The law of reflection:

$$\theta_1 = \theta_1'$$

The law of refraction:

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$ **Snell's Law**

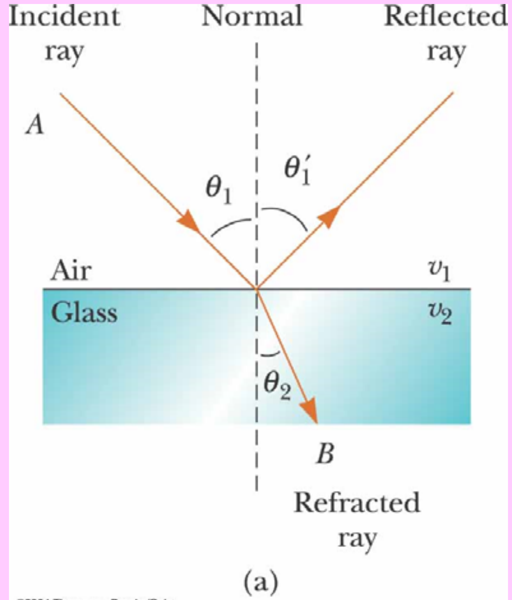
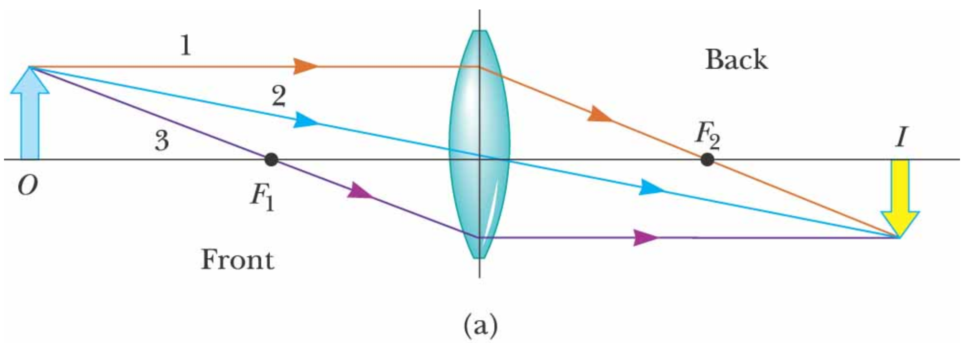
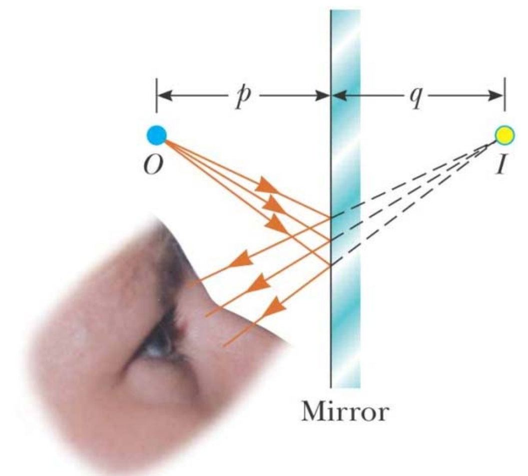


Image formation





Chapter 23

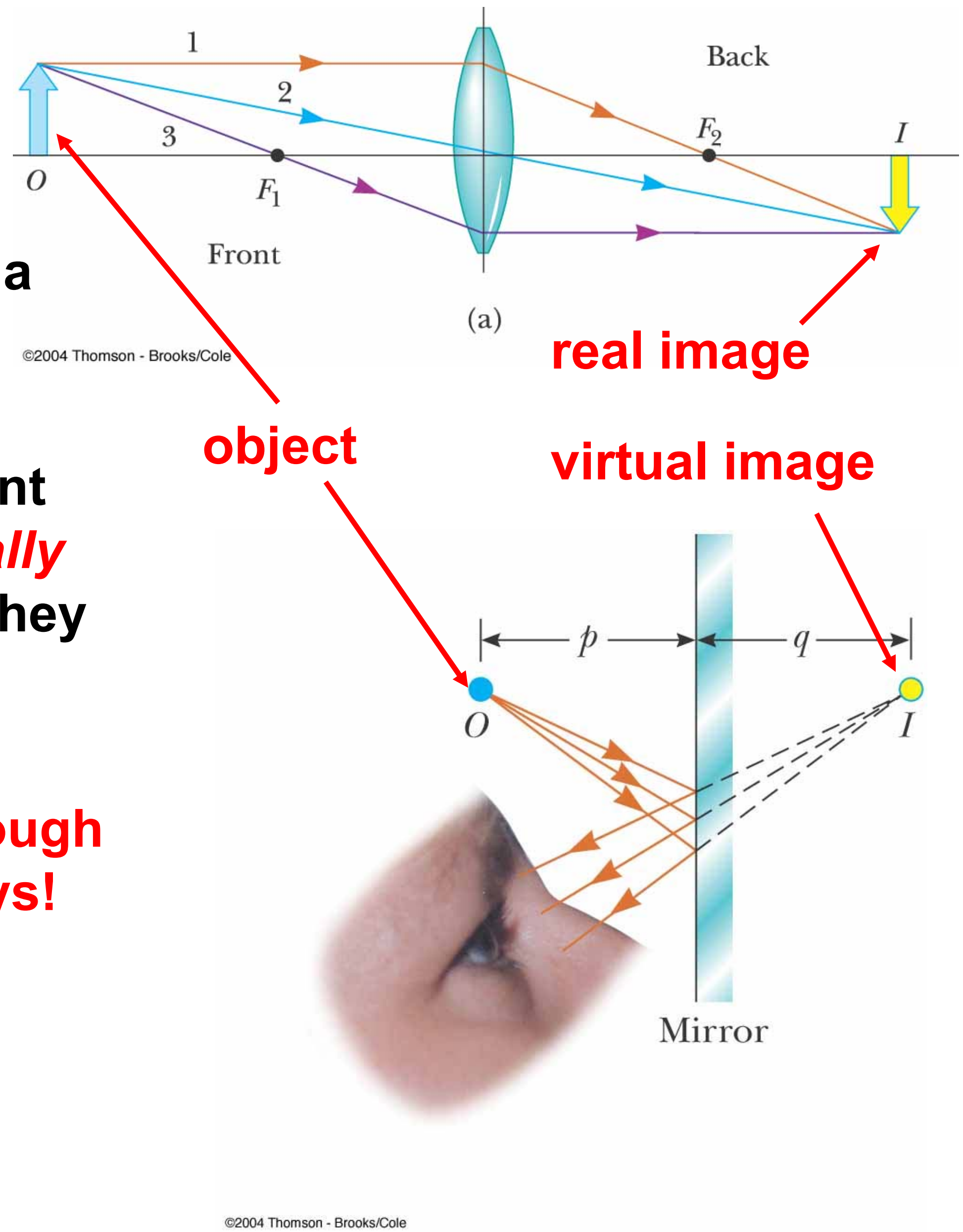
Ray Optics - Applications: Image Formation

- **Images are always located by extending diverging rays back to a point at which they intersect**

- **Images are located either at a point from which the rays of light *actually* diverge or at a point from which they *appear* to diverge**

- **To find the image it is usually enough to find intersection of just two rays!**

