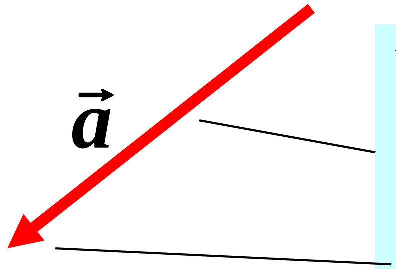
Introduction:

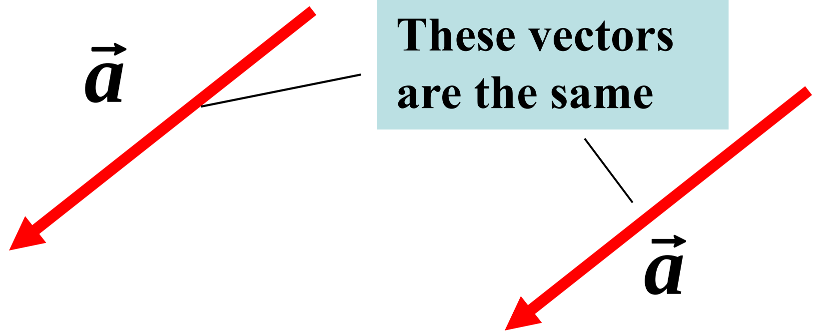
Vectors and Integrals

Vectors



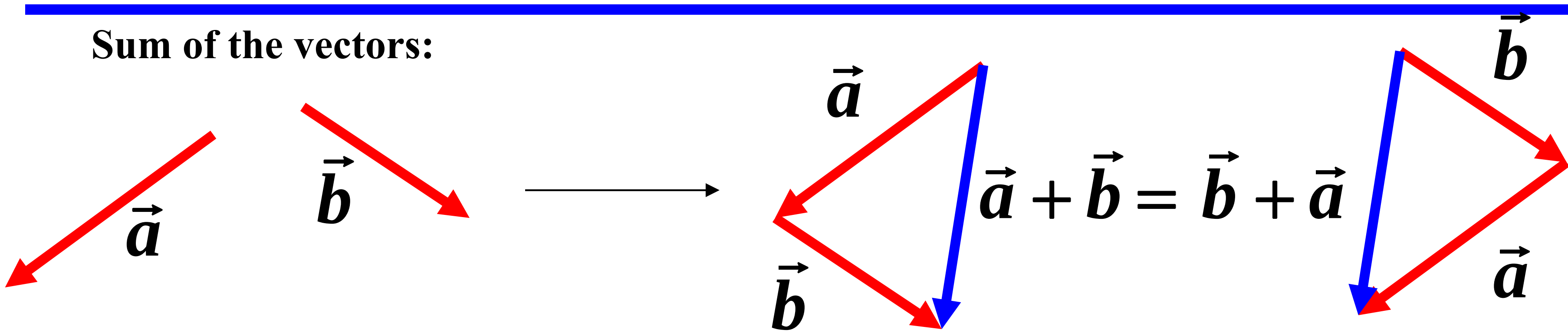
Vectors are characterized by two parameters:

- length (magnitude)
- direction



These vectors are the same

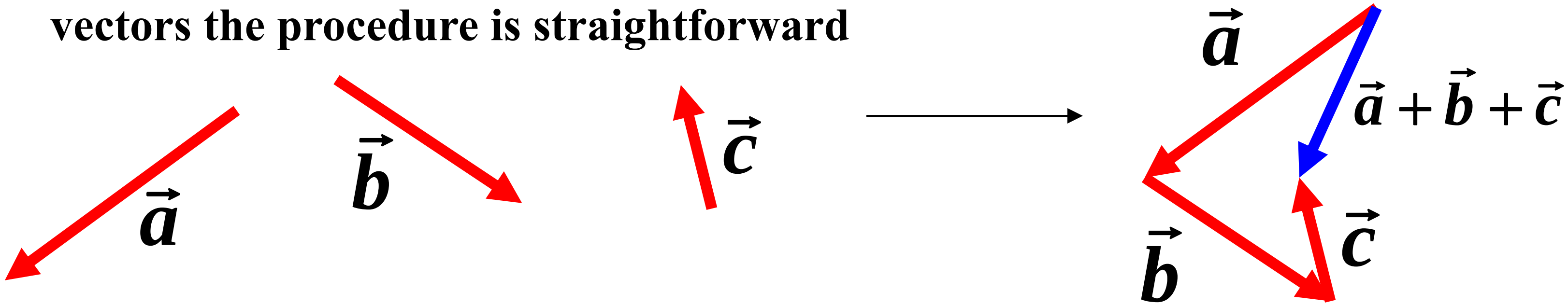
Sum of the vectors:

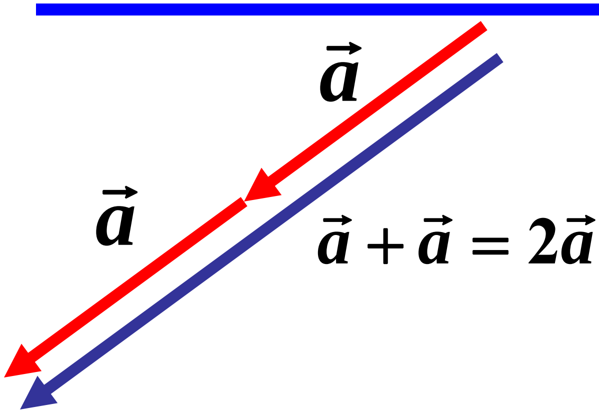


$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

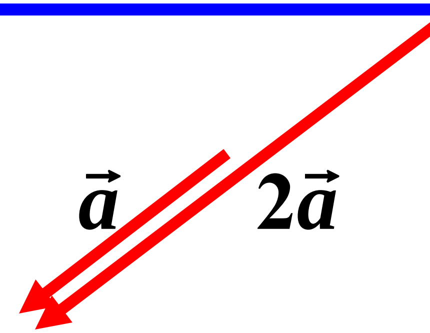
Vectors

Sum of the vectors: for a larger number of vectors the procedure is straightforward

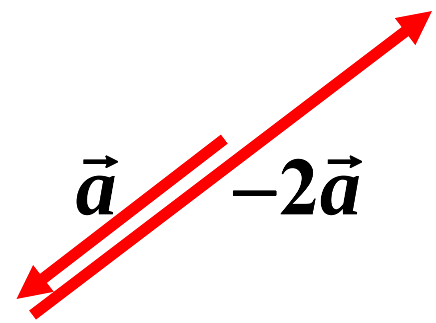




Vector $c\vec{a}$ (where c is the positive number) has the same direction as $\vec{a}$, but its length is c times larger



Vector $c\vec{a}$ (where c is the negative number) has the direction opposite to $\vec{a}$, and c times larger length



Vectors

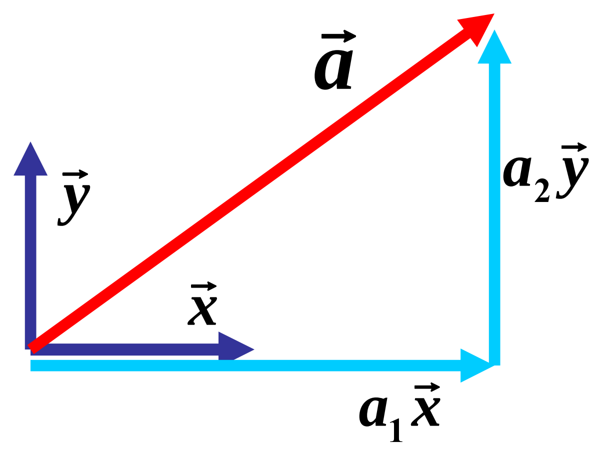
The vectors can be also characterized by a set of numbers (components), i.e.

$$\vec{a} = (a_1, a_2, ...)$$

This means the following: if we introduce some basic vectors, for example $\vec{x}$ and $\vec{y}$ in the plane, then we can write

$$\vec{a} = a_1\vec{x} + a_2\vec{y}$$

$\vec{x}, \vec{y}$ usually have unit magnitude



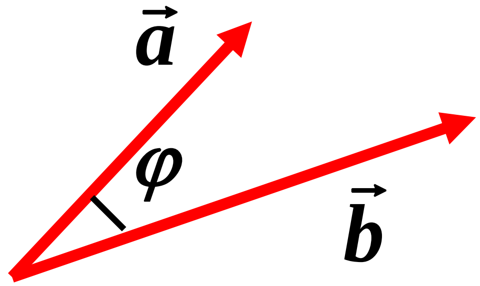
Then the sum of the vectors is the sum of their components:

$$\vec{a} = (a_1, a_2)$$
$$\vec{b} = (b_1, b_2) \longrightarrow \vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2)$$

$$\vec{a} = (a_1, a_2) \longrightarrow c\vec{a} = (ca_1, ca_2)$$

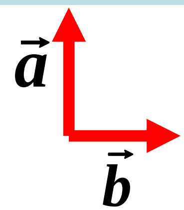
Vectors: Scalar and Vector Product

Scalar Product



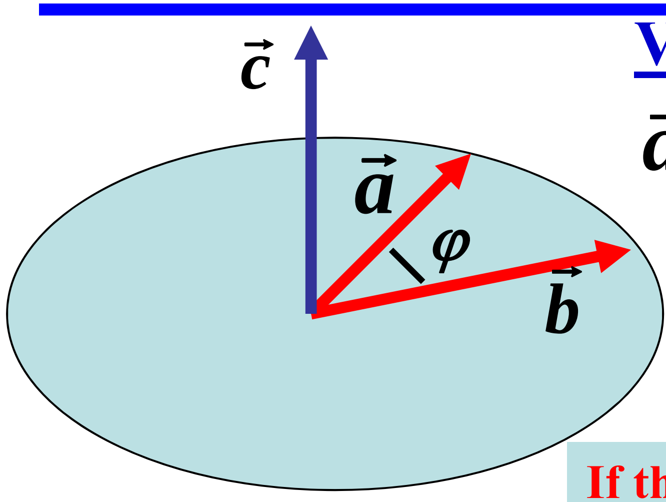
$\vec{a} \cdot \vec{b}$ is the scalar (not vector) $= ab\cos(\varphi)$

If the vectors are orthogonal then the scalar product is 0



$\vec{a} \cdot \vec{b} = 0$

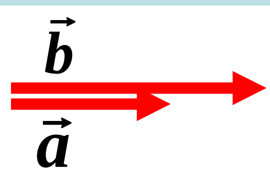
Vector Product



$\vec{a} \times \vec{b} = \vec{c}$ is the VECTOR, the magnitude of which is $ab\sin(\varphi)$

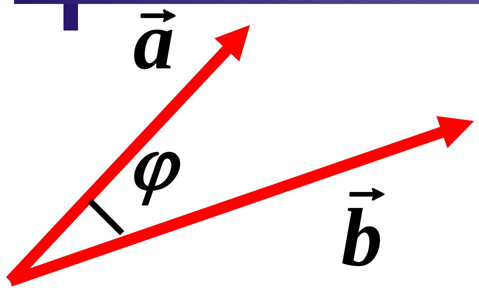
Vector $\vec{c}$ is orthogonal to the plane formed by $\vec{a}$ and $\vec{b}$

