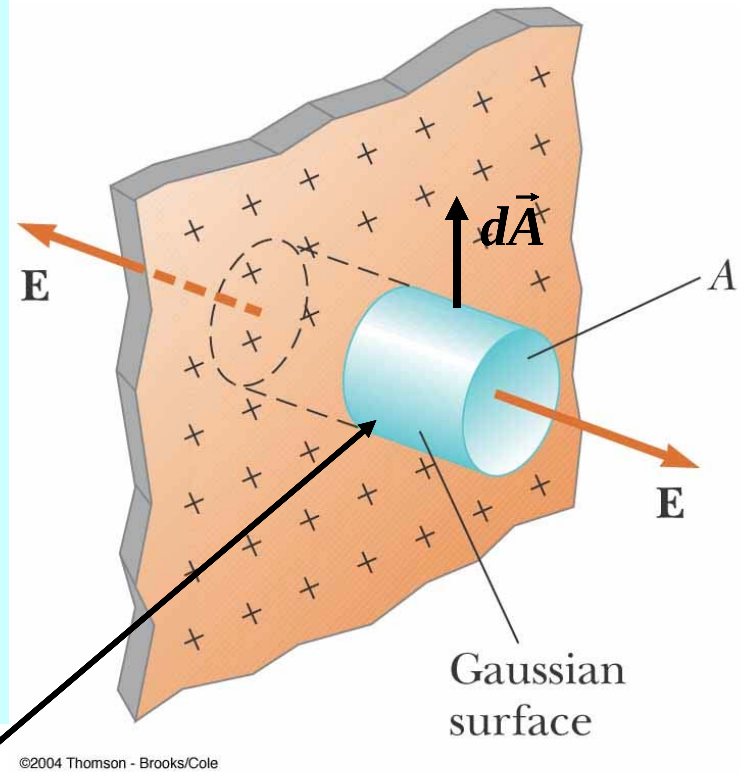
Gauss's Law: Applications

Field due to a plane of charge

- **Symmetry:**

  E must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane

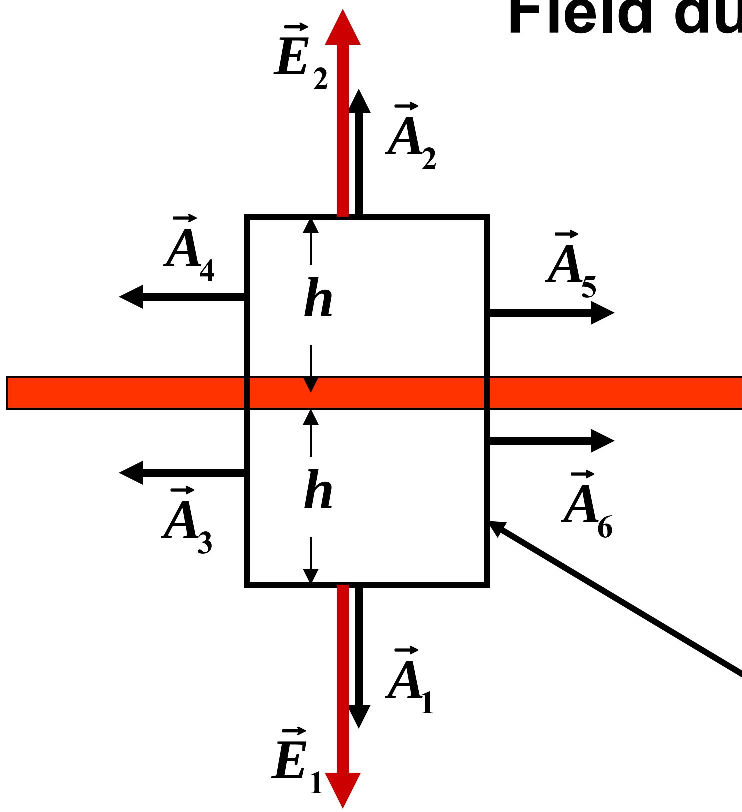
- Choose a small cylinder whose axis is perpendicular to the plane for the gaussian surface

