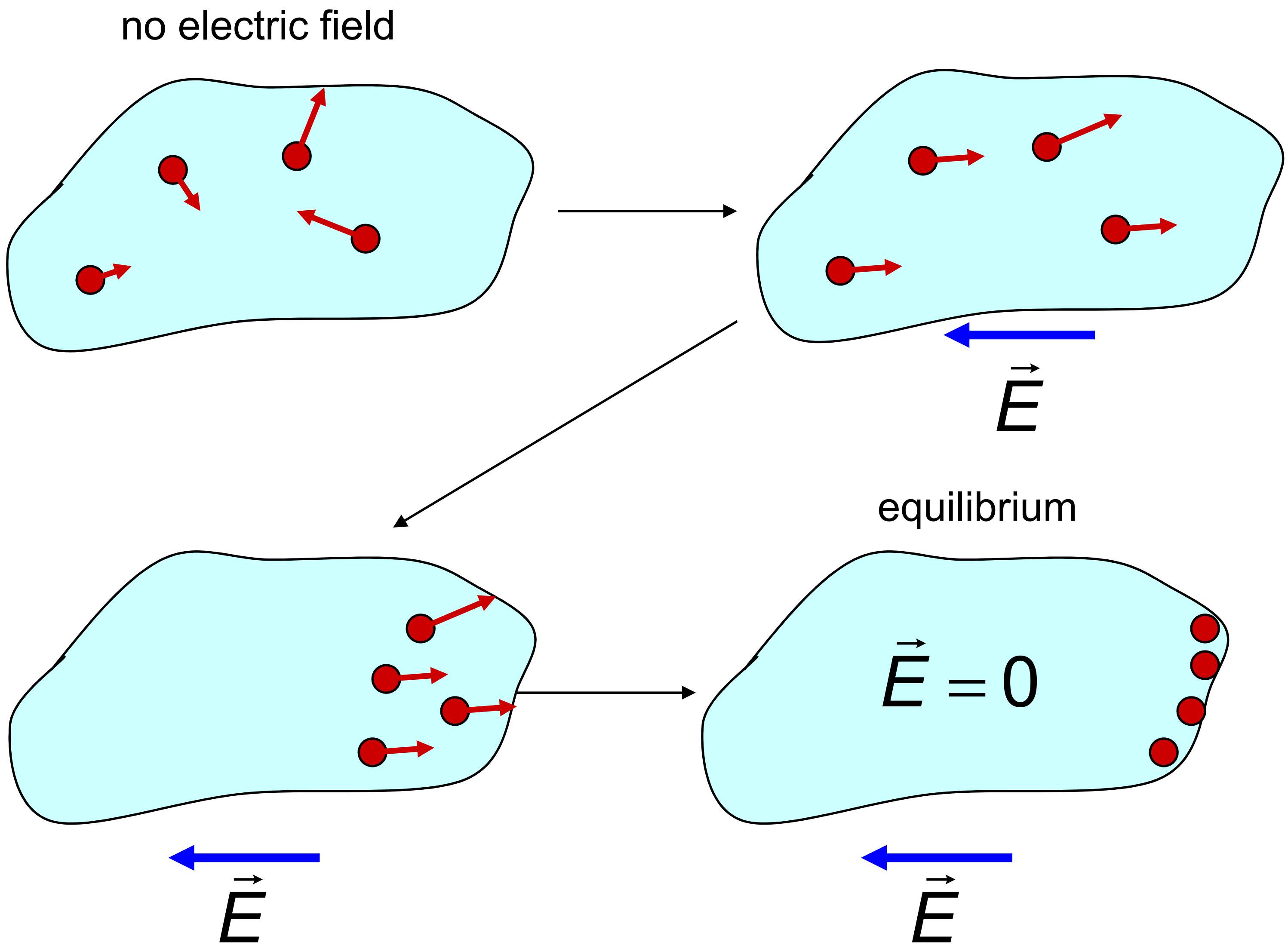
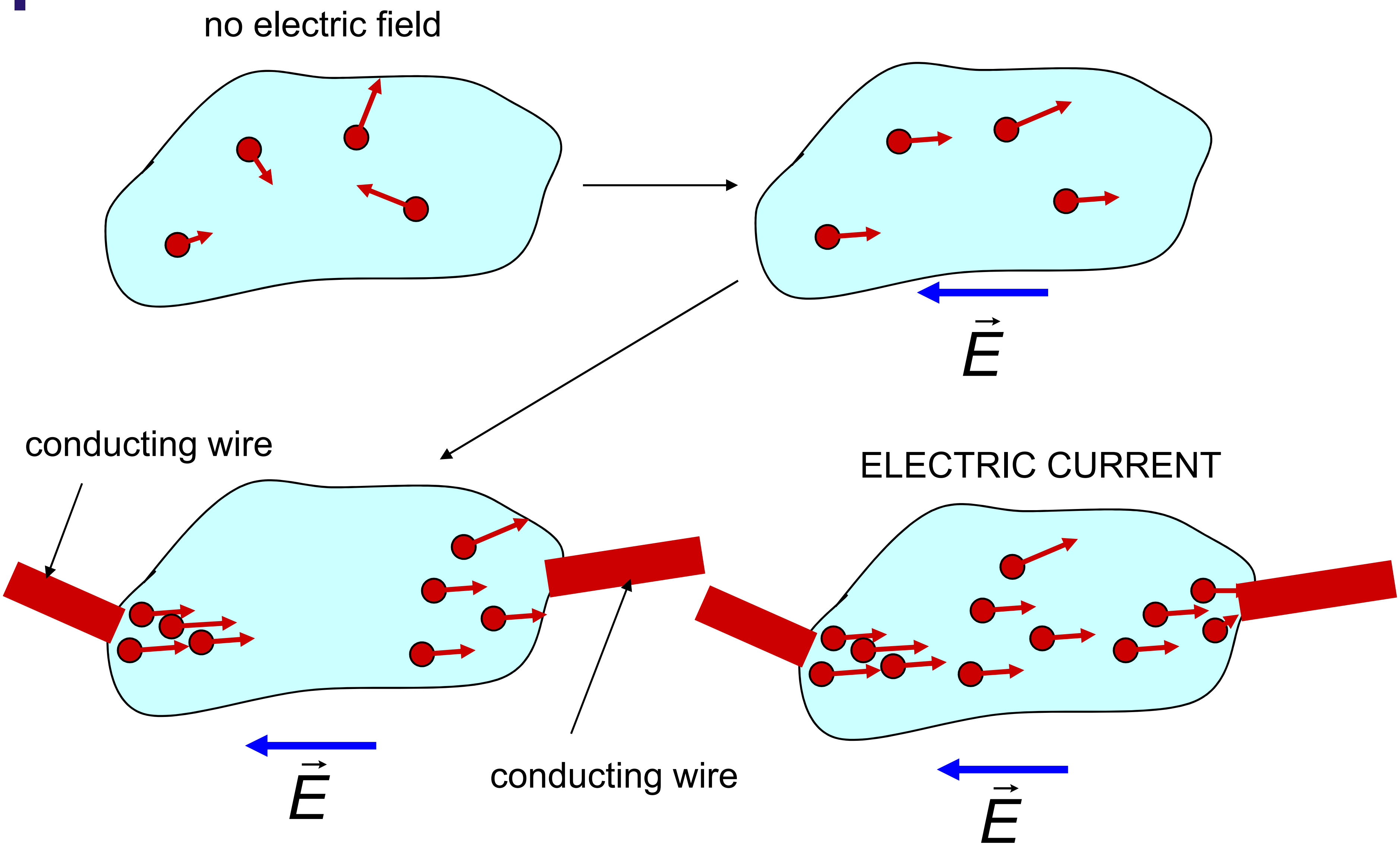
Electrical Current and Resistance