- **Magnification =** $\dfrac{\text{image height}}{\text{object height}}$



Flat Refracting Surface

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$

Snell's Law

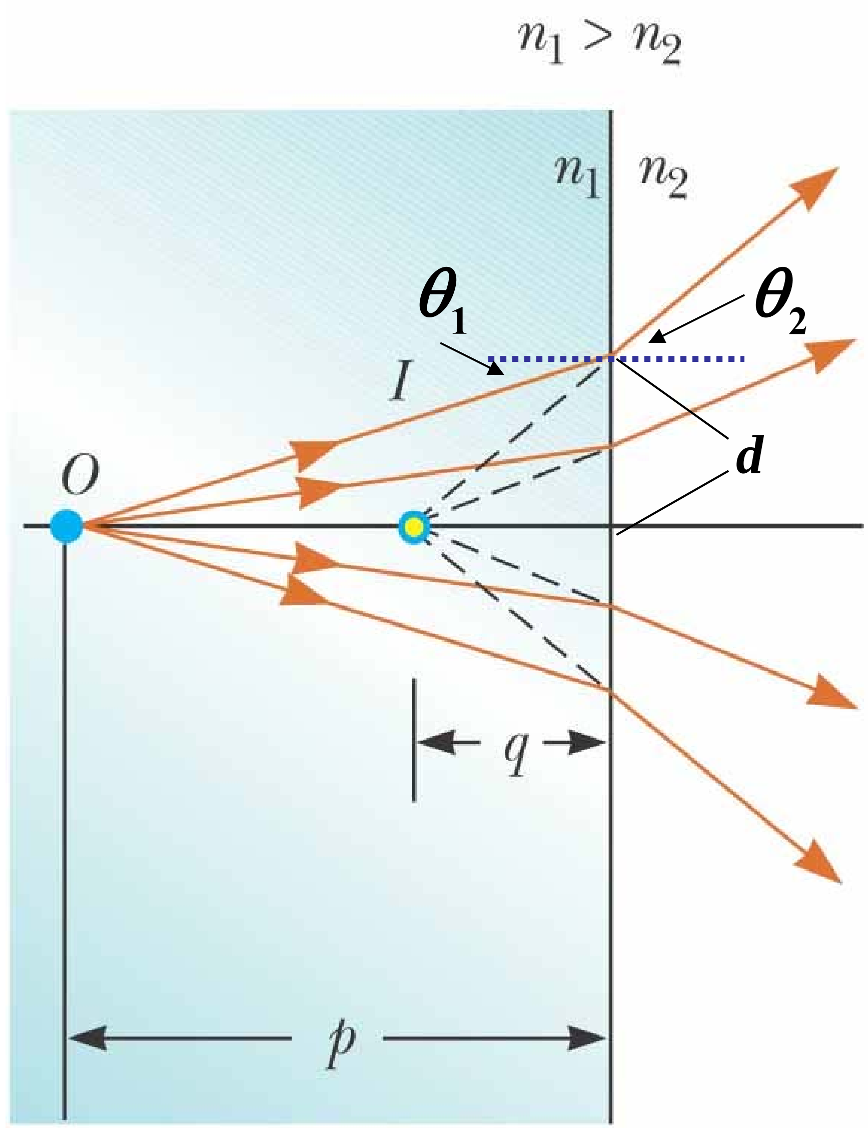
$$\sin\theta_2 \approx \theta_2 \approx \frac{d}{q}$$

$$\sin\theta_1 \approx \theta_1 \approx \frac{d}{p}$$

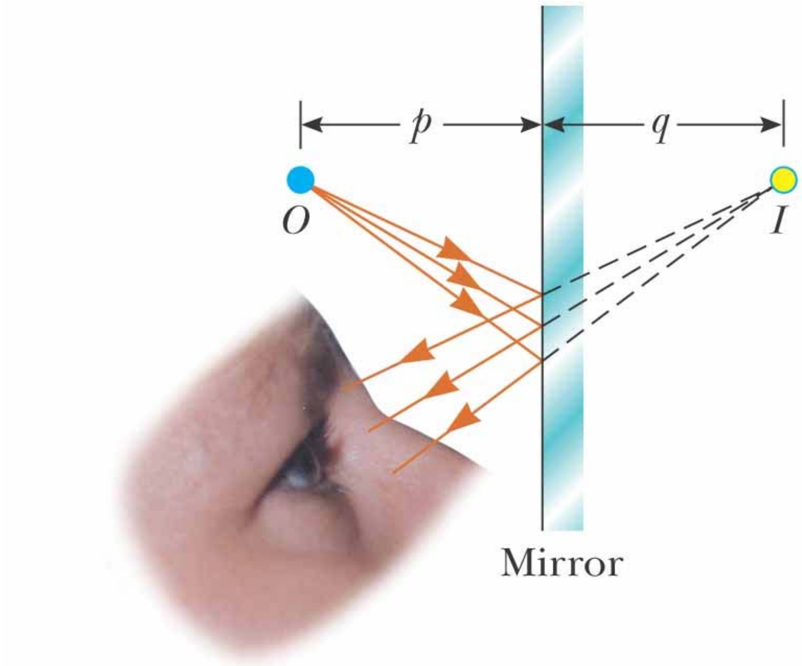
$$n_2 \frac{d}{q} = n_1 \frac{d}{p}$$

$$q = p\frac{n_2}{n_1}$$

Image is always virtual

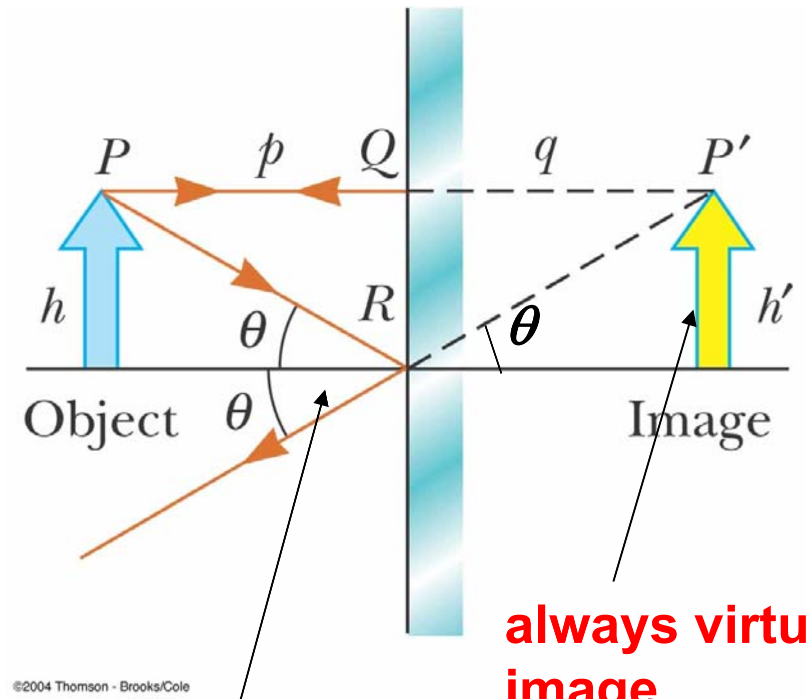


Flat mirror



Flat Mirror

- One ray starts at point *P*, travels to *Q* and reflects back on itself
- Another ray follows the path *PR* and reflects according to the law of reflection
- The triangles *PQR* and *P'QR* are congruent
- $h = h'$ - magnification is 1.