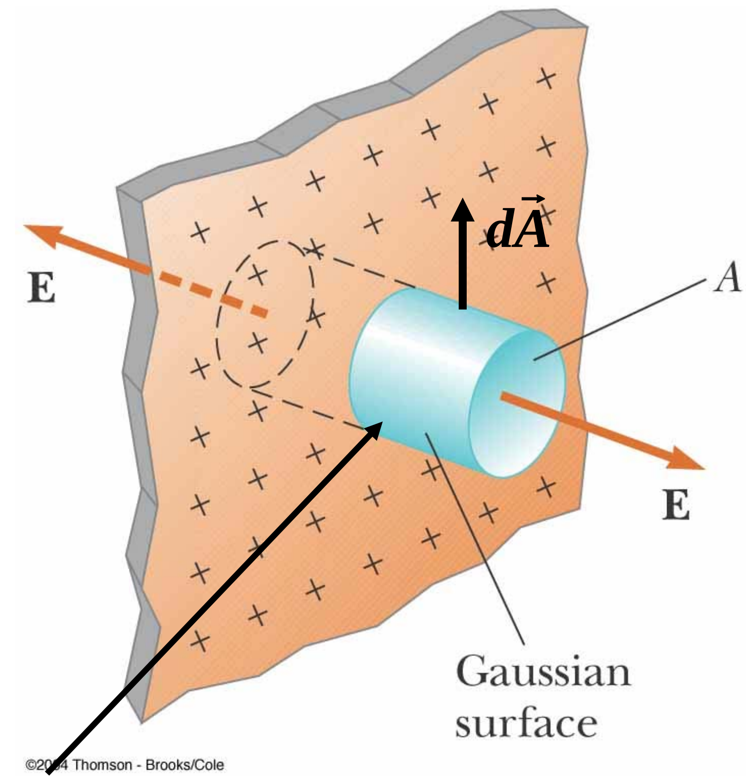


The flux through this surface is **0**

Gauss's Law: Applications

Field due to a plane of charge





The flux through this surface is **0**

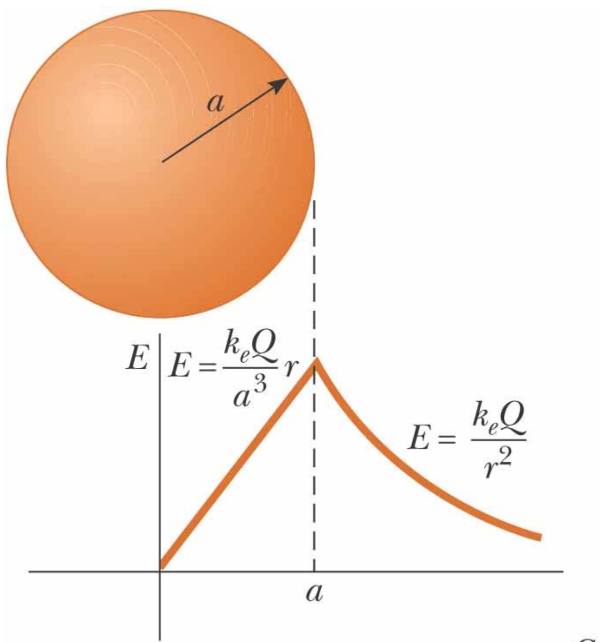
$$E_1 = E_2 = E \qquad A_1 = A_2 = A$$

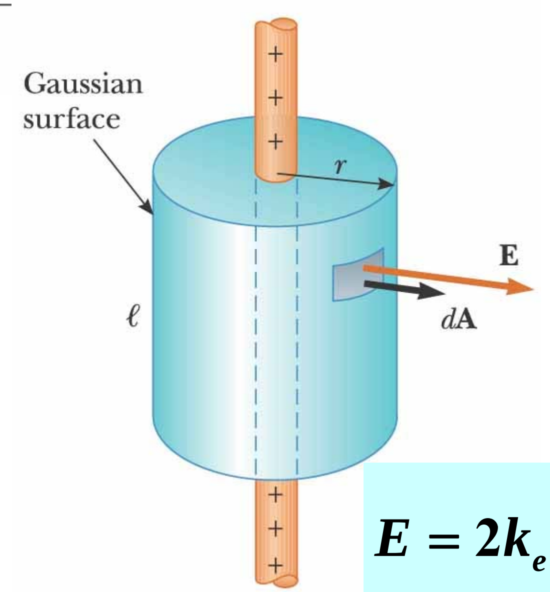
$$\Phi = \vec{E}_1\vec{A}_1 + \vec{E}_2\vec{A}_2 = EA + EA = 2EA$$

$$\Phi = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad 2EA = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

does not depend on ***h***

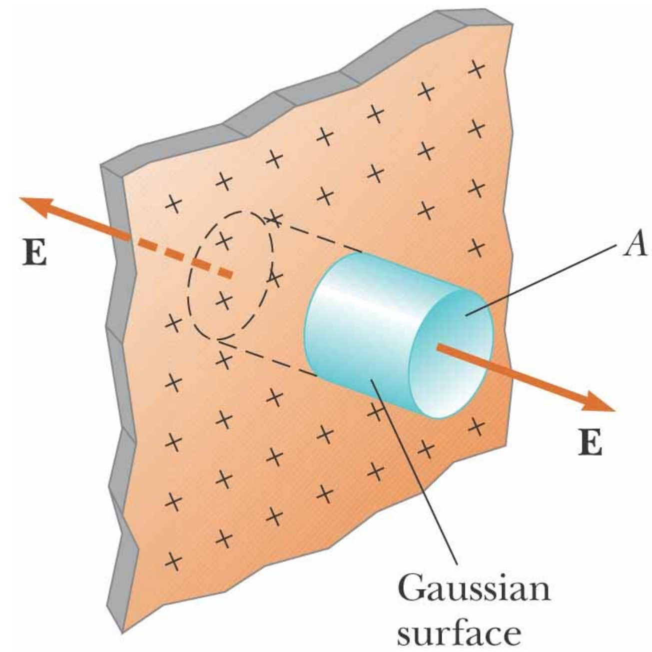
Gauss's Law: Applications





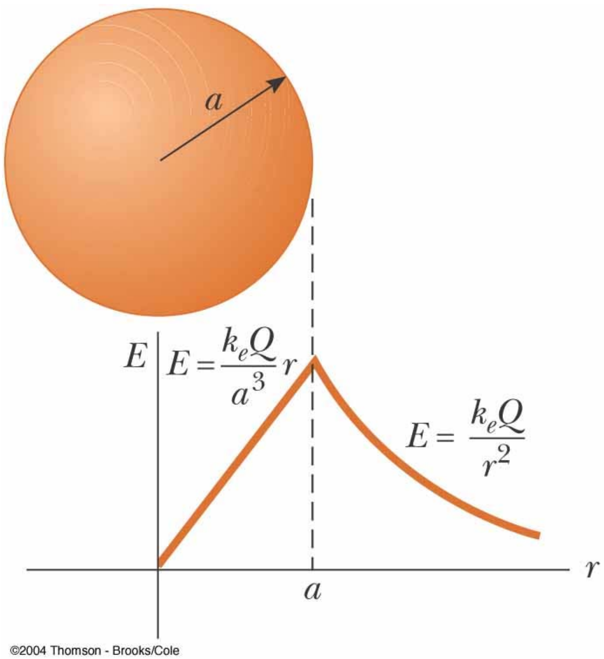
(a)