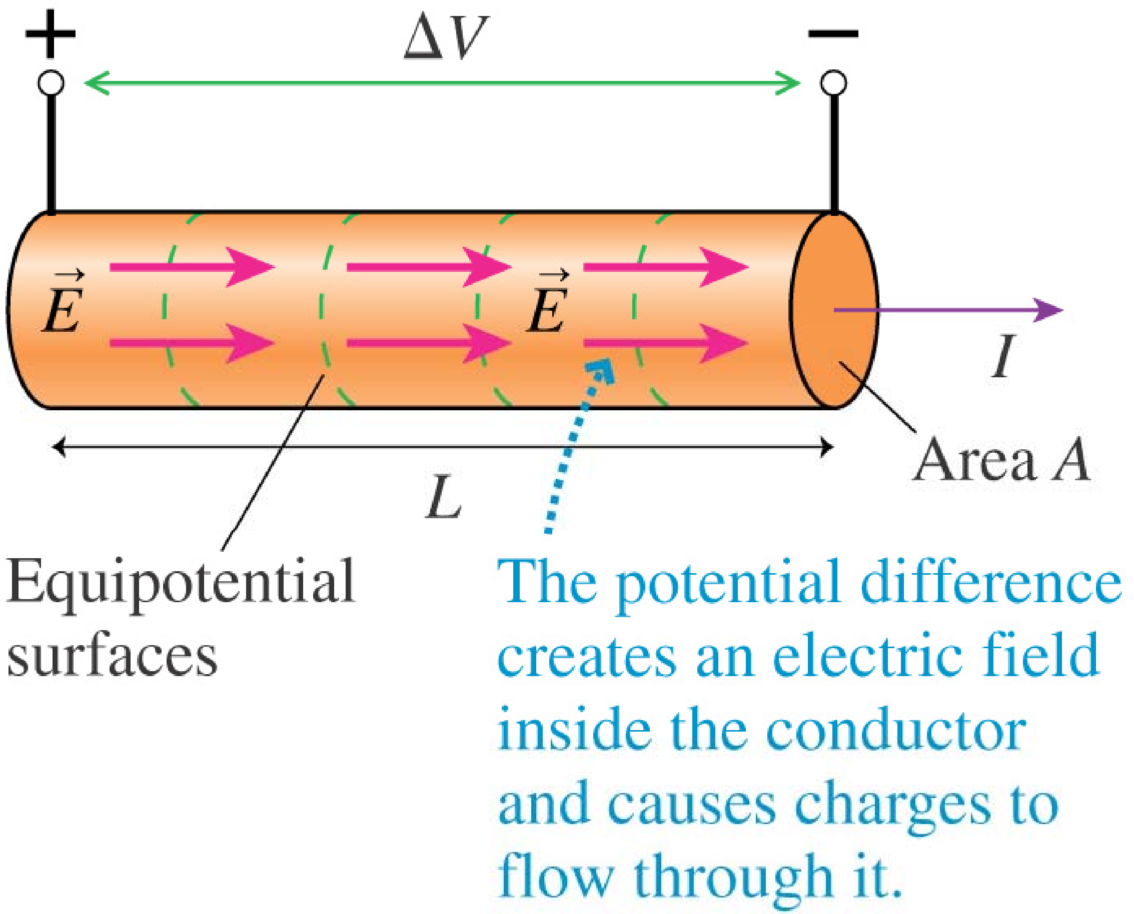
Conductor in Electric Field



Conductor in Electric Field

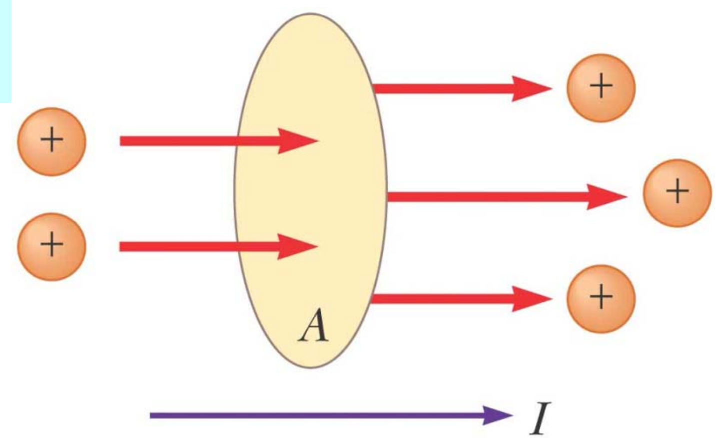


Conductor in Electric Field: Electric current



Electric Current

- **Electric current** is the rate of flow of charge through some region of space
- The SI unit of current is the **ampere (A), 1 A = 1 C / s**



