Chapter 26

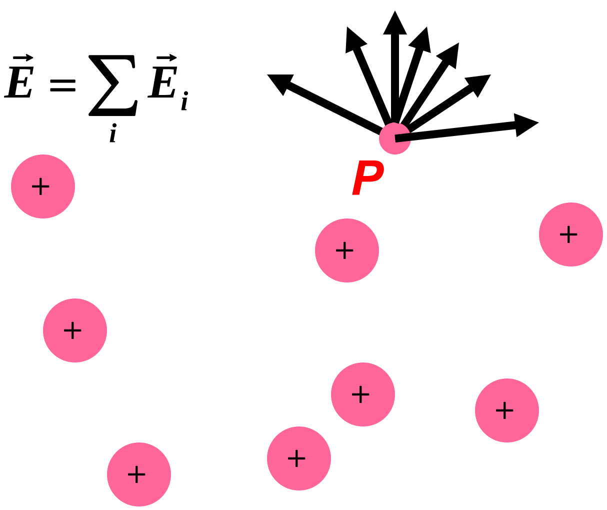
Electricity and Magnetism

Electric Field: Continuous Charge Distribution

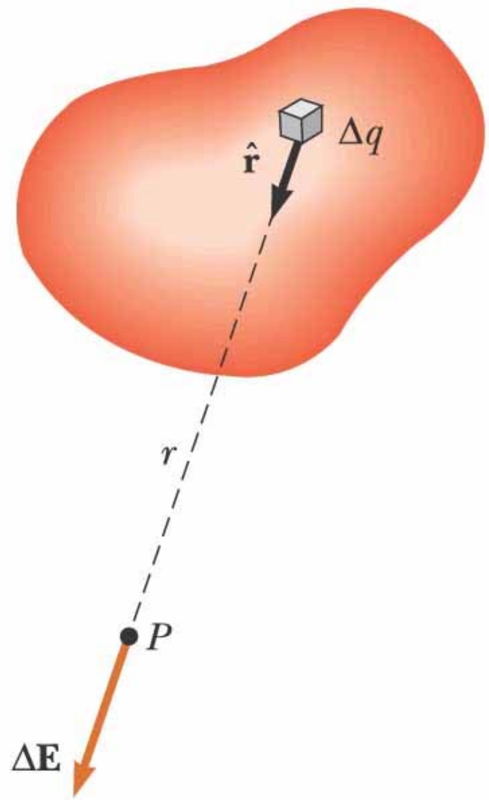
Find electric field at point *P*.

$$\vec{E}_i = k_e \frac{q_i}{r_i^{\,2}} \hat{\vec{r}}_i$$

$$\vec{E} = \sum_i \vec{E}_i$$



Continuous Charge Distribution



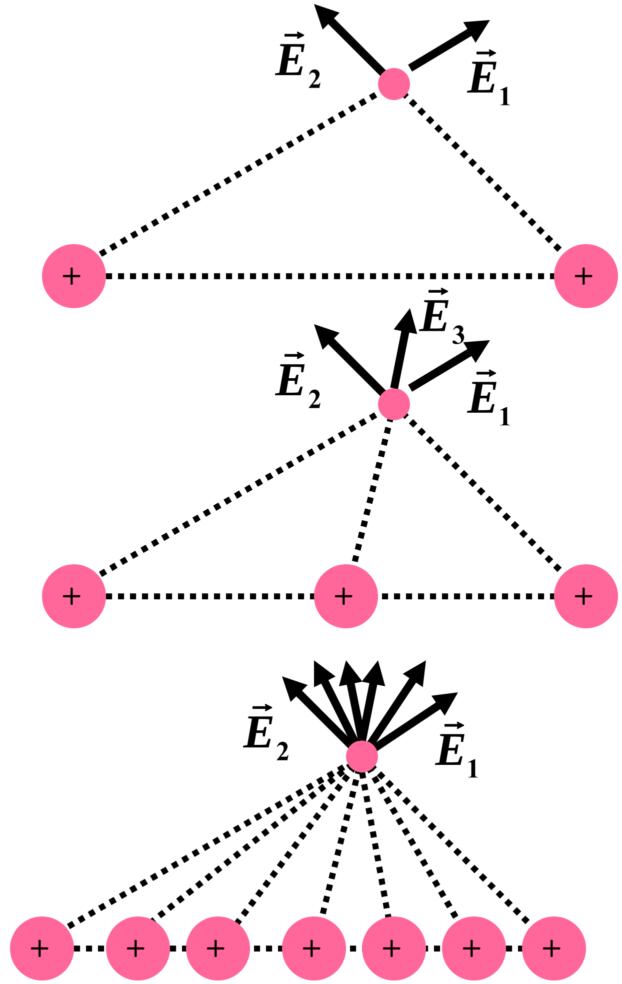
$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

Electric Field: Continuous Charge Distribution

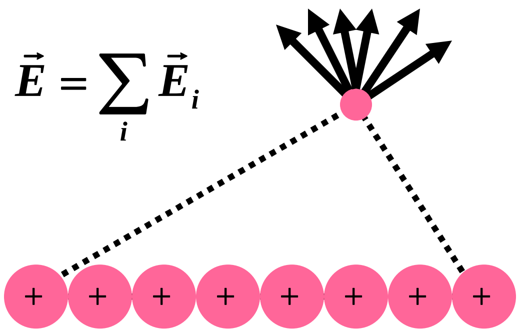
$$\vec{E} = k_e \frac{q}{r^2}\hat{\vec{r}}$$

Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



$$\vec{E} = \sum_i \vec{E}_i$$

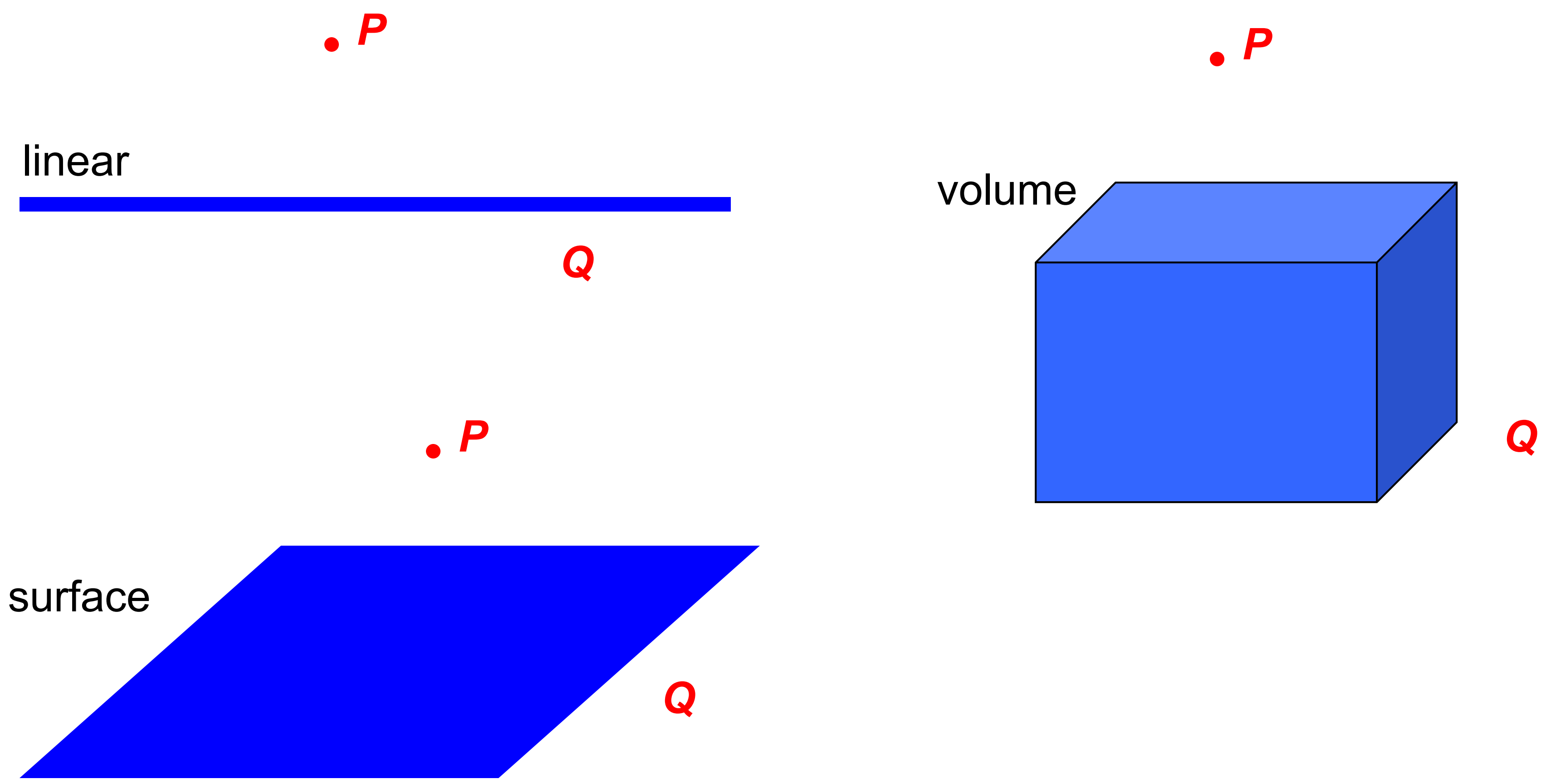


In this situation, the system of charges can be modeled as continuous

The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume

Electric Field: Continuous Charge Distribution

The total electric charge is ***Q***. What is the electric field at point ***P*** ?



Continuous Charge Distribution: Charge Density

The total electric charge is ***Q.***

Linear, length ***L***

Q

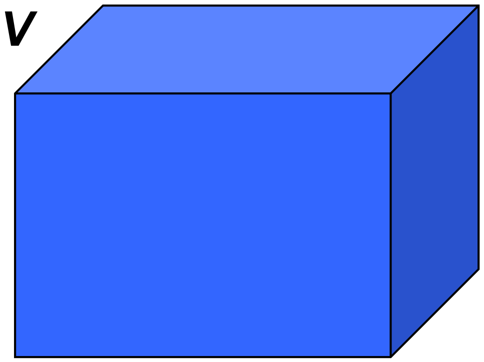
Amount of charge in a small volume ***dl:***

$$dq = \frac{Q}{L} dl = \lambda\, dl$$

$$\lambda = \frac{Q}{L}$$

Linear charge density

Volume ***V***



Q

Amount of charge in a small volume ***dV:***

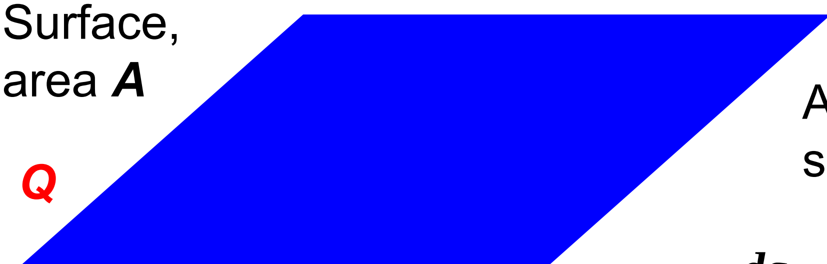
$$dq = \frac{Q}{V} dV = \rho\, dV$$

$$\rho = \frac{Q}{V}$$

Volume charge density

Surface, area ***A***

Q



Amount of charge in a small volume ***dA:***

$$dq = \frac{Q}{A} dA = \sigma\, dA$$

$$\sigma = \frac{Q}{A}$$

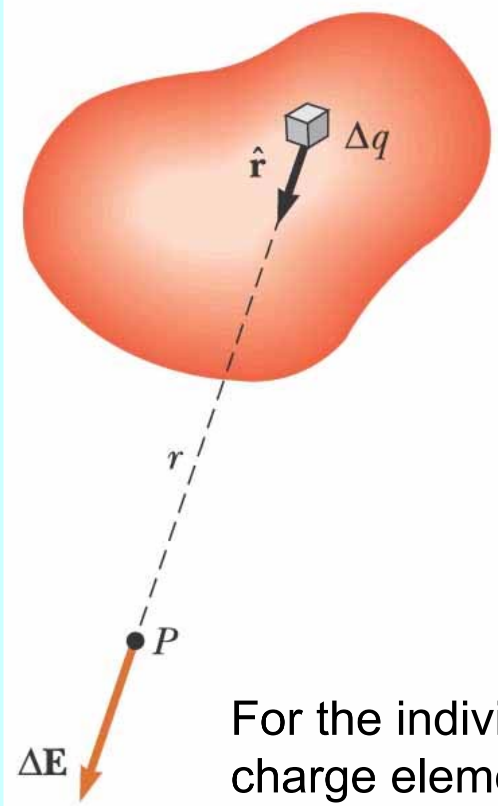
Surface charge density

Electric Field: Continuous Charge Distribution

Procedure:

- Divide the charge distribution into small elements, each of which contains Δq
- Calculate the electric field due to one of these elements at point ***P***
- Evaluate the total field by summing the contributions of all the charge elements