$$E = 2k_e \frac{\lambda}{r}$$



$$E = \frac{\sigma}{2\varepsilon_0}$$

Gauss's Law: Applications



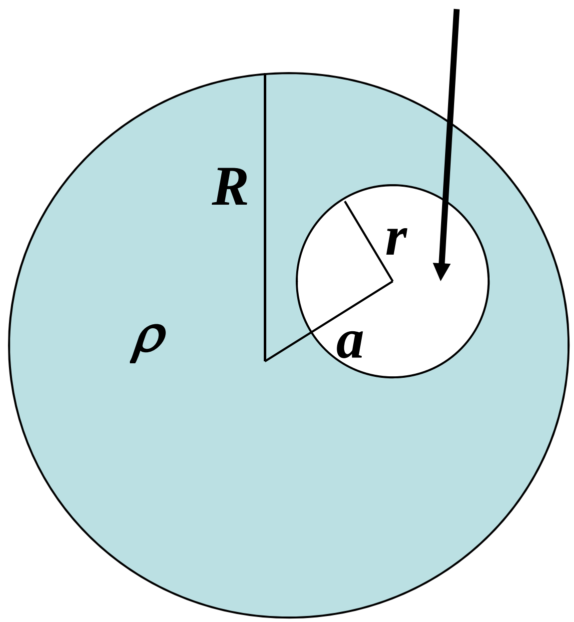
$$Q = \frac{4}{3}\pi a^3 \rho$$

$$E = k_e \frac{Q}{a^3} r = k_e \frac{\frac{4}{3}\pi a^3 \rho}{a^3} r = \frac{4}{3}\pi k_e \rho r$$

$$\vec{E} = \frac{4}{3}\pi k_e \rho \vec{r}$$

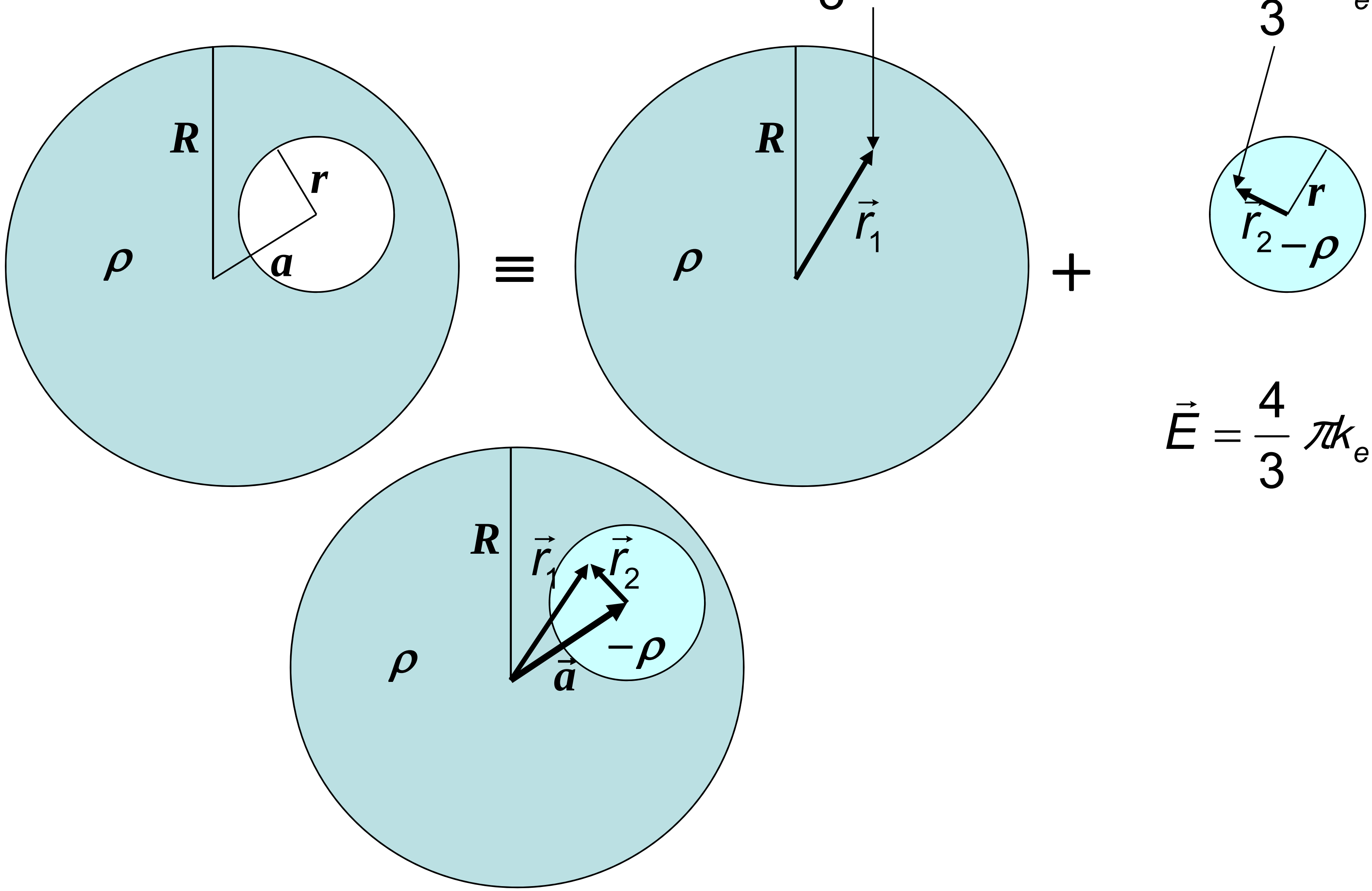
Example

Find electric field inside the hole



$$\vec{E} = \frac{4}{3}\pi k_e \rho \vec{r}_1$$

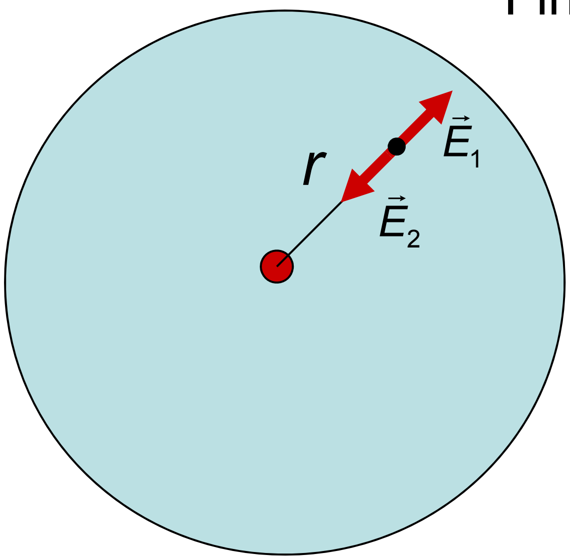
$$\vec{E} = -\frac{4}{3}\pi k_e \rho \vec{r}_2$$



$$\vec{E} = \frac{4}{3}\pi k_e \rho \vec{r}$$

$$\vec{E} = \frac{4}{3}\pi k_e \rho \vec{r}_1 - \frac{4}{3}\pi k_e \rho \vec{r}_2 = \frac{4}{3}\pi k_e \rho (\vec{r}_1 - \vec{r}_2) = \frac{4}{3}\pi k_e \rho \vec{a} = const$$