- Assume charges are moving perpendicular to a surface of area ***A***
- If ΔQ is the amount of charge that passes through ***A*** in time Δt, then the average current is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

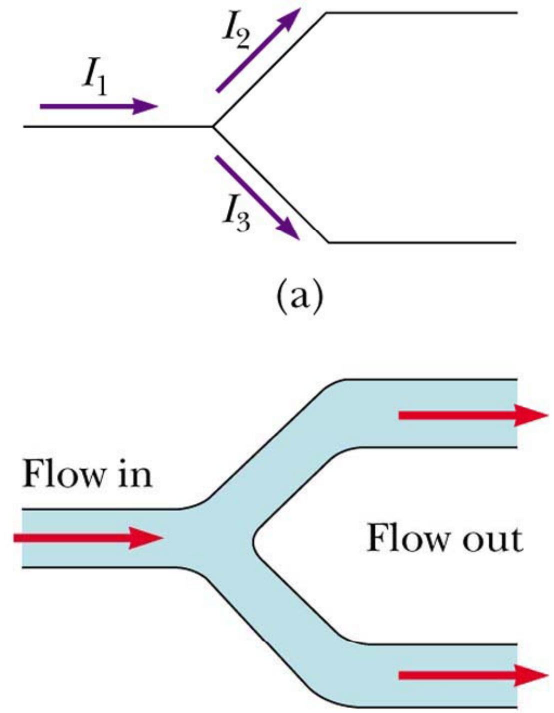
Conservation of current at a junction: Junction Rule

- The first Kirchhoff's rule – Junction Rule:
- The sum of the currents entering any junction must equal the sum of the currents leaving that junction

  \- A statement of Conservation of Charge

$$\sum I_{in} = \sum I_{out}$$

$$I_1 = I_2 + I_3$$

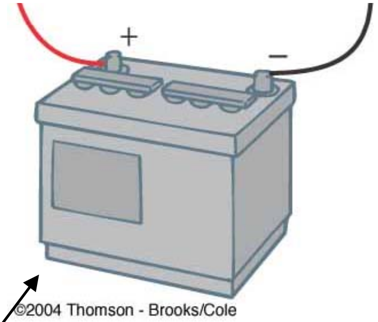


(a)

(b)

Batteries: Voltage

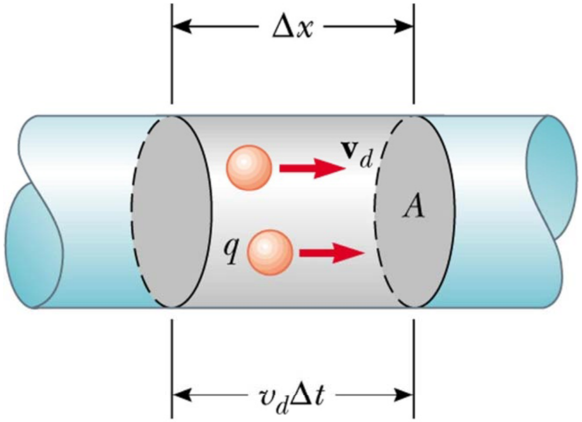
- The battery establishes an electric field in the connecting wires
- This field applies a force on electrons in the wire just outside of the plates
- The force causes the electrons to move onto the negative plate



Battery- produce the fixed voltage – the fixed potential difference

Ohm’s Law: Resistance

Current Density



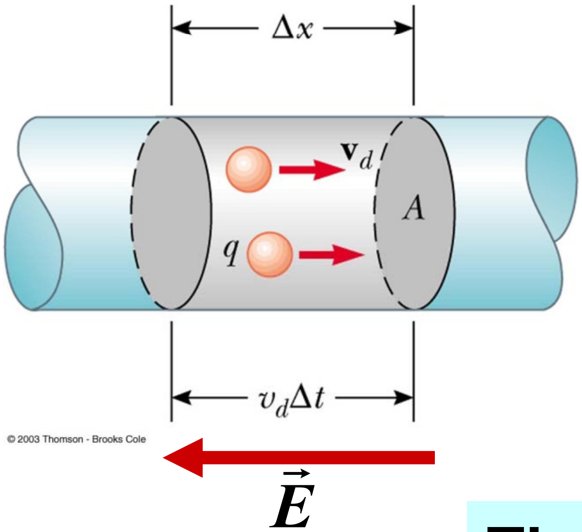
Current density is defined as the current per unit area

$$j = \frac{I}{A}$$

This expression is valid only if the current density is uniform and ***A*** is perpendicular to the direction of the current

j has **SI** units of **A/m^2**

Ohm's Law



Ohm's Law:
Current density is proportional to electric field

$$j = \sigma E$$

The constant of proportionality, σ, is called the conductivity of the conductor.

The conductivity depends only on the material of conductor.

Simplified model of electron motion in conductor gives

$$\sigma = \frac{n\tau q^2}{m}$$

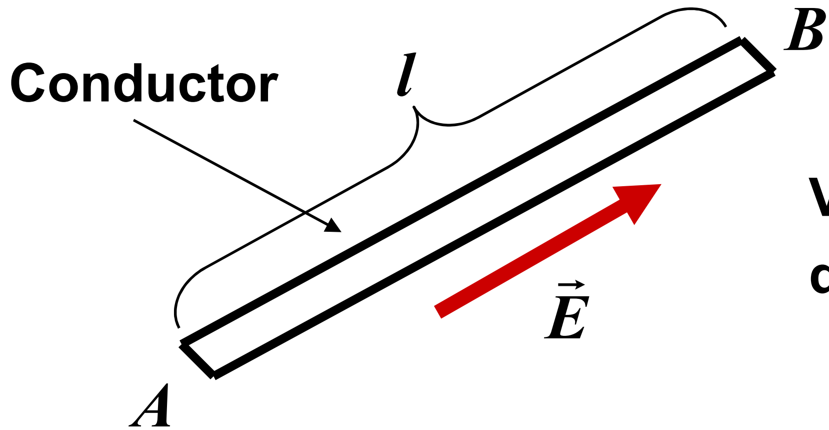
τ **- is the material dependent characteristic of conductor.**

