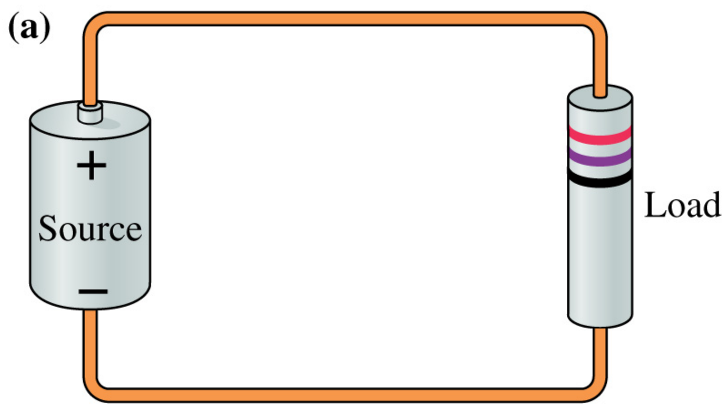
Fundamentals of Circuits: Direct Current (DC)

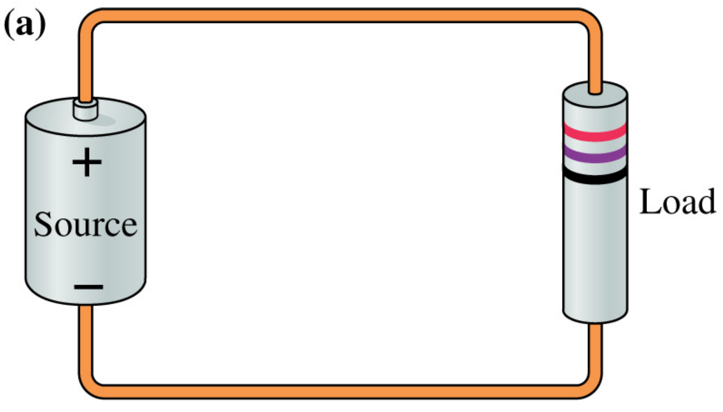
Electrical Circuits: Batteries + Resistors, Capacitors

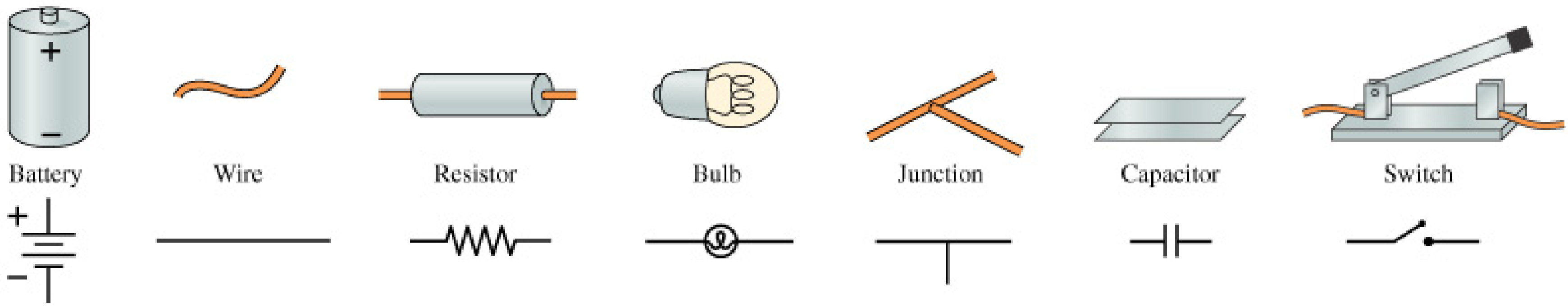
- The battery establishes an electric field in the connecting wires
- This field applies a force on electrons in the wire just outside the terminals of the battery



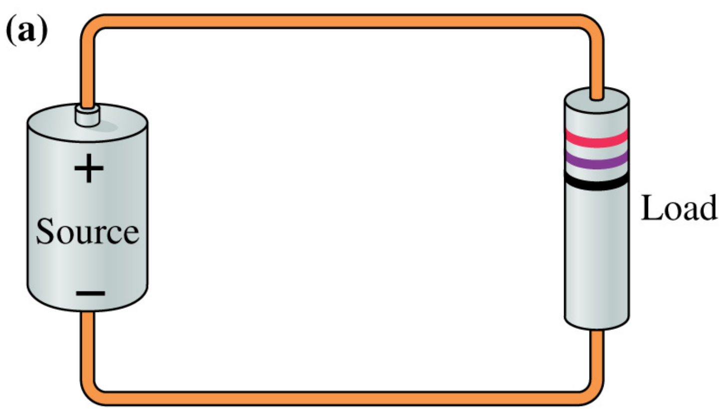
Direct Current

- When the current in a circuit has a constant magnitude and direction, the current is called ***direct current***
- Because the potential difference between the terminals of a battery is constant, **the battery produces direct current**
- The battery is known as a source of **emf** (electromotive force)

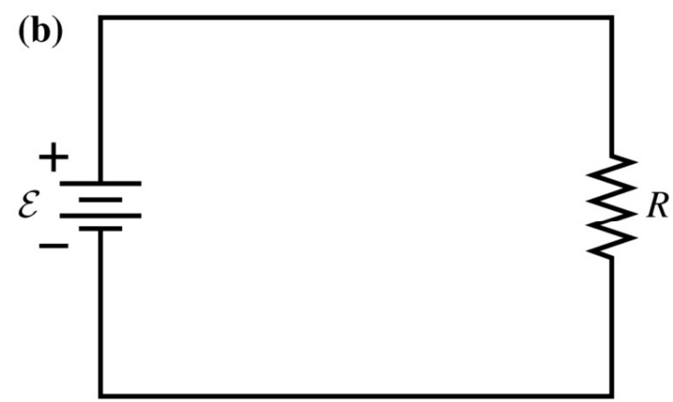


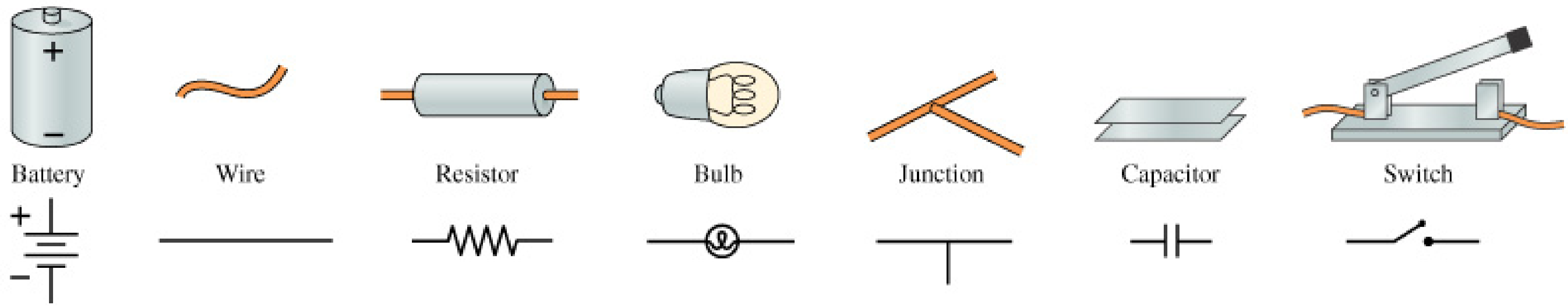


Basic symbols used for electric circuit drawing

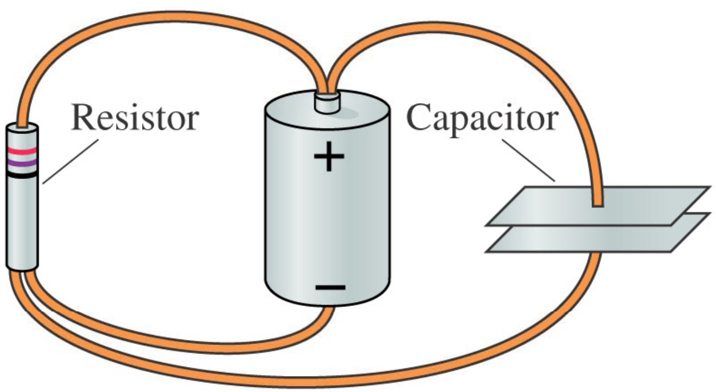


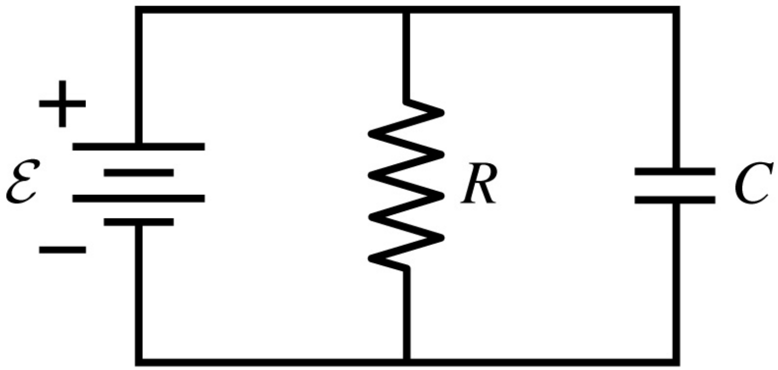
Circuit diagram: abstract picture of the circuit

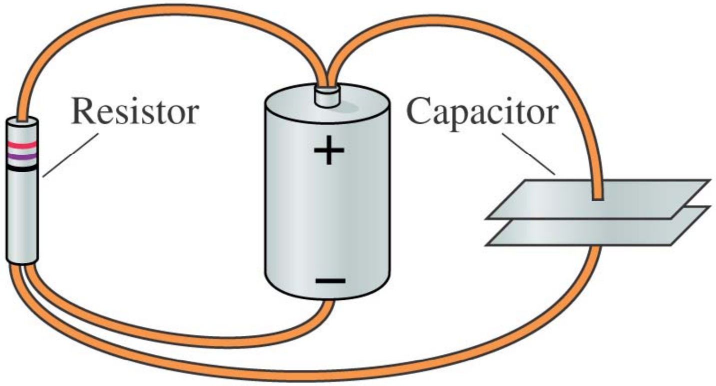


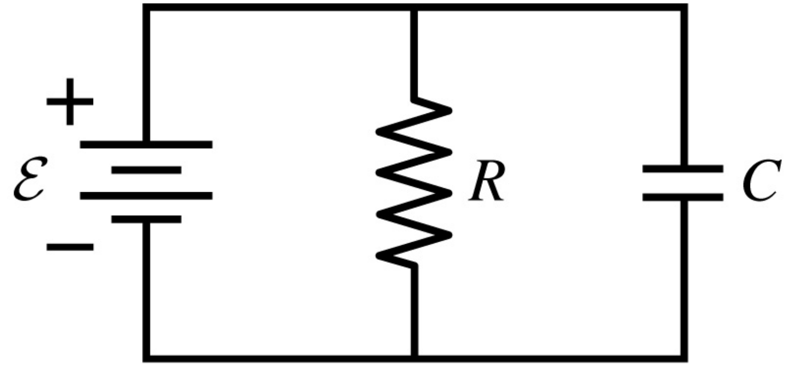


Circuit diagram: abstract picture of the circuit

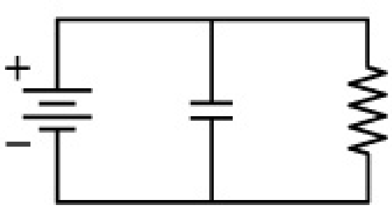


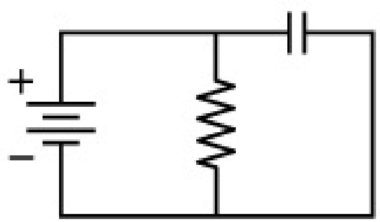


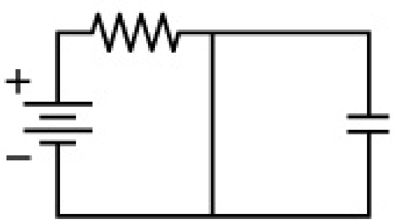


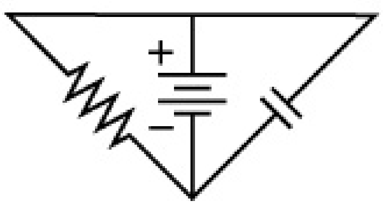


Which of these diagrams represent the same circuit?


(a)


(b)


(c)

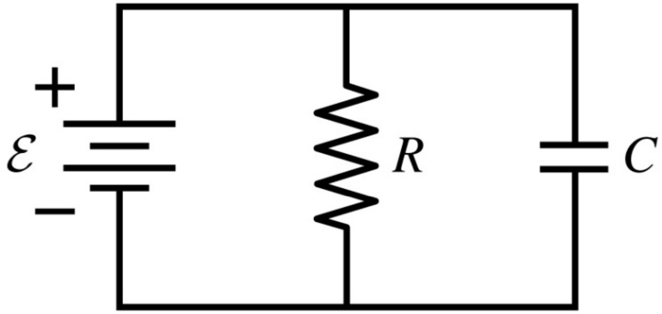

(d)

Analysis of a circuit

Full analysis of the circuit:

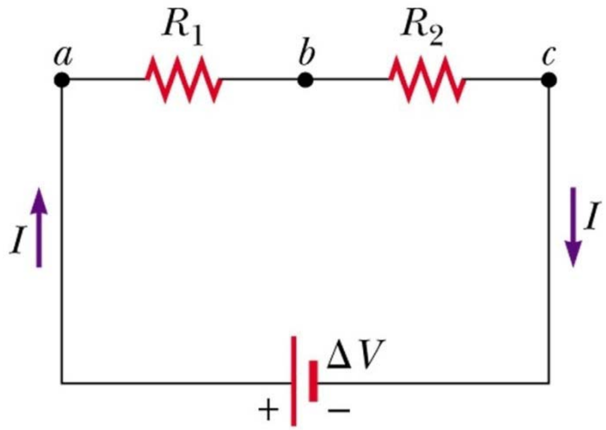
- Find the potential difference across each circuit component;

- Find the current through each circuit component.