Symmetry: take advantage of any symmetry to simplify calculations



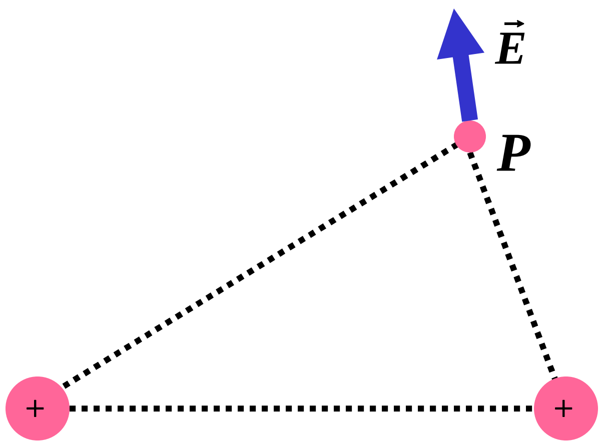
For the individual charge elements

$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

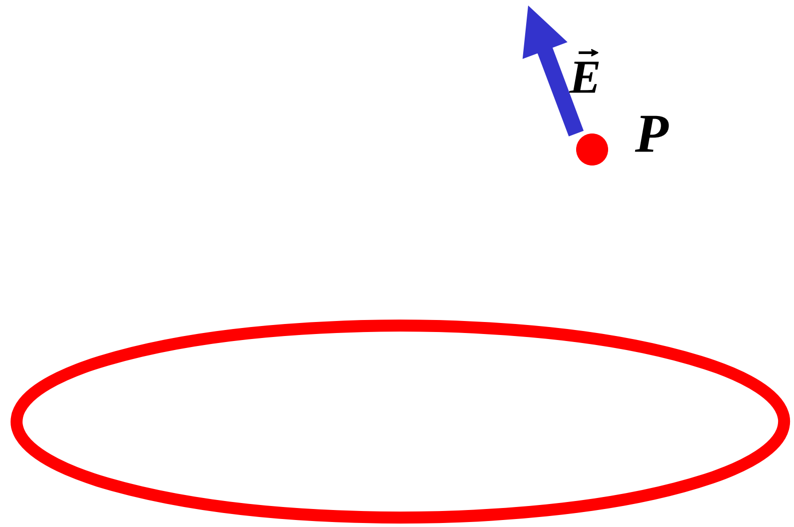
Because the charge distribution is continuous

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Electric Field: Symmetry



no symmetry



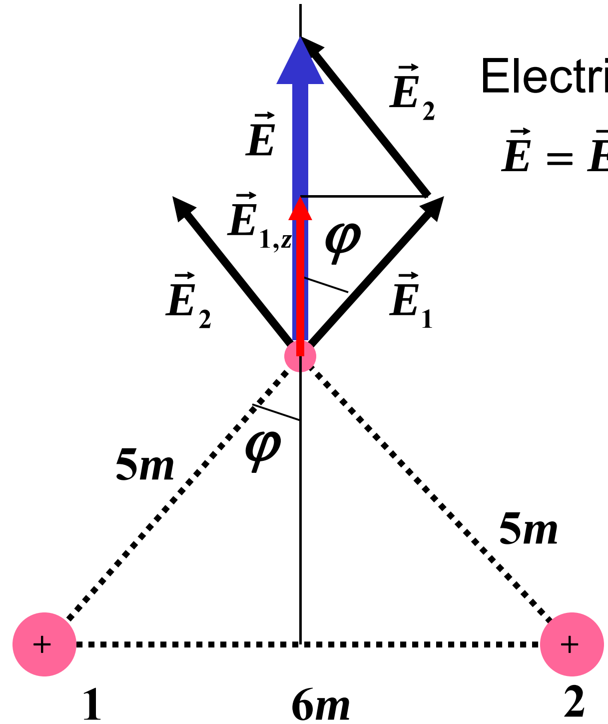
no symmetry

no symmetry

Electric Field: Symmetry

$$\vec{E} = k_e \frac{q}{r^2}\hat{\vec{r}}$$

$q_1 = 10\,\mu C$ $q_2 = 10\,\mu C$

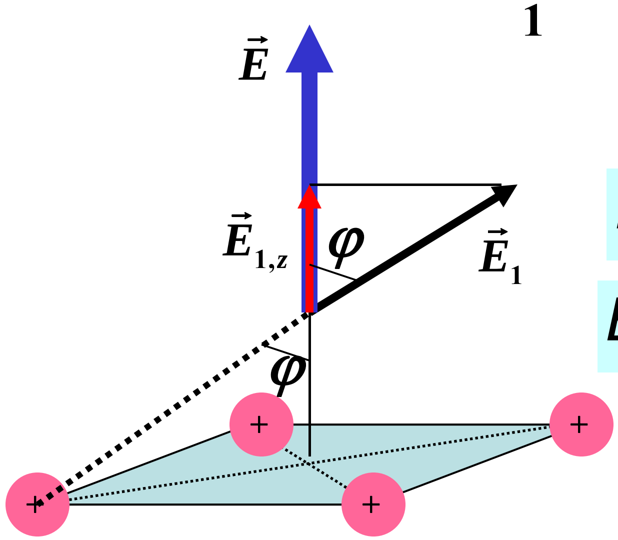


Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E = 2E_1 \cos\varphi$$

$$E = 2E_{1,z} = 2E_{2,z}$$



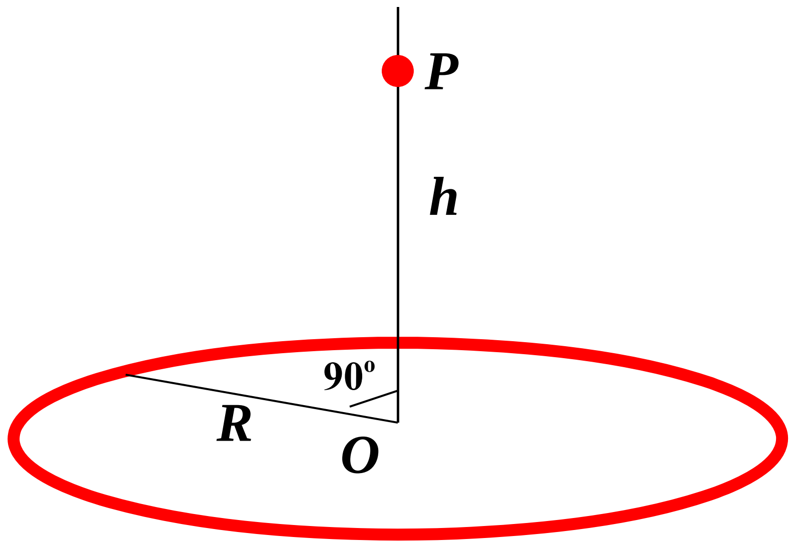
$$E_{1,z} = E_1 \cos\varphi$$

$$E = 4E_{1,z} = 4E_2 \cos\varphi$$

The symmetry gives us the direction of resultant electric field

Electric Field: Continuous Charge Distribution

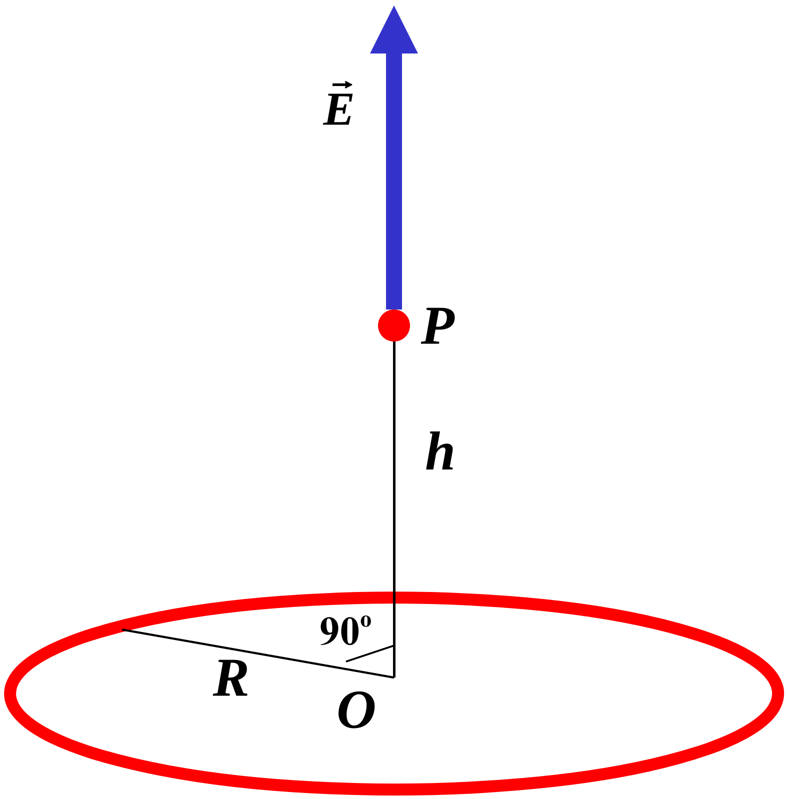
What is the electric field at point *P* ?



λ **- linear charge density**

What is the electric field at point *P* ? λ **- linear charge density**

1. **Symmetry** determines the direction of the electric field.



What is the electric field at point *P* ? λ - linear charge density

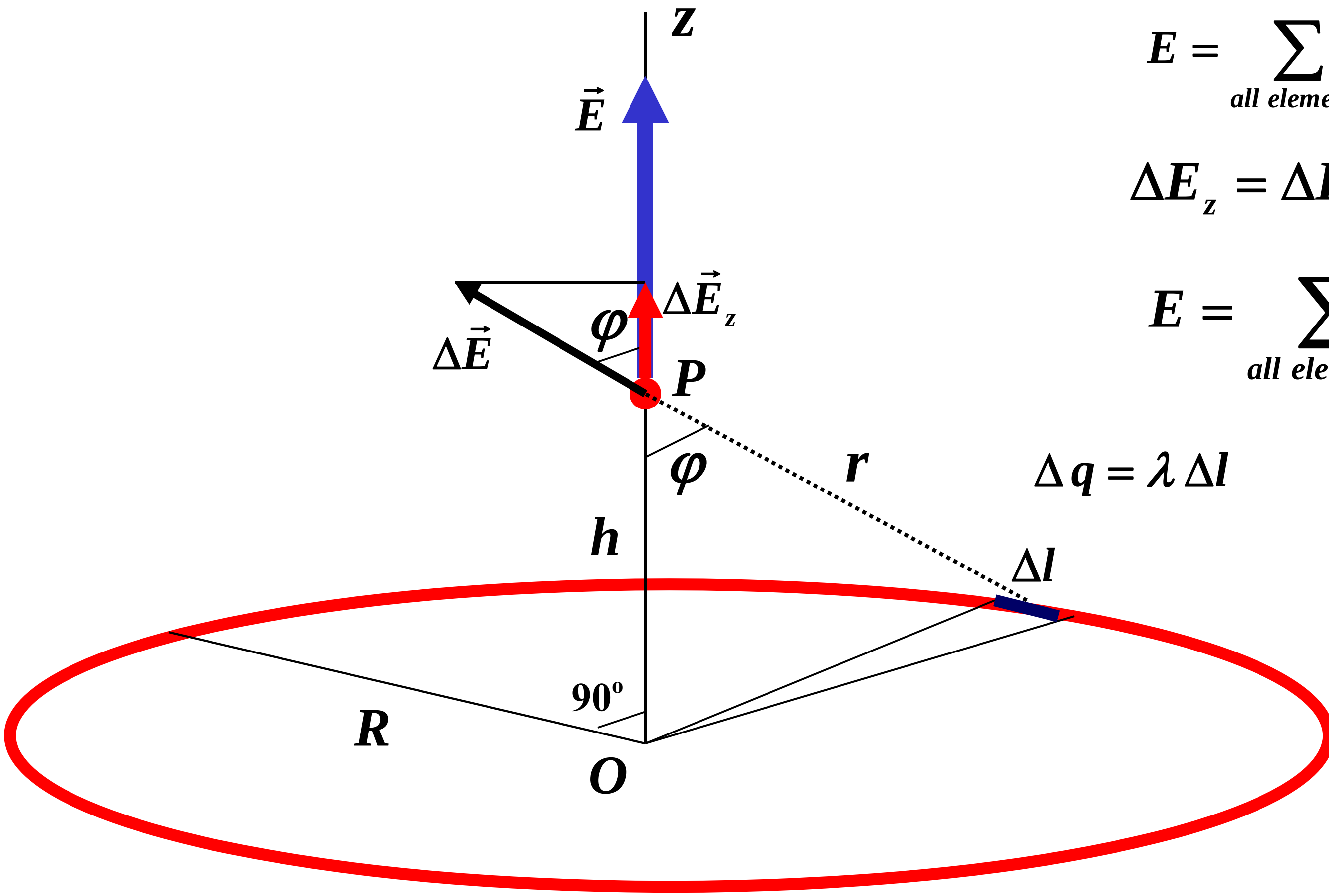
2. Divide the charge distribution into small elements, each of which contains ***Δq***

$$\vec{E} = \sum_{all\ elements} \Delta\vec{E} = \sum_{all\ elements} \Delta\vec{E}_z$$

