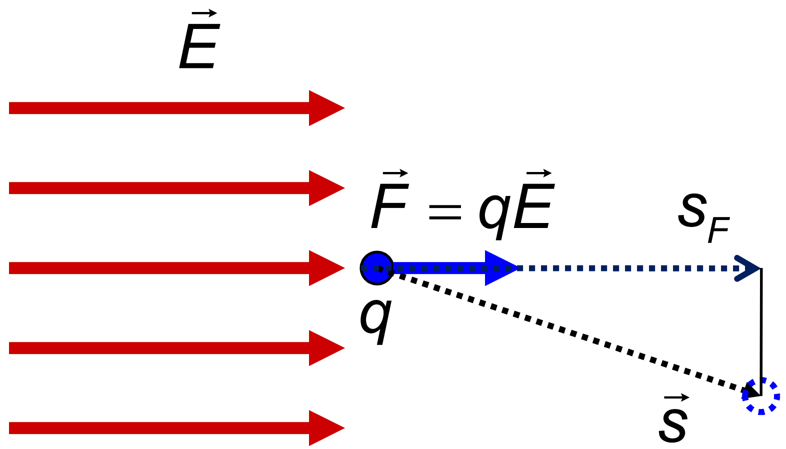
Electric Potential

Reading: Chapter 21

Electric Potential Energy

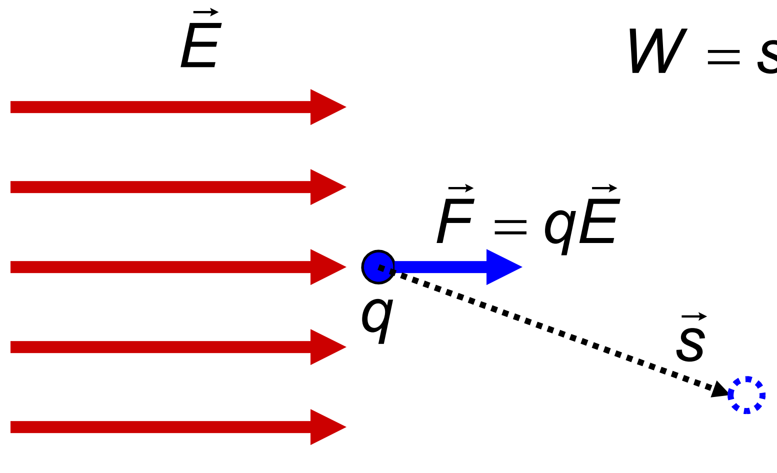
When a test charge is placed in an electric field, it experiences a force



If $\vec{s}$ is an infinitesimal displacement of test charge, then the work done by electric force during the motion of the charge is given by

$$W = s_F F$$

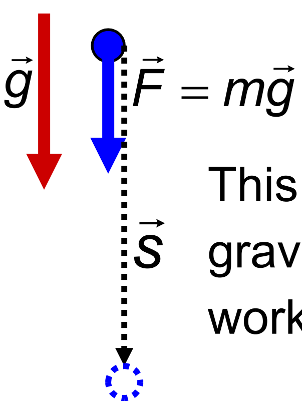
Electric Potential Energy



$$W = s_F F = q\, s_F E$$

This is the work **done by electric field.**

In this case work is positive.