Example



The sphere has a charge ***Q*** and radius ***a.*** The point charge ***–Q/8*** is placed at the center of the sphere. Find all points where electric field is zero.

$$\vec{E}_1 = k_e \frac{Q}{a^3}\vec{r}$$

$$\vec{E}_2 = -k_e \frac{Q}{8r^2}\vec{\mathbf{r}}$$

$$\vec{E}_1 + \vec{E}_2 = 0$$

$$k_e \frac{Q}{a^3} r = k_e \frac{Q}{8r^2}$$

$$r^3 = \frac{a^3}{8}$$

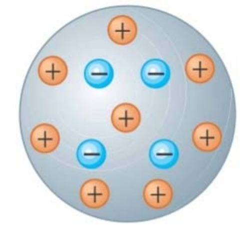
$$r = \frac{a}{2}$$

Conductors in Electric Field

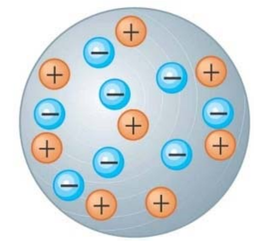
Electric Charges: Conductors and Isolators

➢ Electrical conductors are materials in which some of the electrons are free electrons

- ❑ These electrons can move relatively freely through the material
- ❑ Examples of good conductors include copper, aluminum and silver



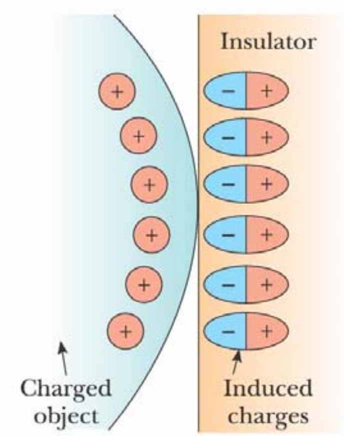
(e)



(a)

➢ Electrical insulators are materials in which all of the electrons are bound to atoms

- ❑ These electrons can not move relatively freely through the material
- ❑ Examples of good insulators include glass, rubber and wood



(a)

➢ Semiconductors are somewhere between insulators and conductors

Electrostatic Equilibrium

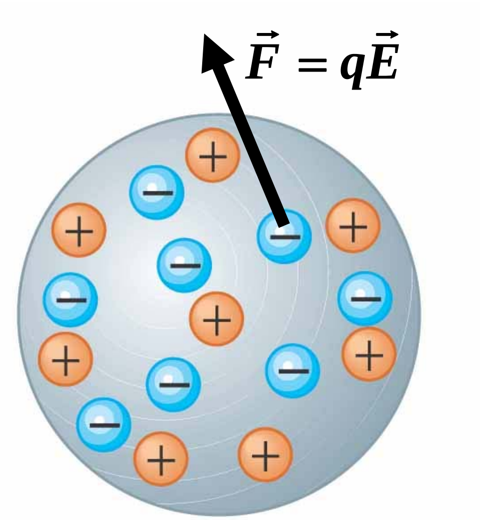
Definition:

when there is no net motion of charge within a conductor, the conductor is said to be in **electrostatic equilibrium**

Because the electrons can move freely through the material

- **no motion** means that there are **no electric forces**
- **no electric forces means that the electric field inside the conductor is 0**

If electric field inside the conductor is not **0**, $\vec{E} \neq 0$ then there is an electric force $\vec{F} = q\vec{E}$ and, from the second Newton's law, there is a motion of free electrons.

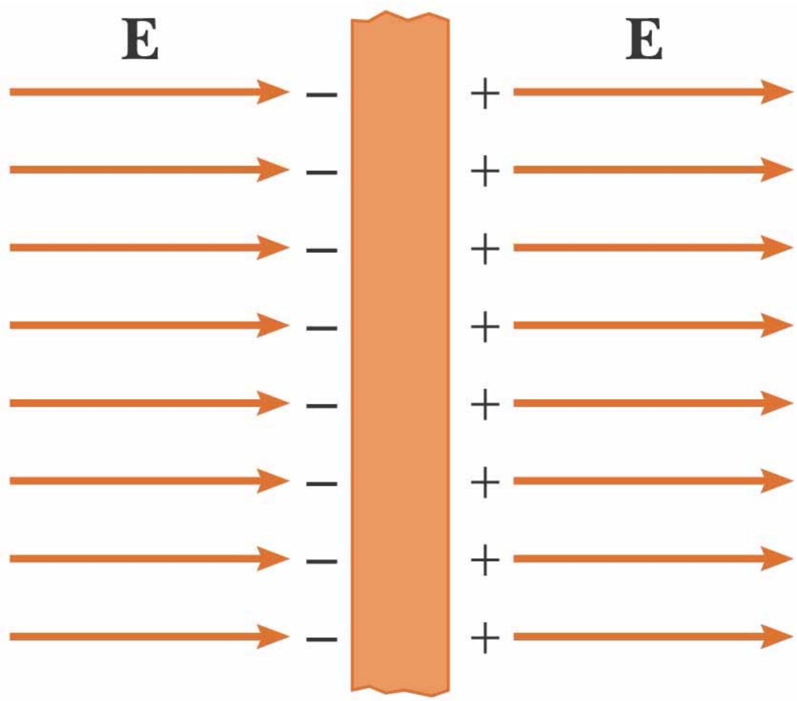


(a)