Ohm's Law

$$j = \sigma E$$

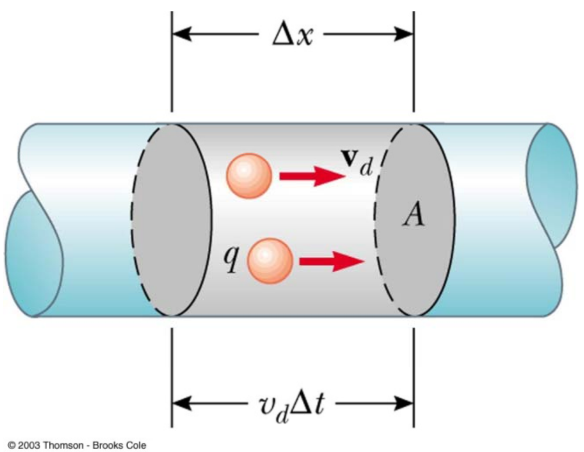
- **Ohm's law** states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current
  - Most metals, but not all, obey Ohm's law
  - Materials that obey Ohm's law are said to be ***ohmic***
  - Materials that do not obey Ohm's law are said to be ***nonohmic***
- Ohm's law is not a fundamental law of nature
- Ohm's law is an empirical relationship valid only for certain materials

Ohm's Law



Voltage across the conductor (potential difference between points A and B)

$$\Delta V = V_B - V_A = El$$



where electric field is the same along the conductor. Then

$$E = \frac{\Delta V}{l}$$

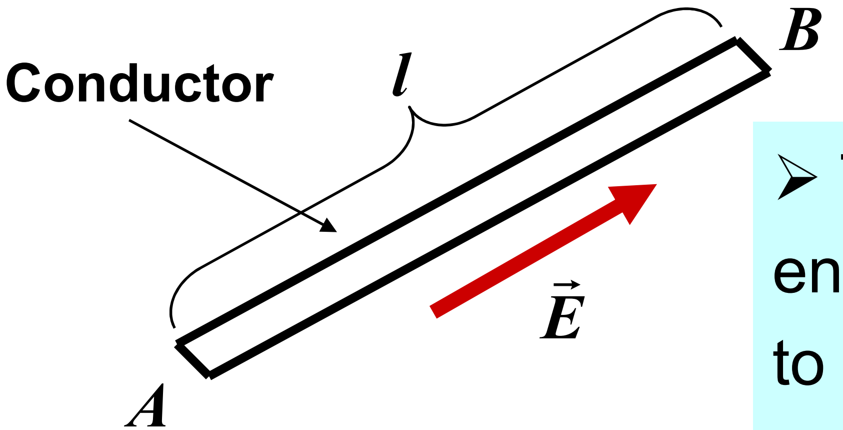
$$j = \frac{I}{A}$$

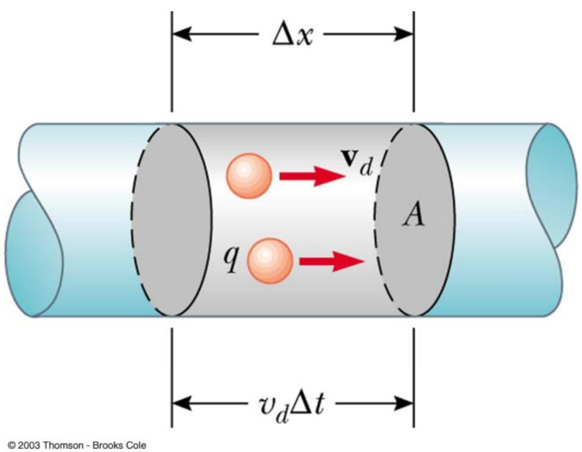
$$\boxed{j = \sigma E}$$

$$E = \frac{\Delta V}{l} = \frac{1}{\sigma} j = \frac{I}{\sigma A}$$

Another form of the Ohm's Law $\longrightarrow$ $\boxed{\Delta V = \frac{l}{\sigma A} I = RI}$

Ohm's Law: Resistance





- The voltage applied across the ends of the conductor is proportional to the current through the conductor
- The constant of proportionality is called the **resistance** of the conductor

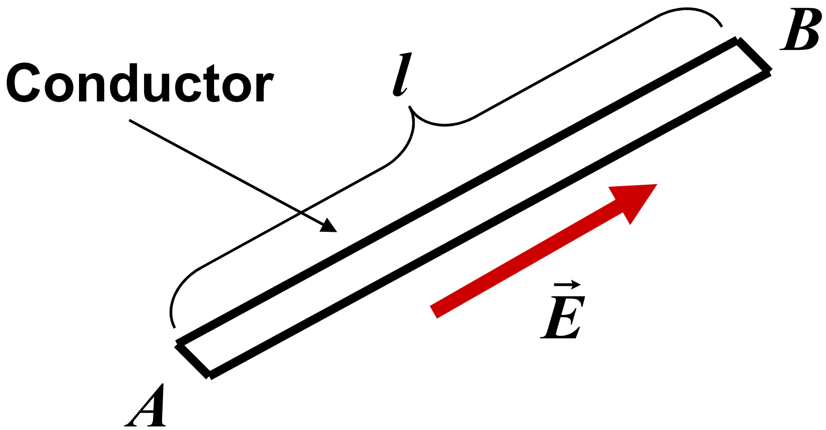
$$\Delta V = RI$$

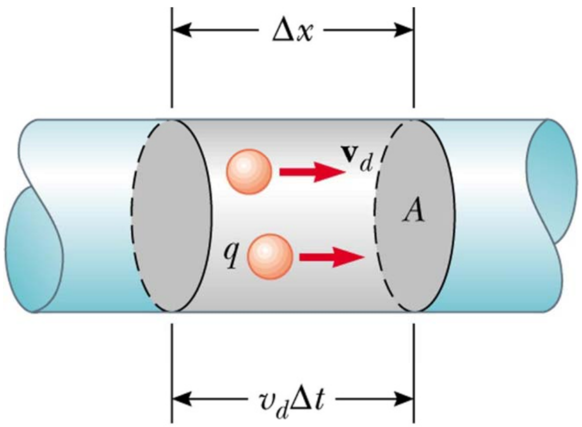
resistance

SI unit of resistance is ***ohm*** (Ω)