always virtual image

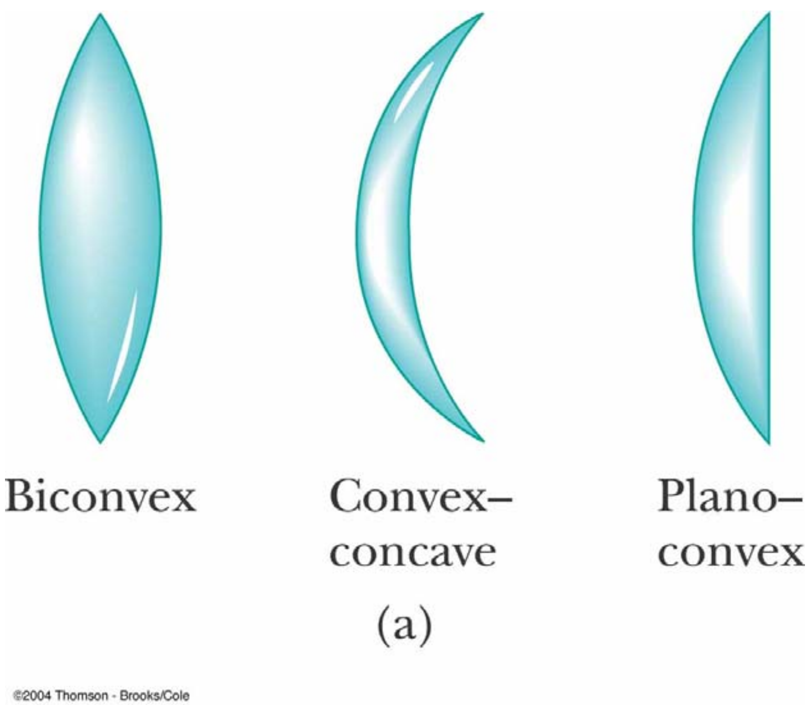
The law of reflection

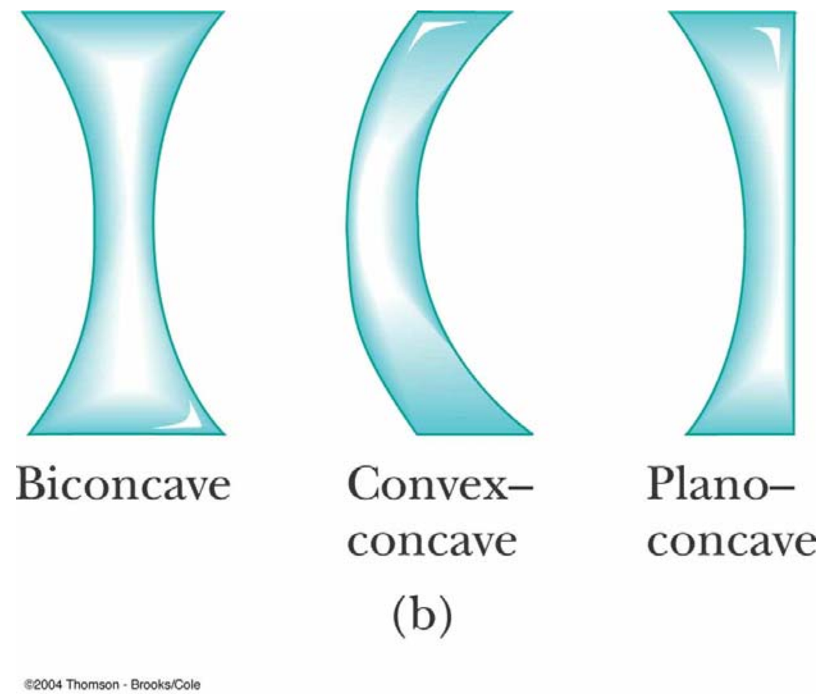
Chapter 23

Geometric Optics - Applications: Thin Lenses

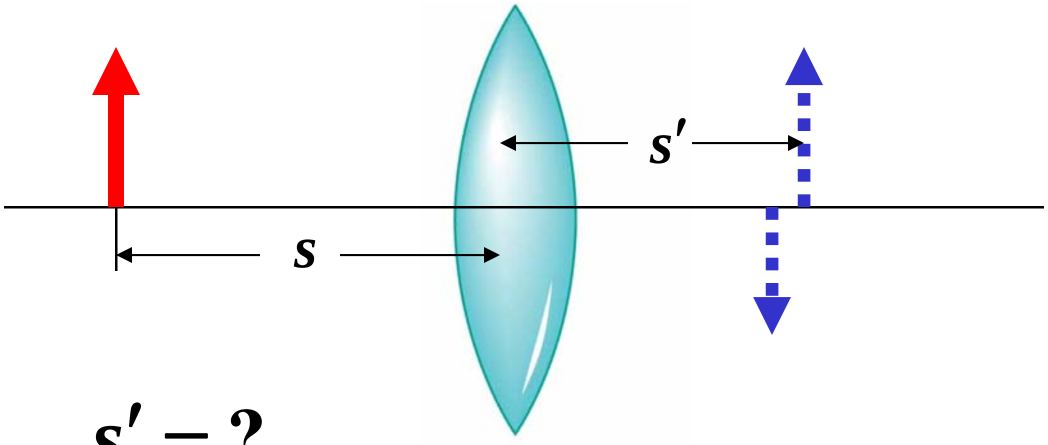
Thin Lenses

"Thin" means that the width is much smaller than the *radius of curvature*





Thin Lenses



$$s' = ?$$

Thin Lens Equation:

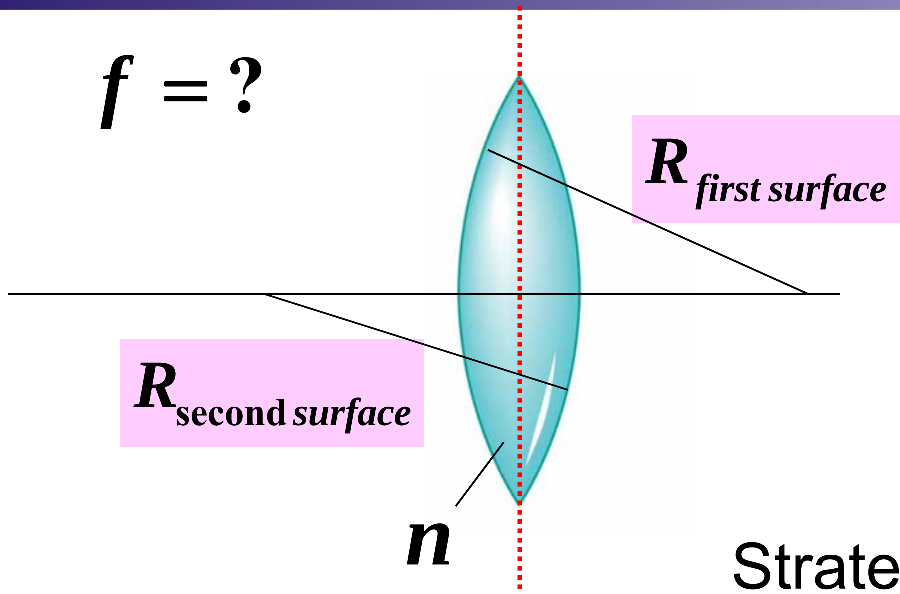
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Object Distance — Image Distance — Focal Length

The thin lens is characterized by only *one* parameter – FOCAL LENGTH.

Thin Lenses: Focal Length

$f = ?$



Strategy of Finding f:

$$\frac{1}{f} = (n-1)\left(s_{first} \frac{1}{R_{first\ surface}} + s_{second} \frac{1}{R_{second\ surface}} \right)$$

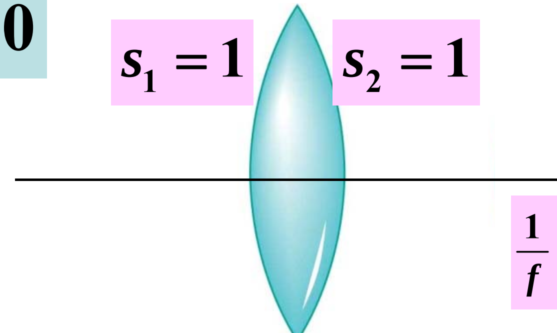
$s_{first} = -1$) $s_{second} = 1$)