Conductor in Electrostatic Equilibrium

- **The electric field is zero everywhere inside the conductor**

- Before the external field is applied, free electrons are distributed throughout the conductor
- When the external field is applied, the electrons redistribute until the magnitude of the internal field equals the magnitude of the external field
- There is a net field of zero inside the conductor



Conductor in Electrostatic Equilibrium

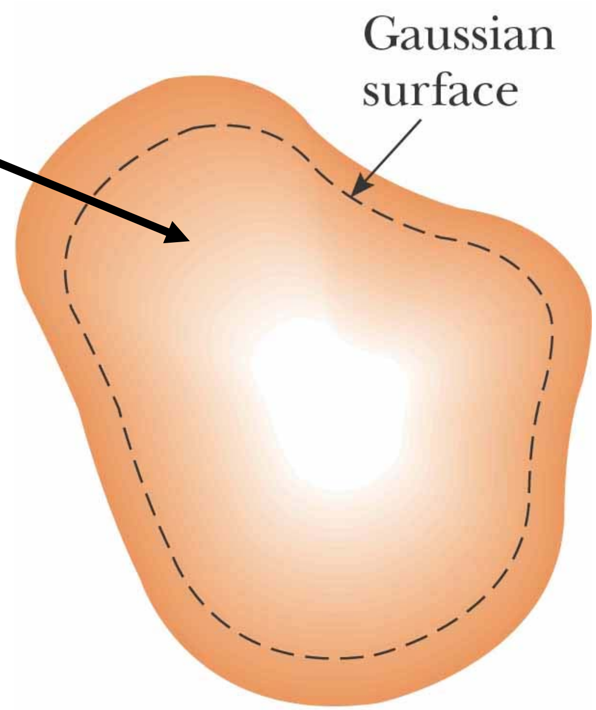
- **If an isolated conductor carries a charge, the charge resides on its surface**

Electric filed is 0,

so the net flux through Gaussian surface is 0

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

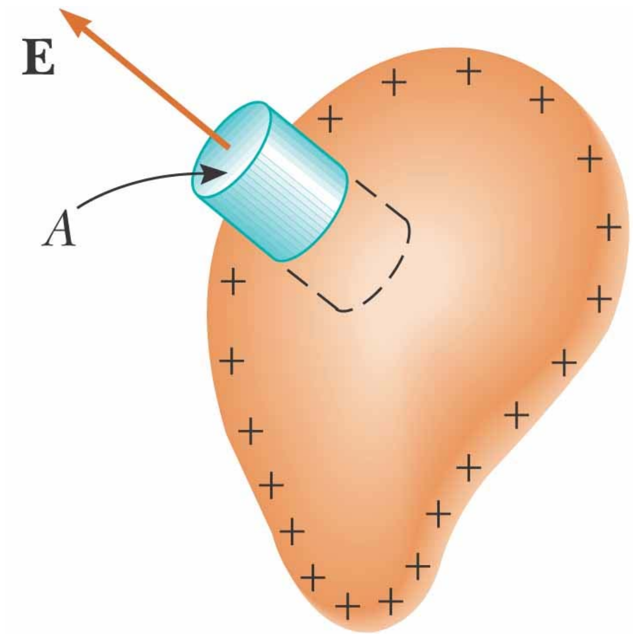
Then $q_{in} = 0$



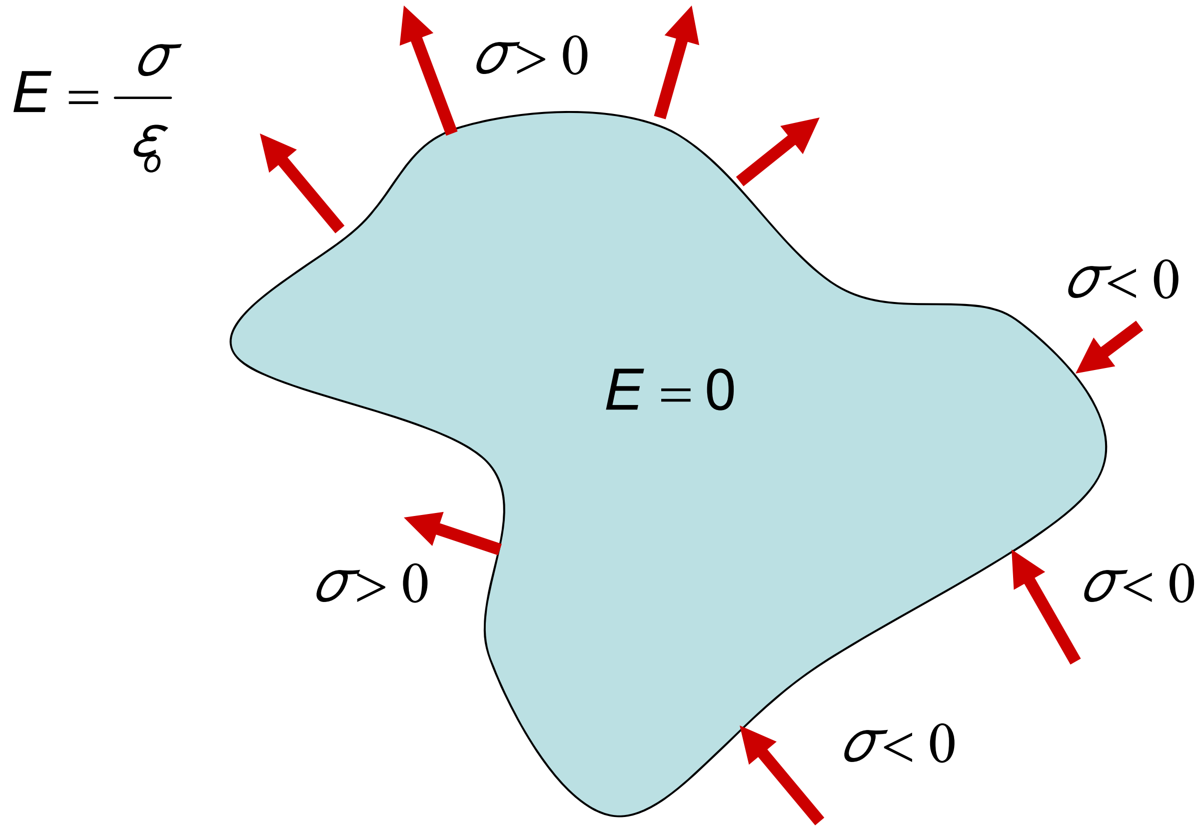
Conductor in Electrostatic Equilibrium

- **The electric field just outside a charged conductor is perpendicular to the surface and has a magnitude of σ/ε_o**