There are two methods of analysis:
(1) Through **equivalent resistors and capacitors** (parallel and series circuits) – easy approach
(2) Through **Kirchhoff's rules** – should be used only if the first approach cannot be applied.

Resistors in Series



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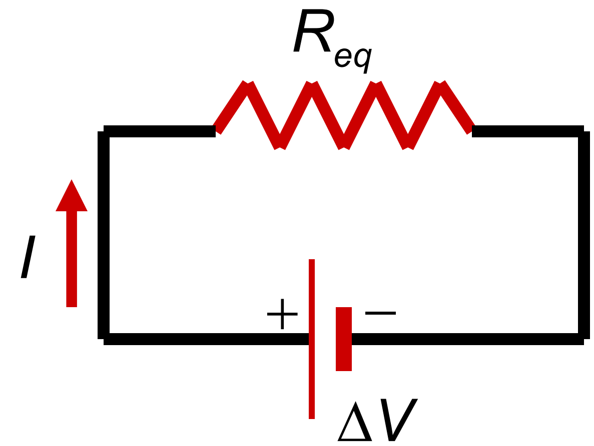


We introduce an equivalent circuit with just one **equivalent resistor** so that the current through the battery is the same as in the original circuit

Then from the Ohm's law we can find the current in the equivalent circuit

Resistors in Series

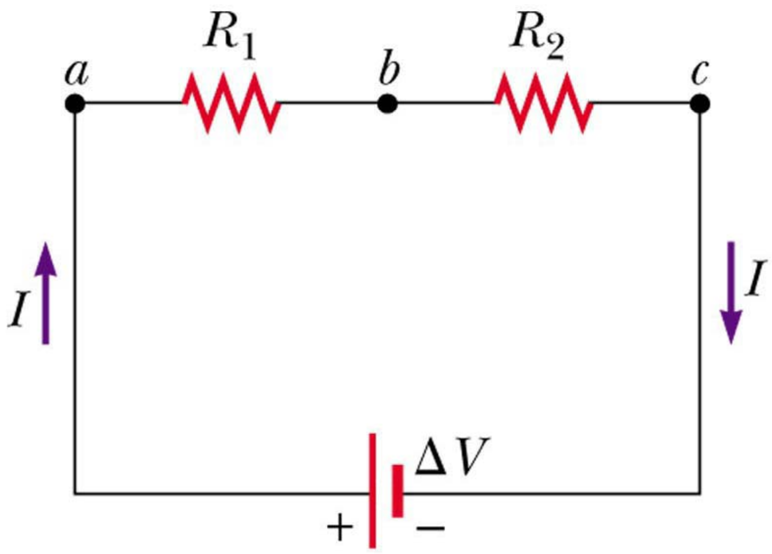
For a series combination of resistors, the currents are the same through all resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval

Ohm's law: $V_c - V_b = IR_2$

$$V_b - V_a = IR_1$$

$$V_c - V_a = V_c - V_b + V_b - V_a =$$
$$= IR_2 + IR_1 = I\left(R_1 + R_2\right)$$

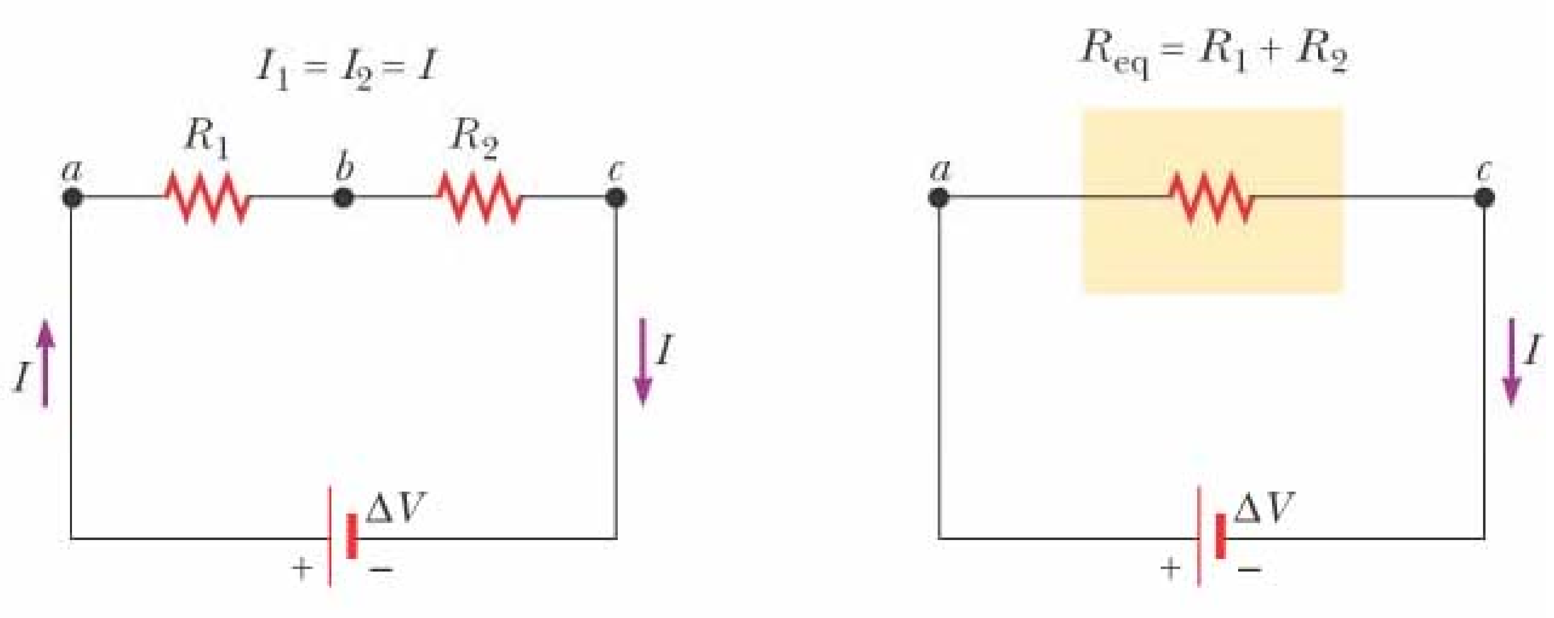
$$R_{eq} = R_1 + R_2$$



(b)

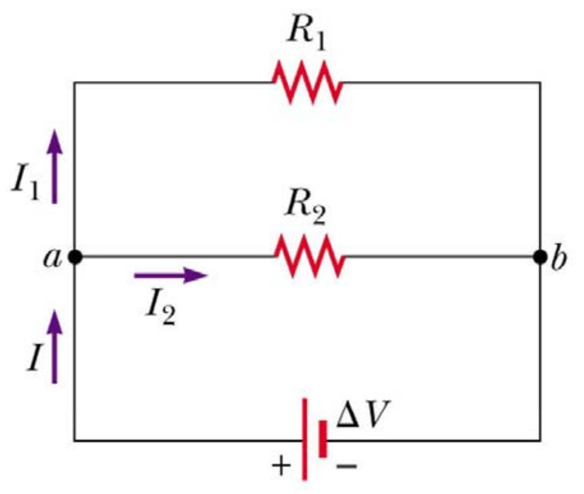
The equivalent resistance has the same effect on the circuit as the original combination of resistors

Resistors in Series



- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance

Resistors in Parallel



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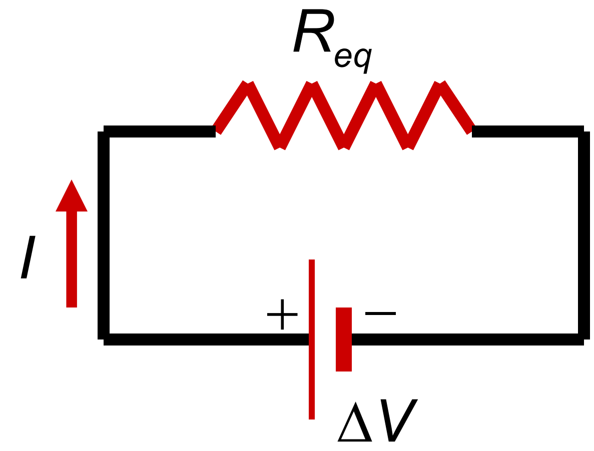


We introduce an equivalent circuit with just one **equivalent resistor** so that the current through the battery is the same as in the original circuit

Then from the Ohm's law we can find the current in the equivalent circuit

Resistors in Parallel

- The potential difference across each resistor is the same because each is connected directly across the battery terminals
- The current, *I*, that enters a point must be equal to the total current leaving that point

$$I = I_1 + I_2$$

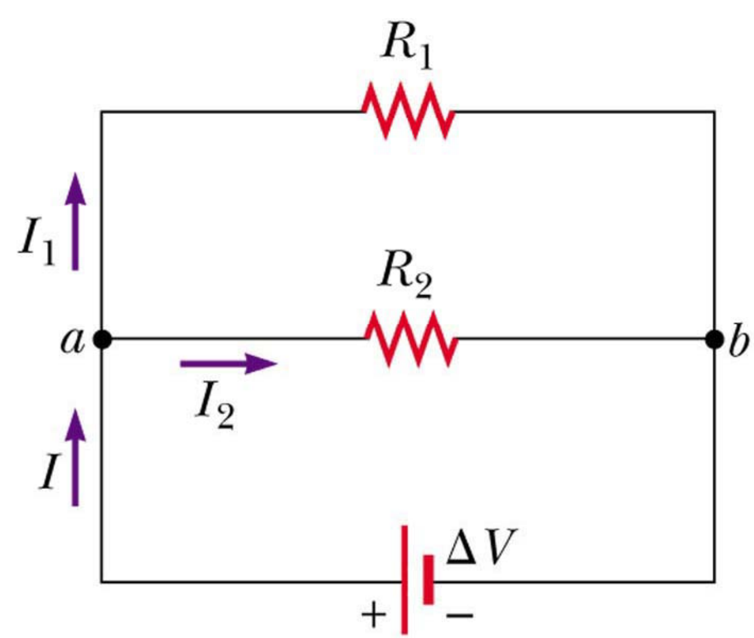
- Consequence of Conservation of Charge

Ohm's law: $V_b - V_a = \Delta V = I_1 R_1$

$$V_b - V_a = \Delta V = I_2 R_2$$

Conservation of Charge: $I = I_1 + I_2$

$$I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$



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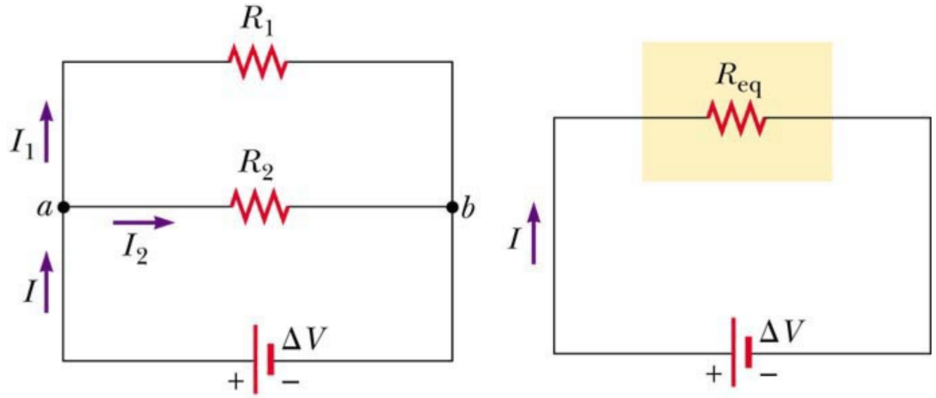
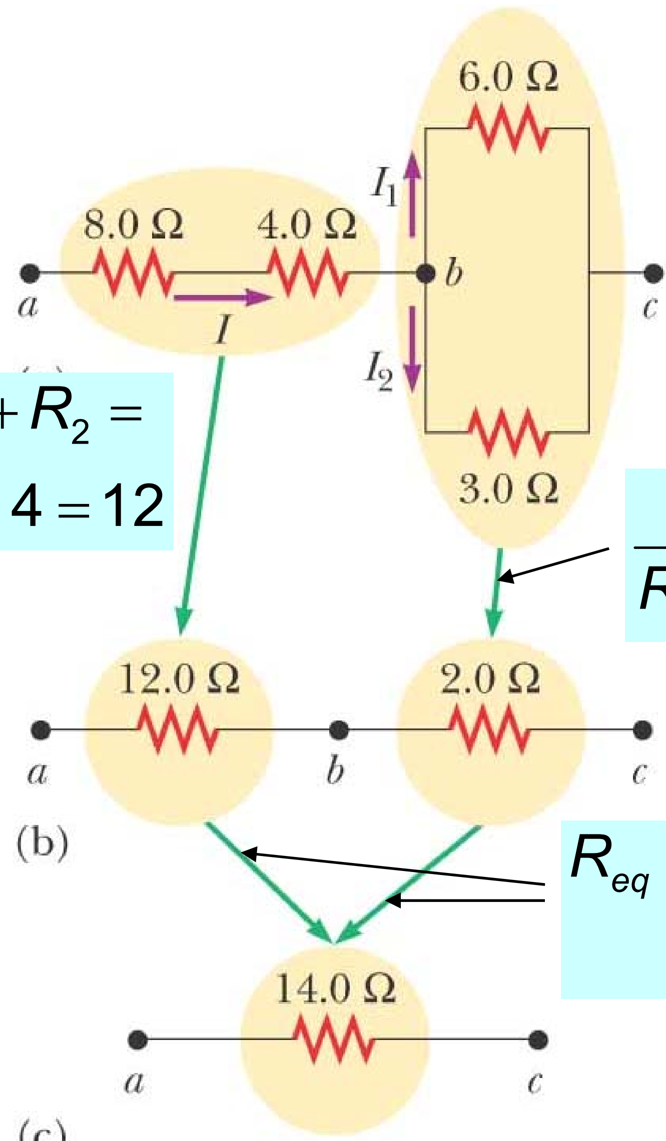
Resistors in Parallel