$s_{first} = 1$ ($s_{second} = -1$ (

Focal Length: Examples

$f > 0$

$s_1 = 1$ $s_2 = 1$

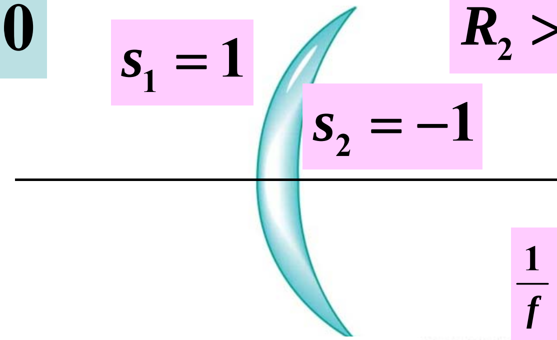


$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$f > 0$

$s_1 = 1$ $s_2 = -1$

$R_2 > R_1$ $\frac{1}{R_2} < \frac{1}{R_1}$

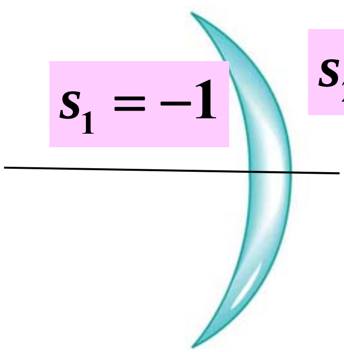


$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$f > 0$

$R_1 > R_2$ $\frac{1}{R_1} < \frac{1}{R_2}$

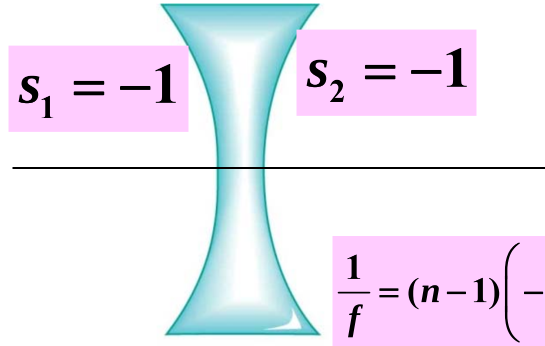
$s_1 = -1$ $s_2 = 1$



$$\frac{1}{f} = (n-1)\left(-\frac{1}{R_1} + \frac{1}{R_2}\right)$$

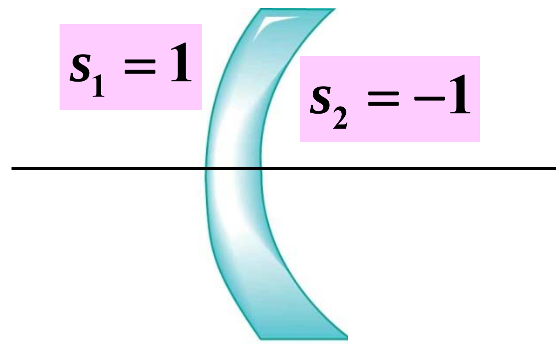
$f < 0$

$s_1 = -1$ $s_2 = -1$



$$\frac{1}{f} = (n-1)\left(-\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$s_1 = 1$ $s_2 = -1$



$R_1 > R_2$

$\frac{1}{R_1} < \frac{1}{R_2}$

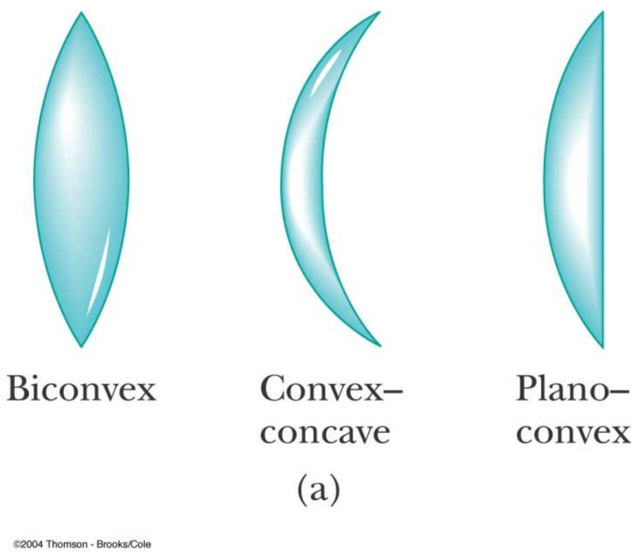
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$f < 0$

Thin Lenses

$f > 0$

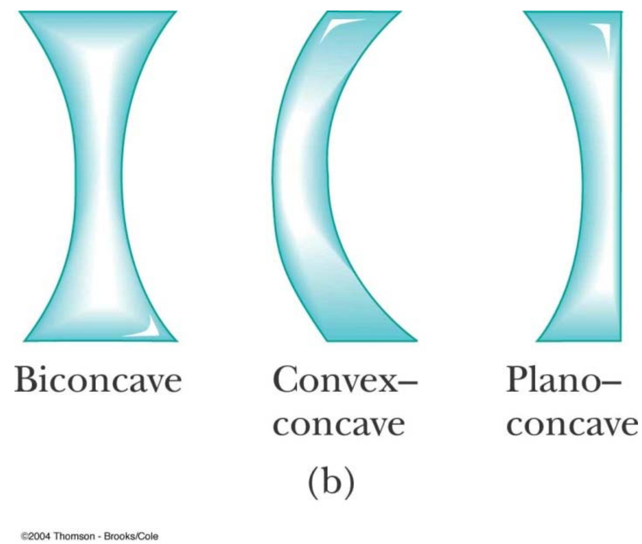
Converging lens



They are thickest in the middle

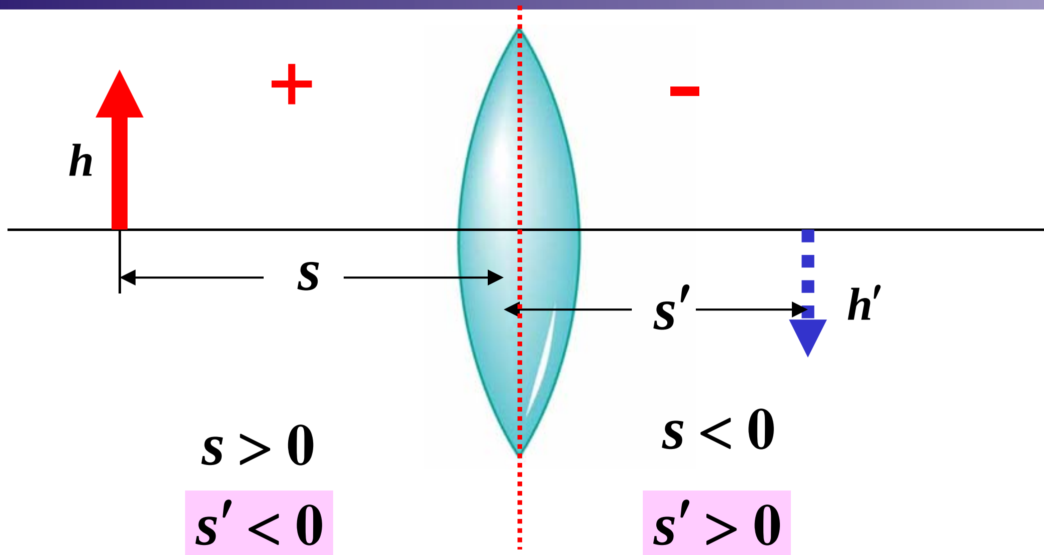
$f < 0$

Diverging lens



They are thickest at the edges

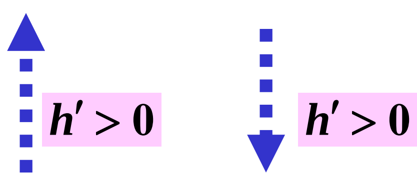
Thin Lenses: Sign Conventions for s, s'