$$E = \sum_{all\ elements} \Delta E_z$$

$$\Delta E_z = \Delta E \cos\varphi = k_e \frac{\Delta q}{r^2} \cos\varphi$$

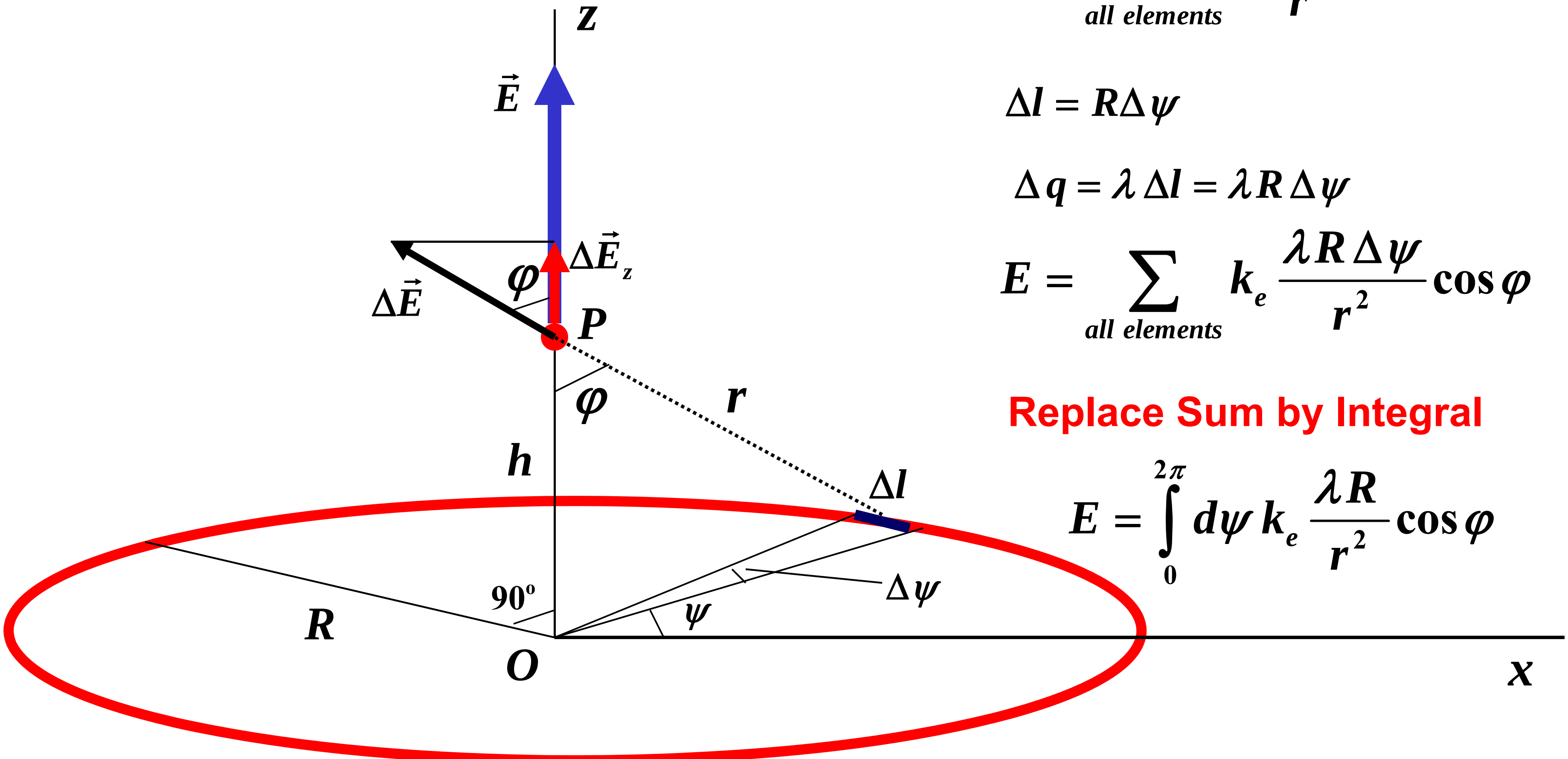
$$E = \sum_{all\ elements} k_e \frac{\Delta q}{r^2} \cos\varphi$$



$$\Delta q = \lambda \Delta l$$

What is the electric field at point *P*? λ **- linear charge density**

3. Evaluate the total field by summing the contributions of all the charge elements ***Δq***



$$E = \sum_{all\ elements} k_e \frac{\Delta q}{r^2} \cos\varphi$$

$$\Delta l = R \Delta\psi$$

$$\Delta q = \lambda\, \Delta l = \lambda R\, \Delta\psi$$

$$E = \sum_{all\ elements} k_e \frac{\lambda R\, \Delta\psi}{r^2} \cos\varphi$$

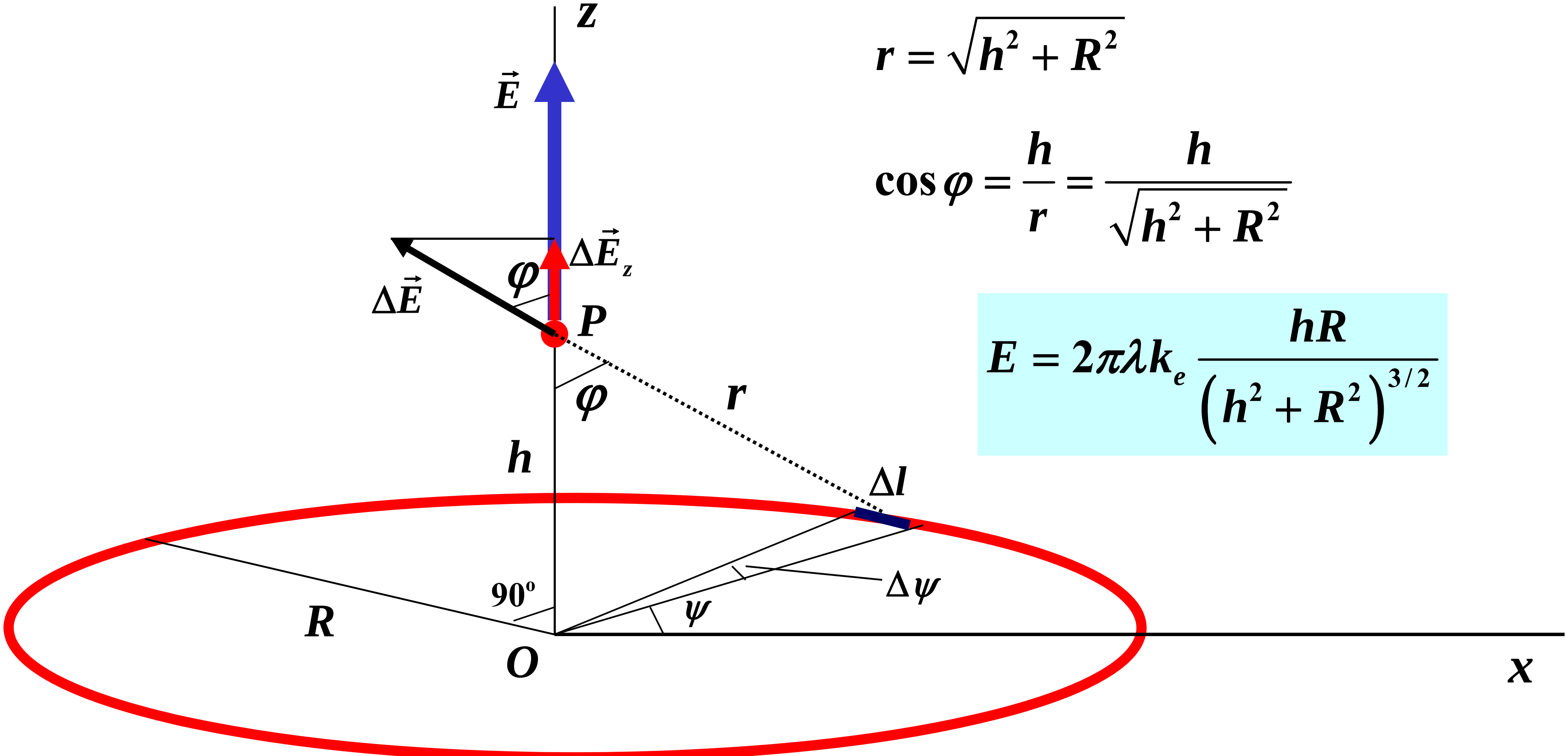
Replace Sum by Integral

$$E = \int_0^{2\pi} d\psi\, k_e \frac{\lambda R}{r^2} \cos\varphi$$

What is the electric field at point *P* ? λ - linear charge density

4. Evaluate the integral

$$E=\int_0^{2\pi} d\psi\, k_e \frac{\lambda R}{r^2}\cos\varphi = k_e \frac{\lambda R}{r^2}\cos\varphi \int_0^{2\pi} d\psi = 2\pi k_e \frac{\lambda R}{r^2}\cos\varphi$$



$$r=\sqrt{h^2+R^2}$$

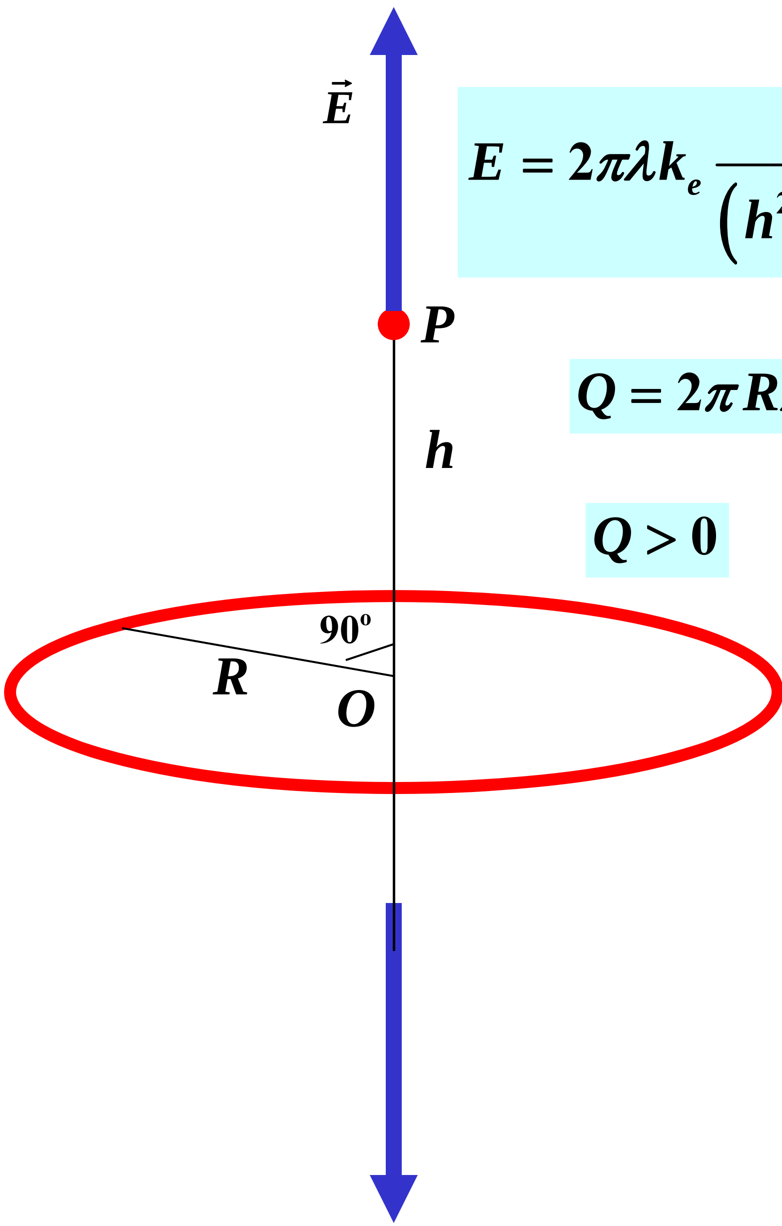
$$\cos\varphi=\frac{h}{r}=\frac{h}{\sqrt{h^2+R^2}}$$

$$E=2\pi\lambda k_e \frac{hR}{\left(h^2+R^2\right)^{3/2}}$$

$$E = 2\pi\lambda k_e \frac{hR}{\left(h^2 + R^2\right)^{3/2}}$$

$$Q = 2\pi R\lambda$$

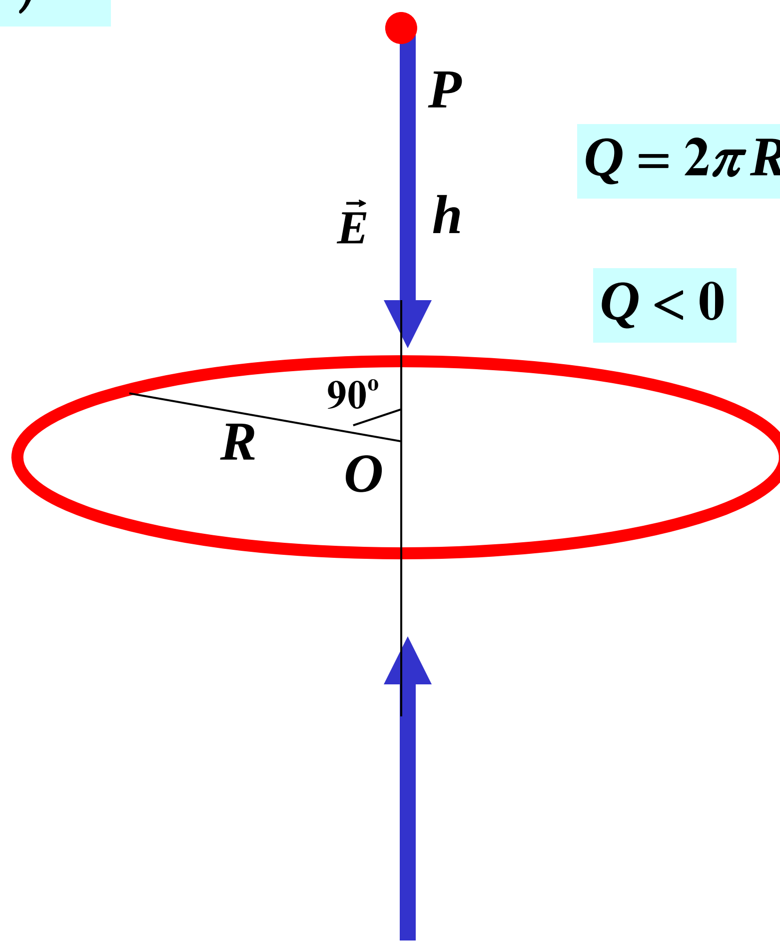
$$Q > 0$$



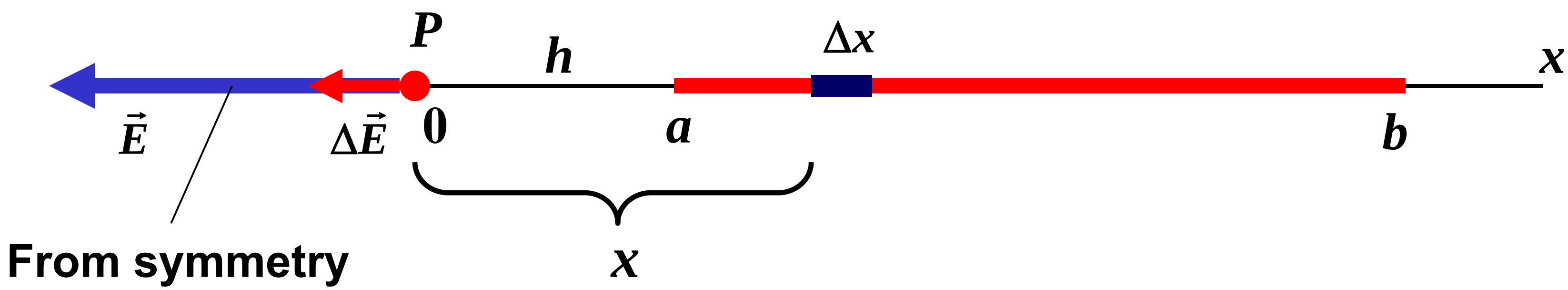
$$E = 2\pi\,|\lambda|\,k_e \frac{hR}{\left(h^2 + R^2\right)^{3/2}}$$