If the vectors have the same direction then vector product is 0



$\vec{a} \times \vec{b} = 0$

Vectors: Scalar Product



Scalar Product

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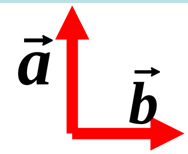
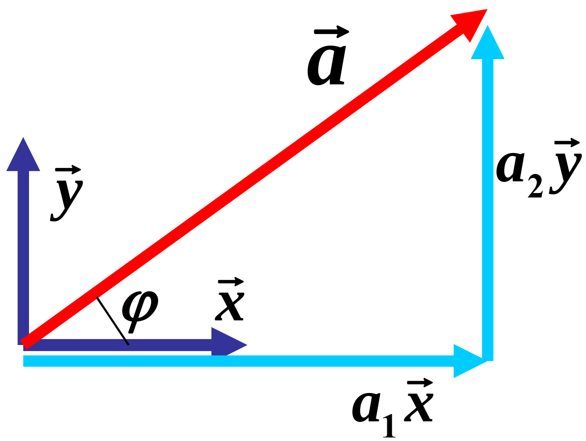


$\vec{a}\cdot\vec{b} = 0$



$\vec{a} = a_1\vec{x} + a_2\vec{y}$

It is straightforward to relate the scalar product of two vectors to their components in orthogonal basis

If the basis vectors $\vec{x},\vec{y}$ are orthogonal and have unit magnitude (length) then we can take the scalar product of vector $\vec{a} = a_1\vec{x} + a_2\vec{y}$ and basis vectors $\vec{x},\vec{y}$:

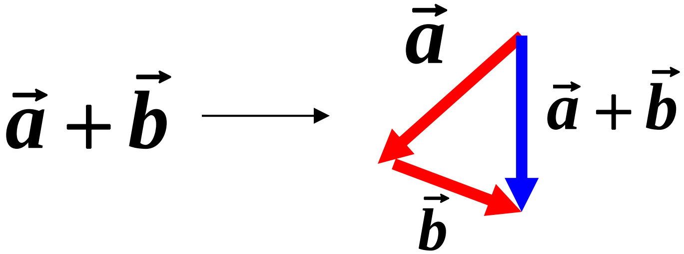
$$a\cos(\varphi) = \vec{a}\cdot\vec{x} = a_1\vec{x}\cdot\vec{x} + a_2\vec{y}\cdot\vec{x} = a_1$$

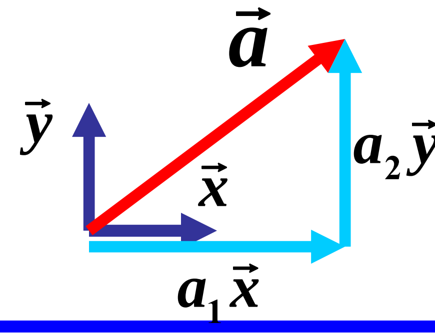
from the definition of the scalar product

=1 (unit magnitude)

=0 (orthogonal)

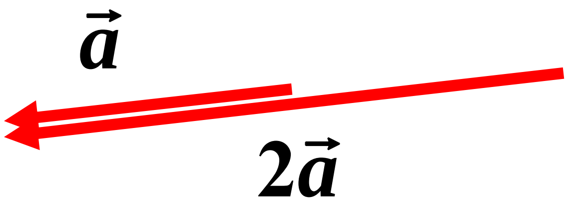
$$a\cos(\pi/2-\varphi) = a\sin(\varphi) = \vec{a}\cdot\vec{y} = a_1\vec{x}\cdot\vec{y} + a_2\vec{y}\cdot\vec{y} = a_2$$

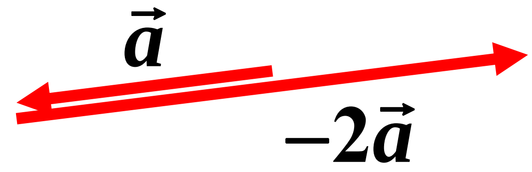


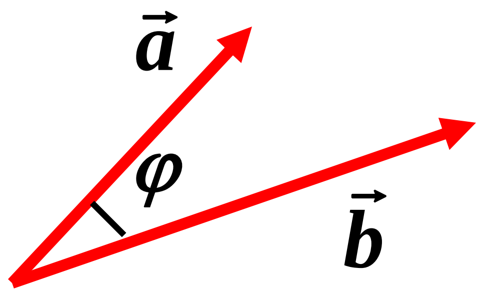


$\vec{a}=(a_1,a_2)$

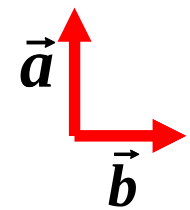
$a_1=\vec{a}\cdot\vec{x}$
$a_2=\vec{a}\cdot\vec{y}$



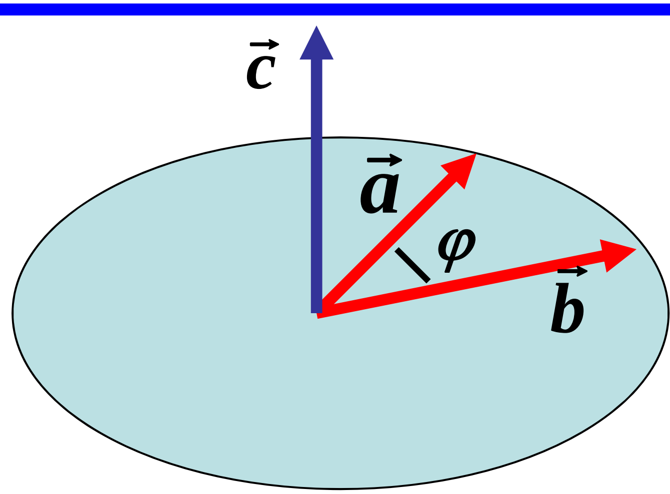




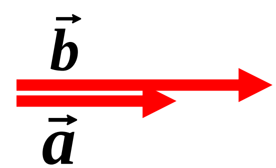
$$\vec{a}\cdot\vec{b}=ab\cos(\varphi)$$



$$\vec{a}\cdot\vec{b}=0$$

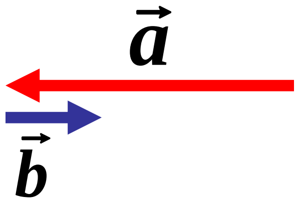


$$\vec{a}\times\vec{b}=\vec{c}$$
$$c=ab\sin(\varphi)$$



$$\vec{a}\times\vec{b}=0$$

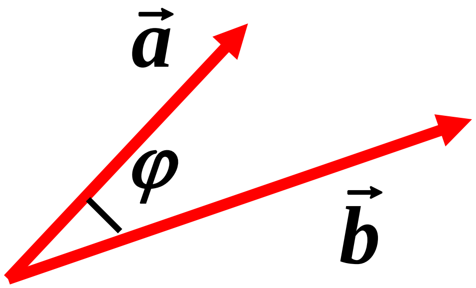
Vectors: Examples



The magnitude of $\vec{a}$ is 5

What is the direction and the magnitude of $\vec{b} = -0.2\vec{a}$

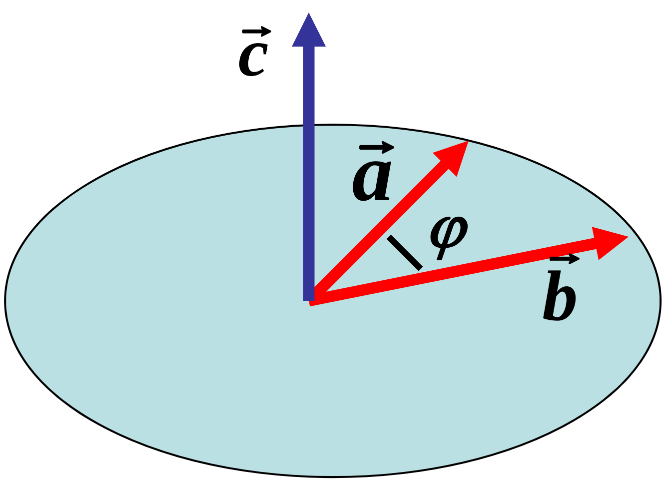
The magnitude of $\vec{b}$ is $b = 0.2 \cdot 5 = 1$, the direction is opposite to $\vec{a}$



The magnitude of $\vec{a}$ is 5, the magnitude of $\vec{b}$ is 2, the angle φ is $\pi / 3$

What is the scalar and vector product of $\vec{a}$ and $\vec{b}$

$$\vec{a} \cdot \vec{b} = 5 \cdot 2 \cos(\pi / 3) = 5$$



$$\vec{a} \times \vec{b} = \vec{c}$$

$$c = 5 \cdot 2 \sin(\pi / 3) = 5\sqrt{3}$$

Integrals

Basic integrals:

$$\int_a^b \frac{dx}{x^n} = \frac{1}{n-1}\left(\frac{1}{a^{n-1}} - \frac{1}{b^{n-1}}\right) \qquad \int_a^b x^n dx = \frac{1}{n+1}\left(b^{n+1} - a^{n+1}\right)$$

You need to recognize these types of integrals.

Examples:

- $\int_a^b \frac{dx}{(x+c)^n}$ → introduce new variable $y = x + c$, $dy = dx$ → $\int_a^b \frac{dx}{(x+c)^n} = \int_{a+c}^{b+c} \frac{dy}{y^n}$

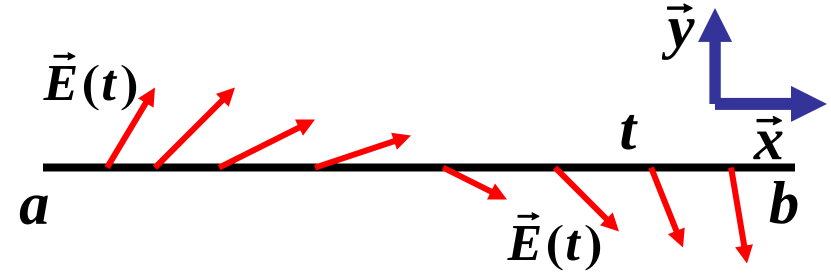
Important: Different Limits in the Integrals

- $\int_a^b \frac{xdx}{(x^2+c)^n}$ → introduce new variable $y = x^2 + c$, $dy = 2xdx$ → $\int_a^b \frac{xdx}{(x^2+c)^n} = \frac{1}{2}\int_{a^2+c}^{b^2+c} \frac{dy}{y^n}$

Integrals

Integrals containing vector functions

$$\int_a^b \vec{E}(t)dt$$



How can we find the values of such integrals?