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Lateral magnification:

$$M = \frac{h'}{h} = -\frac{s'}{s}$$



Thin Lenses: Numerical Strategy

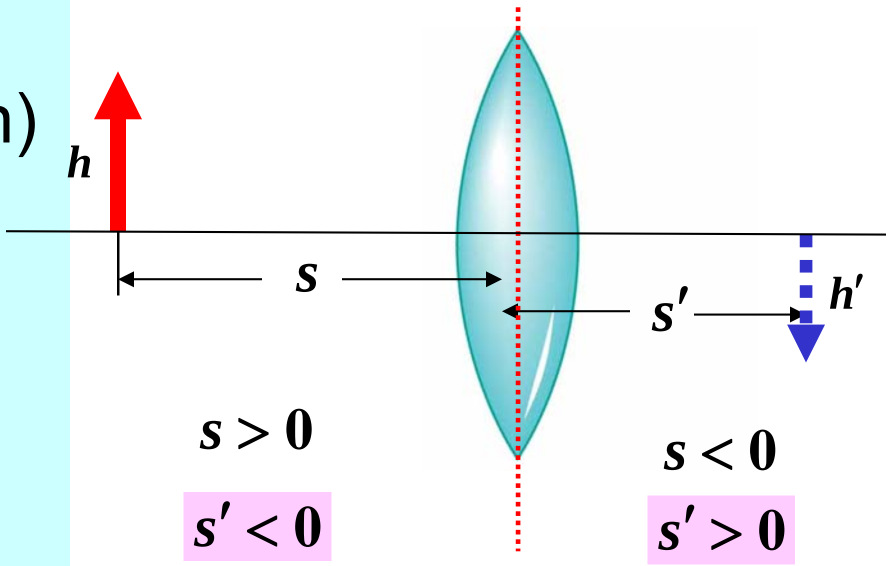
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- Find the focal length ***f***
- From the Thin Lens Equation find ***s'*** (***s*** is known)

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}}$$

- From the sign of ***s'*** find the position of image
- Find magnification

$$M = \frac{h'}{h} = -\frac{s'}{s}$$



Thin Lenses: Focal Points

Thin Lenses: Focal Points: Converging Lenses

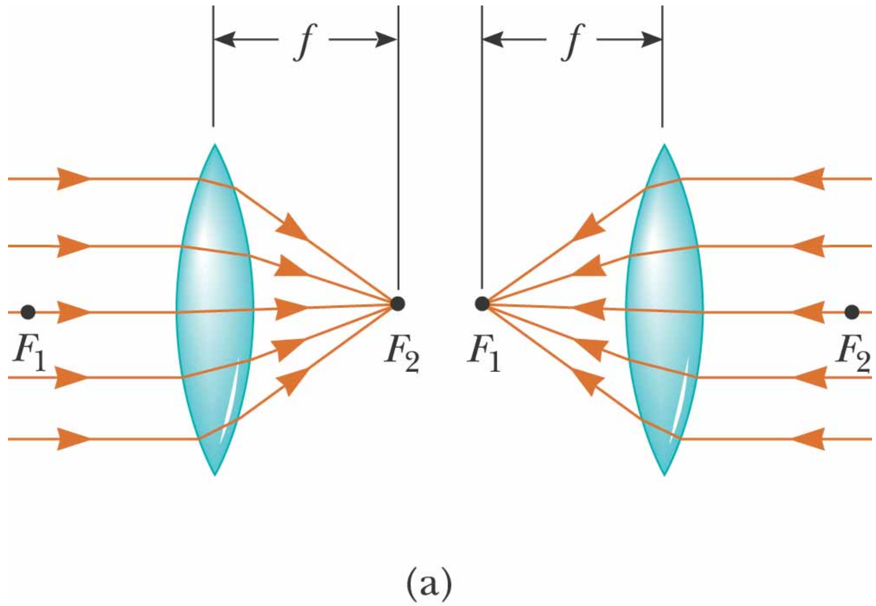
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

- If ***s>> f,*** then

$$\frac{1}{s} << \frac{1}{f}$$

and

$$s' = \frac{1}{\frac{1}{f}-\frac{1}{s}} = f$$



- Because light can travel in either direction through a lens, each lens has two focal points.
- However, there is only one focal length

Thin Lenses: Focal Points: Diverging Lenses

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

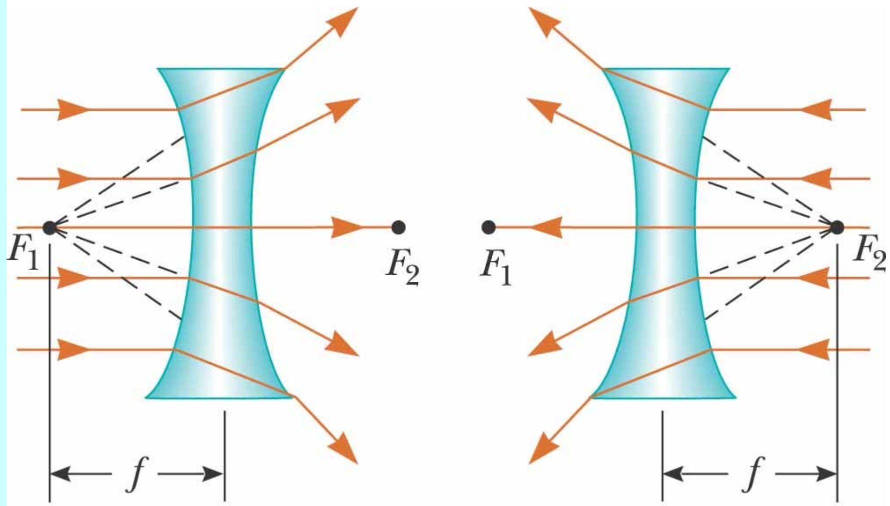
- If ***s>> f,*** then

$$\frac{1}{s} << \frac{1}{f}$$

and

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = f$$

- ***s'*** is negative



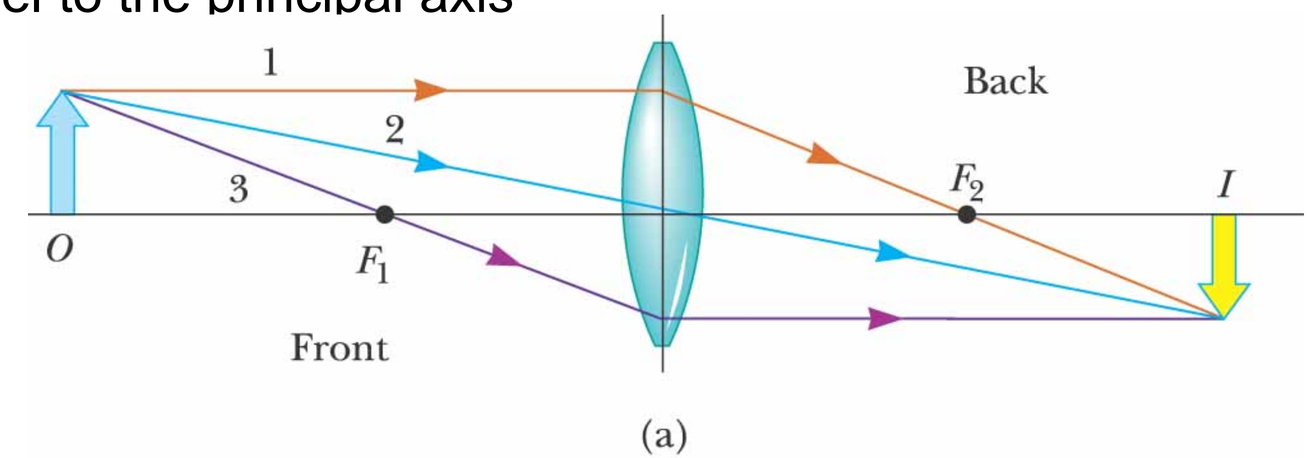
(b)