$$Q = 2\pi R\lambda$$

$$Q < 0$$



What is the electric field at point *P* ? λ - linear charge density



$$E = \sum_{all\ elements} \Delta E$$

$$\Delta E = k_e \frac{\Delta q}{x^2}$$

$$\Delta q = \lambda \, \Delta x$$

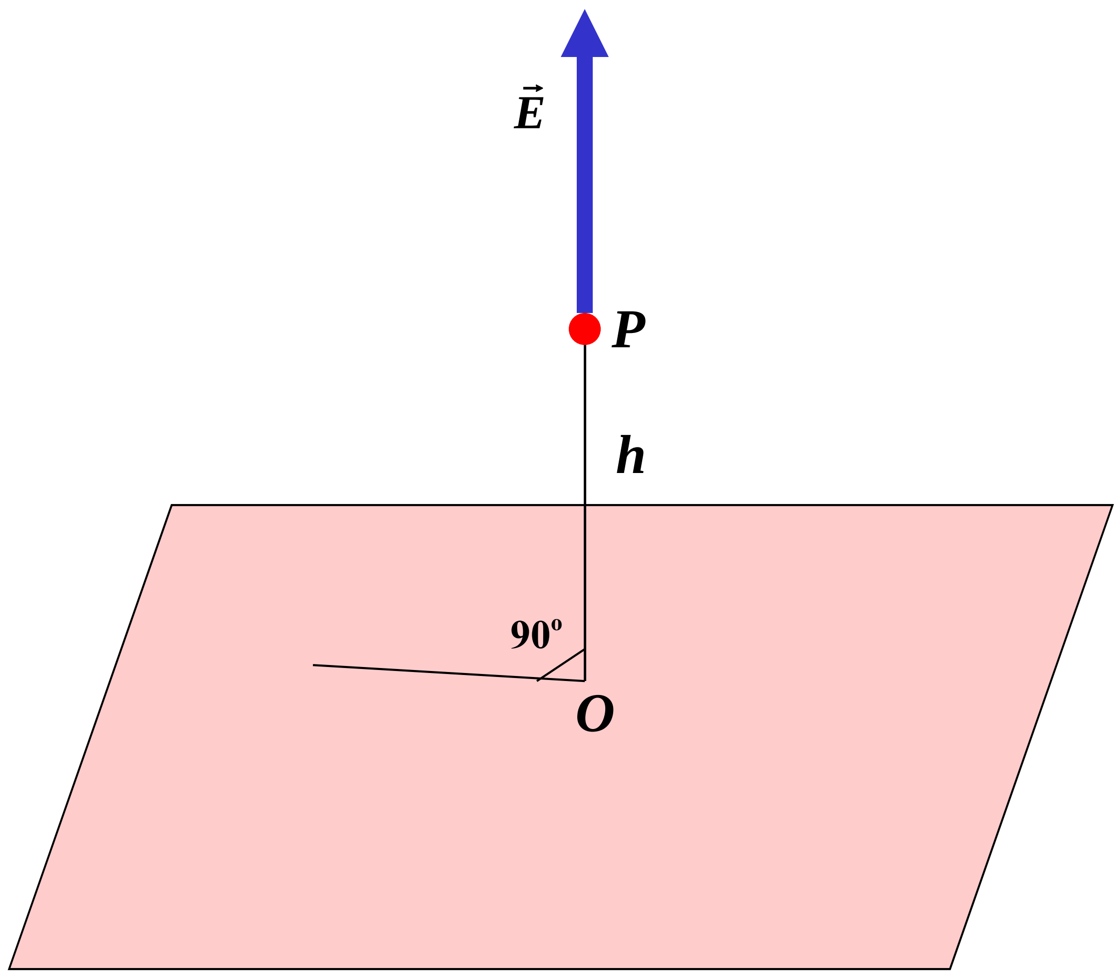
$$E = \sum_{all\ elements} k_e \frac{\Delta q}{x^2} = \sum_{all\ elements} k_e \frac{\lambda \Delta x}{x^2}$$

Replace the Sum by Integral

$$E = k_e \int_a^b dx \frac{\lambda}{x^2} = \lambda k_e \int_a^b \frac{dx}{x^2}$$

What is the electric field at point *P* ? σ - surface charge density

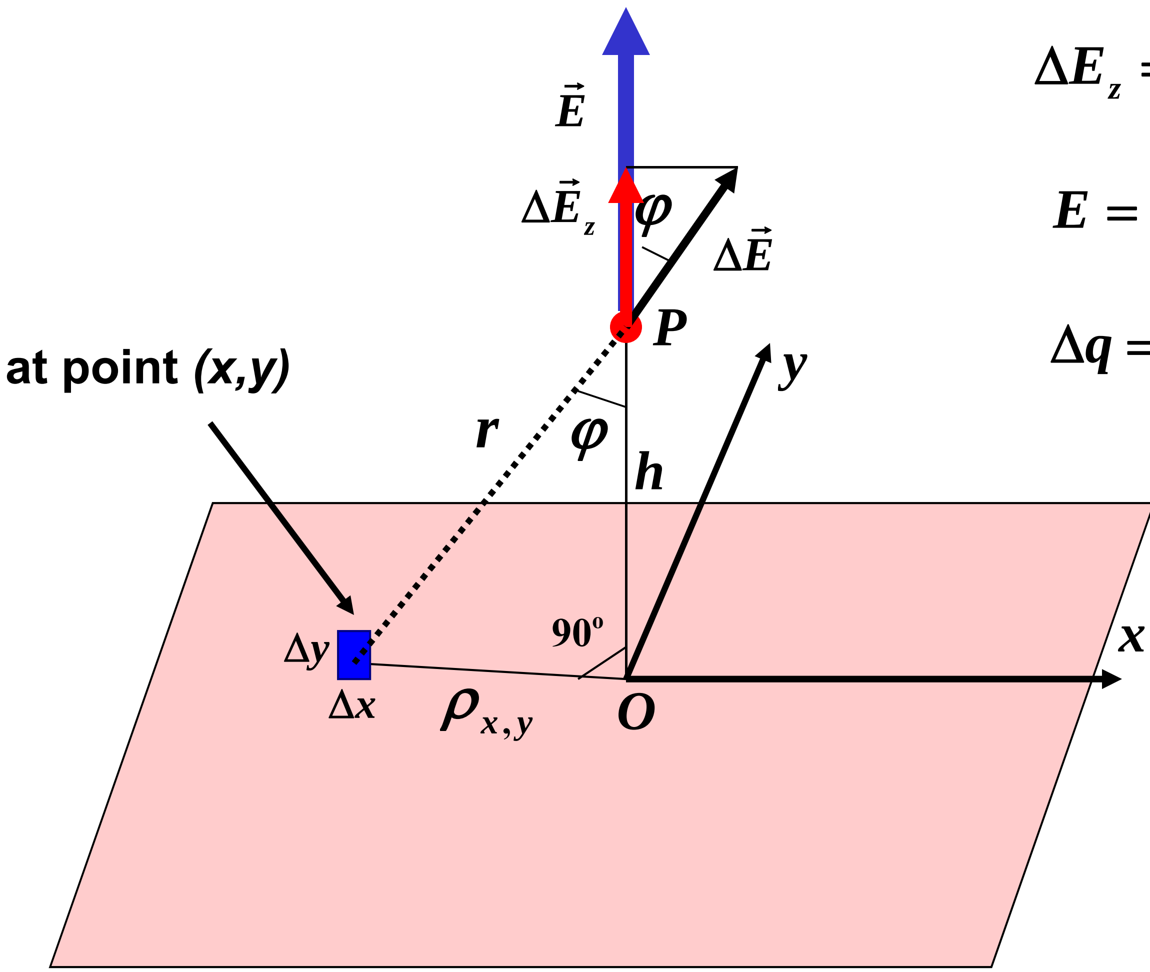
1. **Symmetry** determines the direction of the electric field.



What is the electric field at point ***P*** ? σ - surface charge density

2. Divide the charge distribution into small elements, each of which contains Δq
3. Evaluate the total field by summing the contributions of all the charge elements Δq

Approach A: straightforward



$$E = \sum_{all\ elements} \Delta E_z$$

$$\Delta E_z = \Delta E \cos\varphi = k_e \frac{\Delta q}{r^2} \cos\varphi$$

$$E = \sum_{all\ elements} k_e \frac{\Delta q}{r^2} \cos\varphi$$

$$\Delta q = \sigma \Delta x \Delta y \qquad r = \sqrt{h^2 + \rho_{x,y}^2}$$

$$\cos\varphi = \frac{h}{r} = \frac{h}{\sqrt{h^2 + \rho_{x,y}^2}}$$

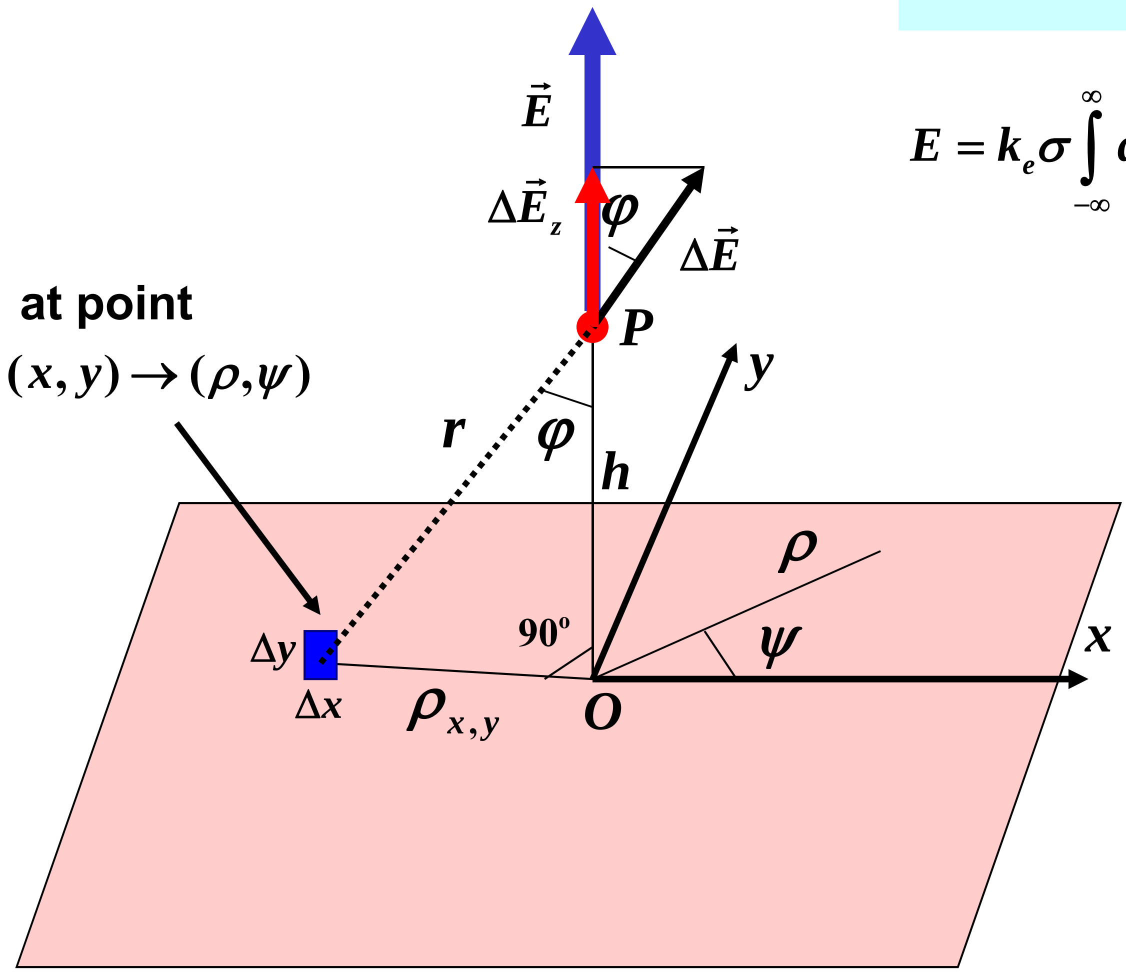
$$E = \sum_{all\ elements} k_e \sigma \Delta x \Delta y \frac{h}{\left(h^2 + \rho_{x,y}^2\right)^{3/2}}$$

Approach A

What is the electric field at point ***P*** ? σ - surface charge density

Replace the Sum by Integral

$$E = \sum_{all\ elements} k_e \sigma \Delta x \Delta y \frac{h}{\left(h^2 + \rho_{x,y}^2\right)^{3/2}}$$



$$E = k_e \sigma \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \frac{h}{\left(h^2 + \rho_{x,y}^2\right)^{3/2}}$$

$$(x,y) \rightarrow (\rho,\psi)$$

$$dxdy = \rho d\rho d\psi$$

$$E = k_e \sigma \int_0^{\infty} d\rho \int_0^{2\pi} d\psi \frac{h\rho}{\left(h^2 + \rho^2\right)^{3/2}}$$

Approach A

What is the electric field at point ***P*** ?