$\int_a^b \vec{E}(t)dt$ - this is the vector, so we can calculate each component of this vector

We can write $\vec{E}(t) = E_1(t)\vec{x} + E_2(t)\vec{y}$, where only scalar functions $E_1(t), E_2(t)$ depend on *t*, but not the basis vectors $\vec{x}, \vec{y}$ then integral takes the form

Then the integral takes the form

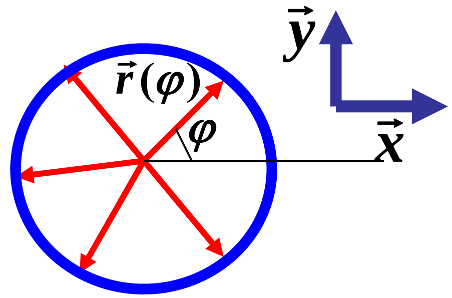
$$\int_a^b \vec{E}(t)dt = \vec{x}\int_a^b E_1(t)dt + \vec{y}\int_a^b E_2(t)dt$$

so now there are two integrals which contain only scalar functions

Integrals

Example:

$$\int_0^{2\pi} \vec{r}(\varphi)\cos(\varphi)d\varphi$$



$\vec{r}(\varphi)$ - along the radius, then we can write the radial vector in terms of radius r

$$\vec{r}(\varphi) = r_1(\varphi)\vec{x} + r_2(\varphi)\vec{y} = r\cos(\varphi)\vec{x} + r\sin(\varphi)\vec{y}$$

Then we have the following expression for the integral

$$\int_0^{2\pi} \vec{r}(\varphi)\cos(\varphi)d\varphi = r\,\vec{x}\int_0^{2\pi}\cos^2(\varphi)d\varphi + r\,\vec{y}\int_0^{2\pi}\cos(\varphi)\sin(\varphi)d\varphi = r\,\pi\,\vec{x}$$

$$= \frac{1}{2}\int_0^{2\pi}[1+\cos(2\varphi)]d\varphi = \frac{2\pi}{2} = \pi$$

$$= \frac{1}{2}\int_0^{2\pi}\sin(2\varphi)d\varphi = 0$$

Chapter 25

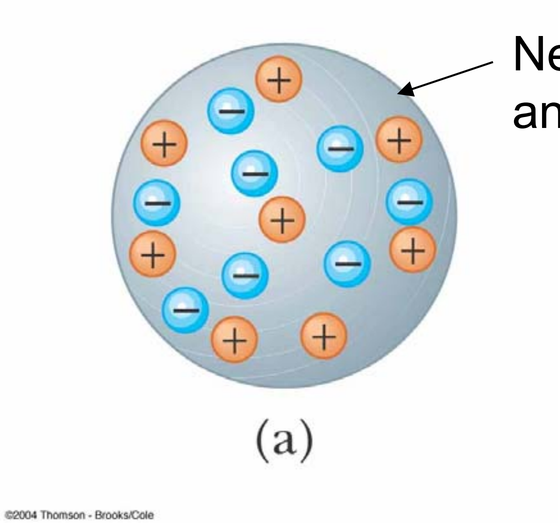
Electricity and Magnetism

Electric Fields: Coulomb's Law

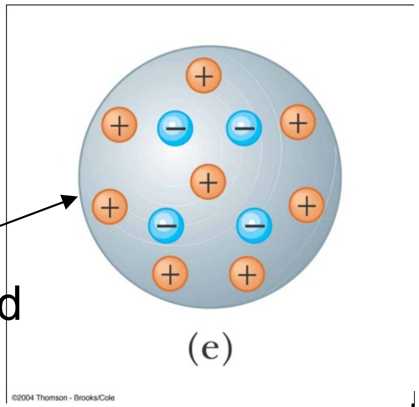
Reading: Chapter 25

Electric Charges

- There are **two kinds** of electric charges
  - Called **positive** and **negative**
    - Negative charges are the type possessed by electrons
    - Positive charges are the type possessed by protons
- Charges of the same sign repel one another and charges with opposite signs attract one another
- Electric charge is always conserved in isolated system



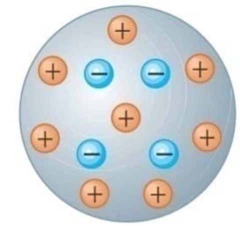
Neutral – equal number of positive and negative charges



Positively charged

Electric Charges: Conductors and Isolators

- Electrical conductors are materials in which some of the electrons are free electrons
  - These electrons can move relatively freely through the material
  - Examples of good conductors include copper, aluminum and silver
- Electrical insulators are materials in which all of the electrons are bound to atoms
  - These electrons can not move relatively freely through the material
  - Examples of good insulators include glass, rubber and wood
- Semiconductors are somewhere between insulators and conductors



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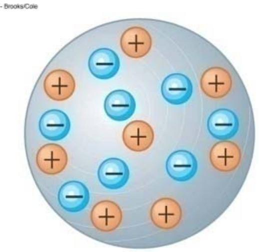
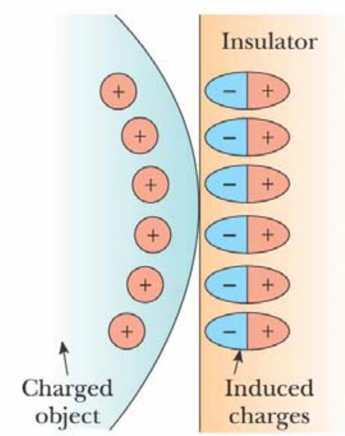
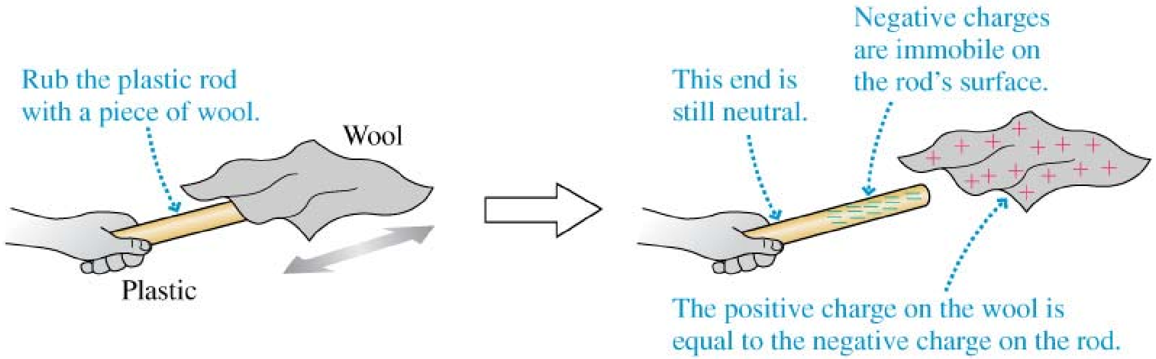


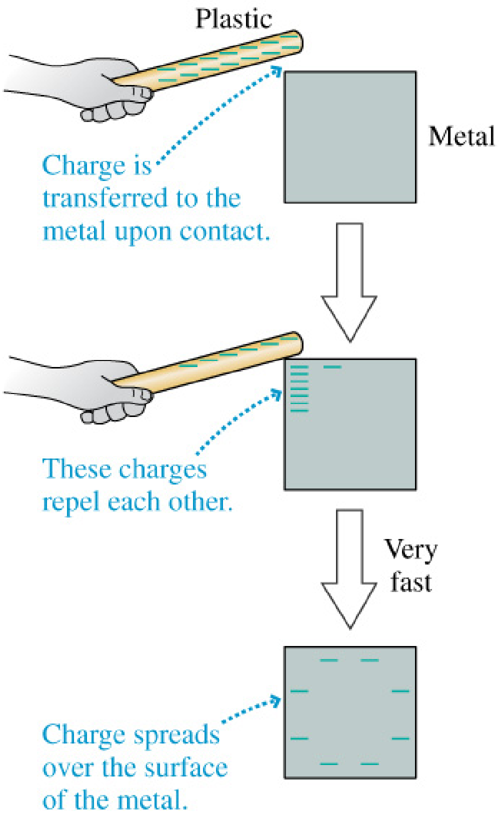
(a)



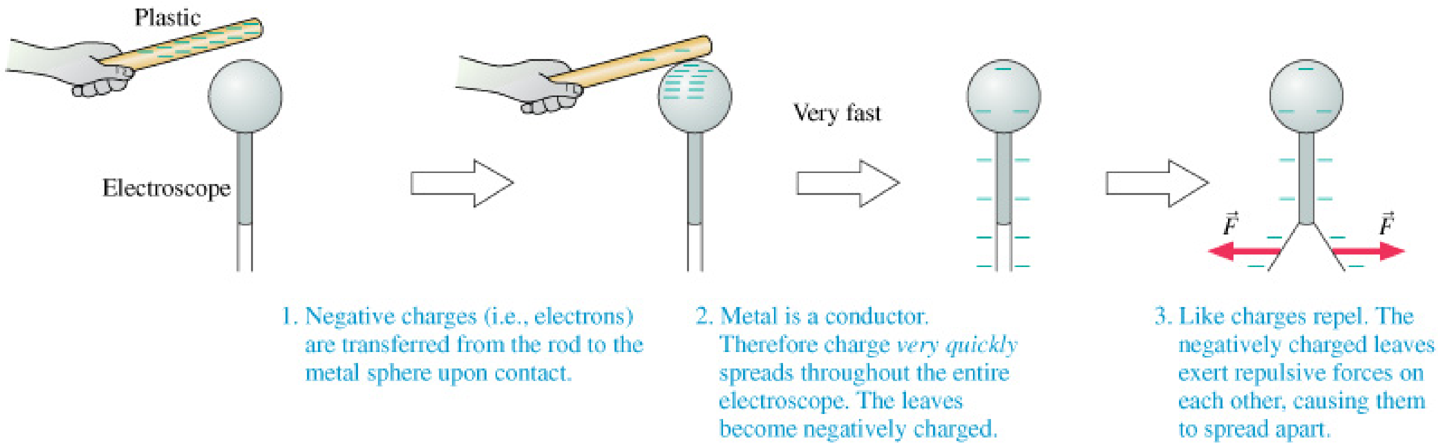
(a)