$1\ \Omega = 1\ \mathrm{V} / \mathrm{A}$

Ohm's Law: Resistance





$$\boxed{\Delta V = RI}$$

resistance

$$R = \frac{l}{\sigma A}$$

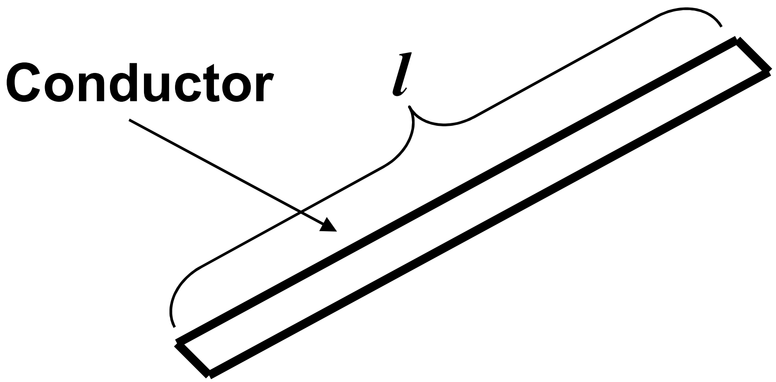
Or

$$R = \frac{\rho l}{A}$$

where $\rho = 1/\sigma$ is the resistivity – the inverse of the conductivity

Resistivity has SI units of ohm-meters (Ω m)

Resistance: Example



$$R = \rho \frac{l}{A}$$

The same amount of material has been used to fabricate the wire with uniform cross-section and length ***l/3***. What is the resistance of the wire?

$$R_1 = \rho \frac{l_1}{A_1}$$

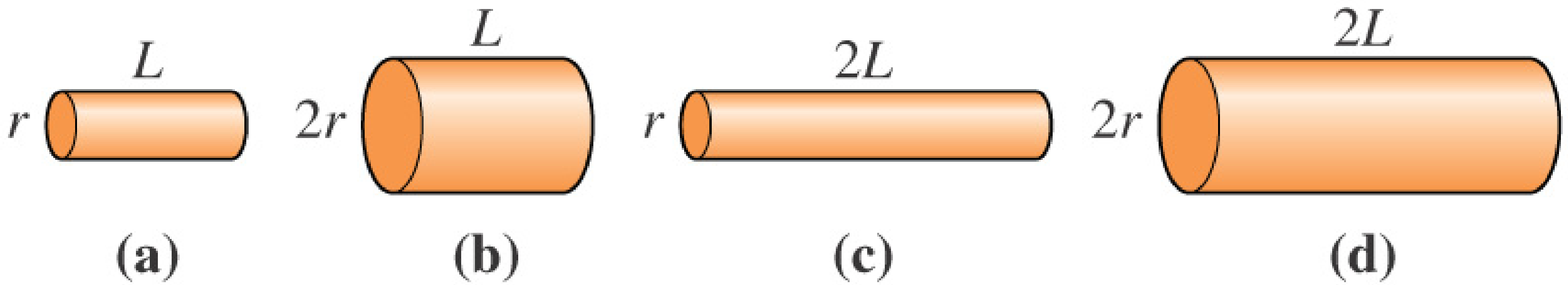
$$l_1 A_1 = lA \qquad\qquad l_1 = l/3$$

$$A_1 = \frac{lA}{l_1} = 3A$$

$$R_1 = \rho \frac{l_1}{A_1} = \rho \frac{l/3}{3A} = \rho \frac{l}{9A} = \frac{R}{9}$$

Resistance: Example

The wires are all made of the same material. Rank in order, from largest to smallest, the resistances of these wires.

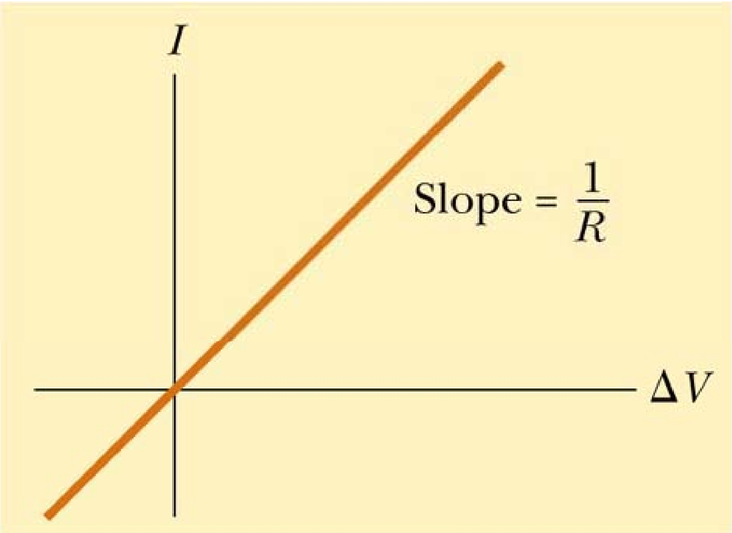


Ohm's Law

$$j = \sigma E \qquad \Delta V = RI$$

– Materials that obey Ohm's law are said to be ***ohmic***

– Materials that do not obey Ohm's law are said to be ***nonohmic***



<u>An ohmic device</u>

➢ The resistance is constant over a wide range of voltages

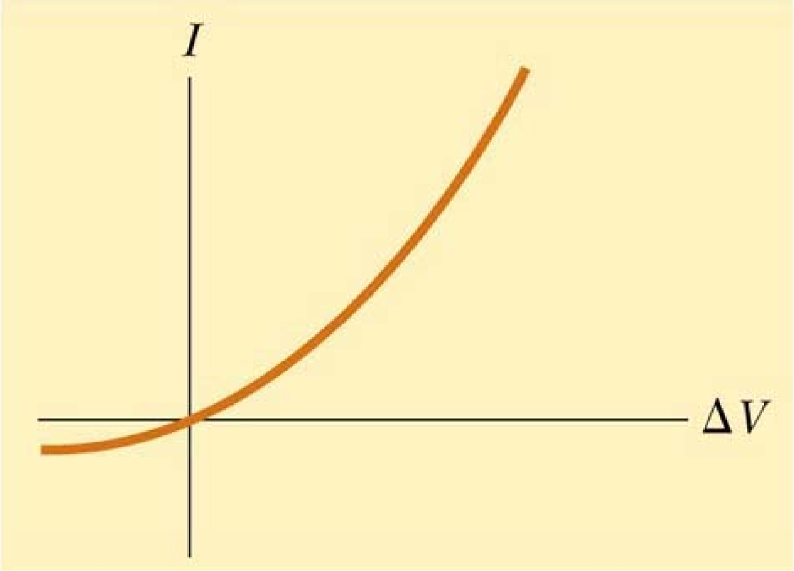
➢ The relationship between current and voltage is linear