σ - surface charge density

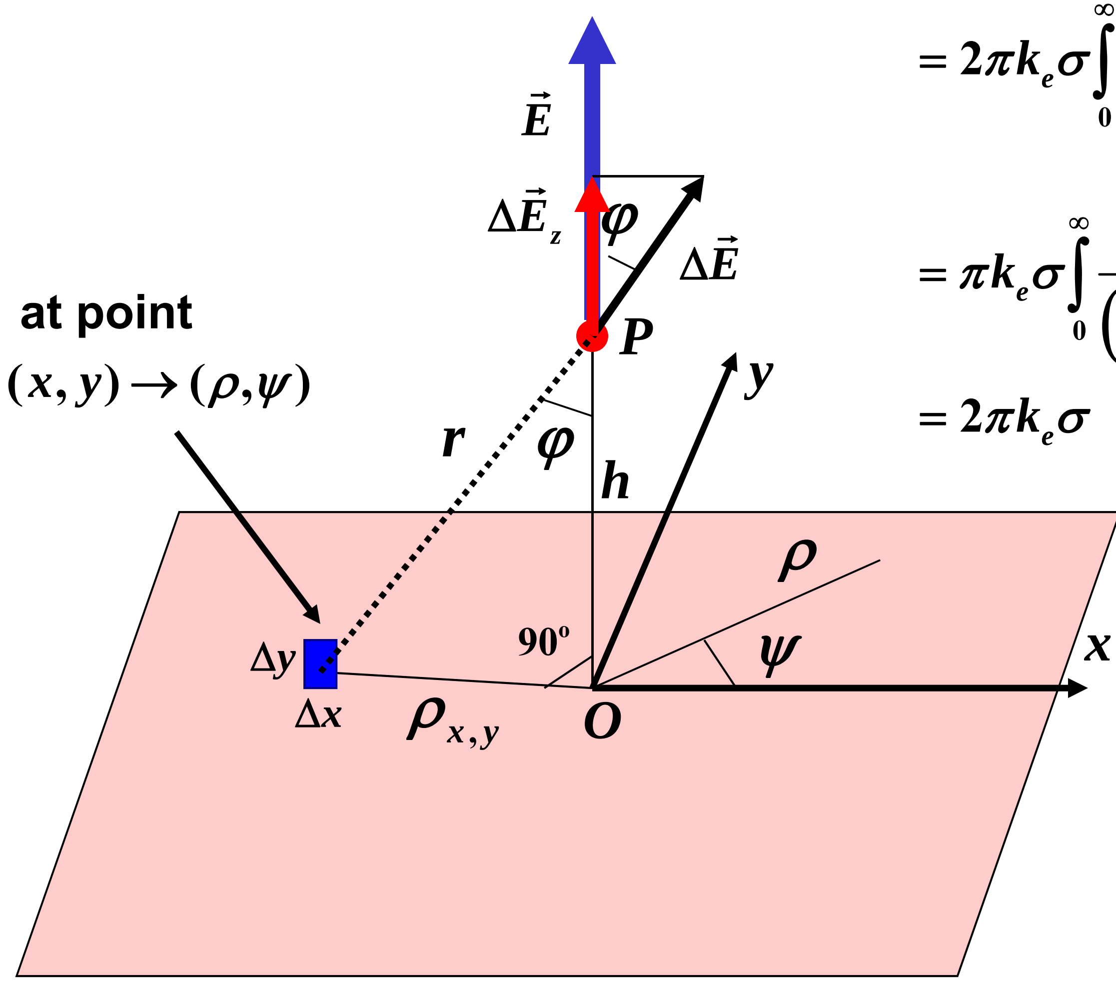
Evaluate the Integral

$$E = k_e\sigma\int_0^\infty d\rho\int_0^{2\pi} d\psi \frac{h\rho}{\left(h^2+\rho^2\right)^{3/2}} =$$

$$= 2\pi k_e\sigma\int_0^\infty d\rho \frac{h\rho}{\left(h^2+\rho^2\right)^{3/2}} =$$

$$= \pi k_e\sigma\int_0^\infty \frac{hdt}{\left(h^2+t\right)^{3/2}} = -2\pi k_e\sigma \frac{h}{\left(h^2+t\right)^{1/2}}\Bigg|_0^\infty =$$

$$= 2\pi k_e\sigma$$

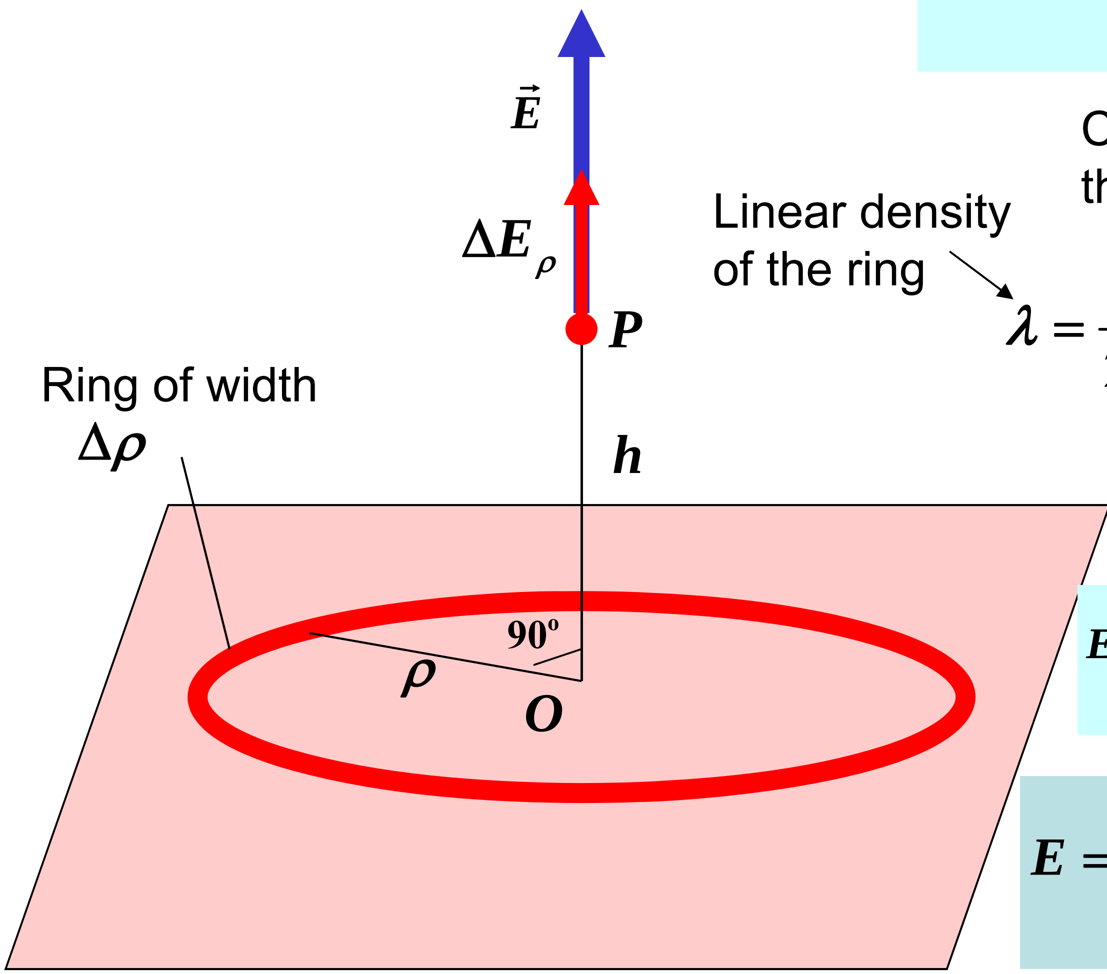


$$E = 2\pi k_e\sigma$$

Approach B

What is the electric field at point ***P*** ?

σ - surface charge density



$$E = \sum_{all\ rings} \Delta E_\rho$$

$$\Delta E_\rho = 2\pi\lambda k_e \frac{h\rho}{\left(h^2+\rho^2\right)^{3/2}}$$

Linear density of the ring

Charge of the ring

$$\lambda = \frac{\Delta Q}{2\pi\rho} = \frac{\sigma\Delta A}{2\pi\rho} = \frac{\sigma 2\pi\rho\Delta\rho}{2\pi\rho} = \sigma\Delta\rho$$

Length of the ring

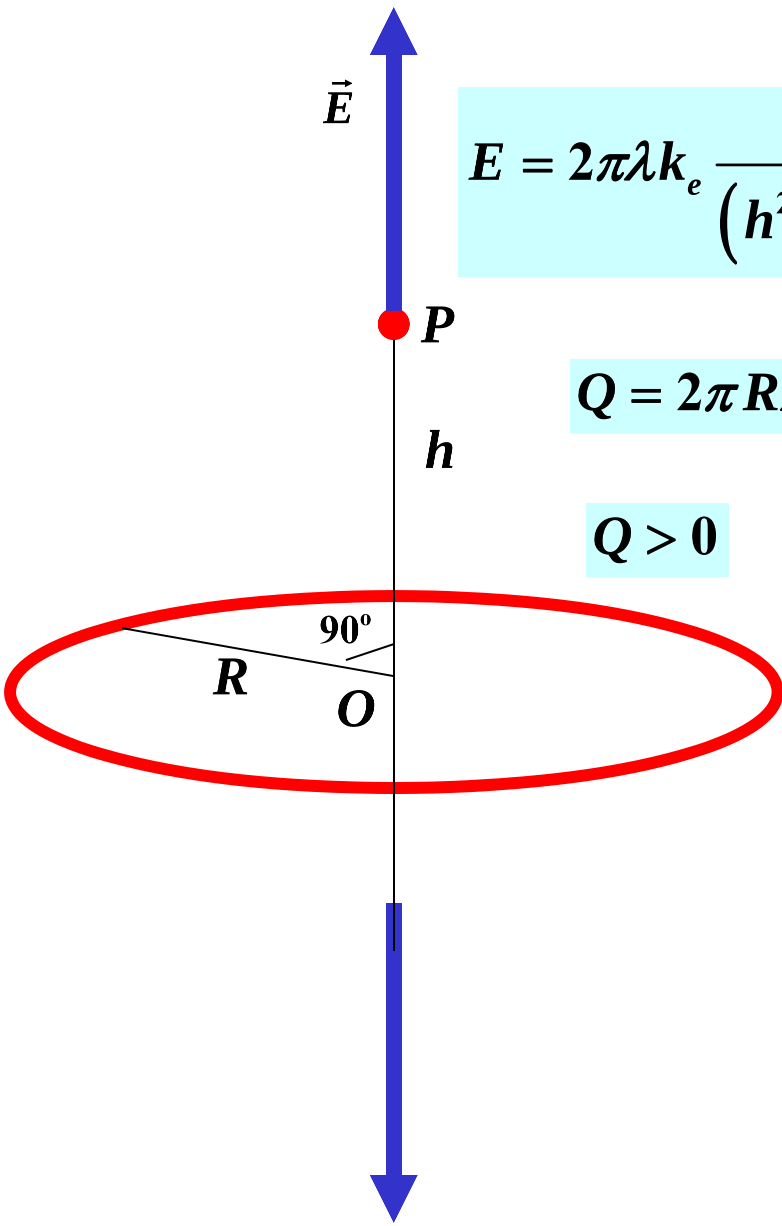
$$E = \sum_{all\ rings} \Delta E_\rho = \sum_{all\ rings} 2\pi\sigma k_e \frac{h\rho\Delta\rho}{\left(h^2+\rho^2\right)^{3/2}}$$

$$E = 2\pi k_e \sigma \int_0^\infty d\rho \frac{h\rho}{\left(h^2+\rho^2\right)^{3/2}} = 2\pi k_e \sigma$$