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- Equivalent Resistance $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$
  - The equivalent is always less than the smallest resistor in the group
- In parallel, each device operates independently of the others so that if one is switched off, the others remain on
- In parallel, all of the devices operate on the same voltage
- The current takes all the paths
  - The lower resistance will have higher currents
  - Even very high resistances will have some currents

Example



$$R_{eq} = R_1 + R_2 = \\ = 8 + 4 = 12$$

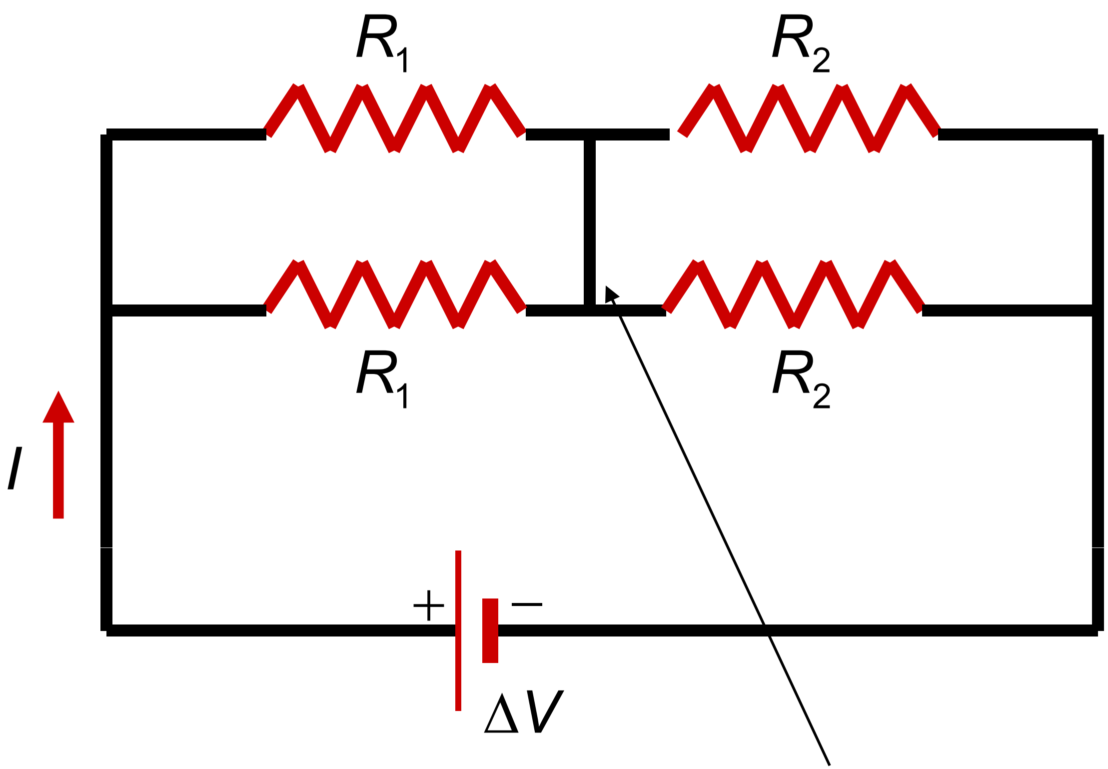
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

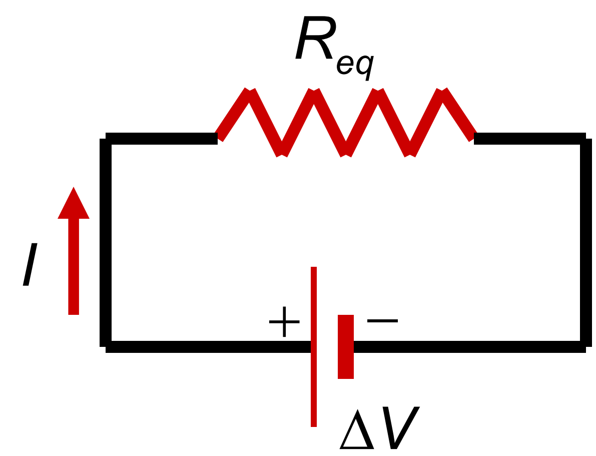
$$R_{eq} = R_1 + R_2 = \\ = 12 + 2 = 14$$

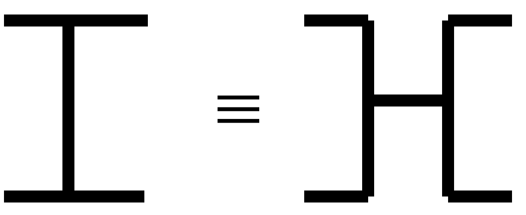
Example

Main question: $R_{eq} = ?$ **or** $I = ?$

$$I = \frac{\Delta V}{R_{eq}}$$





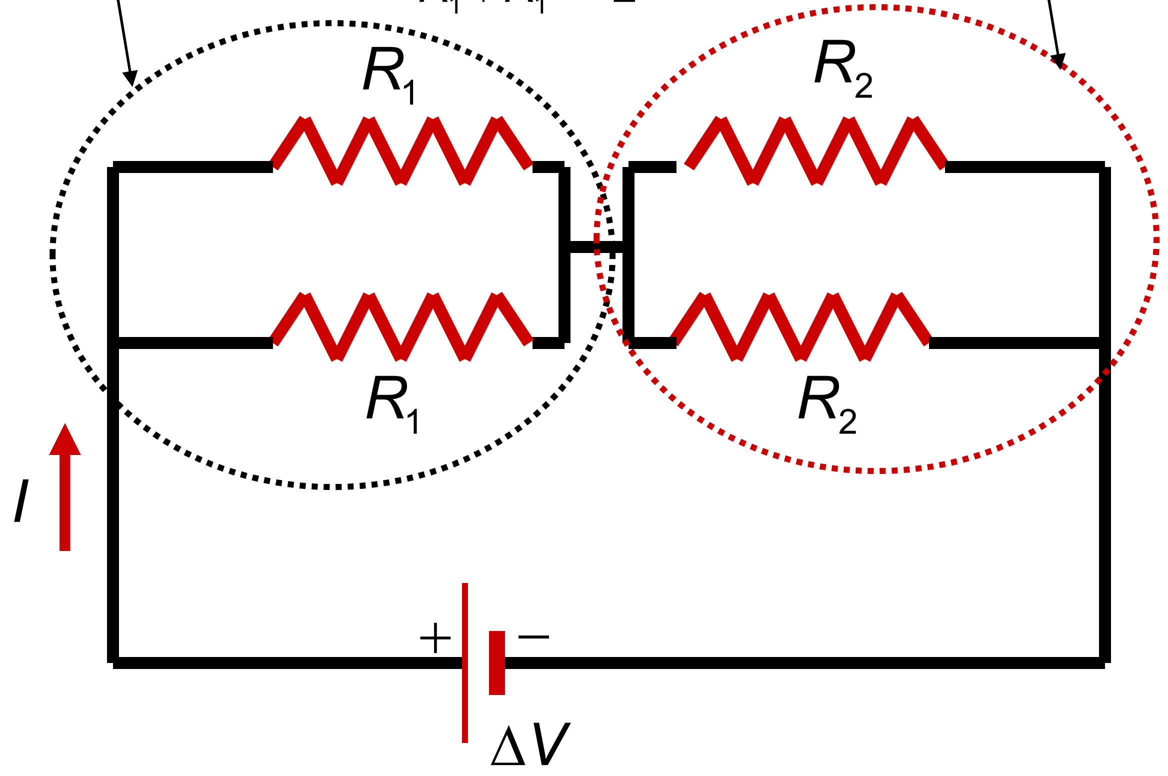


Example

Main question: $R_{eq} = ?$ **or** $I = ?$

in parallel $R_{eq,1} = \frac{R_1 R_1}{R_1 + R_1} = \frac{R_1}{2}$

in parallel $R_{eq,2} = \frac{R_2 R_2}{R_2 + R_2} = \frac{R_2}{2}$



Example

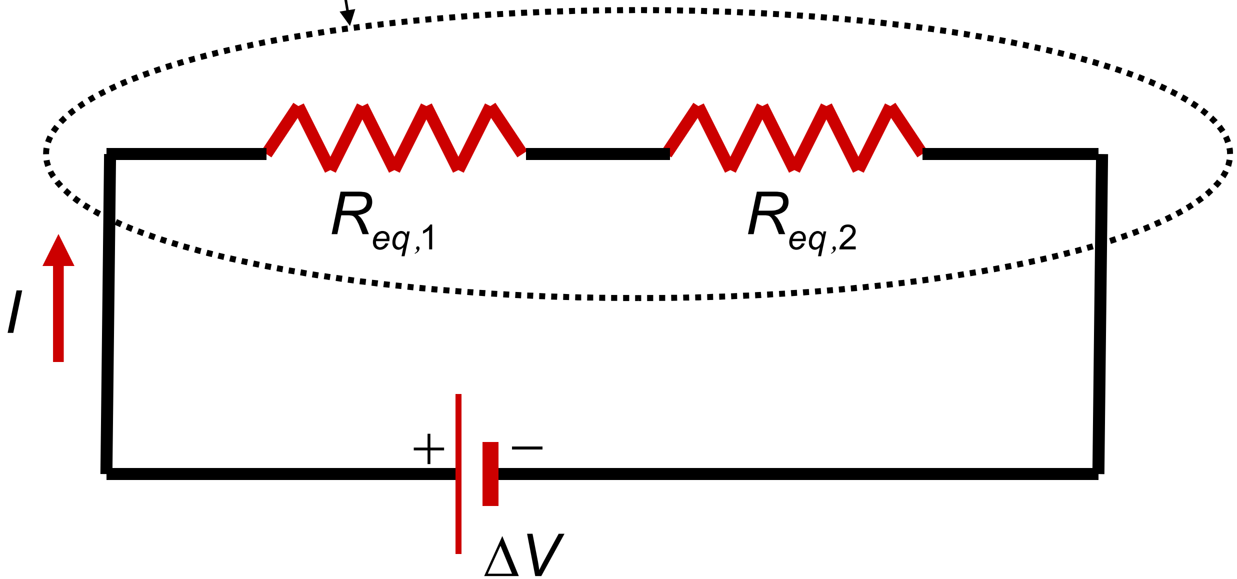
Main question: $R_{eq} = ?$ **or** $I = ?$

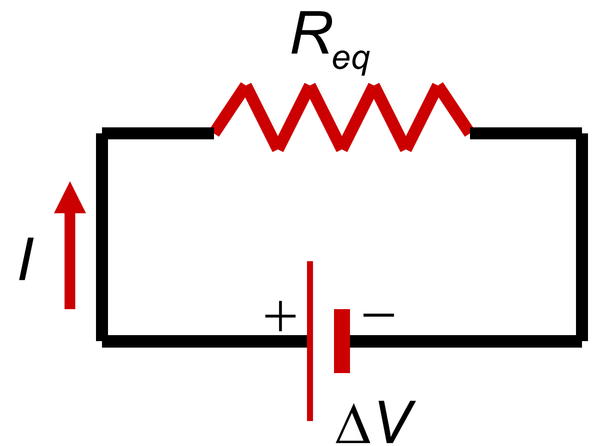
$$R_{eq,1} = \frac{R_1}{2} \qquad R_{eq,2} = \frac{R_2}{2}$$