Thin Lenses: Ray Diagram

Converging Lenses

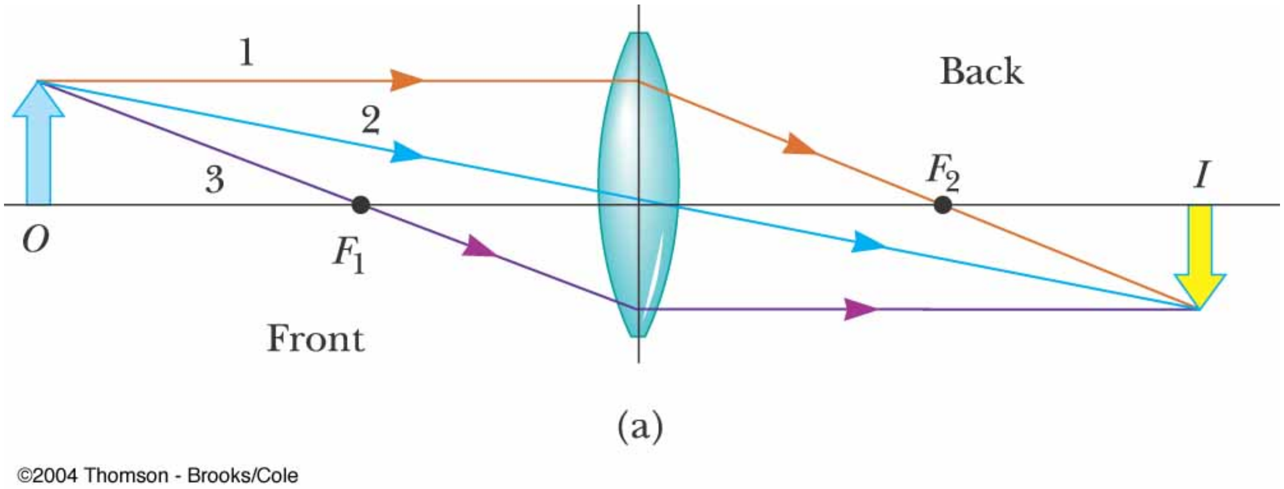
For a converging lens, the following three rays (two is enough) are drawn:

- Ray 1 is drawn parallel to the principal axis and then passes through the focal point on the back side of the lens
- Ray 2 is drawn through the center of the lens and continues in a straight line
- Ray 3 is drawn through the focal point on the front of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis



Converging Lenses: Example 1

$$s > f > 0$$



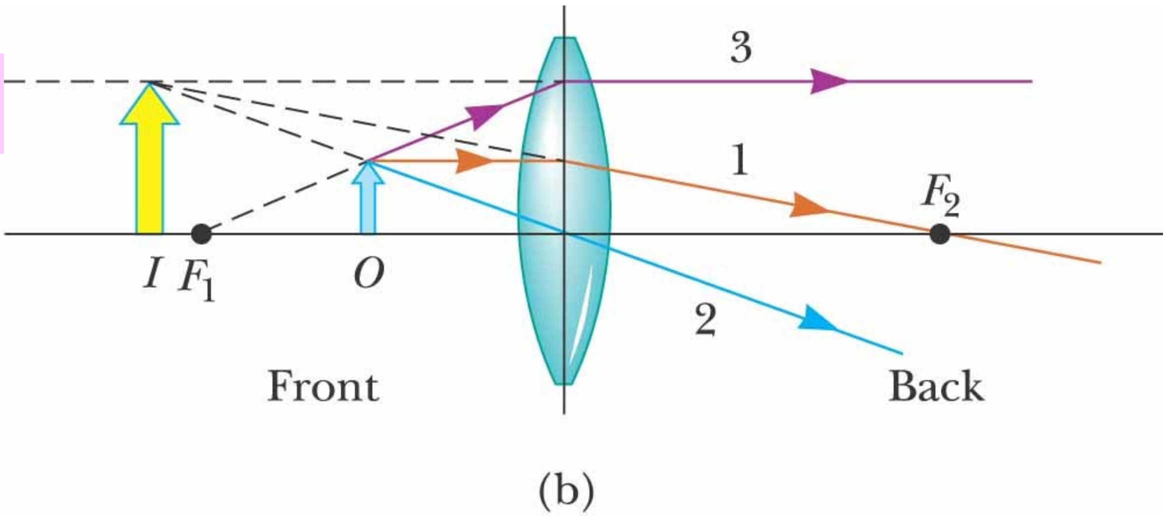
- The image is real
- The image is inverted
- The image is on the back side of the lens

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = \frac{sf}{s - f} > 0$$

$$M = \frac{h'}{h} = -\frac{s'}{s} < 0$$

Converging Lenses: Example 2

$f > s > 0$



(b)

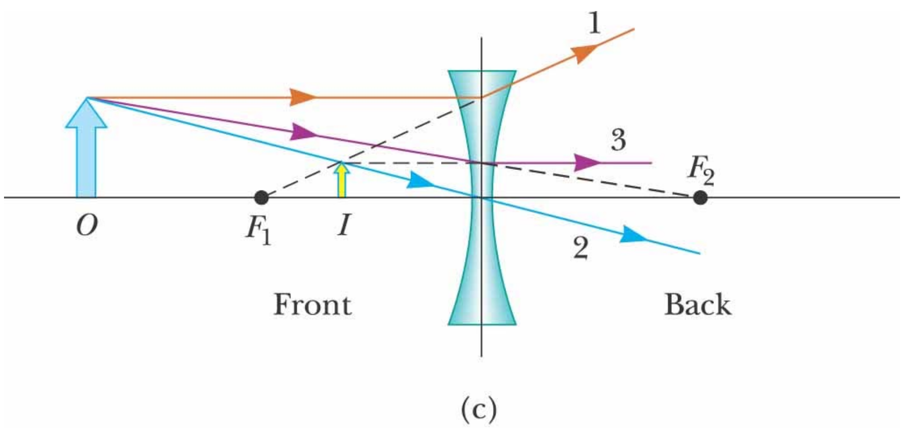
- The image is virtual
- The image is upright
- The image is larger than the object
- The image is on the front side of the lens

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = \frac{sf}{s - f} < 0$$

$$M = \frac{h'}{h} = -\frac{s'}{s} > 0$$

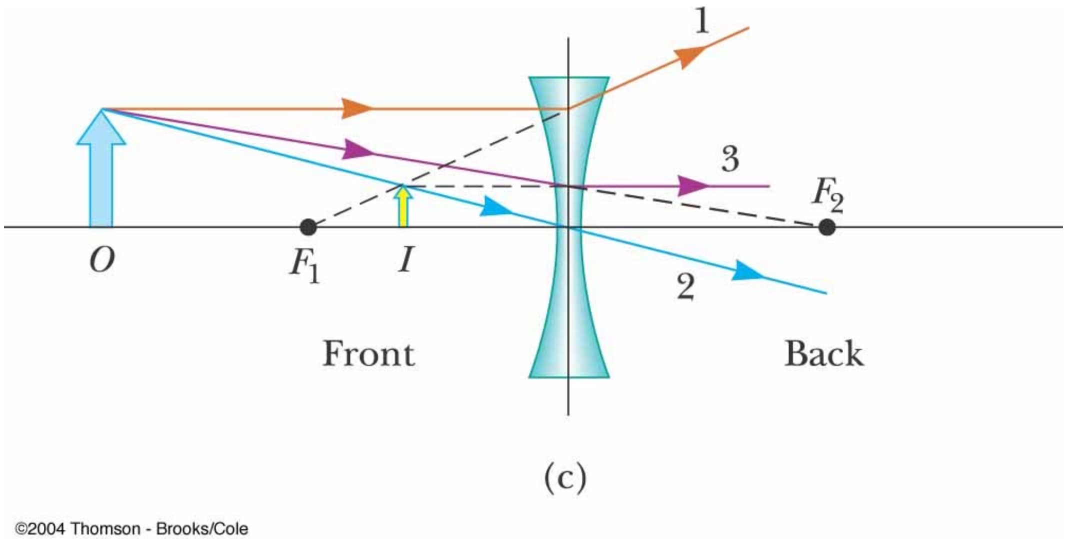
Diverging Lenses

- For a diverging lens, the following three rays (two is enough) are drawn:
  - Ray 1 is drawn parallel to the principal axis and emerges directed away from the focal point on the front side of the lens
  - Ray 2 is drawn through the center of the lens and continues in a straight line
  - Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis



Diverging Lenses: Example

$f < 0$



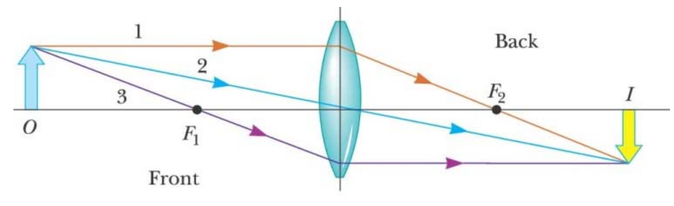
- The image is virtual
- The image is upright
- The image is smaller
- The image is on the front side of the lens

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = \frac{sf}{s - f} < 0$$

$$M = \frac{h'}{h} = -\frac{s'}{s} > 0$$

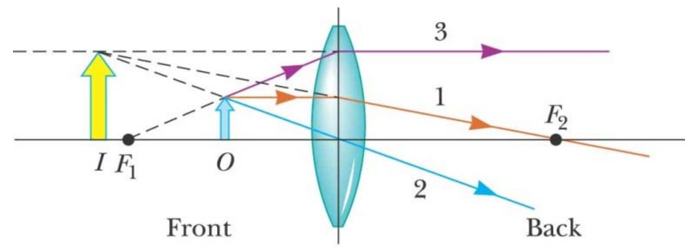
Image Summary

- For a converging lens, when the object distance is greater than the focal length ($s > f$)
  - The image is real and inverted



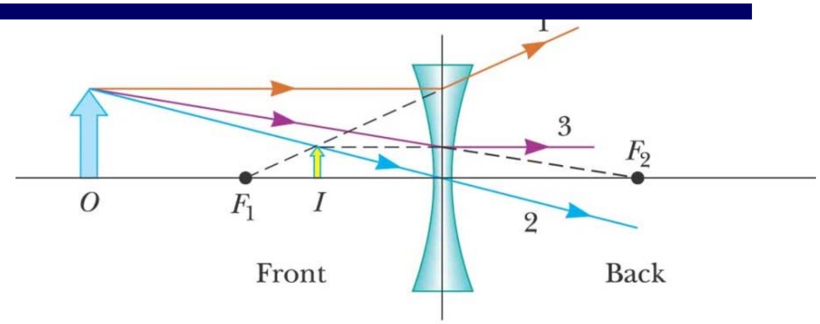
(a)

- For a converging lens, when the object is between the focal point and the lens, ($s < f$)
  - The image is virtual and upright



(b)

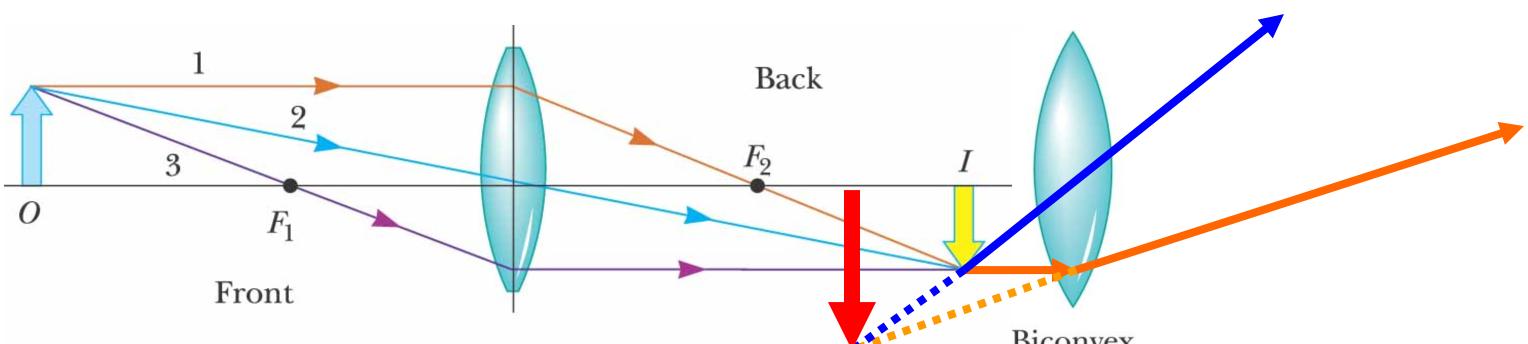
- For a diverging lens, the image is always virtual and upright
  - This is regardless of where the object is placed



(c)

Combination of Two Lenses

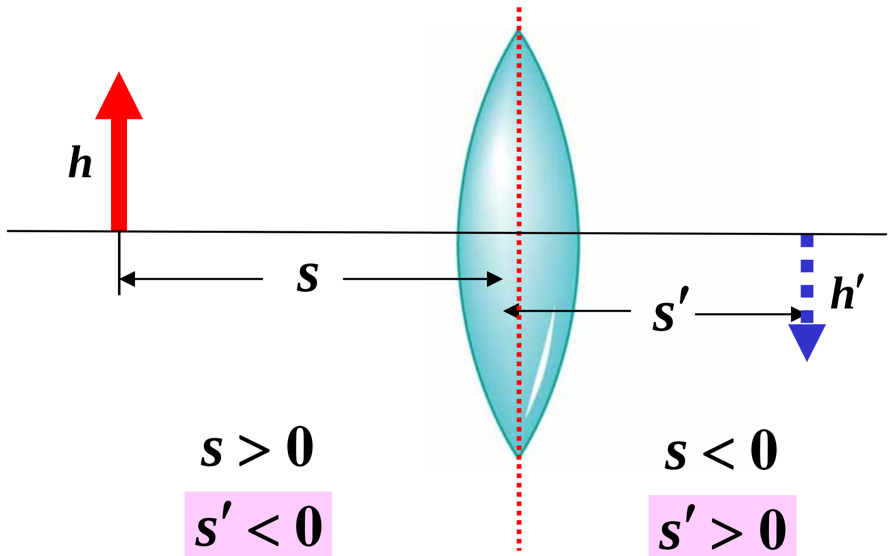
- The image formed by the first lens is located as though the second lens were not present
- *The image of the first lens is treated as the object of the second lens*
- Then a ray diagram is drawn for the second lens
- The image formed by the second lens is the final image of the system
- If the image formed by the first lens lies on the back side of the second lens, then the image is treated as a *virtual object* for the second lens
  - s will be negative
- The overall *magnification* is the product of the magnification of the separate lenses



(a)

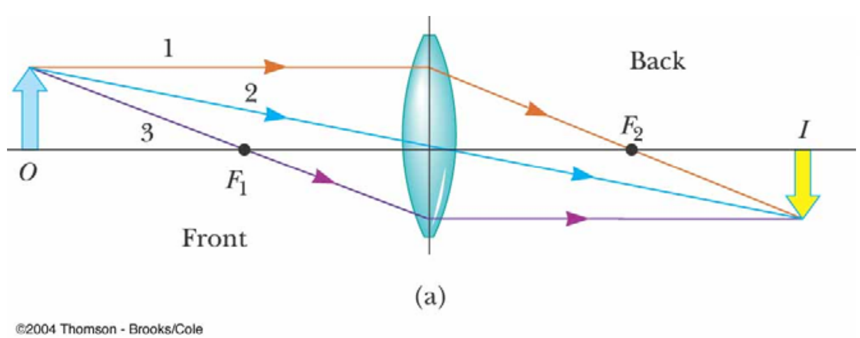
$$\frac{1}{f} = (n-1)\left(s_1 \frac{1}{R_1} + s_2 \frac{1}{R_2}\right)$$