$$E = 2\pi\lambda k_e \frac{hR}{\left(h^2 + R^2\right)^{3/2}}$$

$$Q = 2\pi R\lambda$$

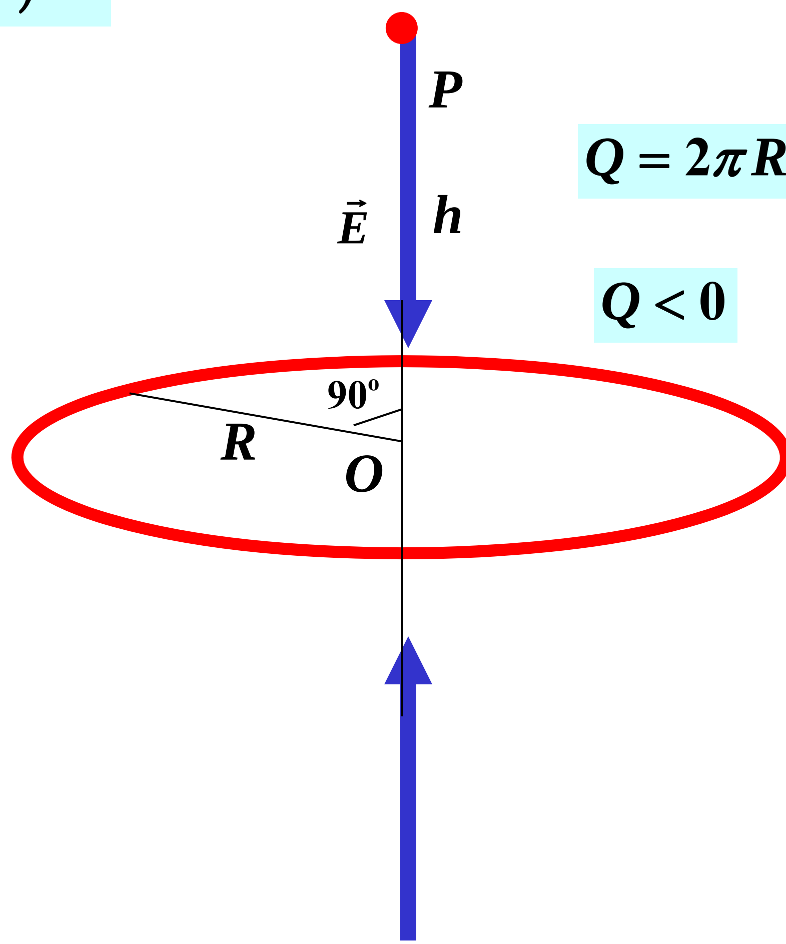
$$Q > 0$$



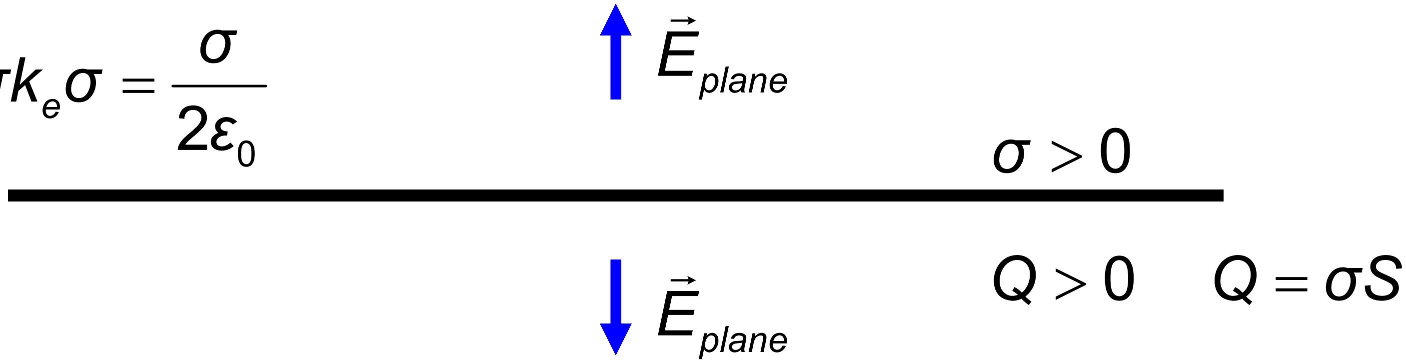
$$E = 2\pi\,|\,\lambda\,|\,k_e \frac{hR}{\left(h^2 + R^2\right)^{3/2}}$$

$$Q = 2\pi R\lambda$$

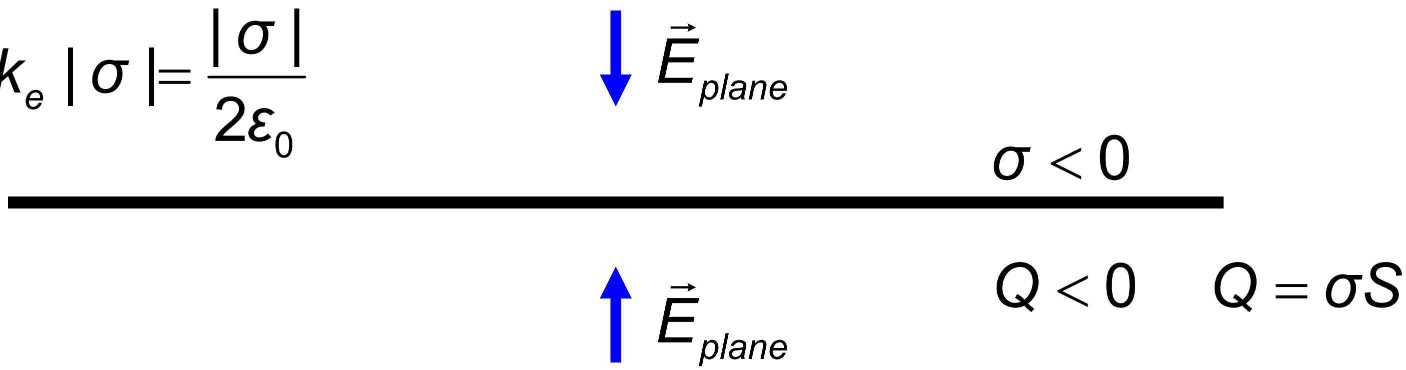
$$Q < 0$$



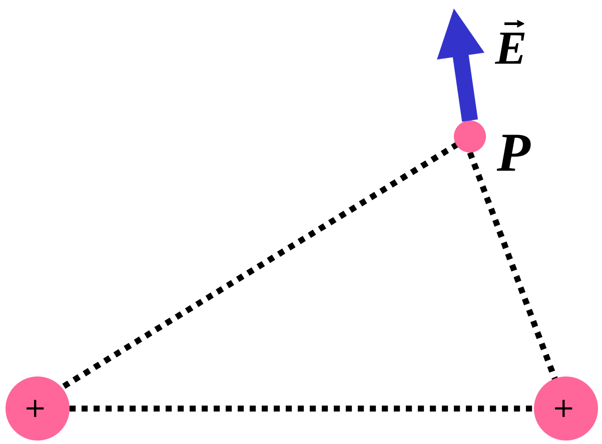
$$E_{plane} = 2\pi k_e \sigma = \frac{\sigma}{2\varepsilon_0}$$



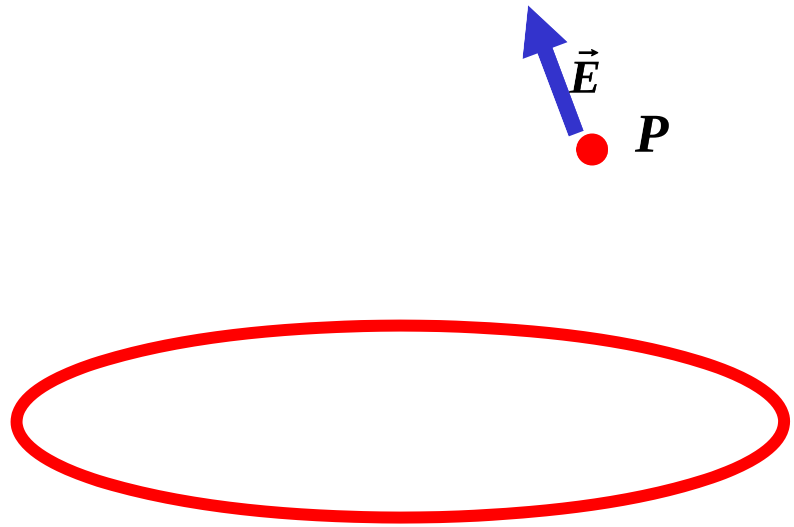
$$E_{plane} = 2\pi k_e |\sigma| = \frac{|\sigma|}{2\varepsilon_0}$$



Electric Field: Symmetry

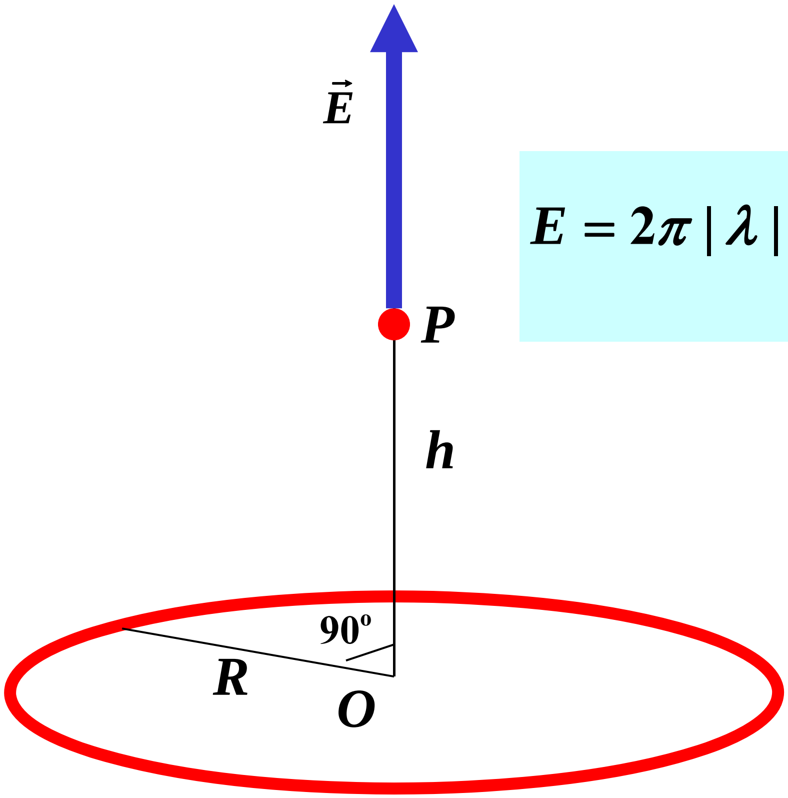


no symmetry

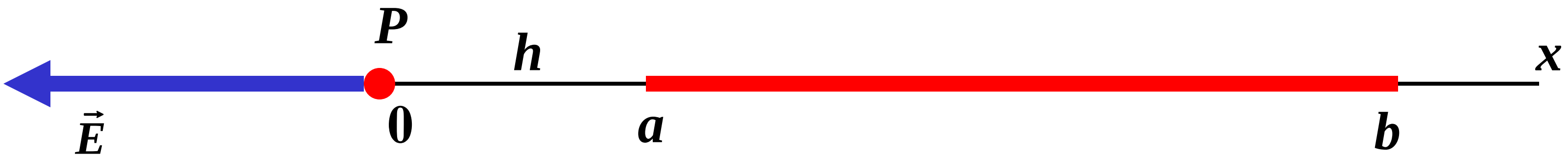


no symmetry

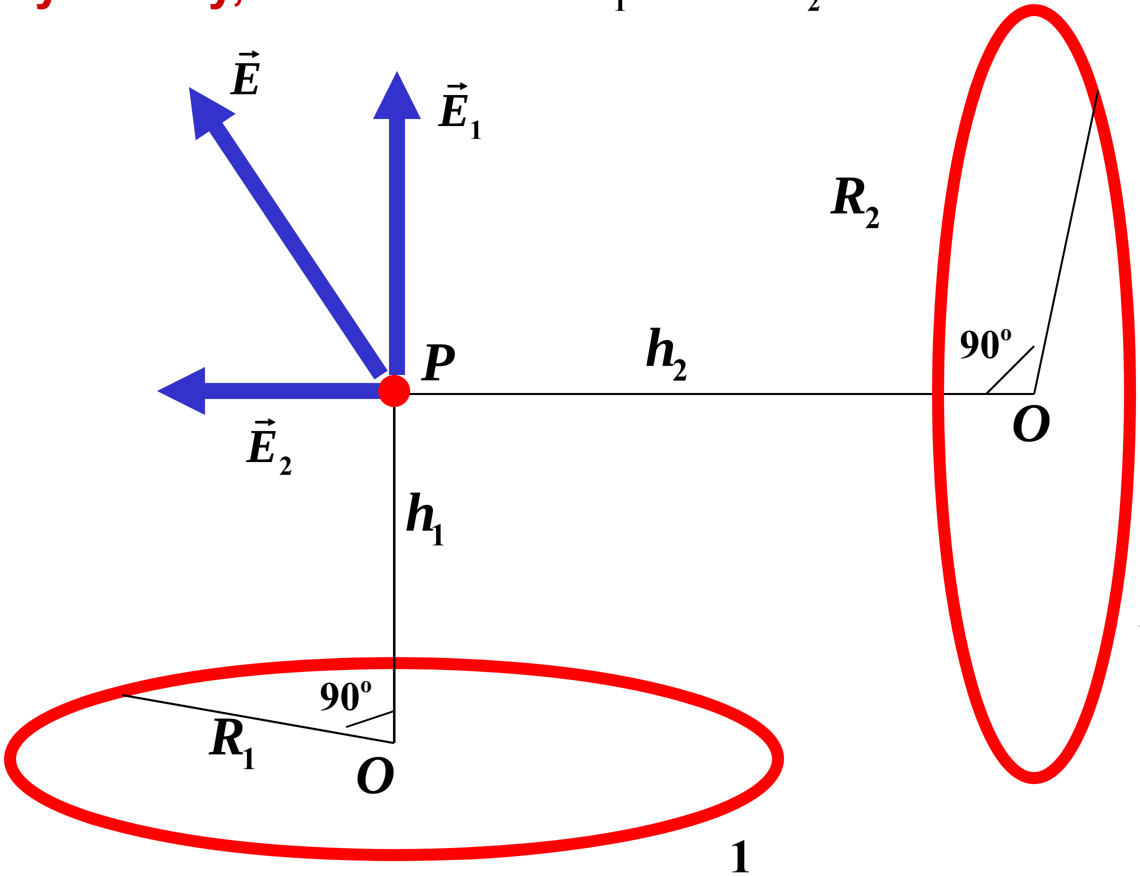
no symmetry



$$E = 2\pi |\lambda| k_e \frac{hR}{\left(h^2 + R^2\right)^{3/2}}$$



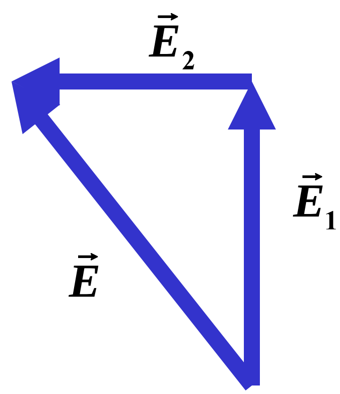
no symmetry, but we know $\vec{E}_1$ and $\vec{E}_2$