- Choose a cylinder as the gaussian surface
- The field must be perpendicular to the surface
  - If there were a parallel component to **E**, charges would experience a force and accelerate along the surface and it would not be in equilibrium
- The net flux through the gaussian surface is through only the flat face outside the conductor
  - The field here is perpendicular to the surface
- Gauss's law: $$\Phi_E = EA = \frac{\sigma A}{\varepsilon_o} \text{ and } E = \frac{\sigma}{\varepsilon_o}$$

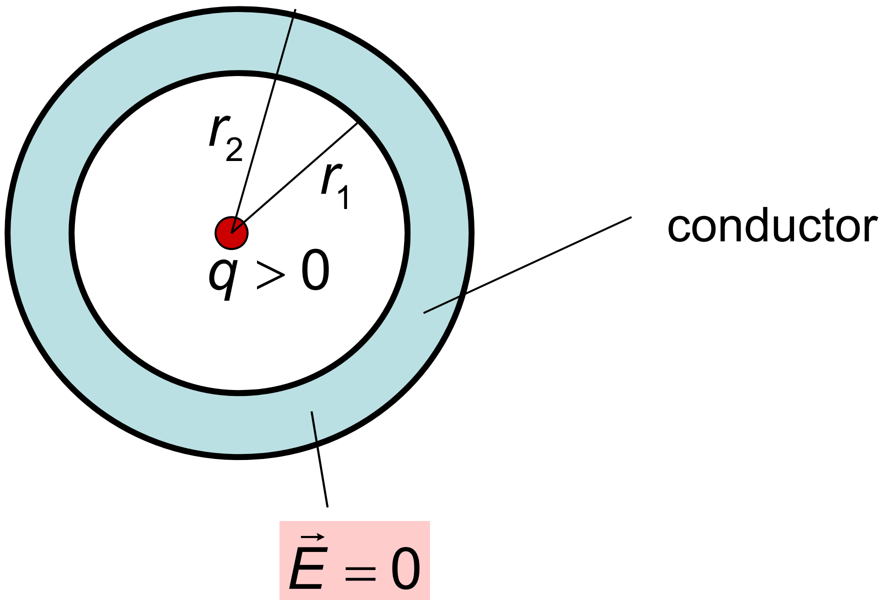


Conductor in Electrostatic Equilibrium



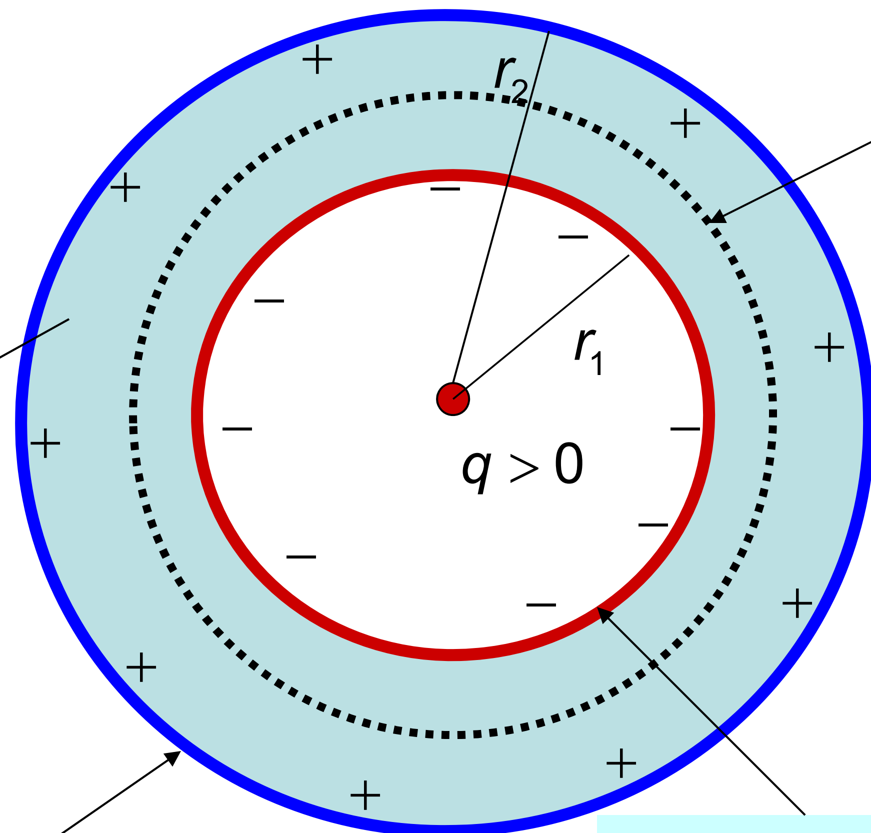
Conductor in Electrostatic Equilibrium: Example