This is very similar to gravitational force: the work done by force is

$$s_F F = mgs_F$$

The change of potential energy is

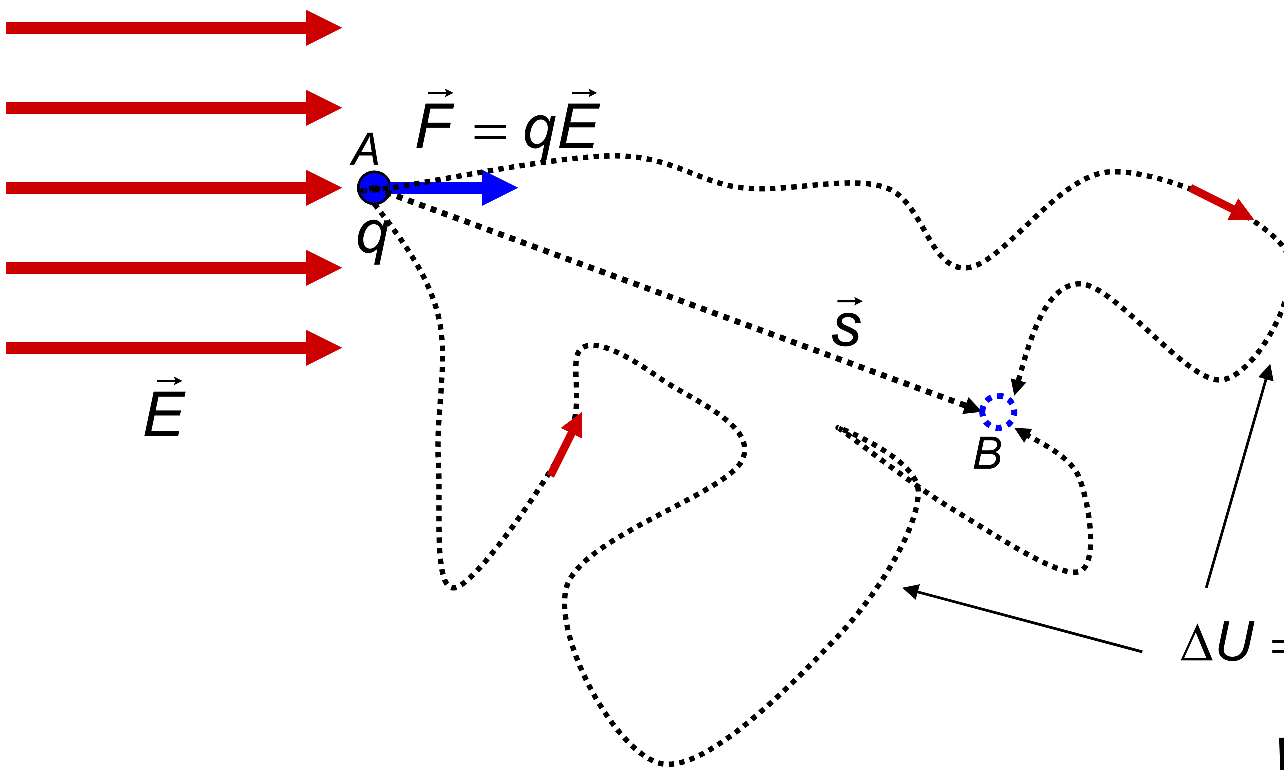
$$\Delta U = -s_F F = -mgs_F$$

Because the positive work is done, the potential energy of charge-field system should decrease. So the change of potential energy is

$$\Delta U = -s_F F = -q\, s_F E$$

minus sign

Electrical Potential Energy



$$\Delta U = U_B - U_A = -Work_{AB}$$

Work is the same for all paths

For all paths: $$U_B = U_A - Work_{AB}$$

The electric force is conservative

Electric Potential

Electric potential is the potential energy per unit charge,

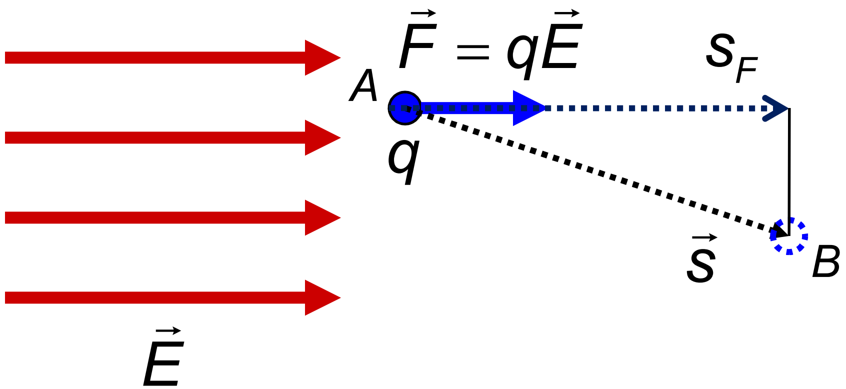
$$V = \frac{U}{q}$$

The potential is **independent** of the value of $\boldsymbol{q}$.

The potential has a value at every point in an electric field

Only the ***difference*** in potential is the meaningful quantity.

$$V_B - V_A = \frac{U_B}{q} - \frac{U_A}{q} = -\frac{Work_{AB}}{q}$$



Electric Potential

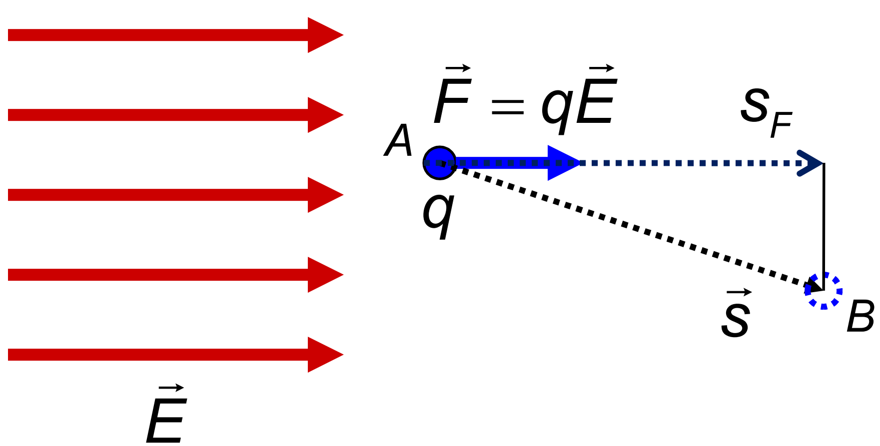
To find the potential at every point

❑ 1. we assume that the potential is equal to **0** at some point, for example at point **A**, $V_A = 0$