in series

$$R_{eq} = R_{eq,1} + R_{eq,2} = \frac{R_1 + R_2}{2}$$

$$I = \frac{\Delta V}{R_{eq}} = \frac{2\Delta V}{R_1 + R_2}$$

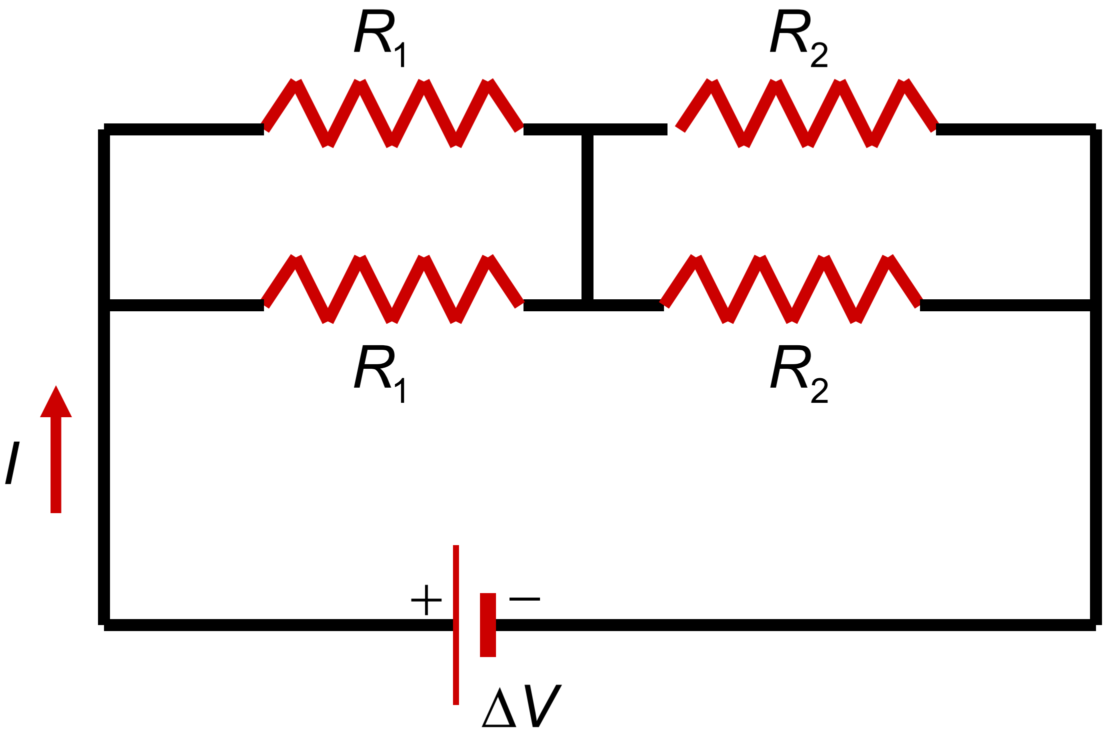


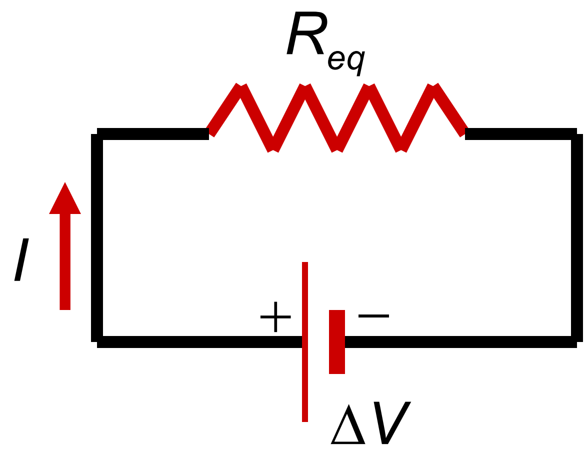


Example

Main question: $R_{eq} = ?$ **or** $I = ?$

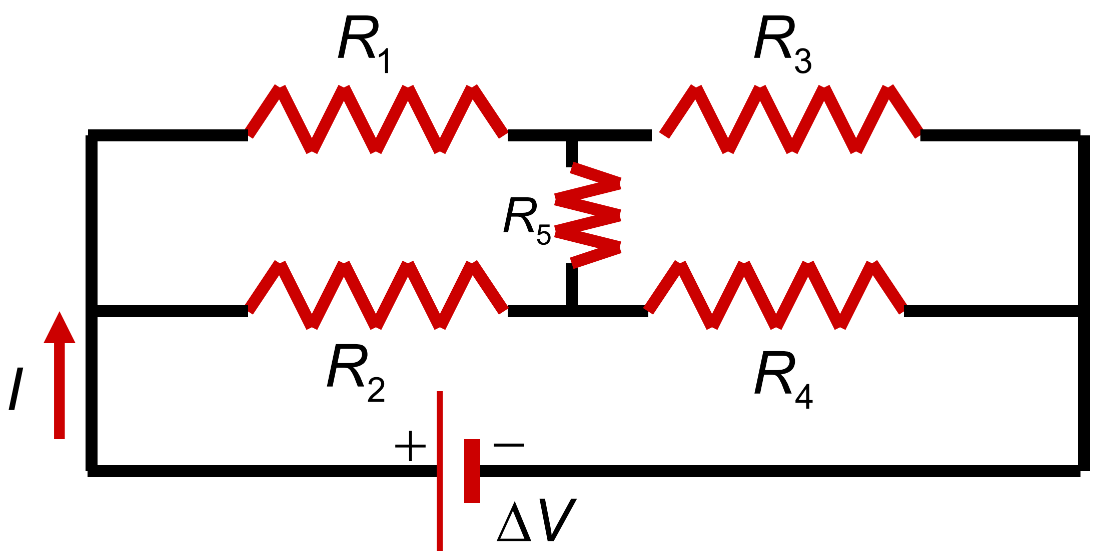
$$I = \frac{\Delta V}{R_{eq}}$$

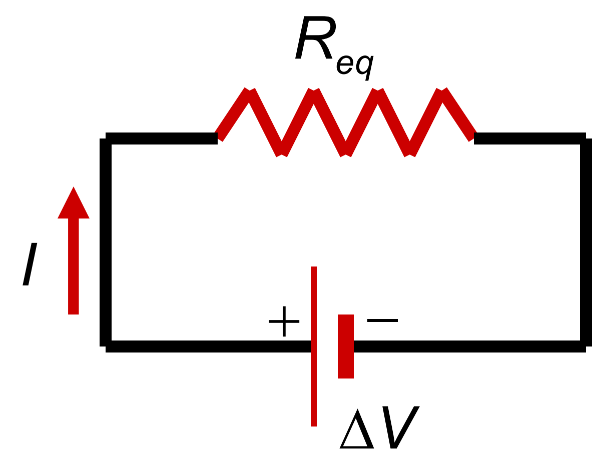




Example

Main question: $R_{eq} = ?$ **or** $I = ?$





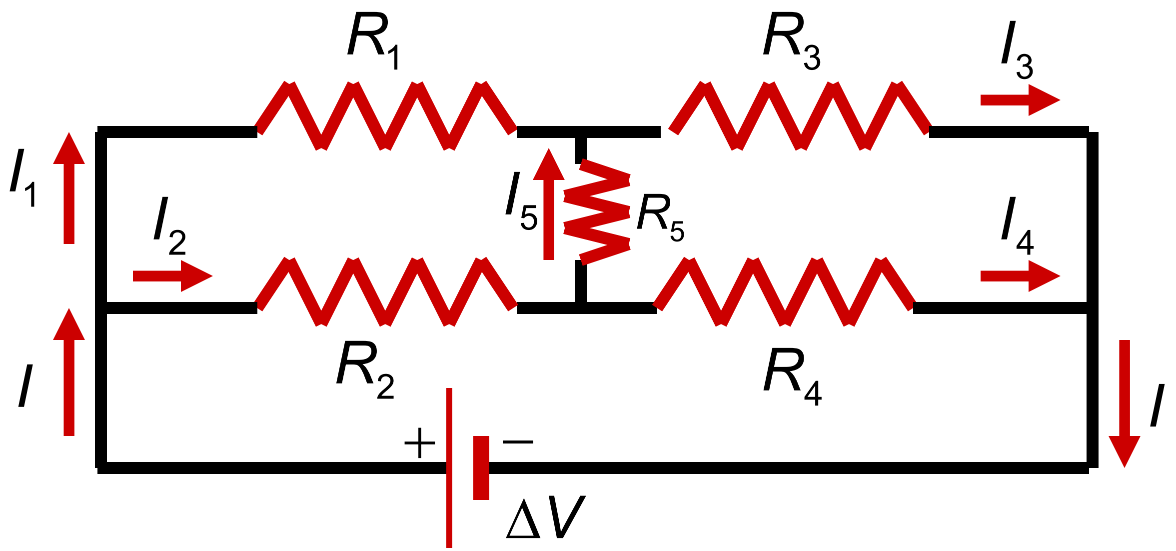
To find R_{eq} **we need to use Kirchhoff's rules.**

Kirchhoff's rules

Kirchhoff's rules

- There are two Kirchhoff's rules

- To formulate the rules we need, at first, to choose the directions of currents through all resistors. If we choose the wrong direction, then after calculation the corresponding current will be negative.



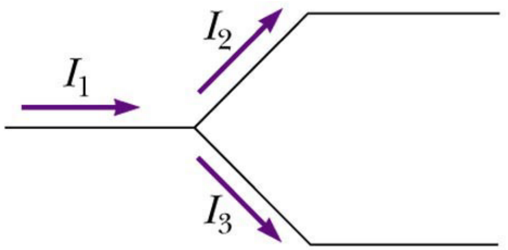
Junction Rule

- The first Kirchhoff's rule – Junction Rule:
- The sum of the currents entering any junction must equal the sum of the currents leaving that junction

  - A statement of Conservation of Charge

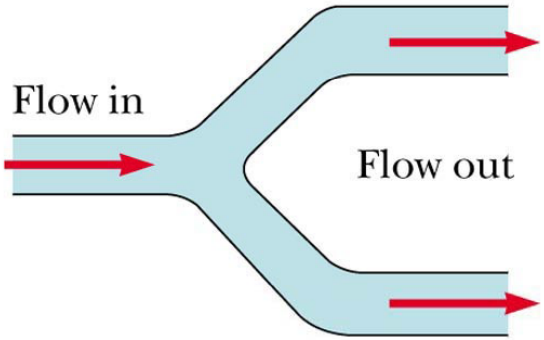
$$\sum I_{in} = \sum I_{out}$$

$$I_1 = I_2 + I_3$$



(a)

In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit

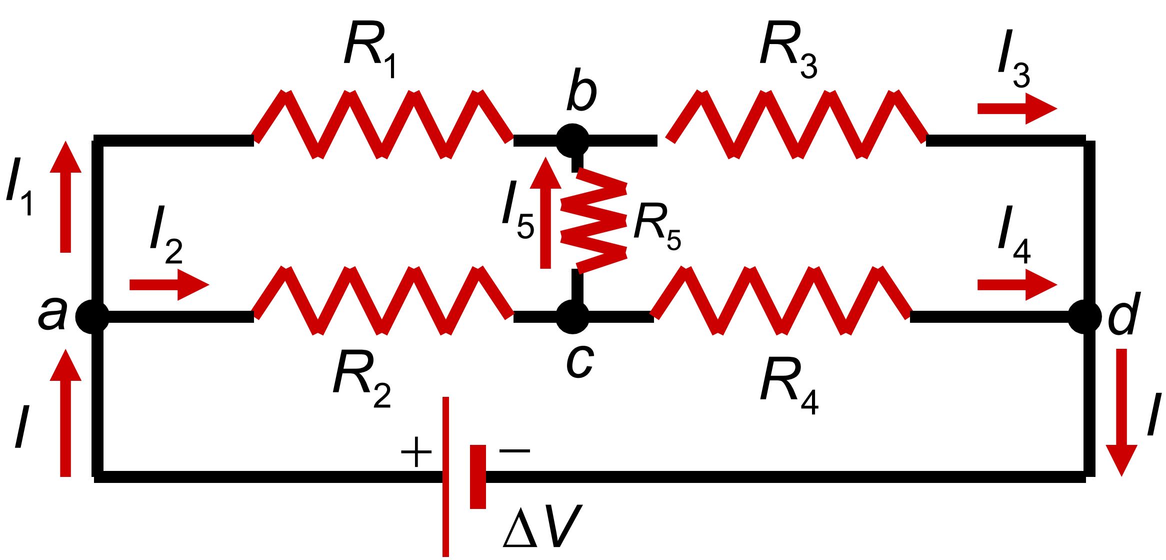


(b)

Junction Rule

- The first Kirchhoff's rule – Junction Rule: $\sum I_{in} = \sum I_{out}$
- In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit

- There are 4 junctions: a, b, c, d.
- We can write the Junction Rule for any three of them



(a) $I = I_1 + I_2$

(b) $I_1 + I_5 = I_3$

(c) $I_2 = I_4 + I_5$

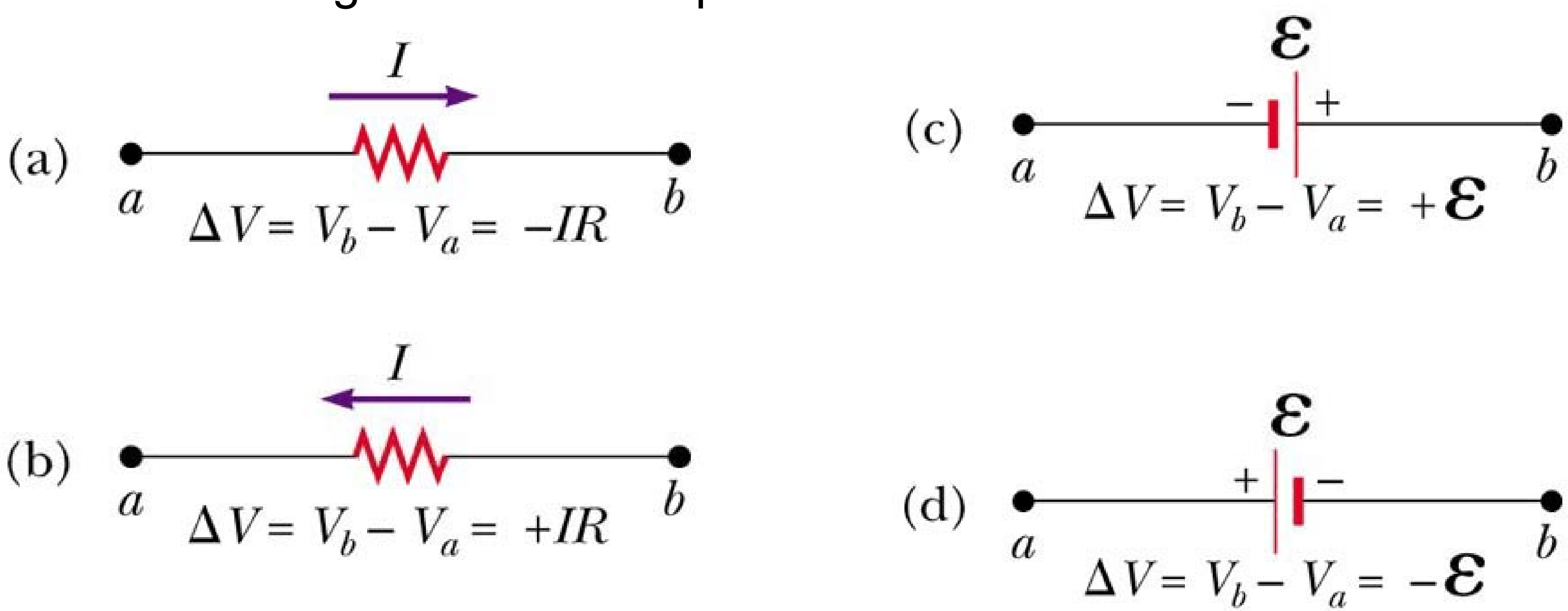
Loop Rule

- The second Kirchhoff's rule – Loop Rule:
  - The sum of the potential differences across all the elements around any closed circuit loop must be zero

    \- A statement of Conservation of Energy

$$\sum_{closed\ loop} \Delta V = 0$$

Traveling around the loop from ***a*** to ***b***



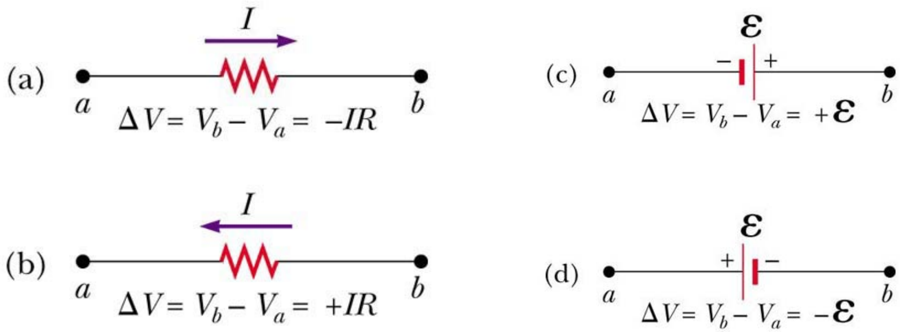
Loop Rule

- The second Kirchhoff's rule – Loop Rule:

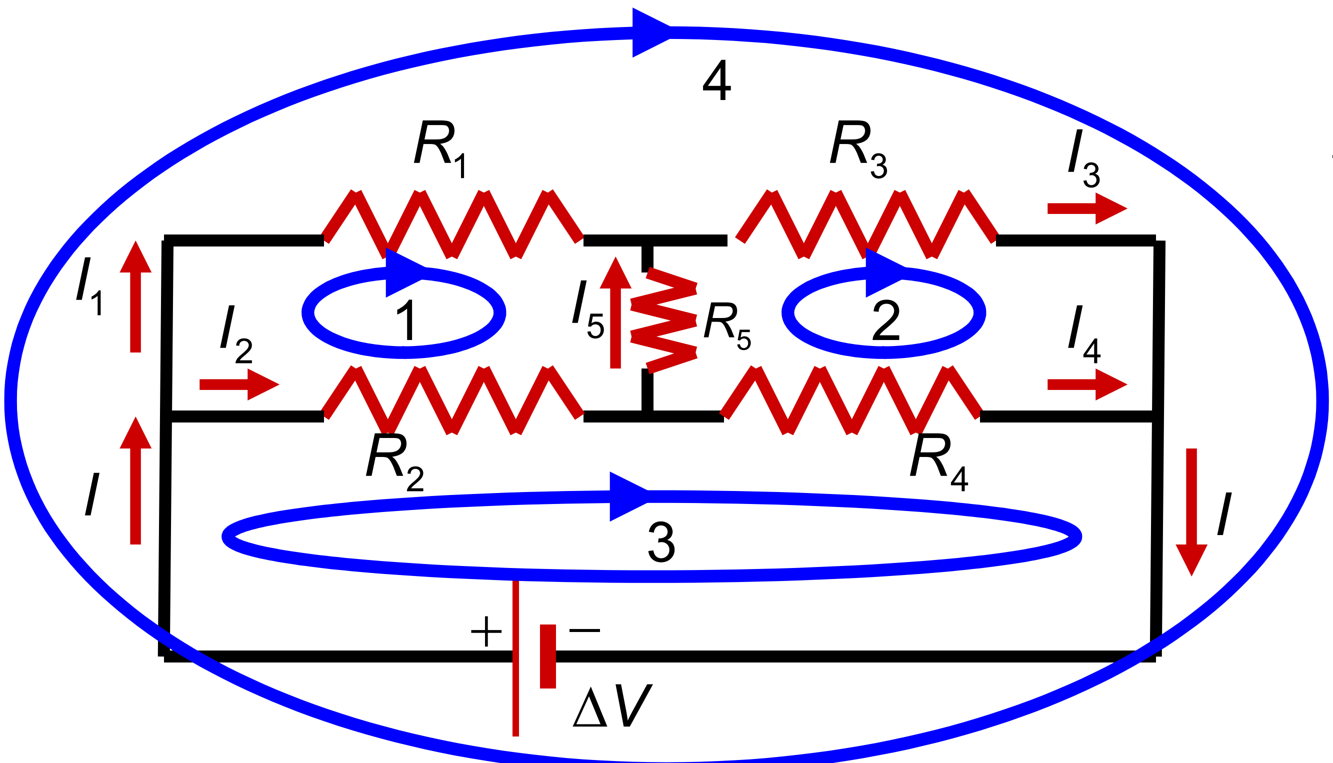
$$\sum_{\textit{closed loop}} \Delta V = 0$$

Loop Rule

- The second Kirchhoff's rule – Loop Rule: $\sum_{closed\ loop} \Delta V = 0$



We need to write the Loop Rule for 3 loops



Loop 1:

$$-I_1R_1 + I_5R_5 + I_2R_2 = 0$$

Loop 2:

$$-I_3R_3 - I_5R_5 + I_4R_4 = 0$$

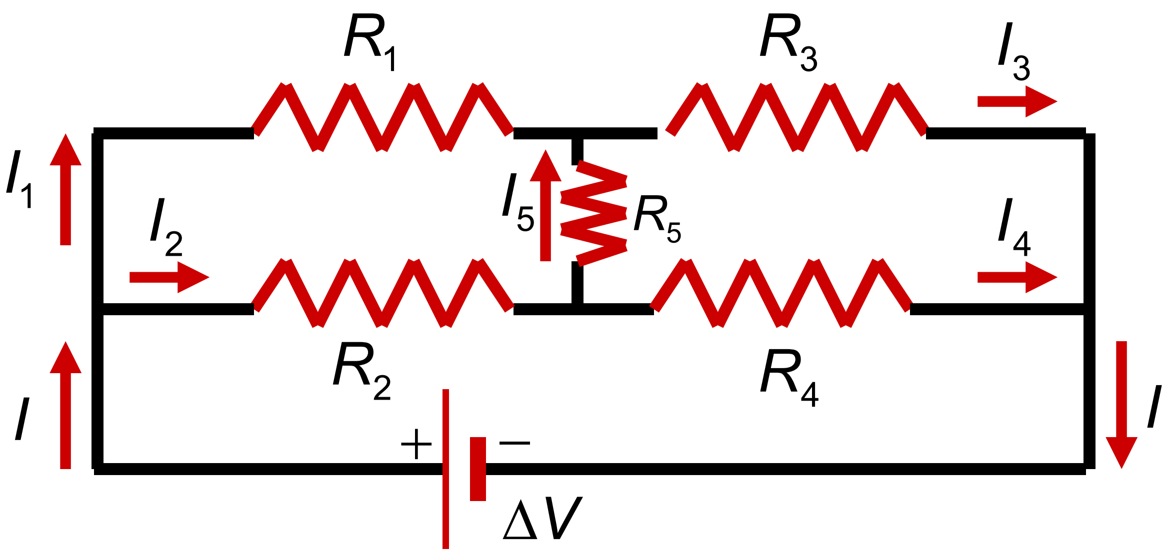
Loop 3:

$$\Delta V - I_2R_2 - I_4R_4 = 0$$

Kirchhoff's Rules

Junction Rule $\sum I_{in} = \sum I_{out}$

- Loop Rule $\sum_{\text{closed loop}} \Delta V = 0$



$$I = I_1 + I_2$$

$$I_1 + I_5 = I_3$$

$$I_2 = I_4 + I_5$$

$$-I_1R_1 + I_5R_5 + I_2R_2 = 0$$

$$-I_3R_3 - I_5R_5 + I_4R_4 = 0$$

$$\Delta V - I_2R_2 - I_4R_4 = 0$$

We have 6 equations and 6 unknown currents.

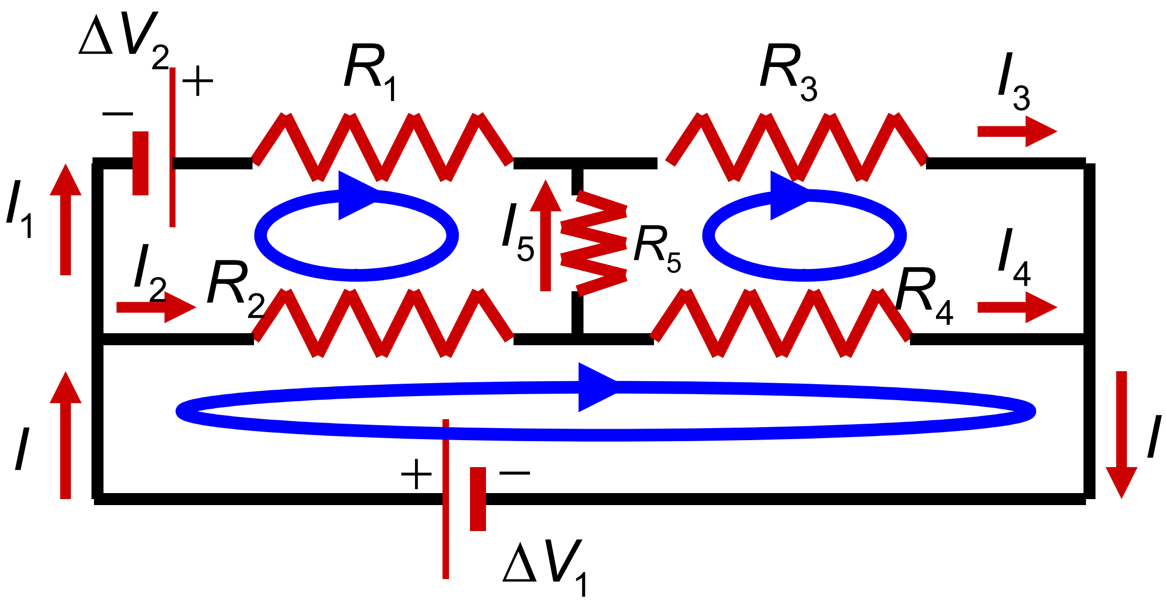
$$R_{eq} = \frac{\Delta V}{I}$$

Kirchhoff's Rules

Junction Rule $\sum I_{in} = \sum I_{out}$

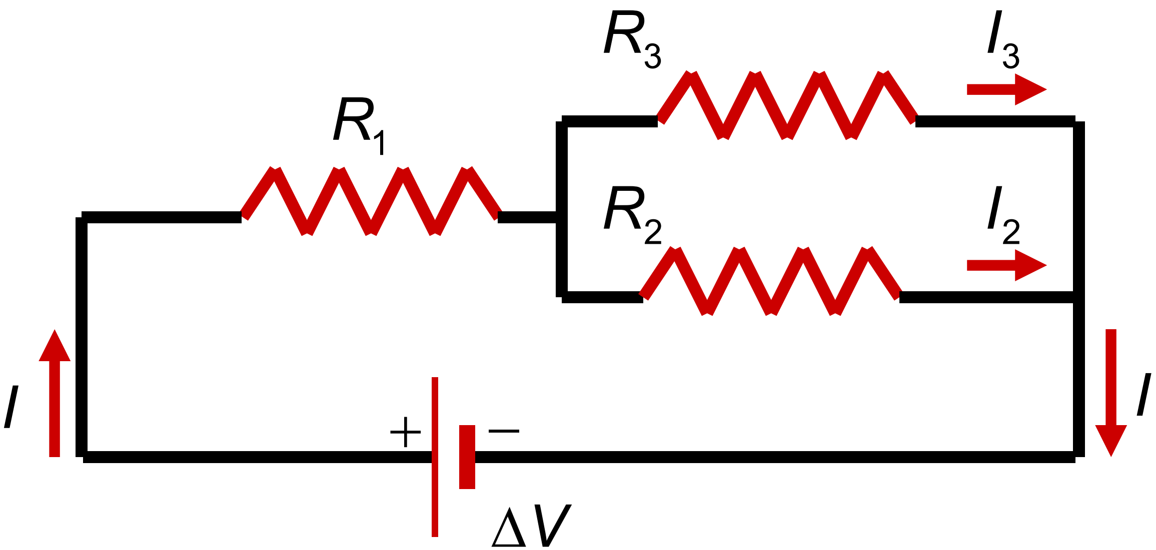
- Loop Rule $\sum_{closed\ loop} \Delta V = 0$

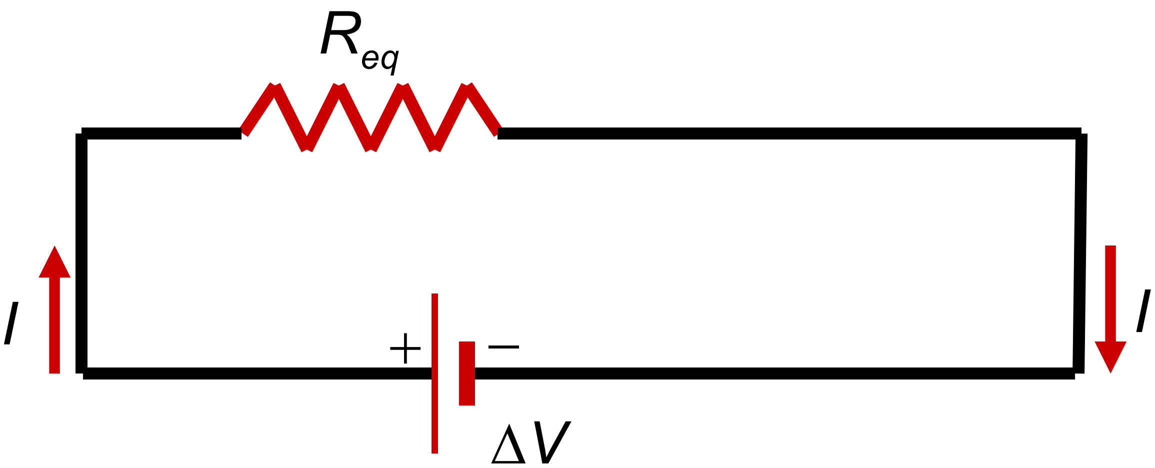
$$I = I_1 + I_2$$
$$I_1 + I_5 = I_3$$
$$I_2 = I_4 + I_5$$
$$-I_1R_1 + I_5R_5 + I_2R_2 + \Delta V_2 = 0$$
$$-I_3R_3 - I_5R_5 + I_4R_4 = 0$$
$$\Delta V_1 - I_2R_2 - I_4R_4 = 0$$



We have 6 equations and 6 unknown currents.