Find electric field if the conductor spherical shell has zero charge



Conductor in Electrostatic Equilibrium: Example

Find electric field if the conductor spherical shell has zero charge



The total charge inside this Gaussian surface is 0, so the electric field is 0

$\vec{E} = 0$

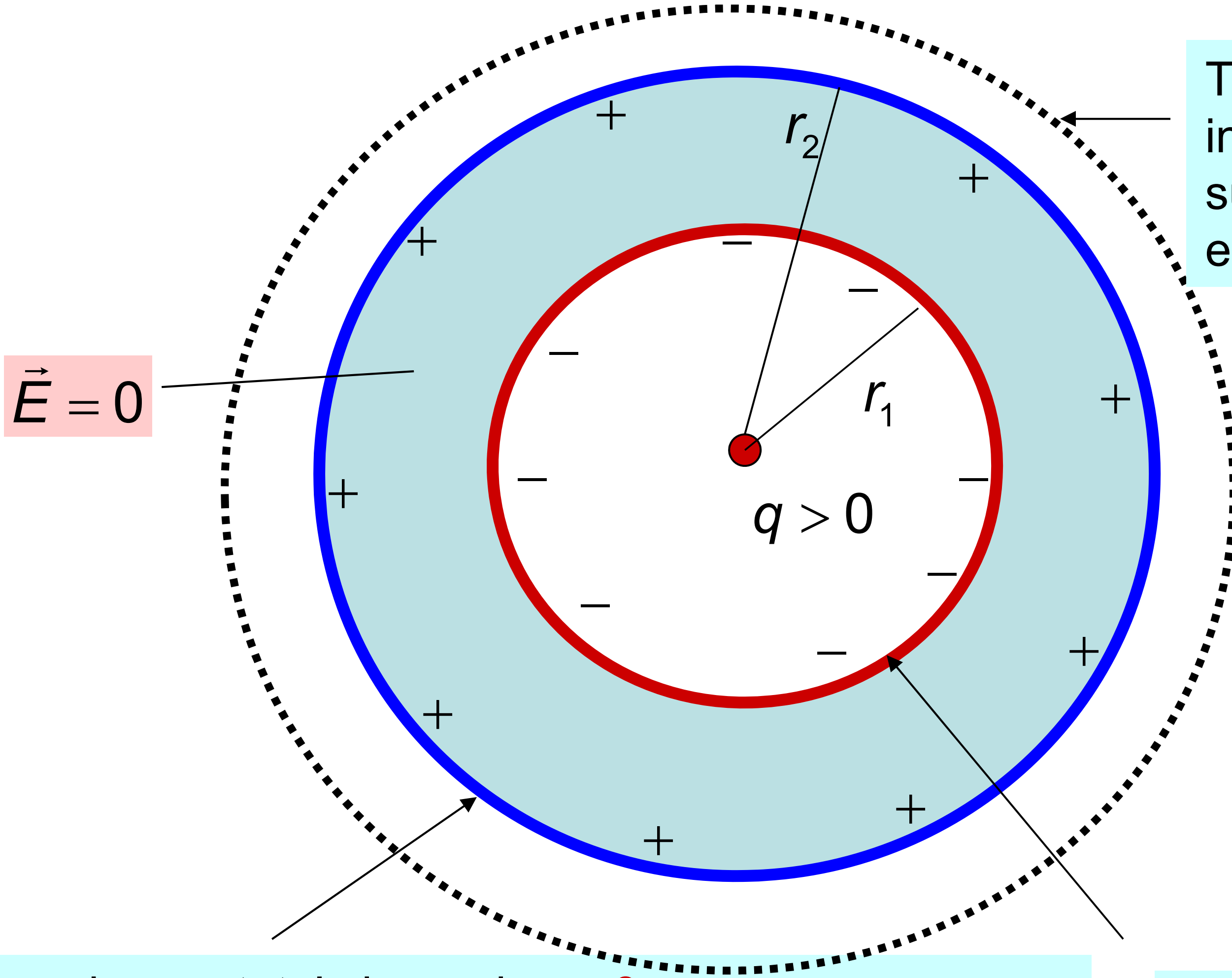
surface charge, total charge is **–q<0**