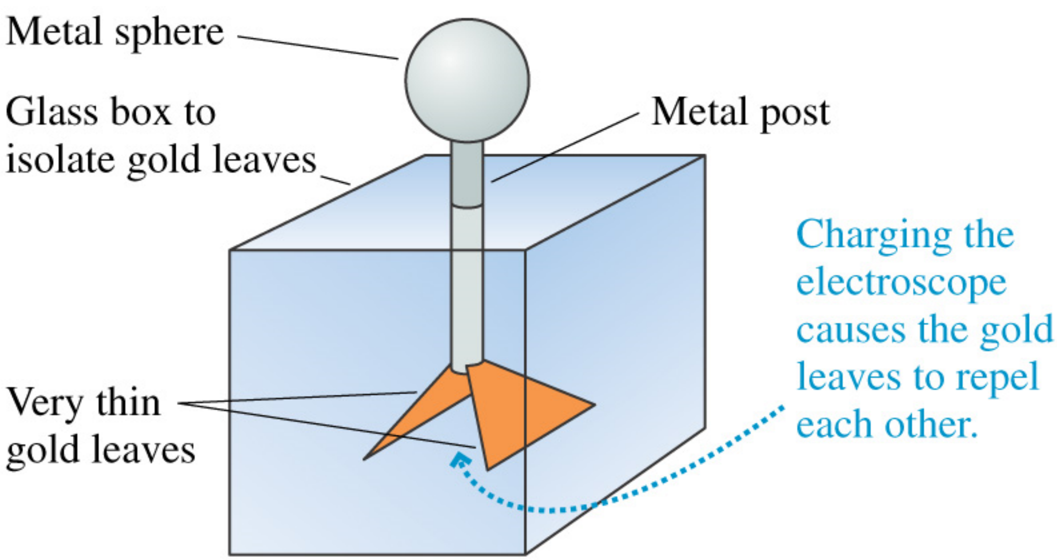
Electric Charges



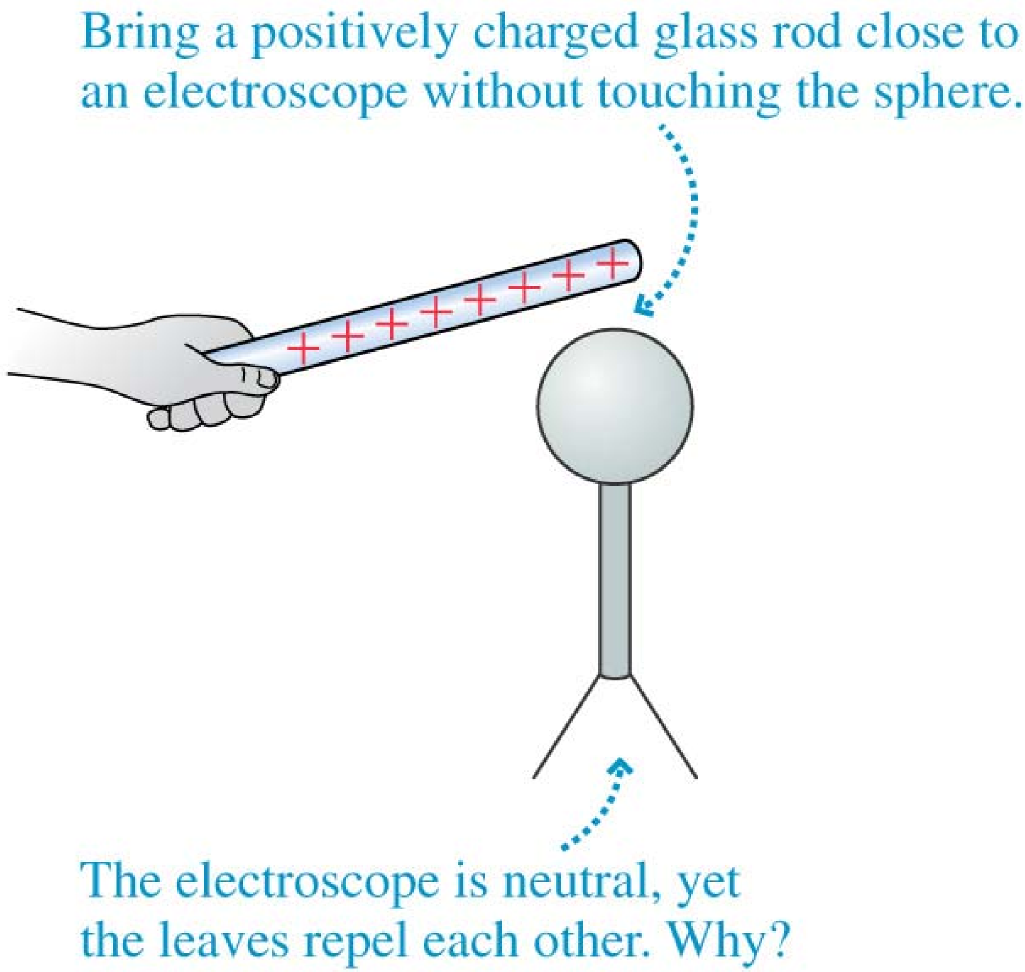


Electric Charges

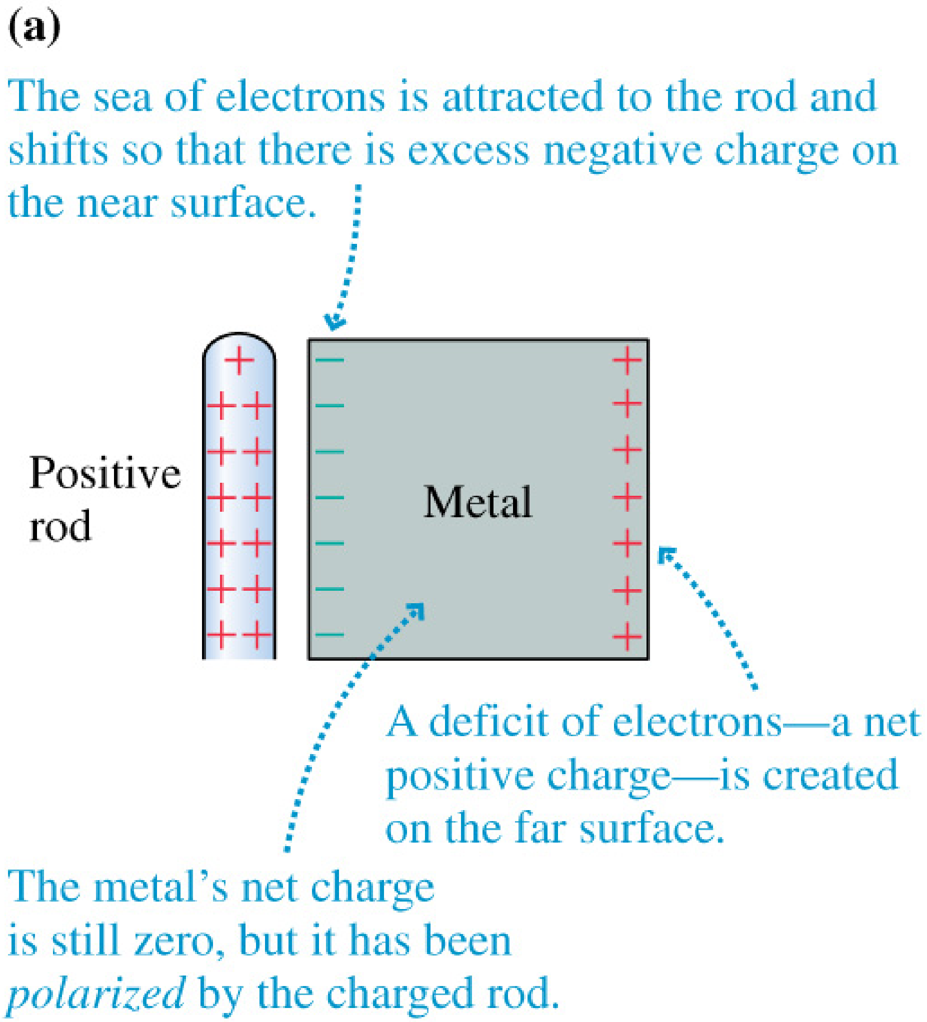




Electric Charges



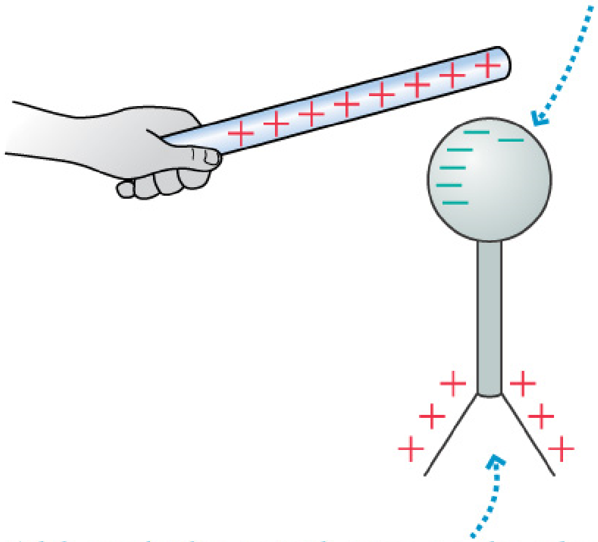
Electric Charges



Electric Charges

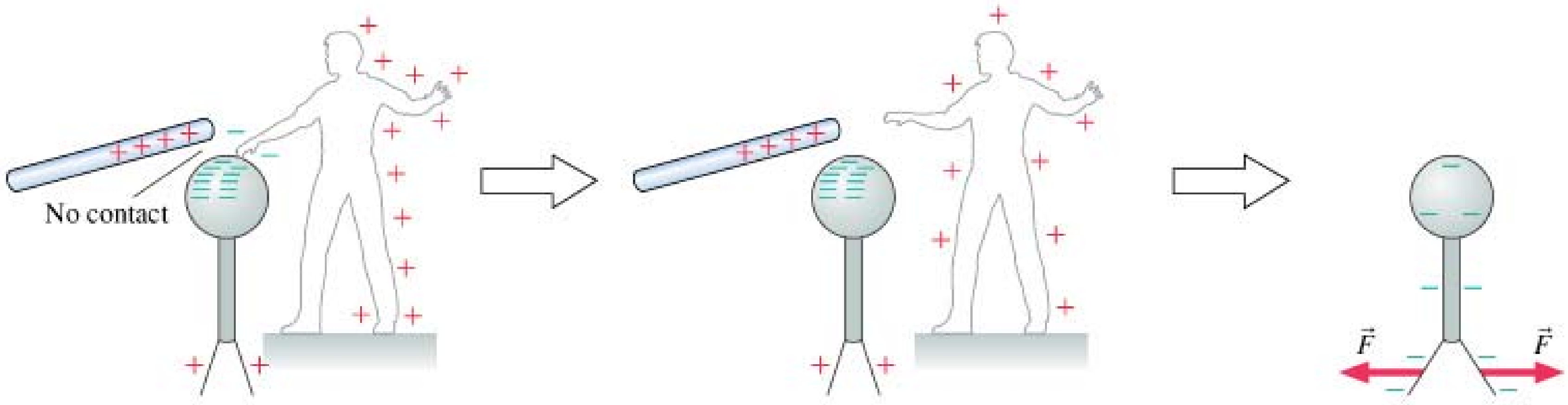
(b)

The electroscope is polarized by the charged rod. The sea of electrons shifts toward the rod.



Although the net charge on the electroscope is still zero, the leaves have excess positive charge and repel each other.

Electric Charges

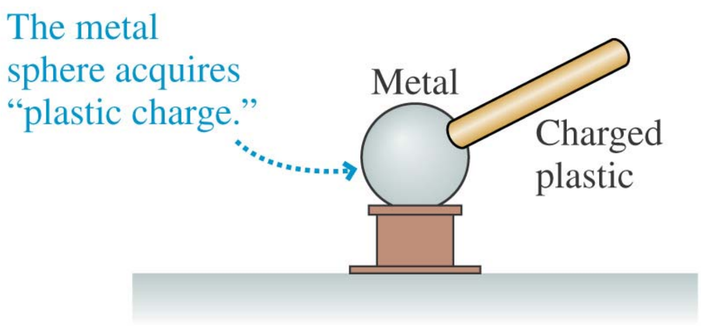


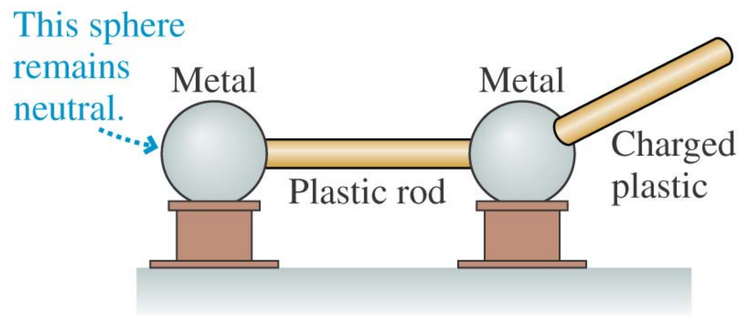
1. The charged rod polarizes the electroscope+person conductor. The leaves repel slightly, due to polarization within the electroscope, but overall the electroscope has an excess of electrons and the person has a deficit of electrons.