➢ The slope is related to the resistance

Ohm's Law

$$j = \sigma E \qquad \Delta V = RI$$

- Materials that obey Ohm's law are said to be ***ohmic***
- Materials that do not obey Ohm's law are said to be ***nonohmic***

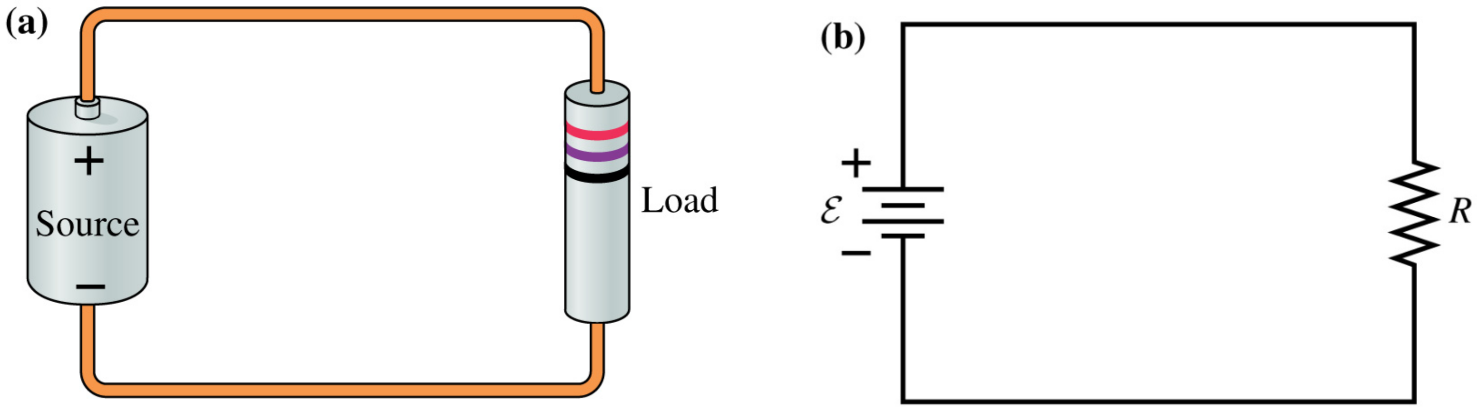


<u>Nonohmic materials</u>

➢ The current-voltage relationship is nonlinear

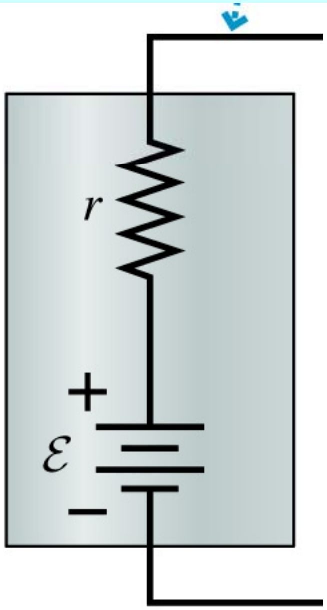
Batteries: EMF (electromotive force)

- The battery establishes an electric field in the connecting wires
- This field applies a force on electrons in the wire just outside of the plates
- The force causes the electrons to move onto the negative plate



Batteries: EMF (electromotive force)

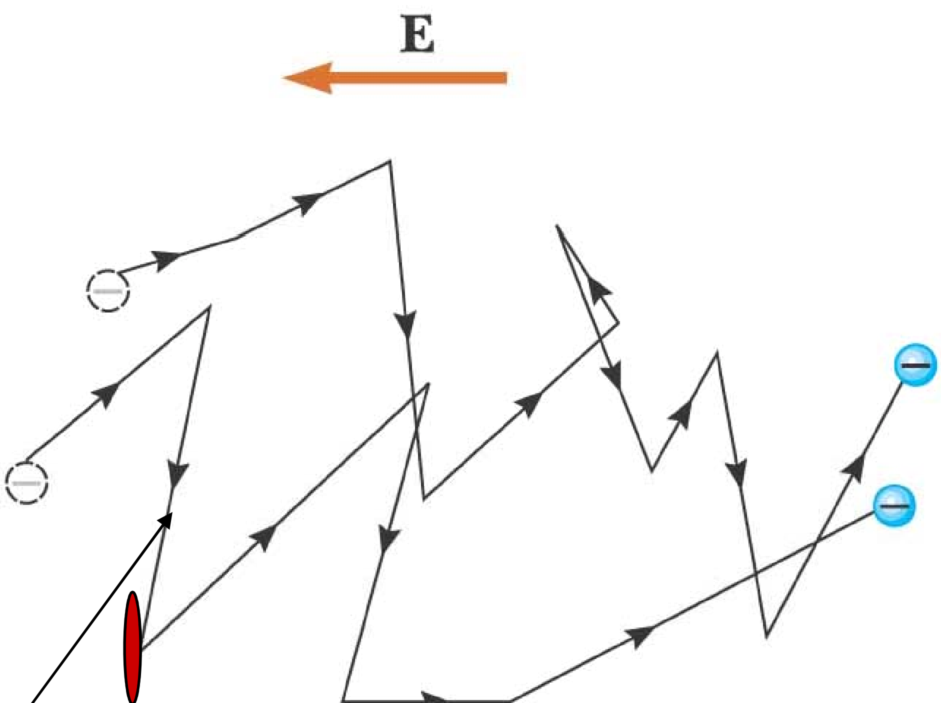
- Electromotive force (EMF) – voltage of the battery
- Internal resistance of the battery