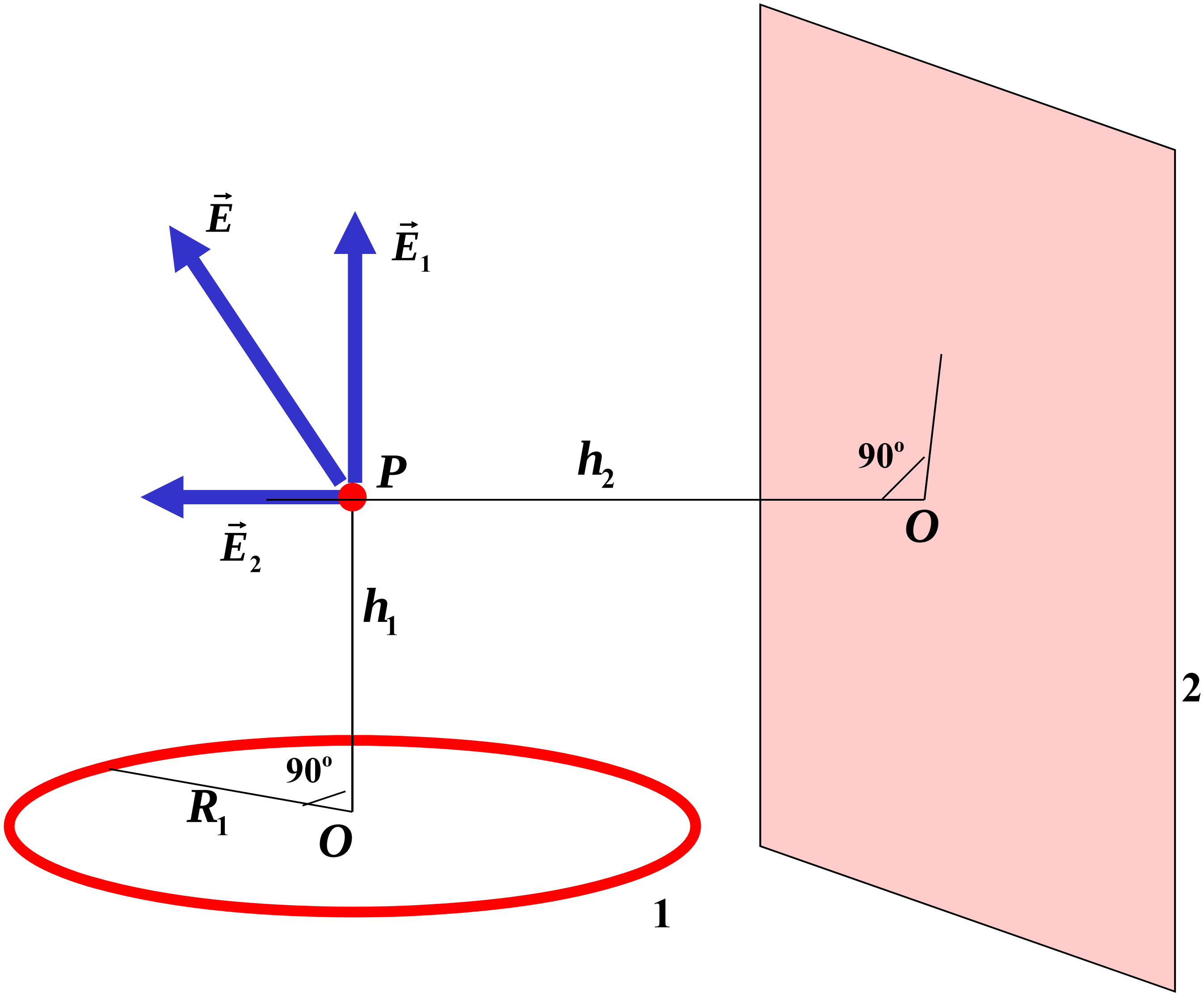
$$E_1 = 2\pi\lambda_1 k_e \frac{h_1 R_1}{\left(h_1^{\,2} + R_1^{\,2}\right)^{3/2}}$$

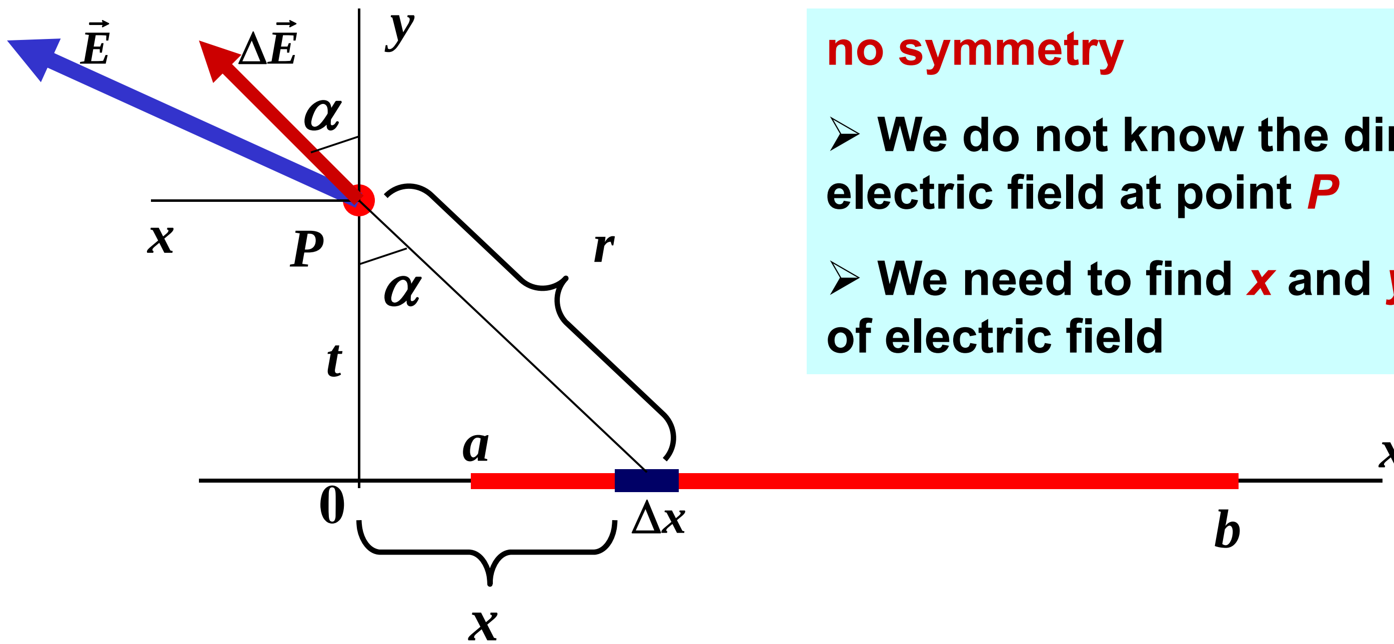
$$E_2 = 2\pi\lambda_2 k_e \frac{h_2 R_2}{\left(h_2^{\,2} + R_2^{\,2}\right)^{3/2}}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



$$E = \sqrt{E_1^{\,2} + E_2^{\,2}}$$


$\vec{E}$
$\vec{E}_1$
P
h_2
90^o
O
$\vec{E}_2$
h_1
90^o
R_1
O
2
1



no symmetry

- We do not know the direction of electric field at point ***P***
- We need to find ***x*** and ***y*** component of electric field

$$\vec{E} = \sum_{all\ elements} \Delta\vec{E}$$

Then

$$E_x = \sum_{all\ elements} \Delta E_x \qquad \Delta E_x = k_e \frac{\Delta q}{r^2} \sin\alpha$$

$$E_y = \sum_{all\ elements} \Delta E_y \qquad \Delta E_x = k_e \frac{\Delta q}{r^2} \cos\alpha$$

$$\Delta q = \lambda \Delta x$$

$$r = \sqrt{t^2 + x^2}$$

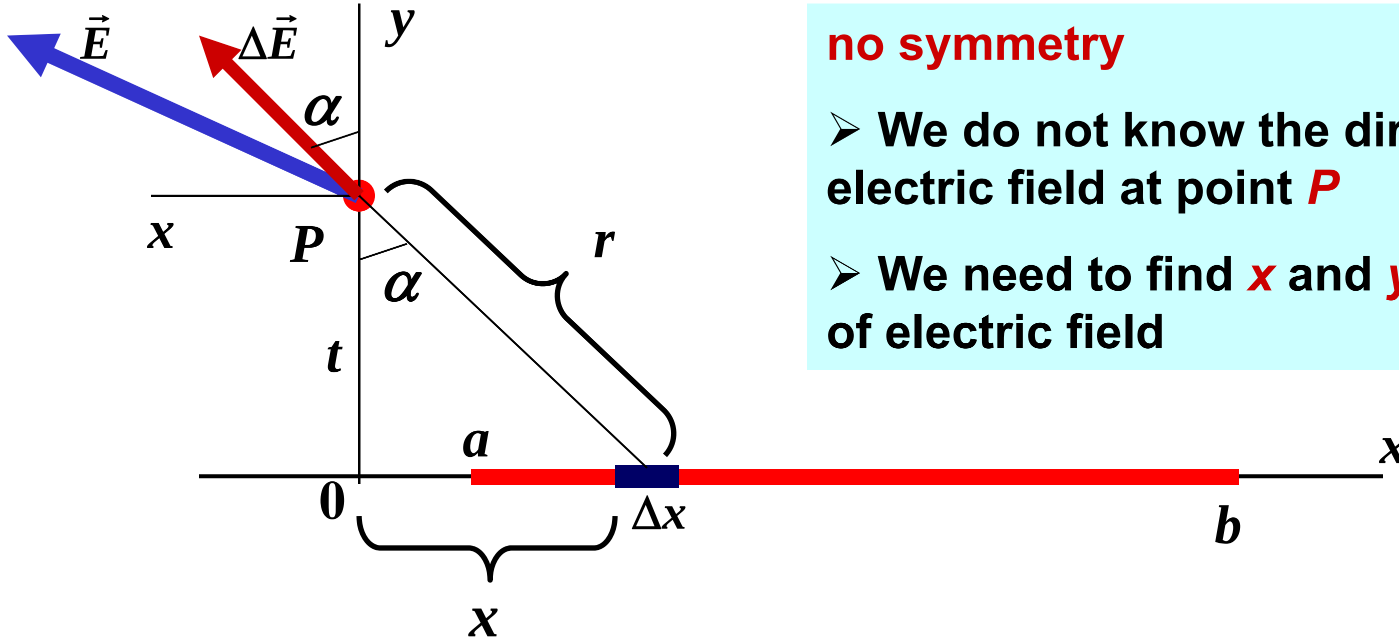
$$\cos\alpha = \frac{t}{r} = \frac{t}{\sqrt{t^2 + x^2}}$$

$$\sin\alpha = \frac{x}{r} = \frac{x}{\sqrt{t^2 + x^2}}$$

Then

$$E_x = \sum_{all\ elements} k_e \frac{\Delta q}{r^2} \sin\alpha = \sum_{all\ elements} k_e \frac{\lambda \Delta x}{x^2 + t^2} \frac{x}{\sqrt{x^2 + t^2}}$$

$$E_y = \sum_{all\ elements} k_e \frac{\Delta q}{r^2} \cos\alpha = \sum_{all\ elements} k_e \frac{\lambda \Delta x}{x^2 + t^2} \frac{t}{\sqrt{x^2 + t^2}}$$