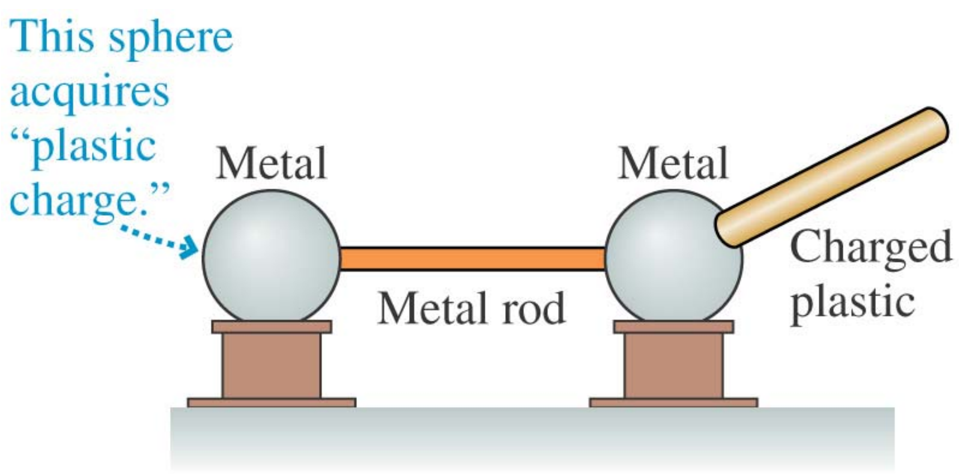
2. The negative charge on the electroscope is isolated when contact is broken.

3. When the rod is removed, the leaves first collapse, as the polarization vanishes, then repel as the excess negative charge spreads out. The electroscope has been *negatively* charged.

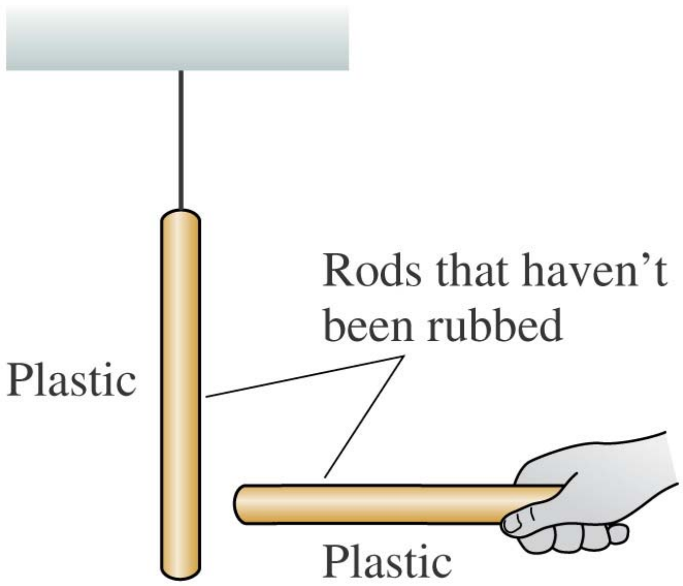
Electric Charges

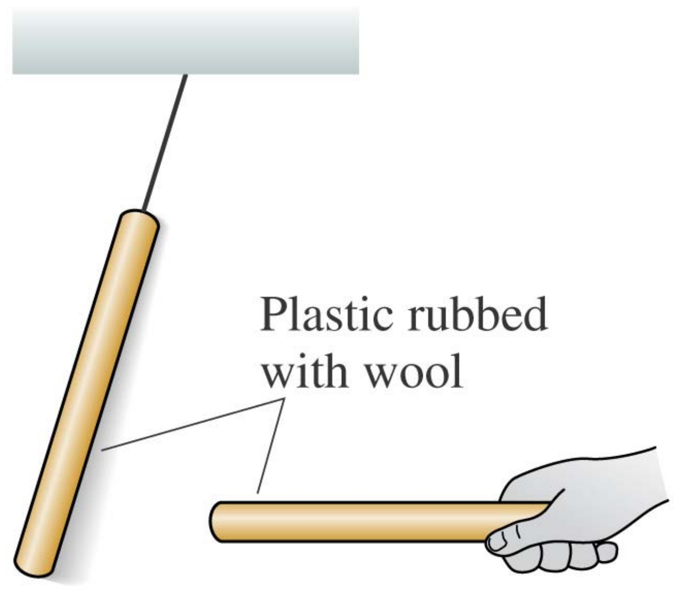


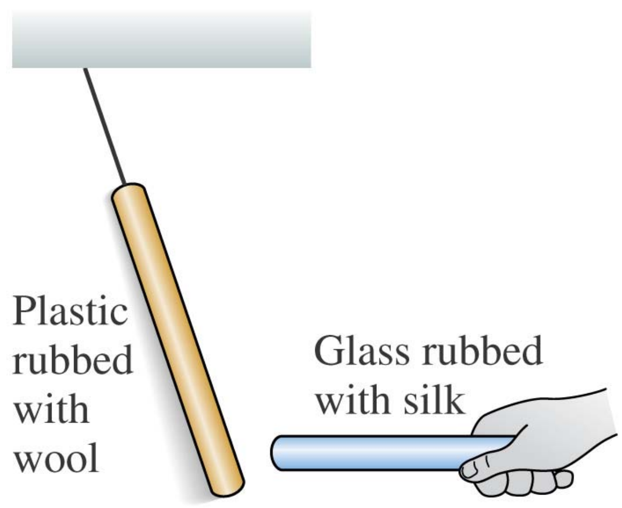




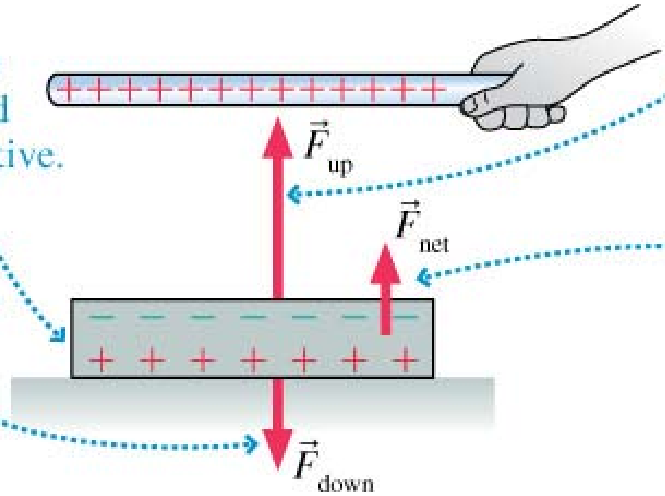
Electric Charges







Electric Charges



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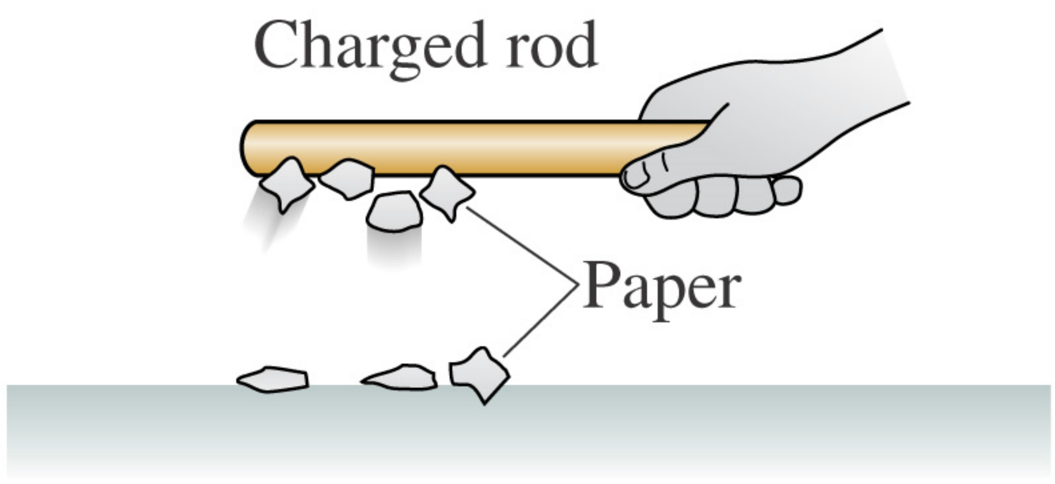
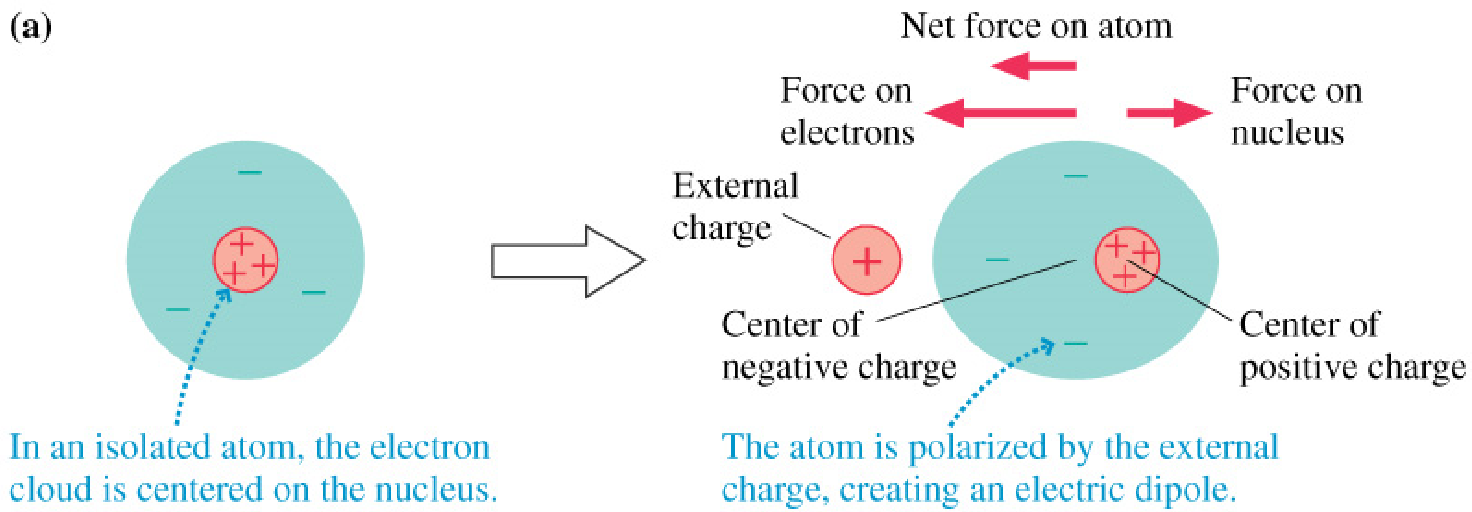
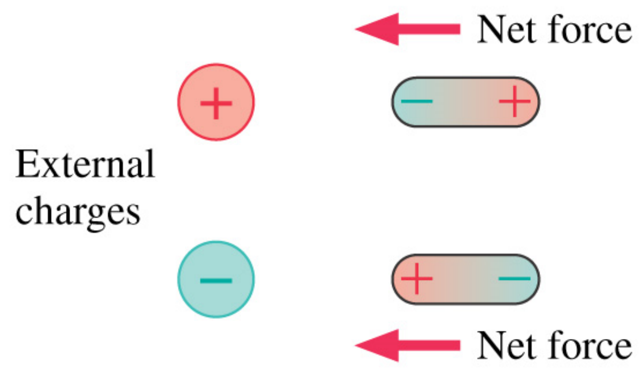