Electric Power

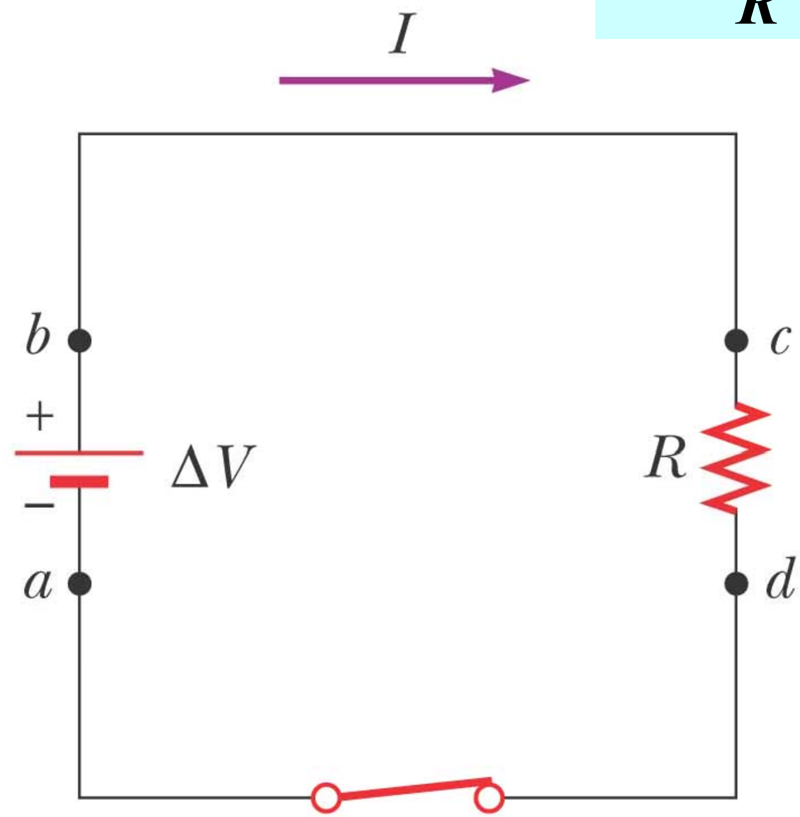
Electrical Power

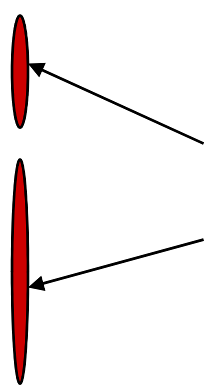
$$I = \frac{\Delta V}{R}$$



$$\vec{v}_f = \vec{v}_i + \frac{q\vec{E}}{m}t$$

(b)





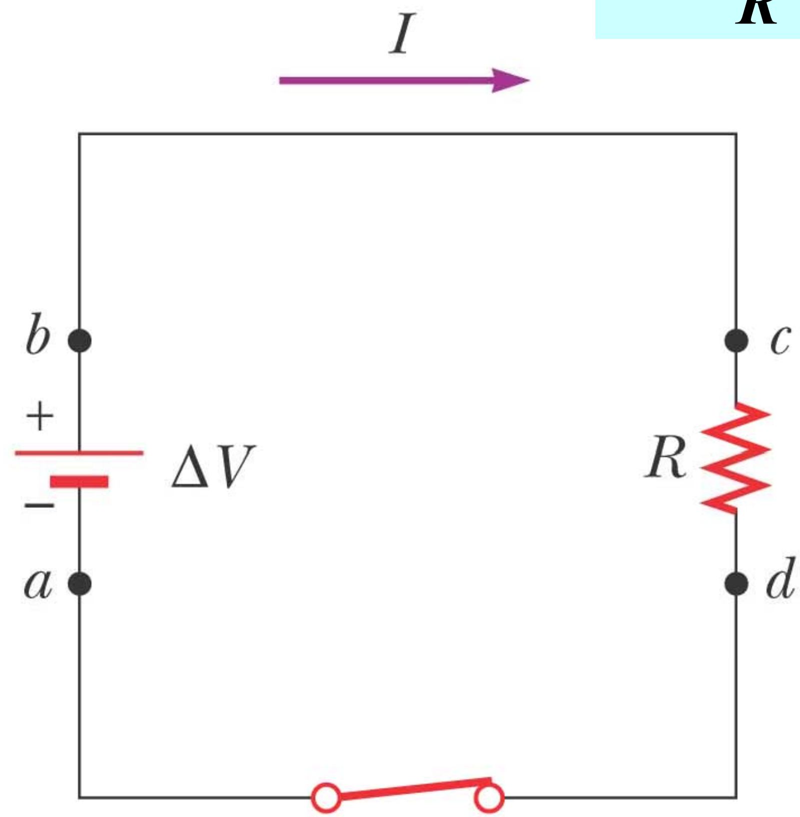
Before the collision

After the collision

Electrical Power

$$I = \frac{\Delta V}{R}$$

- As a charge moves from *a* to *b*, the electric potential energy of the system increases by $Q\Delta V$
  - The chemical energy in the battery must decrease by the same amount