Example

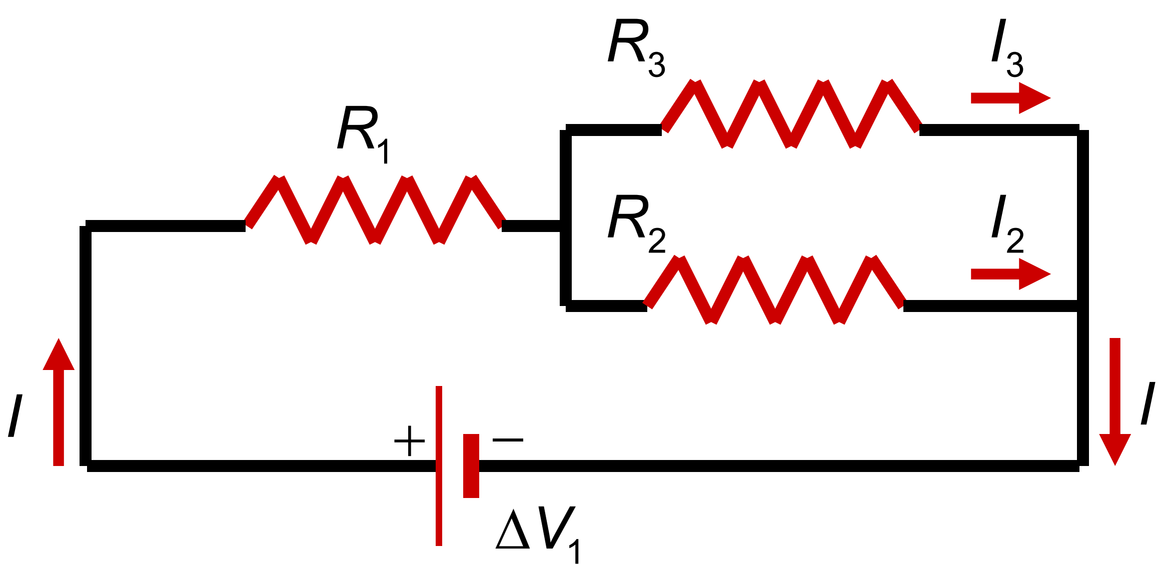




$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$I = \frac{\Delta V}{R_{eq}}$$

Example 1



$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

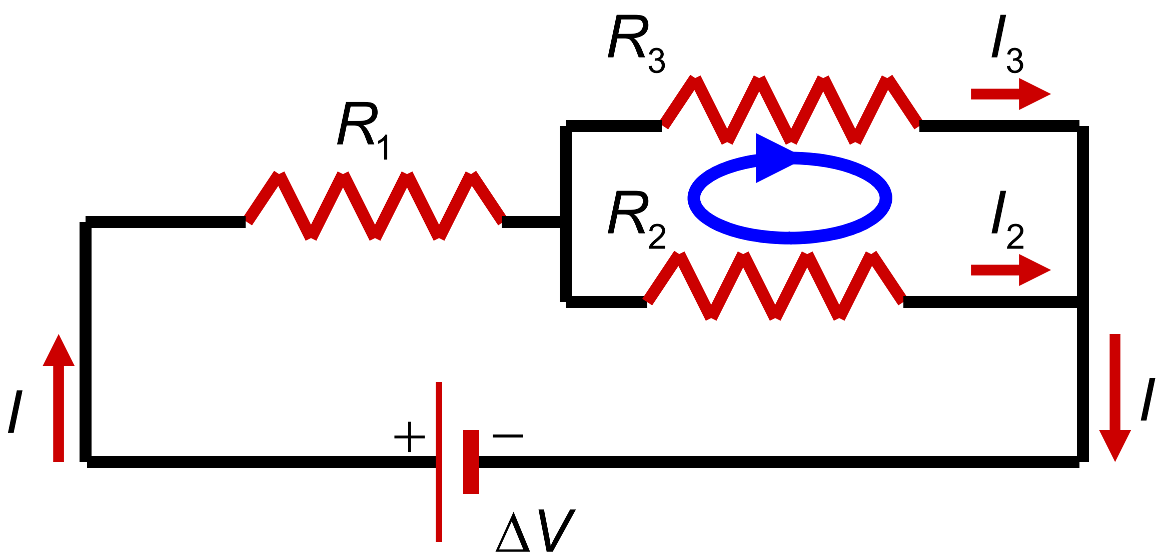
$$I = \frac{\Delta V}{R_{eq}}$$

$$I = I_2 + I_3$$

$$I_2 R_2 = I_3 R_3 \qquad I_2 = I_3 \frac{R_3}{R_2}$$

$$I = I_3\left(1 + \frac{R_3}{R_2}\right) \qquad I_3 = \frac{IR_2}{R_2 + R_3} \qquad I_2 = \frac{IR_3}{R_2 + R_3}$$

Example: solution based on Kirchhoff's Rules



$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

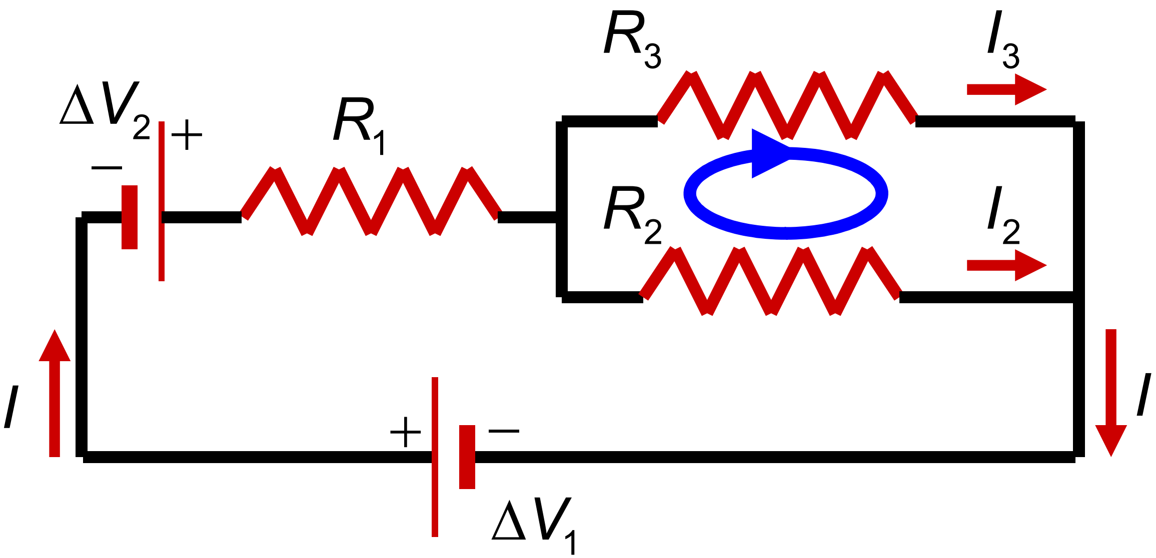
$$\Delta V - I_2 R_2 - I R_1 = 0$$

$$I_2 = I_3 \frac{R_3}{R_2} \qquad I_3 = \frac{I R_2}{R_2 + R_3} \qquad I_2 = \frac{I R_3}{R_2 + R_3}$$

$$\Delta V - \frac{I R_3}{R_2 + R_3} R_2 - I R_1 = 0$$

$$I = \frac{\Delta V}{\frac{R_3 R_2}{R_2 + R_3} + R_1} = \frac{\Delta V}{R_{eq}}$$

Example

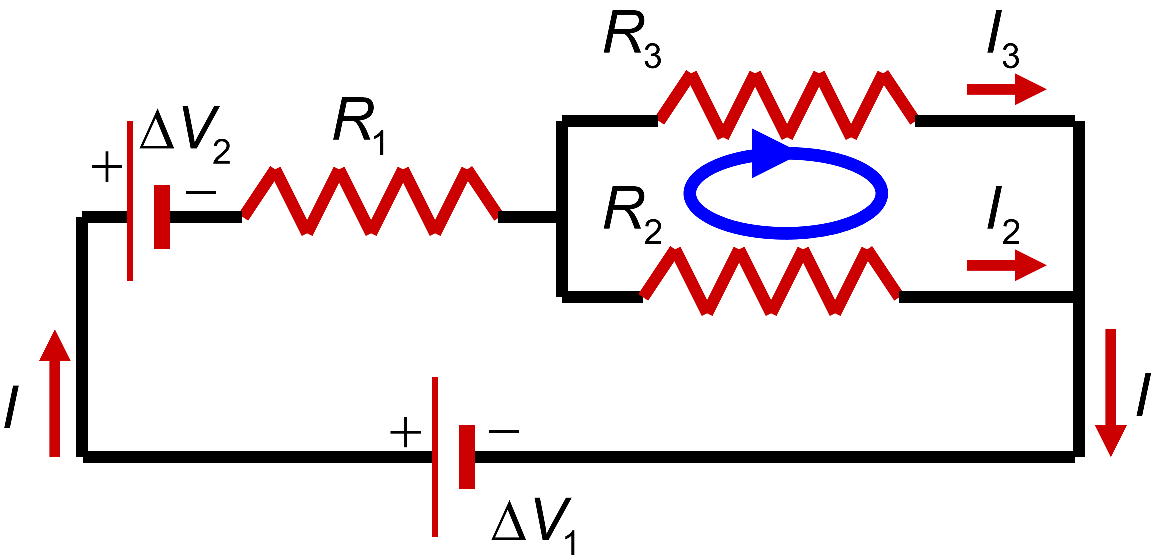


$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

$$\Delta V_1 + \Delta V_2 - I_2 R_2 - I R_1 = 0$$

Example



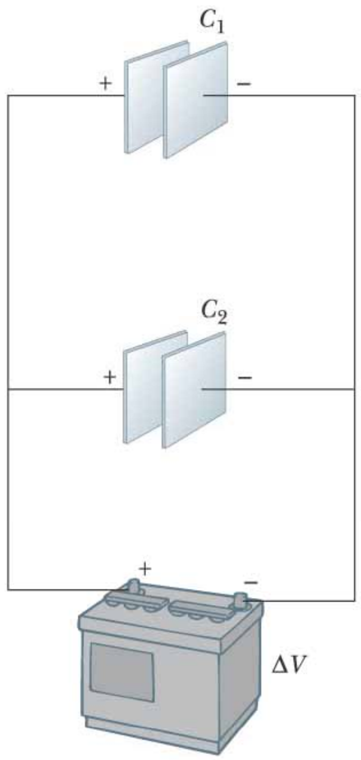
$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

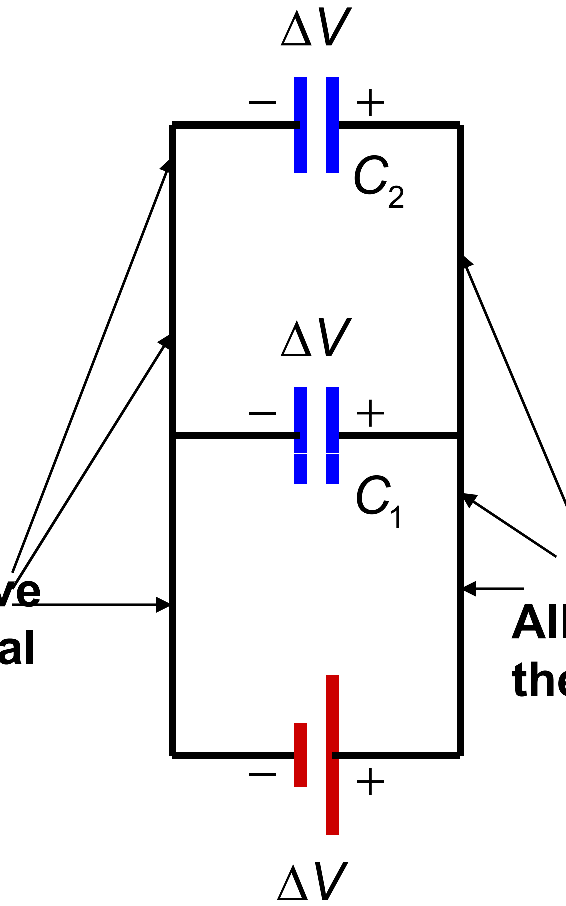
$$\Delta V_1 - \Delta V_2 - I_2 R_2 - I R_1 = 0$$

Electrical circuits with capacitors

Capacitors in Parallel



(a)



All the points have the same potential

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The capacitors 1 and 2 have the same potential difference ΔV

Then the charge of capacitor 1 is $Q_1 = C_1 \Delta V$

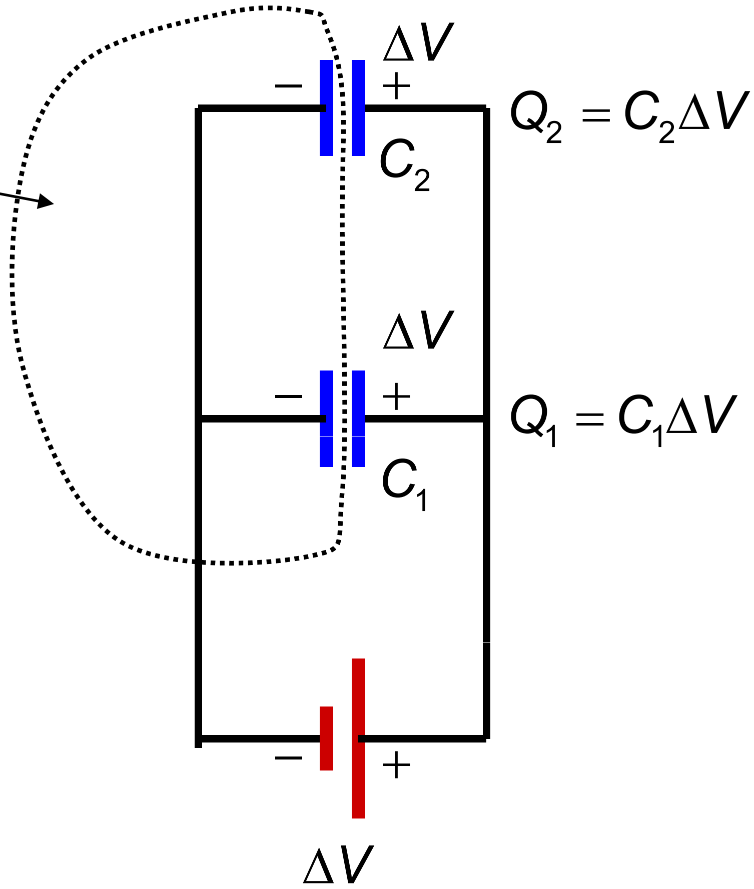
The charge of capacitor 2 is $Q_2 = C_2 \Delta V$