Electric Charges

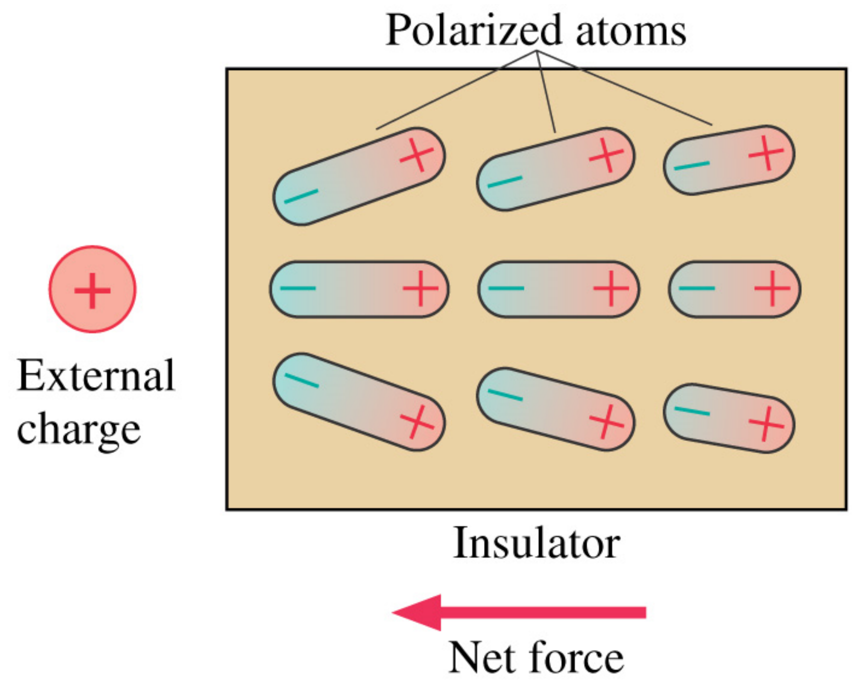


Electric Charges

(b)



Electric dipoles can be created by either positive or negative charges. In both cases, there is an attractive net force toward the external charge.



Electric Charges

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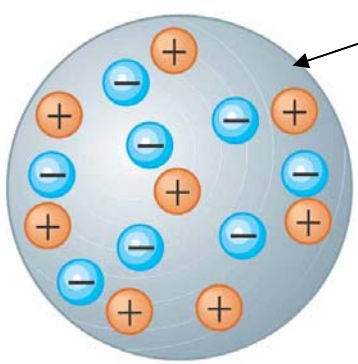
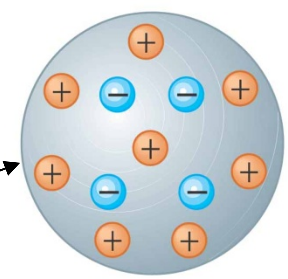


(a)

Neutral – equal number of positive and negative charges

Positively charged

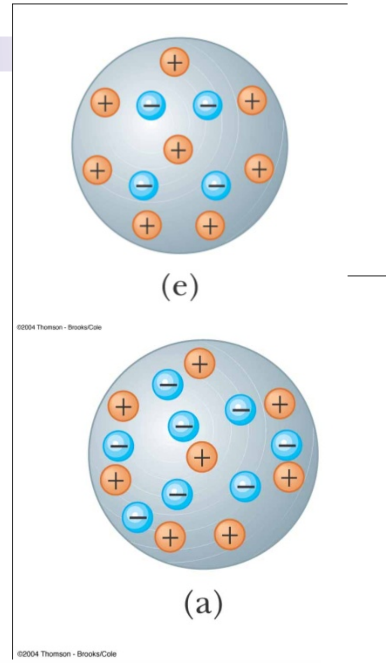


(e)

Electric Charges: Conductors and Isolators

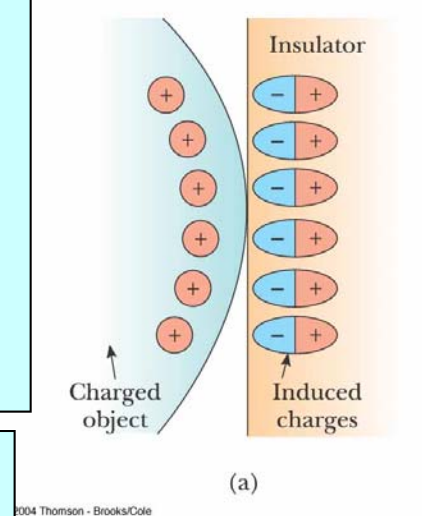
➢ Electrical conductors are materials in which some of the electrons are free electrons

- ❑ These electrons can move relatively freely through the material
- ❑ Examples of good conductors include copper, aluminum and silver



➢ Electrical insulators are materials in which all of the electrons are bound to atoms

- ❑ These electrons can not move relatively freely through the material
- ❑ Examples of good insulators include glass, rubber and wood



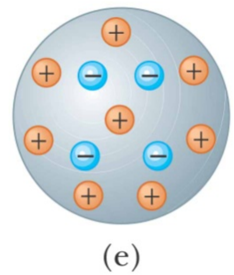
➢ Semiconductors are somewhere between insulators and conductors

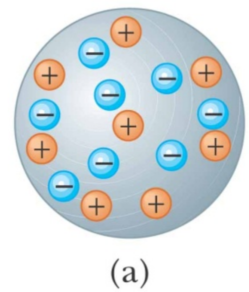
Conservation of Charge

Electric charge is always conserved in isolated system

Two identical sphere

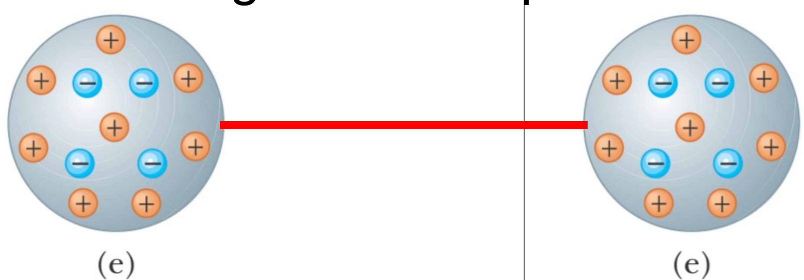
$q_1 = 1\mu C$





$q_2 = -2\,\mu C$

They are connected by conducting wire. What is the electric charge of each sphere?

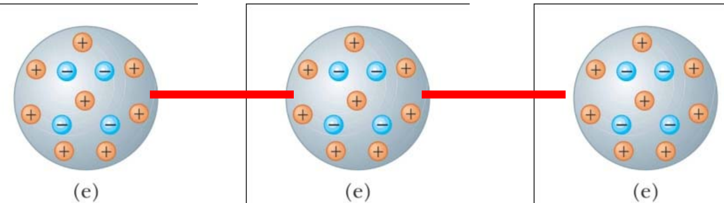


The same charge ***q***. Then the conservation of charge means that :

$$2q = q_1 + q_2$$

$$q = \frac{q_1 + q_2}{2} = \frac{1-2}{2}\mu C = -0.5\mu C$$

For three spheres: $q_1 = 1\mu C$ $q_2 = -2\,\mu C$ $q_3 = 3\,\mu C$



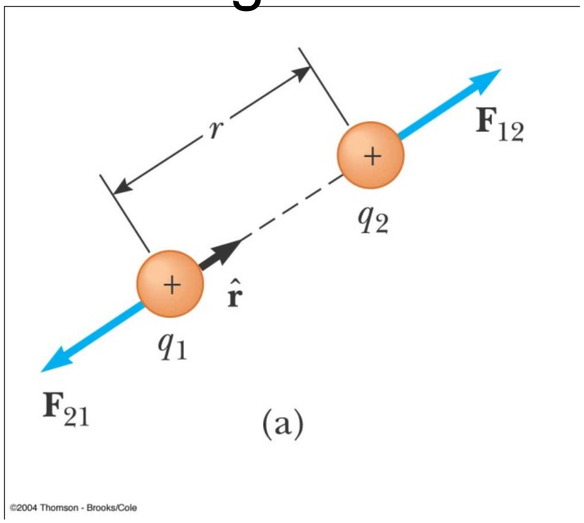
$$3q = q_1 + q_2 + q_3$$

$$q = \frac{q_1 + q_2 + q_3}{3} = \frac{1-2+3}{3}\mu C = 1\mu C$$

Coulomb's Law

- Mathematically, the force between two electric charges:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\vec{r}}$$



- The SI unit of charge is the **coulomb** (C)
- k_e is called the **Coulomb constant**
  - k_e = 8.9875 x 10^9 ***N·m²/C²*** = $1/(4\pi e_o)$
  - e_o is the **permittivity of free space**
  - e_o = 8.8542 x 10^{-12} C^2 / N·m^2

Electric charge:

- ❑ electron e = -1.6 x 10^{-19} C
- ❑ proton e = 1.6 x 10^{-19} C

Coulomb's Law

$$F_{12} = F_{21} = k_e \frac{|q_1||q_2|}{r^2}$$

Direction depends on the sign of the product

$$q_1 q_2$$

$$q_1 q_2 > 0$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

opposite directions, the same magnitude

$$q_1 q_2 > 0$$

$$q_1 q_2 < 0$$

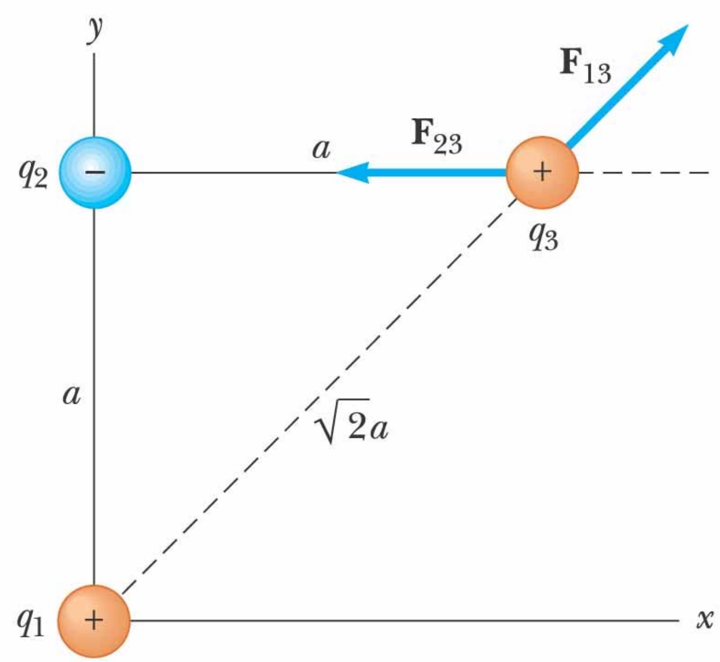
The force is attractive if the charges are of opposite sign
The force is repulsive if the charges are of like sign

Magnitude:

$$F_{12} = F_{21} = k_e \frac{|q_1||q_2|}{r^2}$$

Coulomb's Law: Superposition Principle

- The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$
- The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$
- The *resultant force* exerted on q_3 is the vector sum of $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$

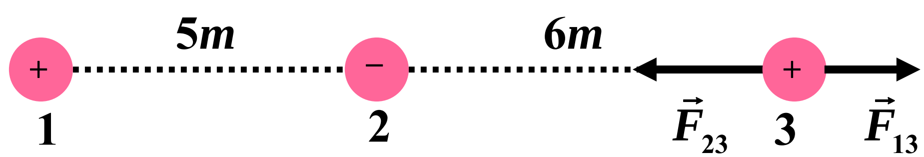


Coulomb's Law

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{\vec{r}}_{21}$$

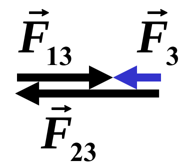
$q_1 = 1\mu C$ $q_2 = -2\mu C$

$q_3 = 3\mu C$



Resultant force:

$$\vec{F}_3 = \vec{F}_{21} + \vec{F}_{31}$$



Magnitude:

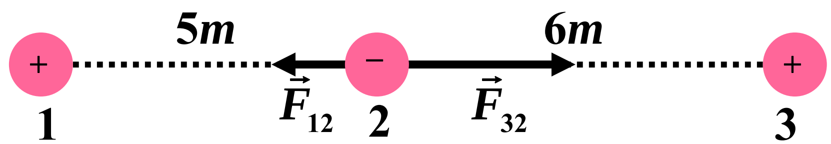
$$F_{23} = k_e \frac{|q_3||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6} \cdot 3 \cdot 10^{-6}}{6^2} N = 15 \cdot 10^{-4} N$$

$$F_{13} = k_e \frac{|q_3||q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 3 \cdot 10^{-6}}{11^2} N = 2.2 \cdot 10^{-4} N$$

$$F_3 = F_{23} - F_{13} = (15 \cdot 10^{-4} - 2.2 \cdot 10^{-4}) N = 12.8 \cdot 10^{-4} N$$

Coulomb's Law

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{\vec{r}}_{21}$$

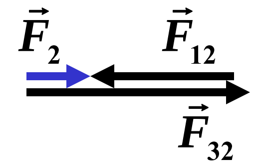


$q_1 = 1\mu C$ $q_2 = -2\mu C$

$q_3 = 3\mu C$

Resultant force:

$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{32}$$



Magnitude:

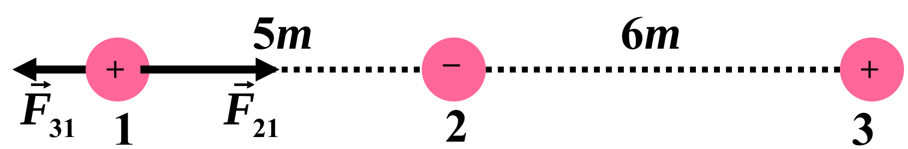
$$F_{32} = k_e \frac{|q_3||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6} \cdot 3 \cdot 10^{-6}}{6^2} N = 15 \cdot 10^{-4} N$$

$$F_{12} = k_e \frac{|q_1||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 2 \cdot 10^{-6}}{5^2} N = 7.2 \cdot 10^{-4} N$$

$$F_2 = F_{32} - F_{12} = (15 \cdot 10^{-4} - 7.2 \cdot 10^{-4}) N = 7.8 \cdot 10^{-4} N$$

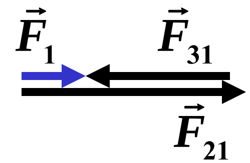
Coulomb's Law

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{\vec{r}}_{21}$$



Resultant force:

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$



$q_1 = 1\mu C$ $q_2 = -2\mu C$

$q_3 = 3\mu C$

Magnitude:

$$F_{31} = k_e \frac{|q_3||q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 3 \cdot 10^{-6}}{11^2} N = 2.2 \cdot 10^{-4} N$$

$$F_{21} = k_e \frac{|q_1||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 2 \cdot 10^{-6}}{5^2} N = 7.2 \cdot 10^{-4} N$$

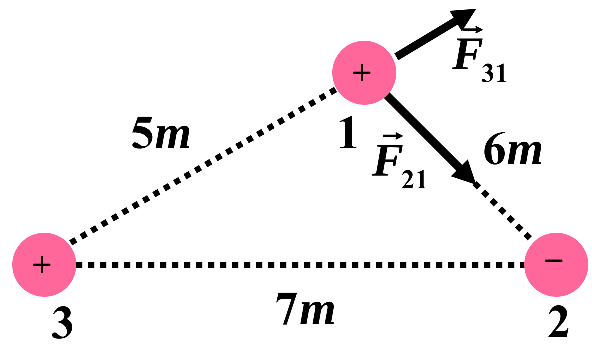
$$F_1 = F_{21} - F_{31} = (7.2 \cdot 10^{-4} - 2.2 \cdot 10^{-4}) N = 5 \cdot 10^{-4} N$$

Coulomb's Law

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{\vec{r}}_{21}$$

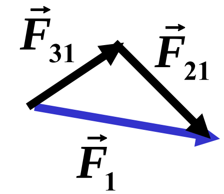
$q_1 = 1\mu C$ $q_2 = -2\mu C$

$q_3 = 3\mu C$



Resultant force:

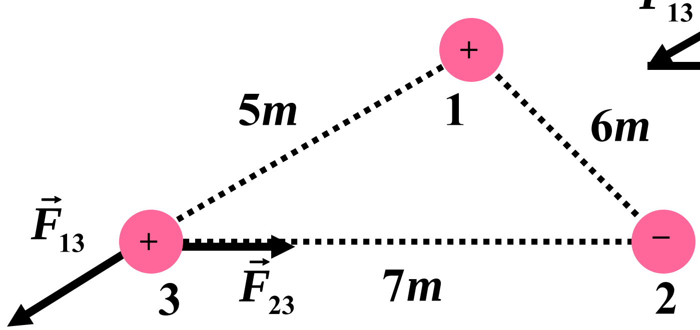
$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$

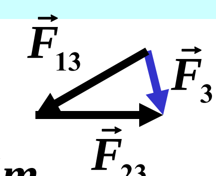


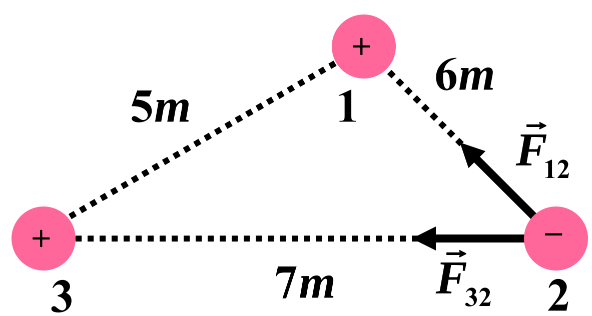
Magnitude:

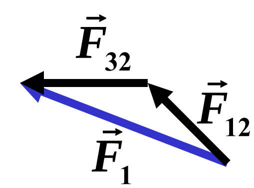
$$F_{21} = k_e \frac{|q_1||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 2 \cdot 10^{-6}}{6^2} N = 5 \cdot 10^{-4} N$$

$$F_{31} = k_e \frac{|q_3||q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 3 \cdot 10^{-6}}{5^2} N = 1.1 \cdot 10^{-3} N$$









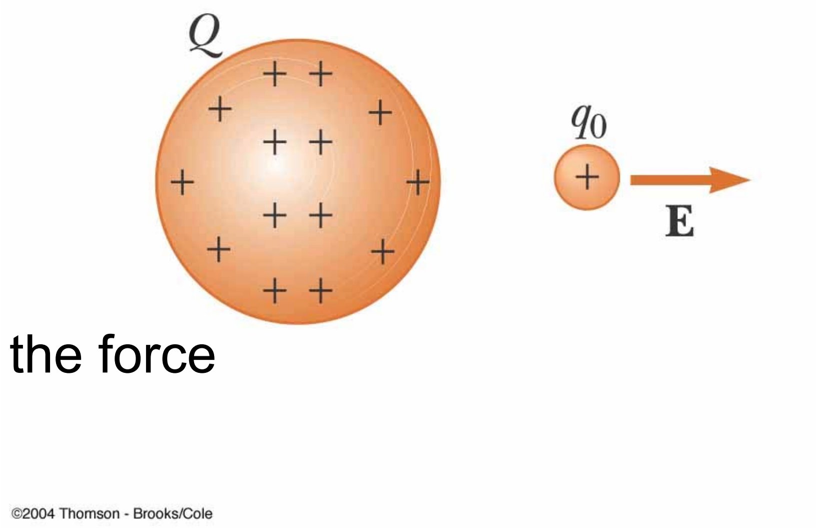
Chapter 25

Electric Field

Electric Field

- An **electric field** is said to exist in the region of space around a charged object
  - This charged object is the **source charge**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it.
- The electric field is defined as the electric force on the test charge per unit charge

$$\vec{E} = \frac{\vec{F}}{q_0}$$



- If you know the electric field you can find the force

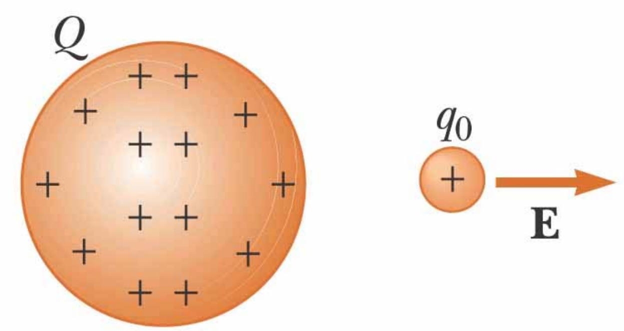
$$\vec{F} = q\vec{E}$$

If *q* is positive, F and E are in the same direction
If *q* is negative, F and E are in opposite directions

Electric Field

- The direction of **E** is that of the force on a positive test charge
- The SI units of **E** are N/C

$$\vec{E} = \frac{\vec{F}}{q_0}$$



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Coulomb's Law:

$$\vec{F} = k_e \frac{qq_0}{r^2}\hat{\vec{r}}$$

Then

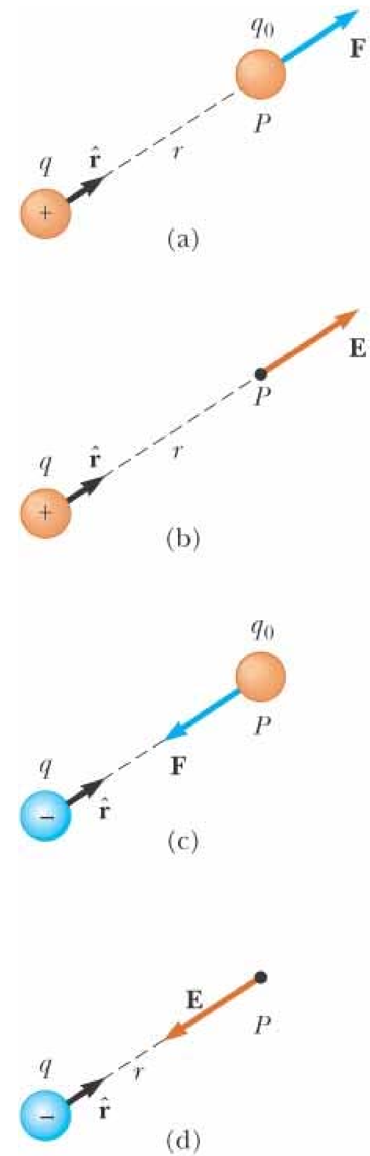
$$\vec{E} = \frac{\vec{F}}{q_0} = k_e \frac{q}{r^2}\hat{\vec{r}}$$

Electric Field

- *q* is positive, **F** is directed away from *q*
   - The direction of **E** is also away from the positive source charge
- *q* is negative, **F** is directed toward *q*
   - **E** is also toward the negative source charge

$$\vec{F} = k_e \frac{q q_0}{r^2} \hat{\vec{r}}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = k_e \frac{q}{r^2} \hat{\vec{r}}$$