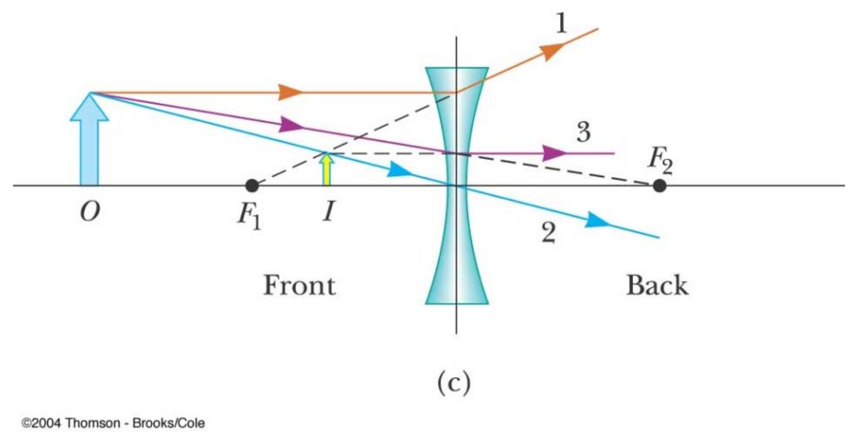
$f > 0$ $\qquad$ $f < 0$



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$M = \frac{h'}{h} = -\frac{s'}{s}$$





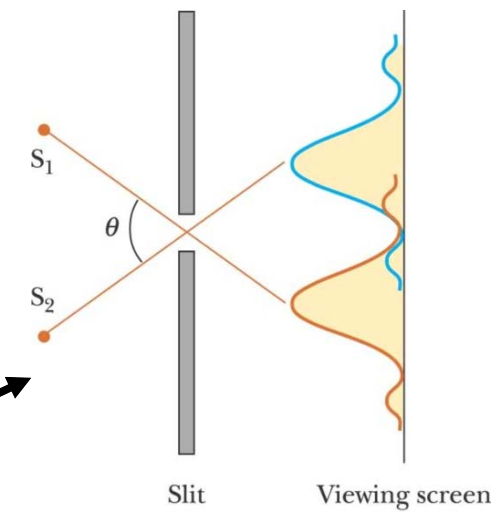
Resolution

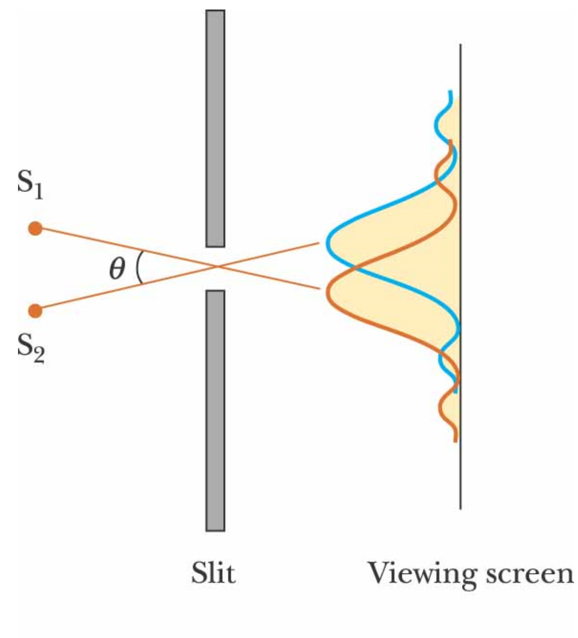
Resolution

- The ability of optical systems to distinguish between closely spaced objects
- If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished

The images are said to be *resolved*

- If the two sources are close together, the two central maxima overlap and the images are not resolved





Resolution, Rayleigh's Criterion

Rayleigh's criterion:

When the *central maximum* of one image falls on the *first minimum* of another image, the images are said to be just resolved

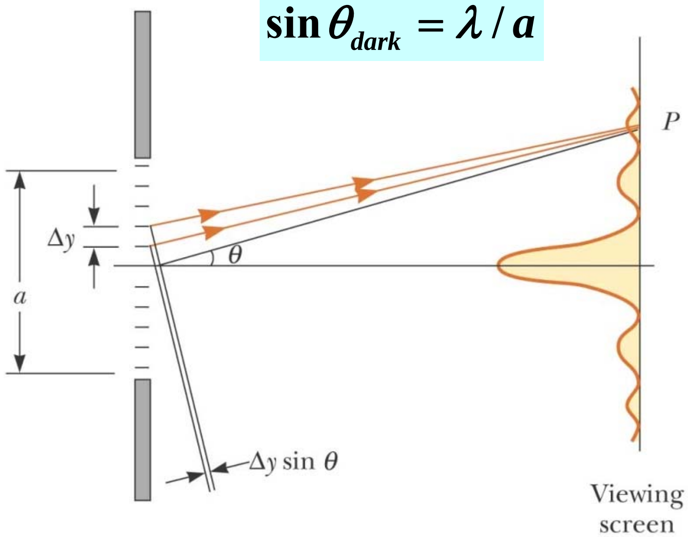
Resolution of a slit:

- Since **$\lambda << a$** in most situations, **$\sin\theta$** is very small and **$\sin\theta \sim \theta$**
- Therefore, the limiting angle (in rad) of resolution for a slit of width ***a*** is

$$\theta_{\min} = \theta_{dark} = \lambda / a$$

- To be resolved, the angle subtended by the two sources must be greater than $\theta_{\min}$

$$\sin\theta_{dark} = \lambda / a$$



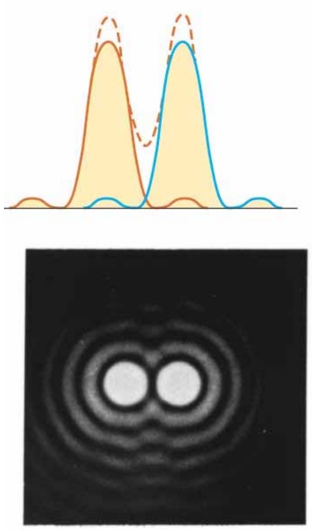
Resolution: Circular Aperture

- The diffraction pattern of a circular aperture consists of a central bright disk surrounded by progressively fainter bright and dark rings
- The limiting angle of resolution of the circular aperture is

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

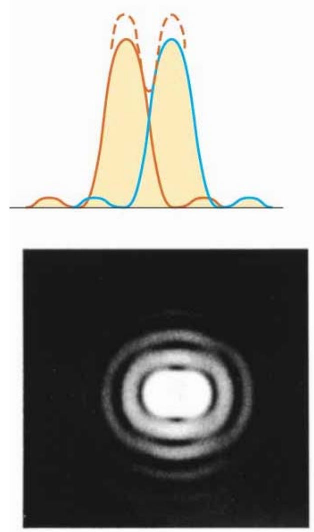
  - *D* is the diameter of the aperture

The images are well resolved



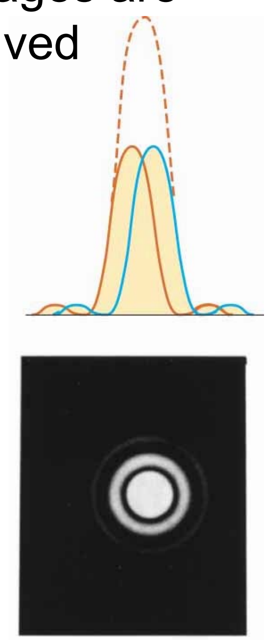
(a)

The images are just resolved



(b)

The images are unresolved



(c)