Capacitors in Parallel

The total charge is

$$Q = Q_1 + Q_2$$

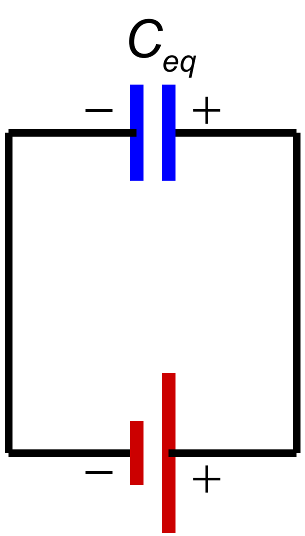
$$Q = C_1\Delta V + C_2\Delta V = (C_1 + C_2)\Delta V$$



This relation is equivalent to the following one

$$Q = C_{eq}\Delta V$$

$$C_{eq} = C_1 + C_2$$

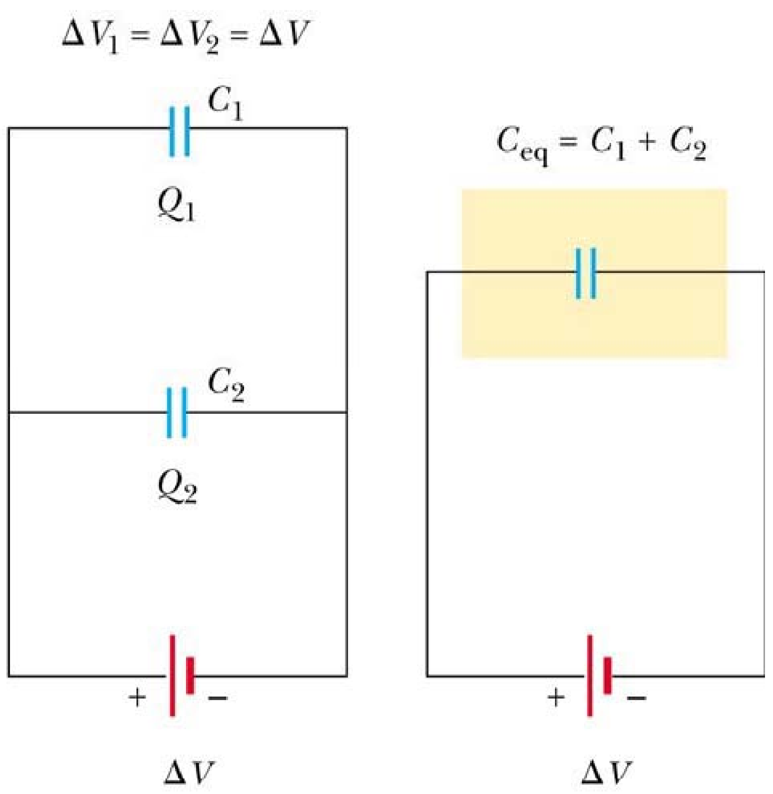


Capacitors in Parallel

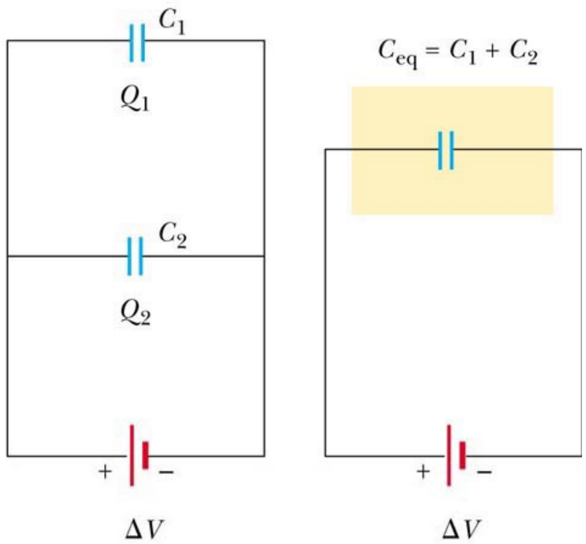
- ➢ The capacitors can be replaced with one capacitor with a capacitance of C_{eq}

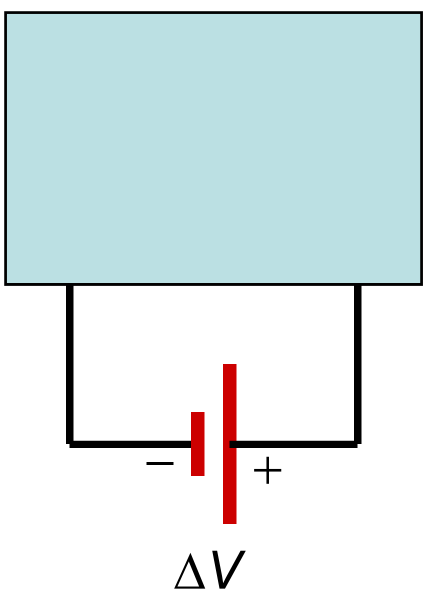
- ➢ The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors

$$Q = C_{eq}\Delta V$$



Capacitors

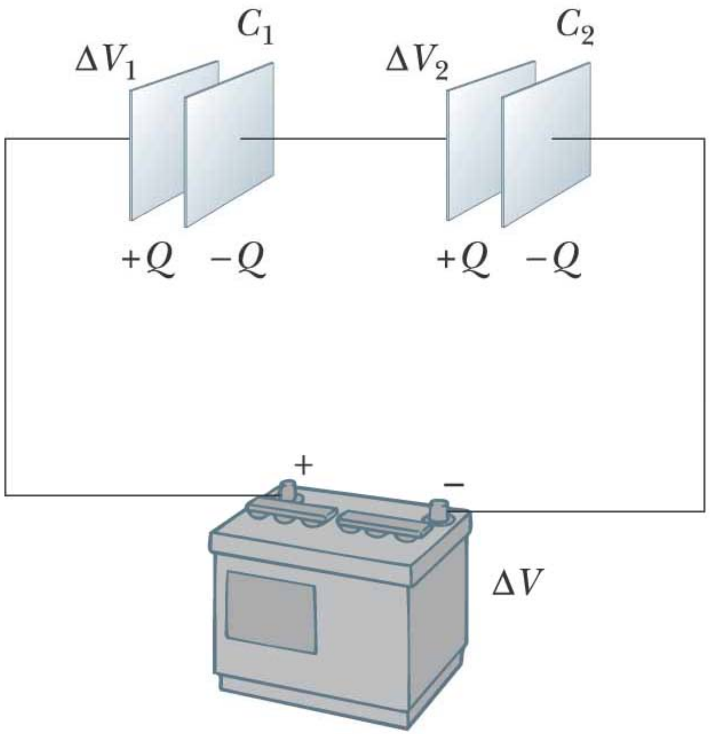




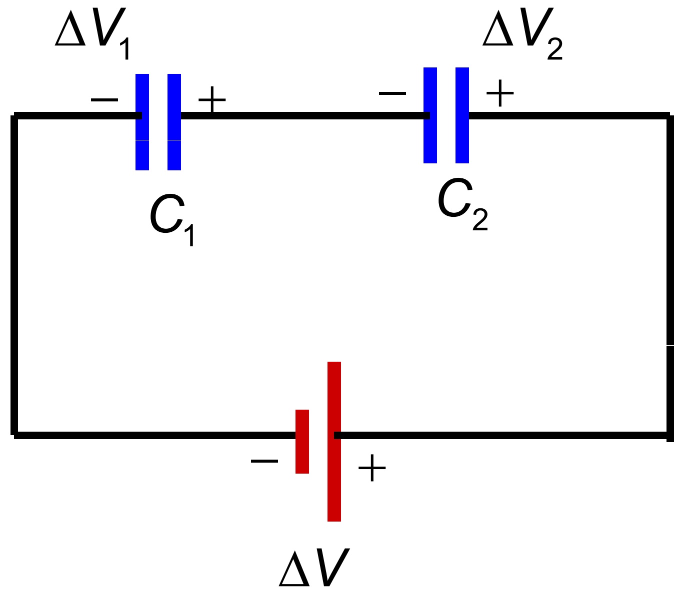
The equivalence means that

$$Q = C_{eq}\Delta V$$

Capacitors in Series



(a)



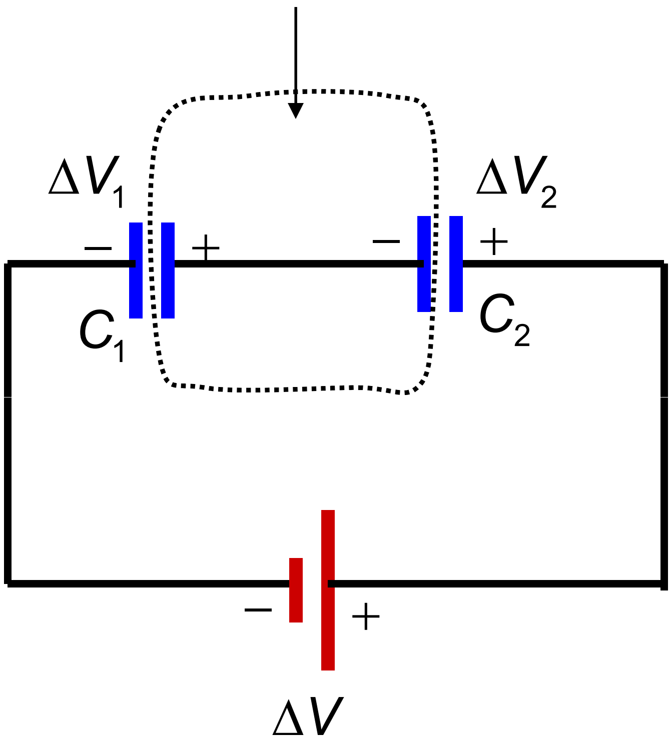
$$\Delta V = \Delta V_1 + \Delta V_2$$

Capacitors in Series

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

The total charge is equal to 0

$$Q_1 = Q_2 = Q$$



$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

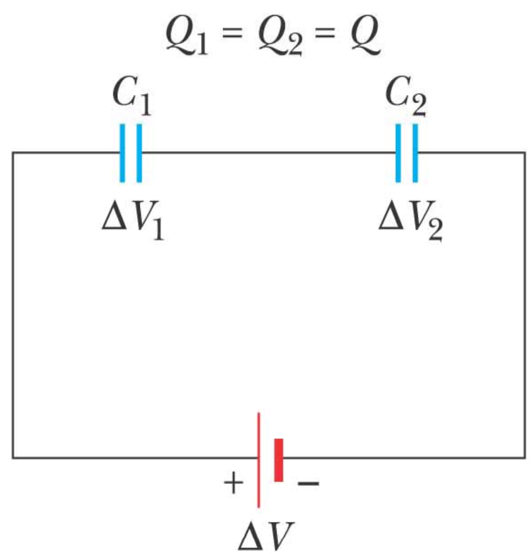
$$\Delta V = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

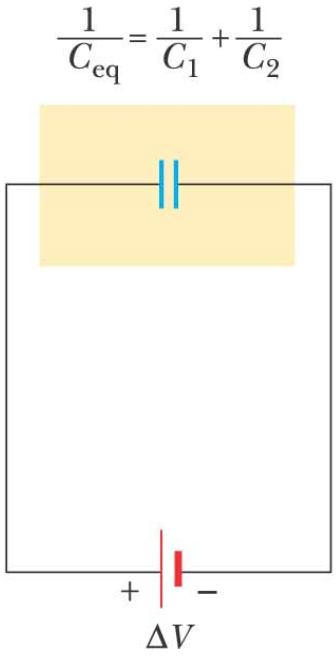
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in Series

- An equivalent capacitor can be found that performs the same function as the series combination
- The potential differences add up to the battery voltage



(b)

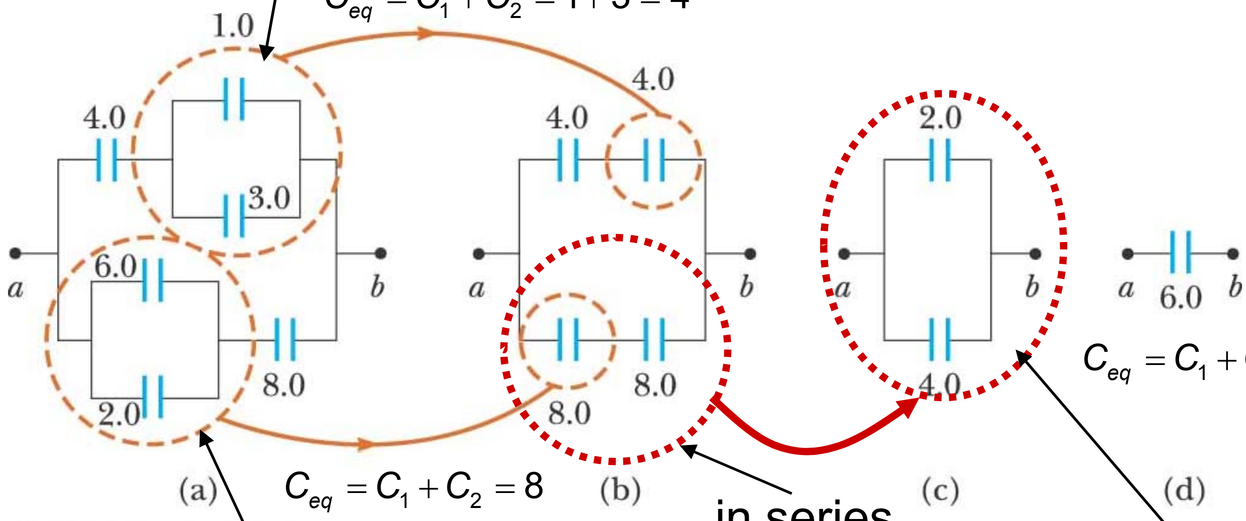


(c)