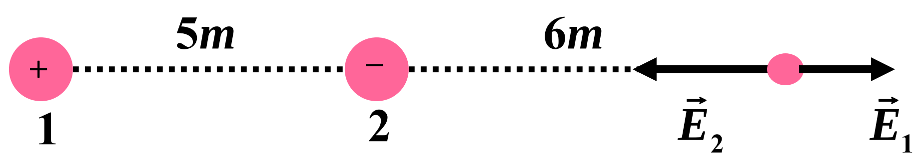
Electric Field: Superposition Principle

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of electric fields of all the charges

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\vec{r}}_i$$

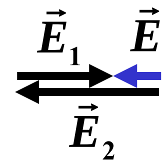
Electric Field

$$\vec{E} = k_e \frac{q}{r^2}\hat{\vec{r}}$$



Electric Field:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



$q_1 = 1\mu C$ $q_2 = -2\mu C$

Magnitude:

$$E_2 = k_e \frac{|q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6}}{6^2} N/C = 5 \cdot 10^2 N/C$$

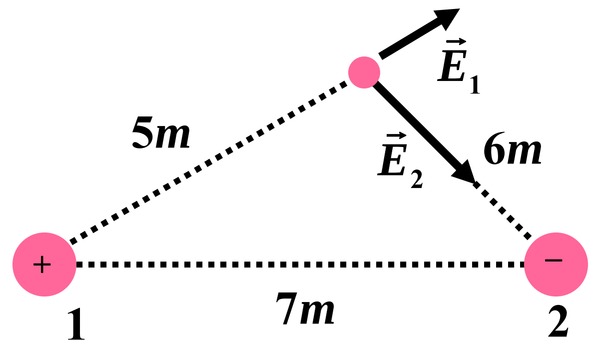
$$E_1 = k_e \frac{|q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6}}{11^2} N/C = 0.7 \cdot 10^2 N/C$$

$$E = E_2 - E_1 = (5 \cdot 10^2 - 0.7 \cdot 10^2) N/C = 4.3 \cdot 10^2 N/C$$

Electric Field

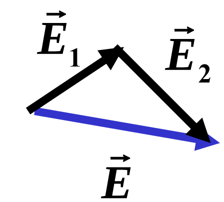
$$\vec{E} = k_e \frac{q}{r^2}\hat{\vec{r}}$$

$q_1 = 1\mu C$ $q_2 = -2\mu C$



Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



Magnitude:

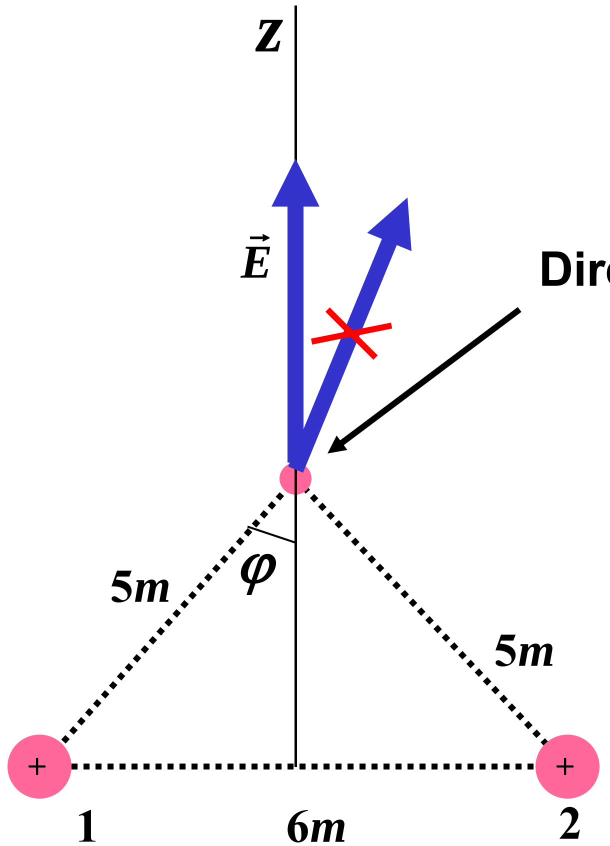
$$E_2 = k_e \frac{|q_2|}{r^2} = 8.9875\cdot 10^9 \frac{2\cdot 10^{-6}}{6^2} N/C = 5\cdot 10^2 N/C$$

$$E_1 = k_e \frac{|q_1|}{r^2} = 8.9875\cdot 10^9 \frac{10^{-6}}{5^2} N/C = 0.37\cdot 10^2 N/C$$

Electric Field

$$\vec{E} = k_e \frac{q}{r^2} \hat{\vec{r}}$$

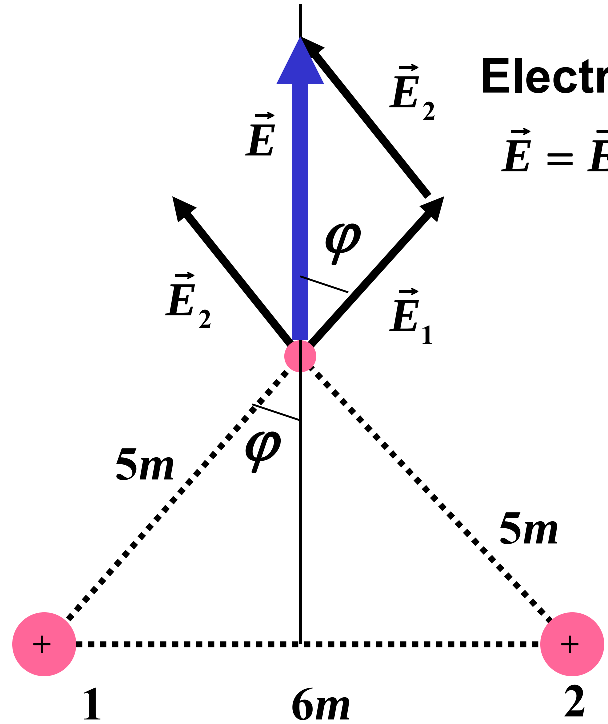
$q_1 = 10\,\mu C$ $q_2 = 10\mu C$



Electric Field

$$\vec{E} = k_e \frac{q}{r^2}\hat{\vec{r}}$$

$q_1 = 10\,\mu C$ $q_2 = 10\mu C$



Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Magnitude:

$$E_2 = E_1 = k_e \frac{|q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10 \cdot 10^{-6}}{5^2} N/C = 3.6 \cdot 10^3 N/C$$

$$E = 2E_1 \cos\varphi$$

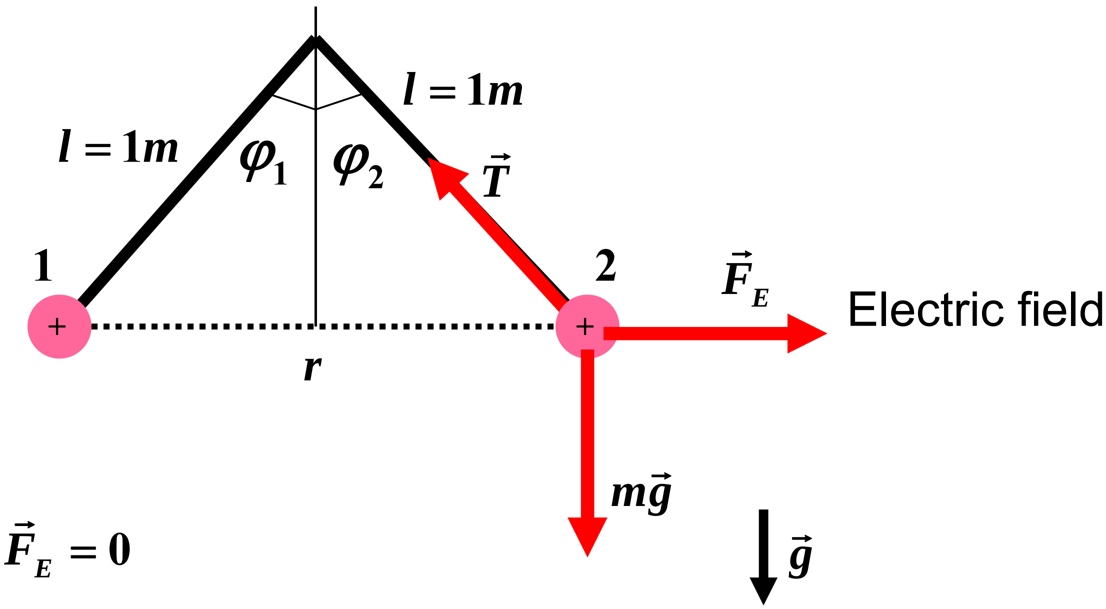
$$\cos\varphi = \frac{\sqrt{5^2 - 3^2}}{5} = \frac{4}{5}$$

$$E = \frac{8}{5}E_1$$

Example

$q_1 = 10\,\mu C$ $q_2 = 10\mu C$

$$m_1 = m_2 = m$$



$$\vec{T} + m\vec{g} + \vec{F}_E = 0$$

$$\vec{F}_E = k_e \frac{q_1 q_2}{r^2}\hat{\vec{r}}$$

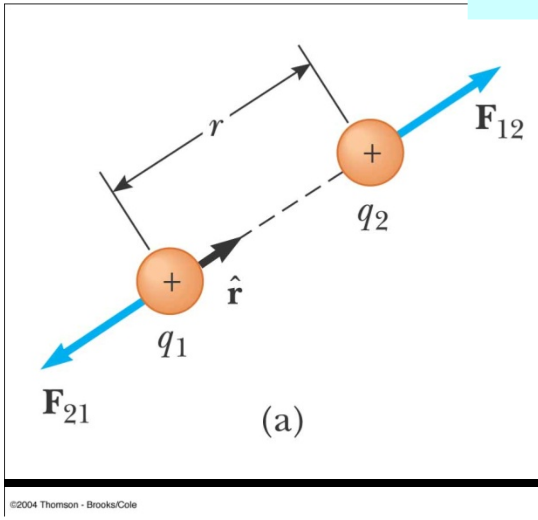
$$T\cos\varphi_2 = mg \qquad T\sin\varphi_2 = F_E$$

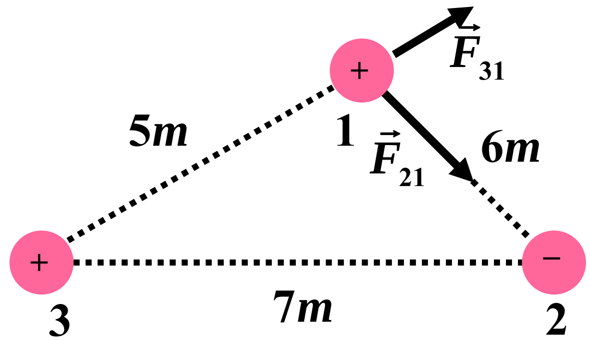
$$\tan\varphi_2 = \frac{F_E}{mg} = k_e \frac{q_1 q_2}{r^2 mg}$$

$$\tan\varphi_1 = \frac{F_E}{mg} = k_e \frac{q_1 q_2}{r^2 mg}$$

$$\varphi_2 = \varphi_1$$

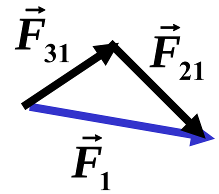
Coulomb's Law: $$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\vec{r}}$$





Resultant force:

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$



$$\vec{E} = k_e \frac{q}{r^2}\hat{\vec{r}}$$

$$\vec{F} = q\vec{E}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\vec{r}}_i$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$