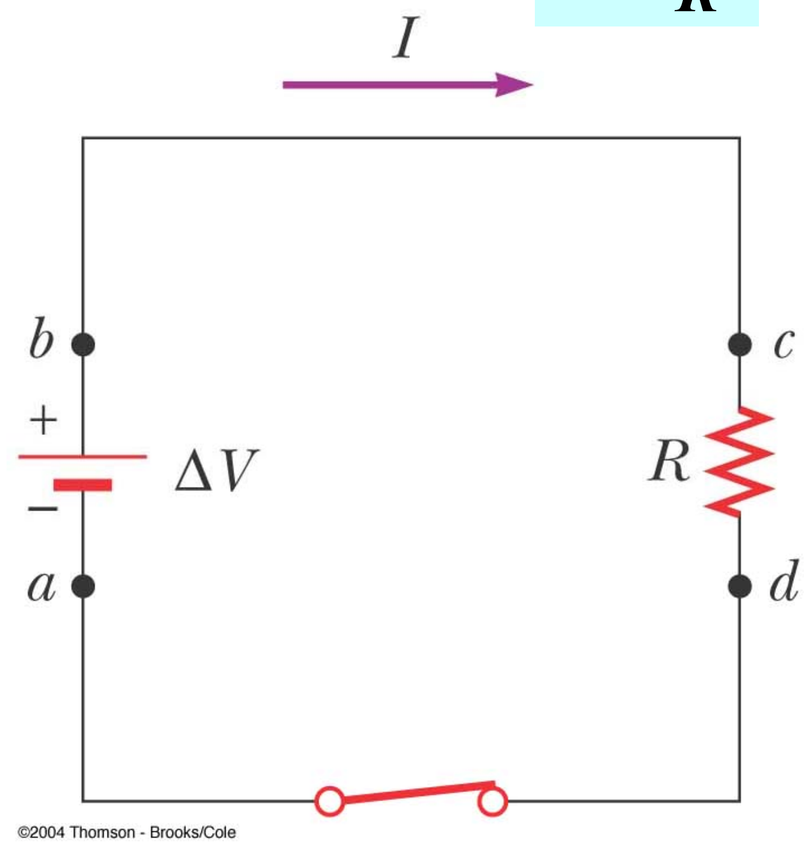


- As the charge moves through the resistor (*c* to *d*), the system loses this electric potential energy during collisions of the electrons with the atoms of the resistor
- This energy is transformed into internal energy in the resistor

Electrical Power

$$I = \frac{\Delta V}{R}$$

➢ The **power** is the rate at which the energy is delivered to the resistor



$\Delta U = Q\Delta V$ - the energy delivered to the resistor when charge **Q** moves from **a** to **b** (or from **c** to **d**)

The power:

$$P = \frac{\Delta U}{\Delta t} = \frac{Q}{\Delta t}\Delta V = I\Delta V$$

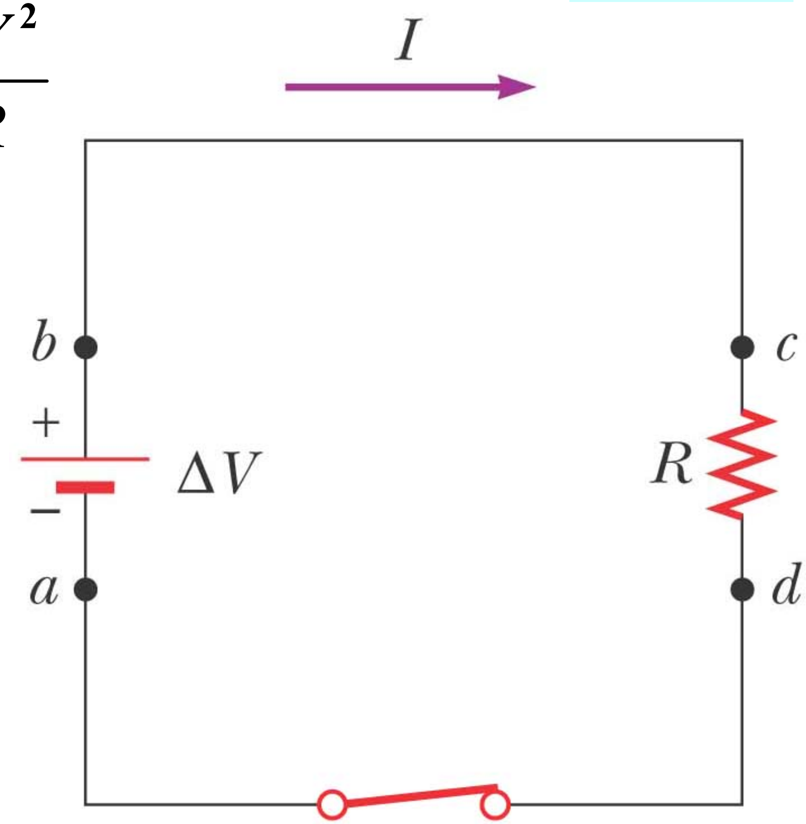
$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

Units: ***I*** is in A, ***R*** is in Ω, ***V*** is in V, and ***P*** is in W (watt)

Electrical Power

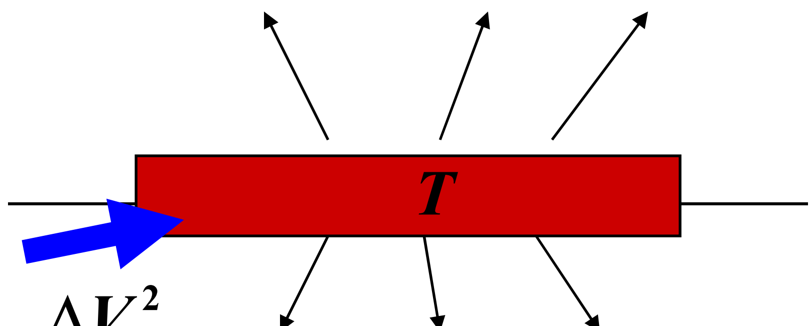
$$I = \frac{\Delta V}{R}$$

The power: $P = I\Delta V = I^2 R = \dfrac{\Delta V^2}{R}$



$$P = \frac{\Delta V^2}{R(T)}$$ **Will increase the temperature of conductor**

Electromagnetic waves (light), $P_{EMW}(T)$



$$P = \frac{\Delta V^2}{R(T)}$$

Heat transfer to air $P_{air}(T) = \alpha(T - T_0)$

$$P = \frac{\Delta V^2}{R(T)} = P_{EMW}(T) + \alpha(T - T_0)$$

Power: Example

A 1000-W heating coil designed to operate from 110 V is made of Nichrome wire 0.5 mm in diameter. Assuming that the resistivity of the Nichrome remains constant at its 20 C value, find the length of wire used.

$$\rho_N = 1.5 \cdot 10^{-6} \Omega \cdot m \qquad R = \rho_N \frac{l}{A} \qquad A = \pi \frac{d^2}{4}$$

$$P = I \Delta V = I^2 R = \frac{U^2}{R}$$

$$R = \frac{U^2}{P}$$

$$l = A \frac{R}{\rho_N} = A \frac{U^2}{\rho_N P} = \pi \frac{d^2}{4} \frac{U^2}{\rho_N P} = \frac{3.14 \cdot 0.5^2 \cdot 10^{-6} \cdot 110^2}{4 \cdot 1.5 \cdot 10^{-6} \cdot 1000} m = 1.58 m$$