Example

in parallel

$$C_{eq} = C_1 + C_2 = 1 + 3 = 4$$



$$C_{eq} = C_1 + C_2 = 8$$

$$C_{eq} = C_1 + C_2 = 6$$

in series

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \cdot 8}{8 + 8} = 4$$

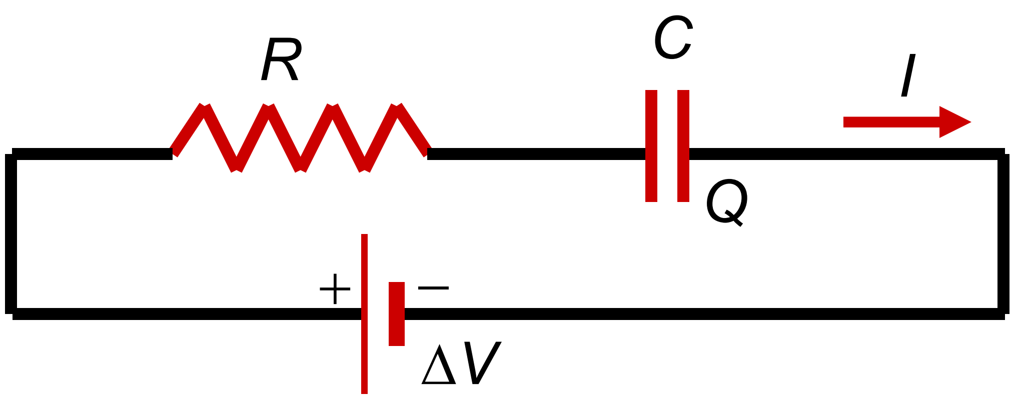
in parallel

in parallel

RC circuits

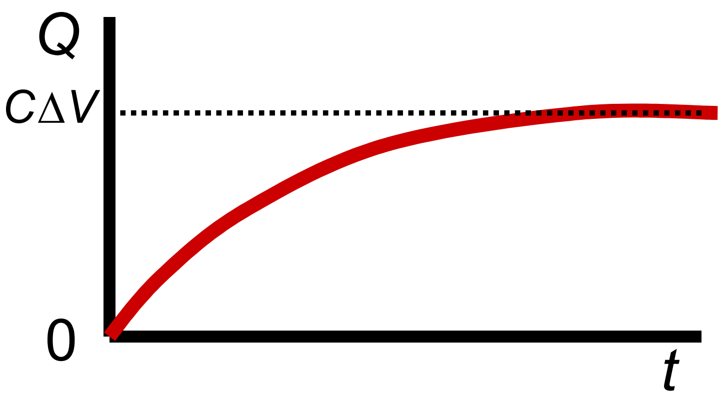
RC circuit

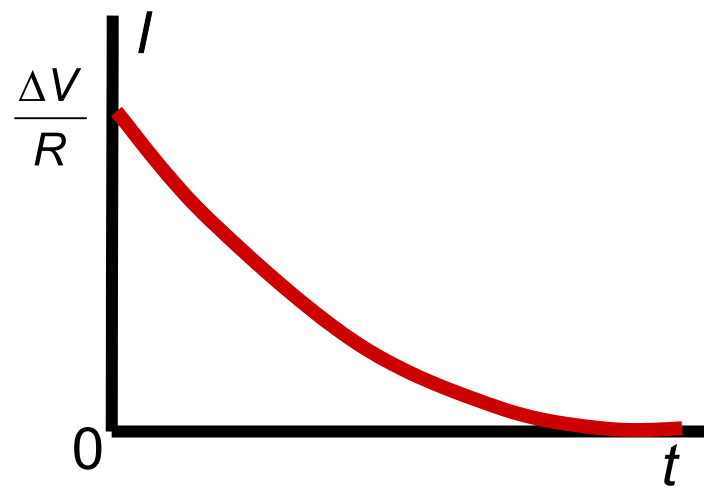
- A direct current circuit may contain capacitors and resistors, the current will vary with time
- When the circuit is completed, the capacitor starts to charge
- The capacitor continues to charge until it reaches its maximum charge $Q = C\Delta V$
- Once the capacitor is fully charged, the current in the circuit is zero

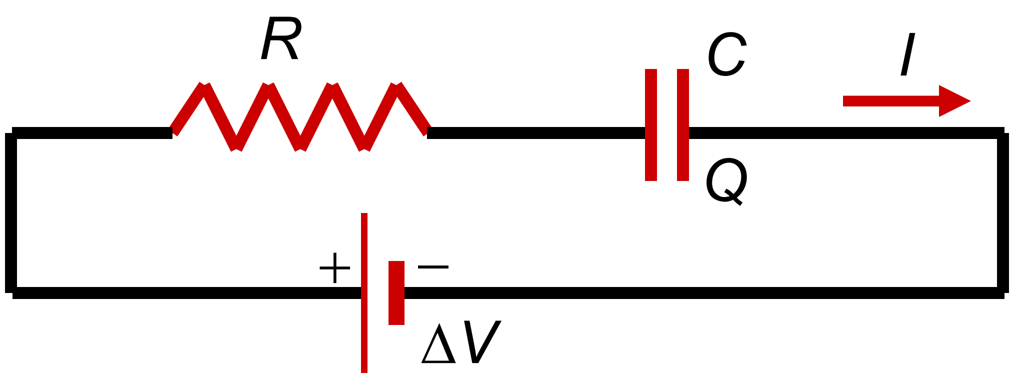


RC circuit

- As the plates are being charged, the potential difference across the capacitor increases
- At the instant the switch is closed, the charge on the capacitor is zero
- Once the maximum charge is reached, the current in the circuit is zero

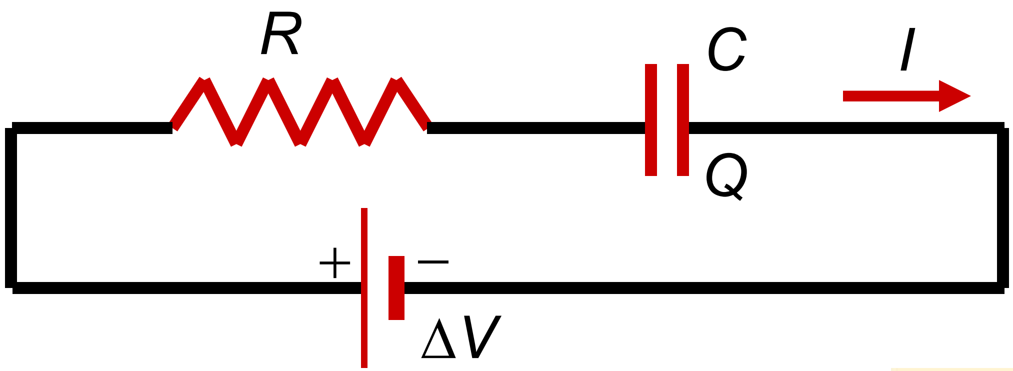






$$\Delta V = I(t)R + \frac{Q(t)}{C}$$

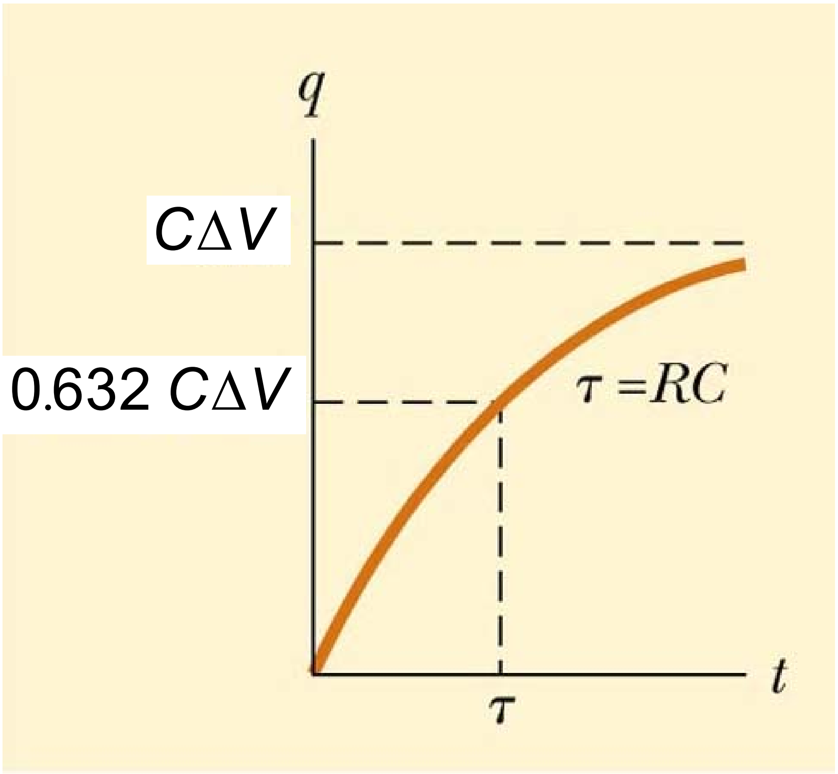
RC circuit



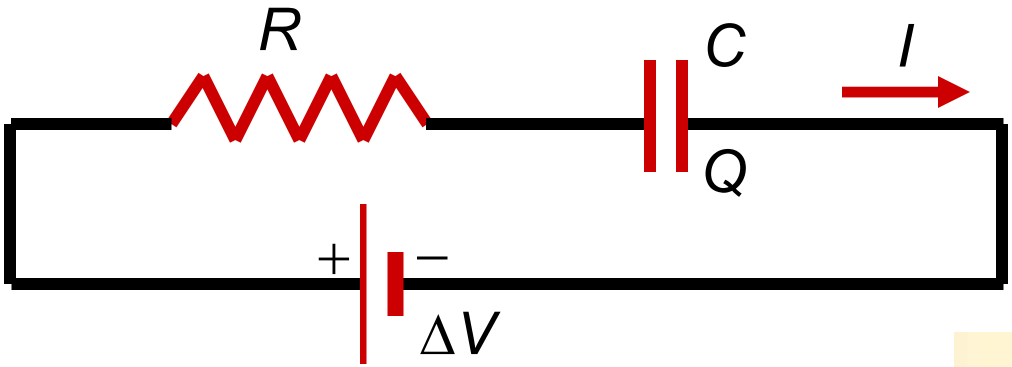
$$\Delta V = I(t)R + \frac{Q(t)}{C}$$

$$Q(t) = C\Delta V\left(1 - e^{-t/RC}\right)$$

$$I(t) = \frac{\Delta V}{R} e^{-t/RC}$$



RC circuit: time constant



$$Q(t) = C\Delta V\left(1 - e^{-t/RC}\right)$$

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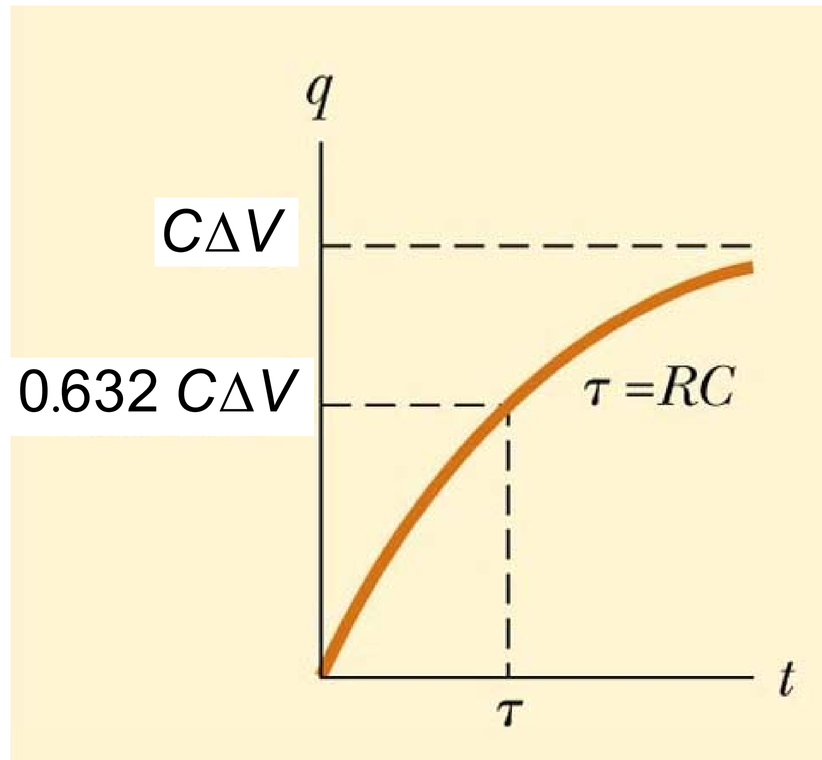
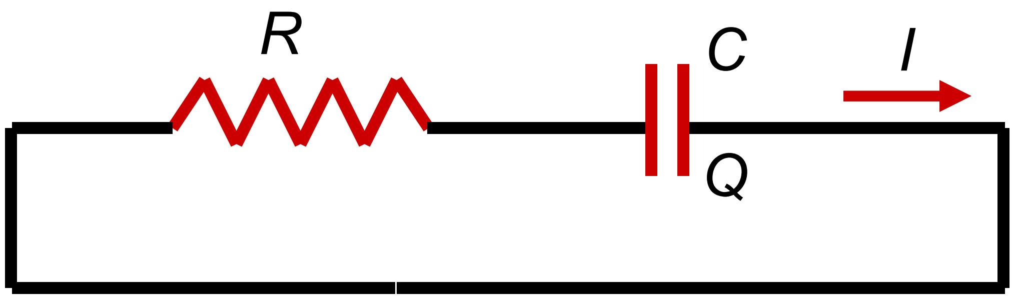


- The time constant represents the time required for the charge to increase from zero to 63.2% of its maximum
- $\tau = RC$ has unit of time

RC circuit



$$q(t) = Qe^{-t/RC}$$

$$I(t) = \frac{Q}{RC} e^{-t/RC}$$

- When a charged capacitor is placed in the circuit, it can be discharged
- The charge decreases exponentially with characteristic time $\tau = RC$