surface charge, total charge is **q>0**

This is because the total charge of the conductor is **0!!!**

Conductor in Electrostatic Equilibrium: Example

Find electric field if the conductor spherical shell has zero charge



The total charge inside this Gaussian surface is q, so the electric field is

$$E = k_e \frac{q}{r^2}$$

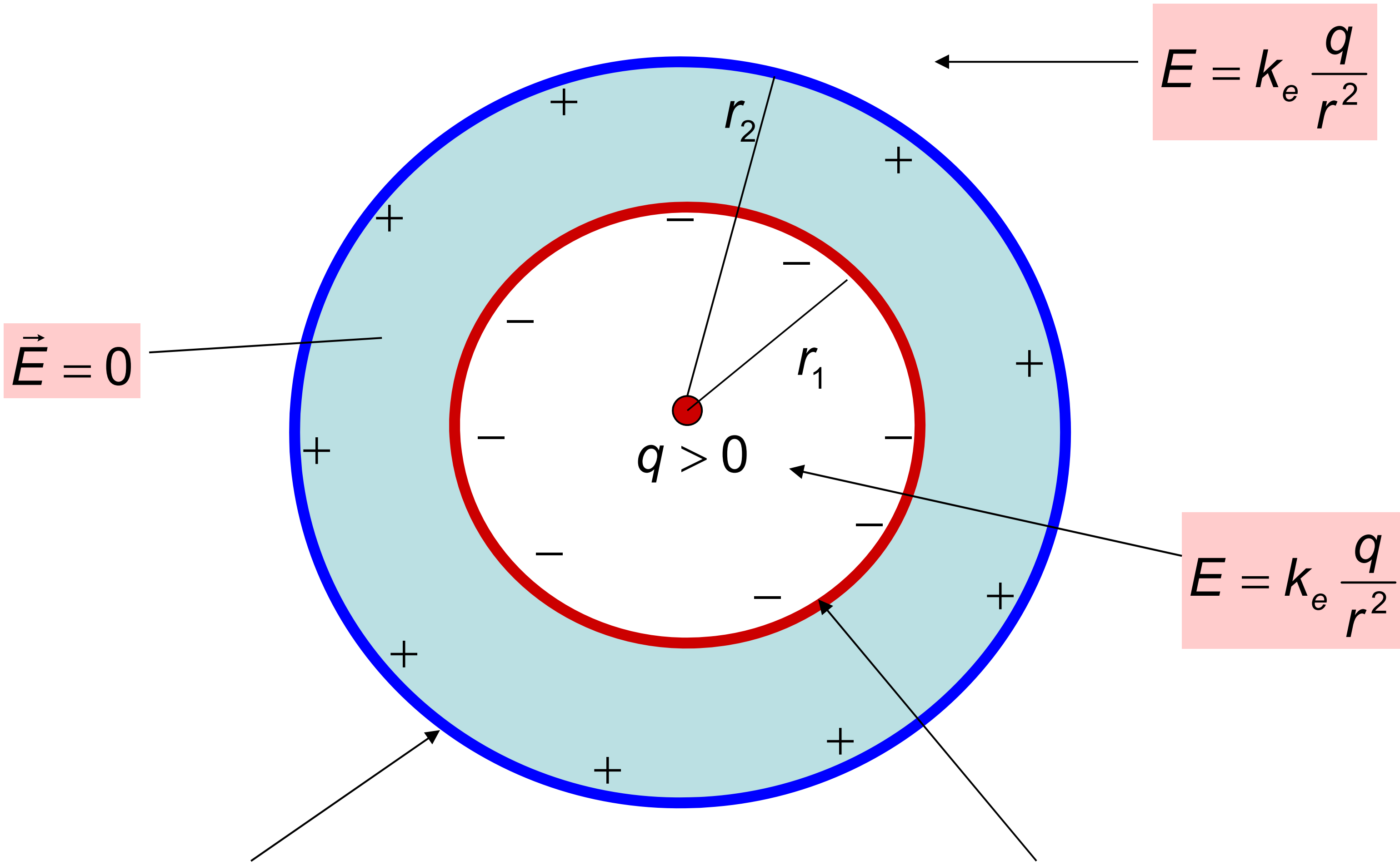
surface charge, total charge is **q>0**

This is because the total charge of the conductor is **0!!!**

total charge is **–q<0**

Conductor in Electrostatic Equilibrium: Example

Find electric field if the conductor spherical shell has zero charge



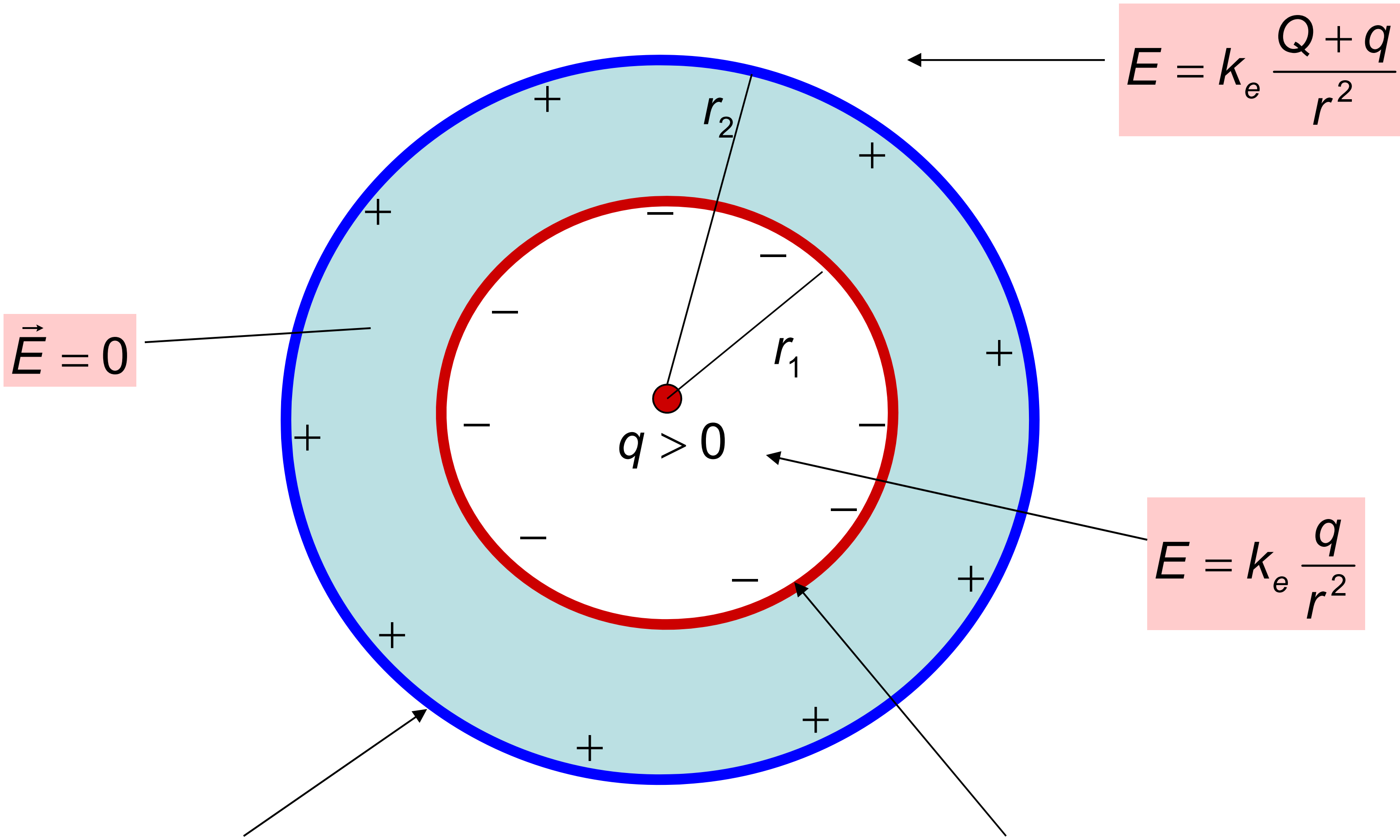
surface charge, total charge is **q>0**

This is because the total charge of the conductor is **0!!!**

total charge is **–q<0**

Conductor in Electrostatic Equilibrium: Example

Find electric field if the charge of the conductor spherical shell is ***Q***



surface charge, total charge is **Q+q>0**

This is because the total charge of the conductor is **Q!!!**

total charge is **–q<0**