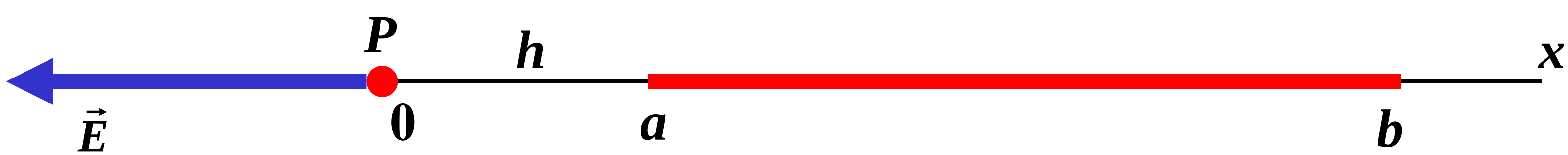
no symmetry

- We do not know the direction of electric field at point ***P***
- We need to find ***x*** and ***y*** component of electric field

$$E_x = \sum_{\text{all elements}} k_e \frac{\lambda \Delta x}{x^2+t^2} \frac{x}{\sqrt{x^2+t^2}} = \int_a^b k_e \frac{\lambda dx}{x^2+t^2} \frac{x}{\sqrt{x^2+t^2}} = \lambda k_e \int_a^b \frac{x dx}{(x^2+t^2)^{3/2}}$$

$$E_y = \sum_{\text{all elements}} k_e \frac{\lambda \Delta x}{x^2+t^2} \frac{t}{\sqrt{x^2+h^2}} = \int_a^b k_e \frac{\lambda dx}{x^2+t^2} \frac{t}{\sqrt{x^2+t^2}} = \lambda k_e t \int_a^b \frac{dx}{(x^2+t^2)^{3/2}}$$

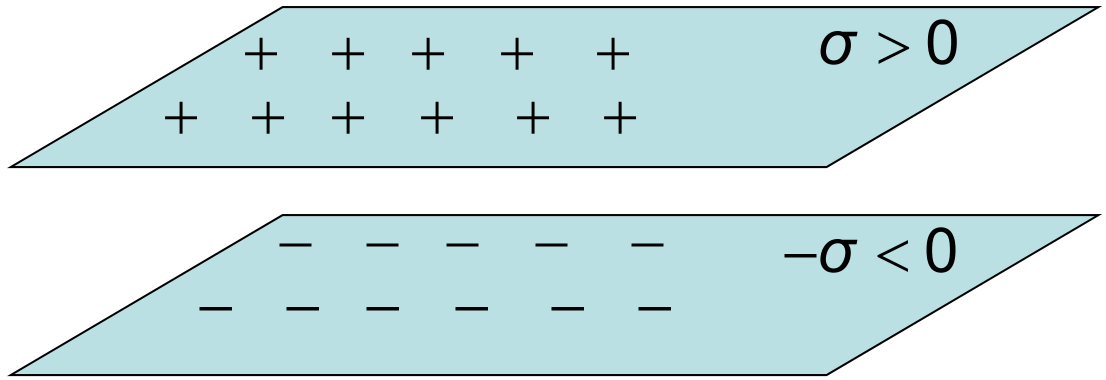
$E_y = 0$

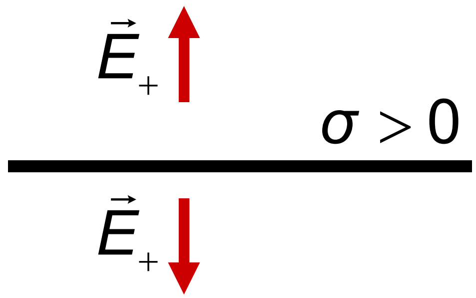


the symmetry tells us that one of the component is **0**, so we do not need to calculate it.

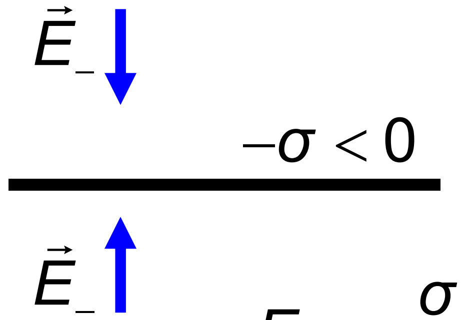
Important Example

Find electric field





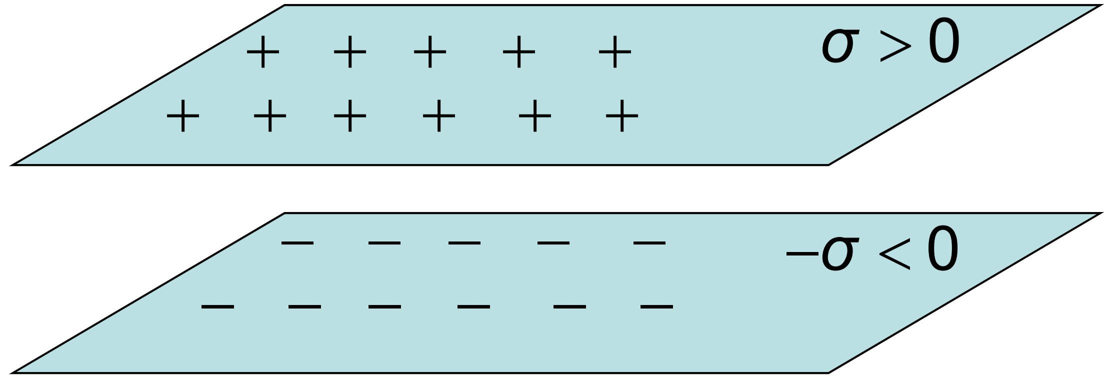
$$E_+ = \frac{\sigma}{2\varepsilon_0}$$



$$E_- = \frac{\sigma}{2\varepsilon_0}$$

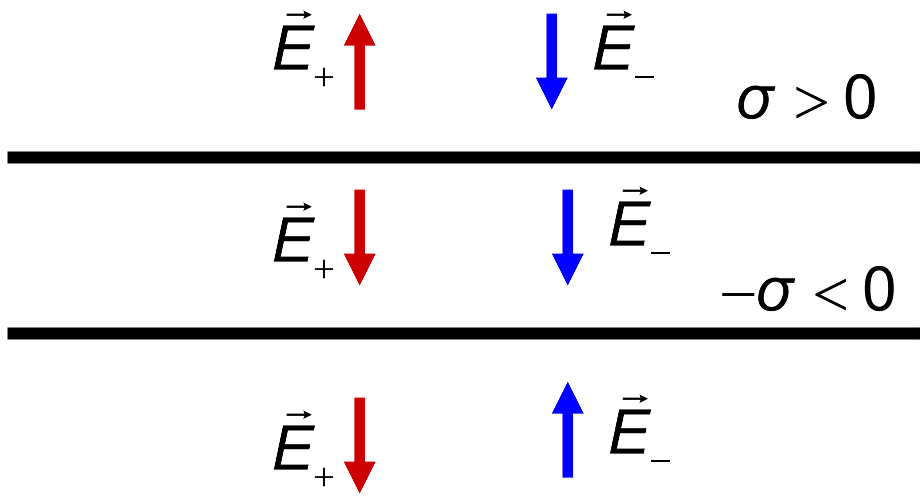
Important Example

Find electric field



$$E_{+} = \frac{\sigma}{2\varepsilon_0} \qquad E_{-} = \frac{\sigma}{2\varepsilon_0}$$

$$\vec{E} = \vec{E}_{+} + \vec{E}_{-}$$

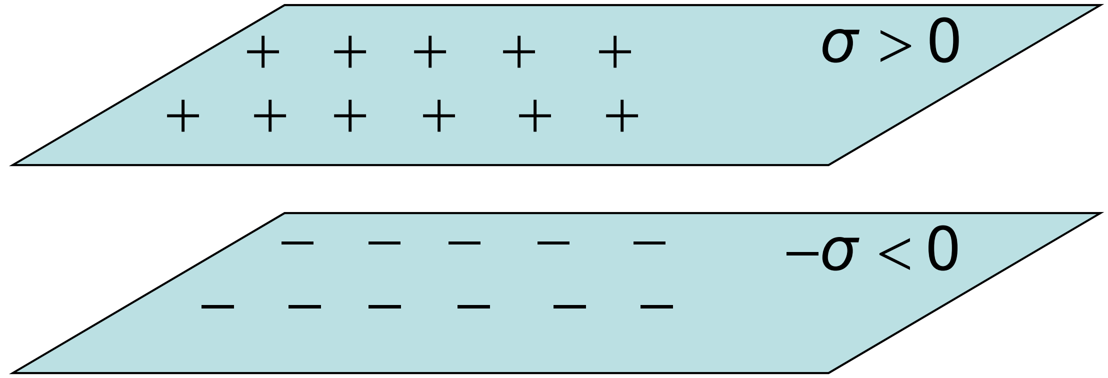


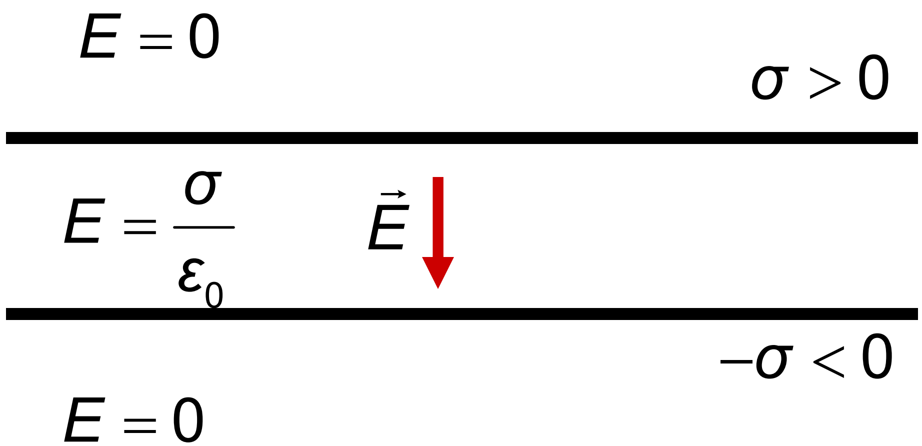
$$E = E_{+} - E_{-} = 0$$

$$E = E_{+} + E_{-} = \frac{\sigma}{\varepsilon_0}$$

$$E = E_{+} - E_{-} = 0$$

Important Example





Motion of Charged Particle

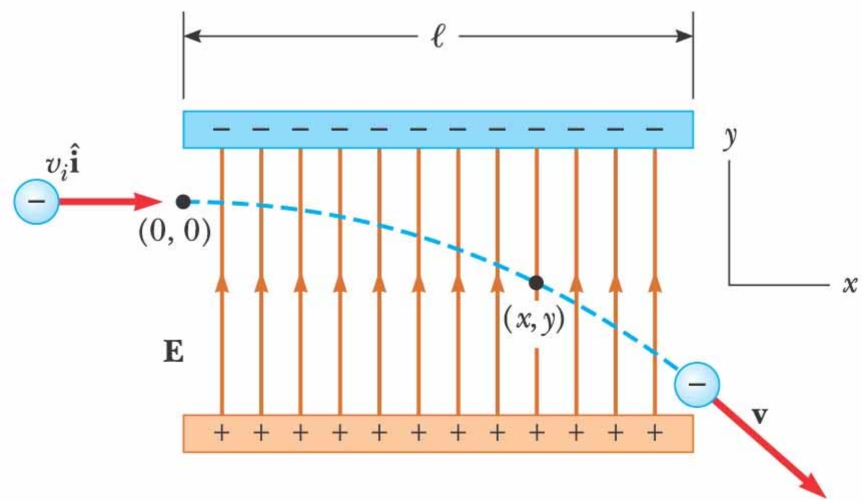
Motion of Charged Particle

- When a charged particle is placed in an electric field, it experiences an electrical force
- If this is the only force on the particle, it must be the net force
- The net force will cause the particle to accelerate according to Newton's second law

$\vec{F} = q\vec{E}$ - Coulomb's law

$\vec{F} = m\vec{a}$ - Newton's second law

$$\vec{a} = \frac{q}{m}\vec{E}$$



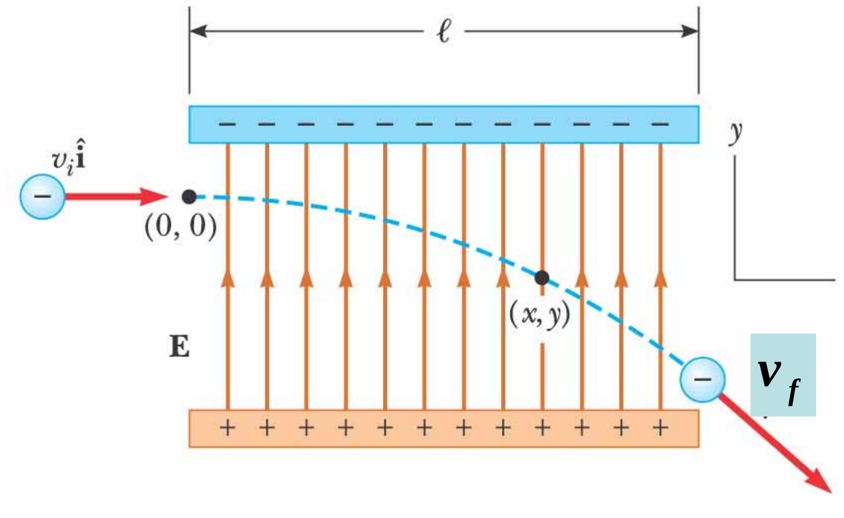
Motion of Charged Particle

What is the final velocity?

$$\vec{F} = q\vec{E}$$ - Coulomb's law

$$\vec{F} = m\vec{a}$$ - Newton's second law

$$a_y = -\frac{|q|}{m}E$$



Motion in ***x*** – with constant velocity v_0

Motion in ***x*** – with constant acceleration $a_y = -\frac{|q|}{m}E$

$$t = \frac{l}{v_0}$$ - travel time

After time ***t*** the velocity in ***y*** direction becomes

$$v_y = a_y t = -\frac{|q|}{m}Et$$ then $$v_f = \sqrt{v_0^2 + \left(\frac{q}{m}Et\right)^2}$$