❑ 2. we find the potential at any point B from the expression

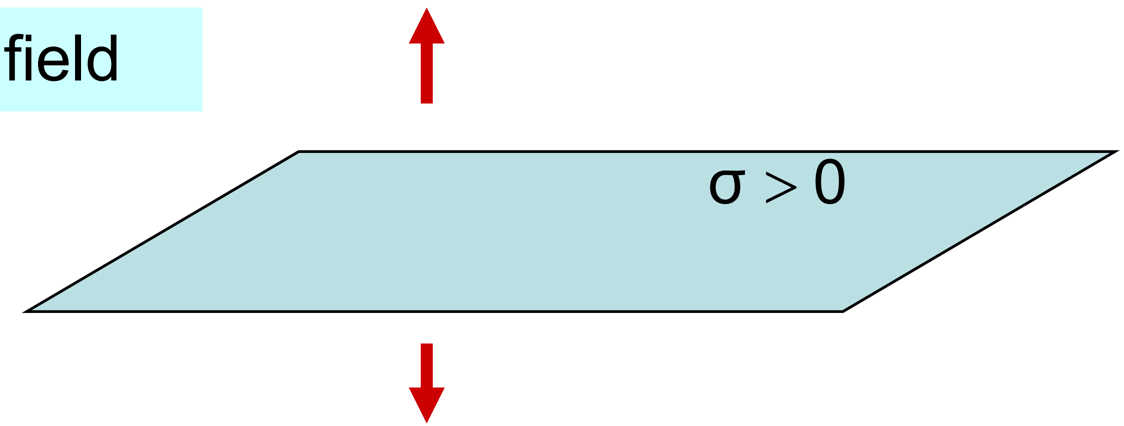
$$V_B = V_A - \frac{Work_{AB}}{q} = -\frac{Work_{AB}}{q}$$

$$V_B = -s_F E$$



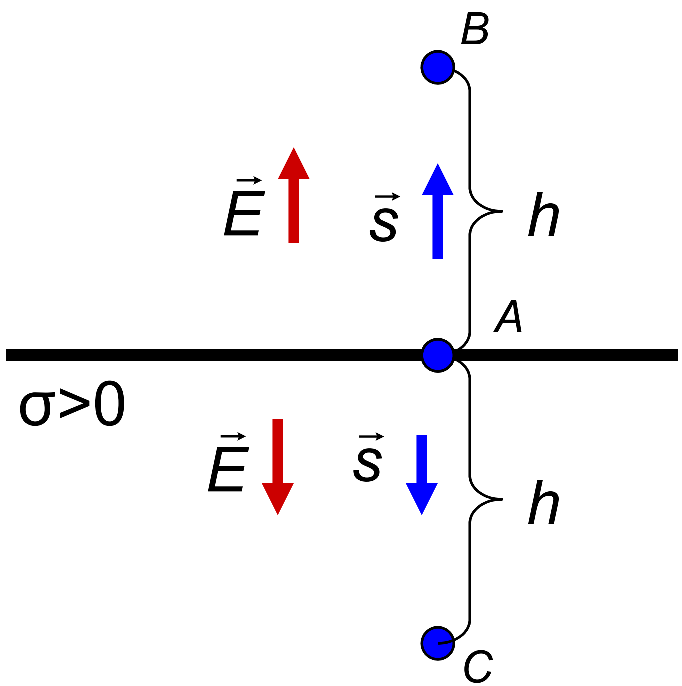
Electric Potential: Example

Plane: Uniform electric field



$$V_A = 0$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

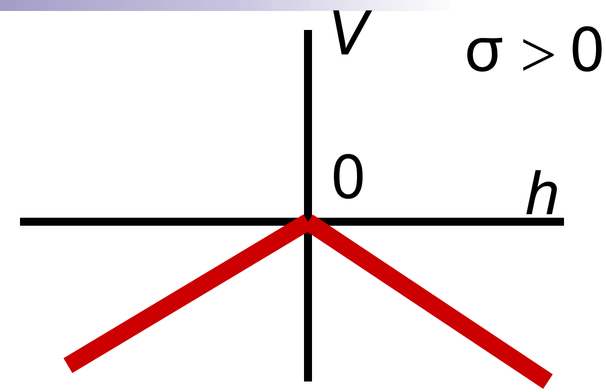


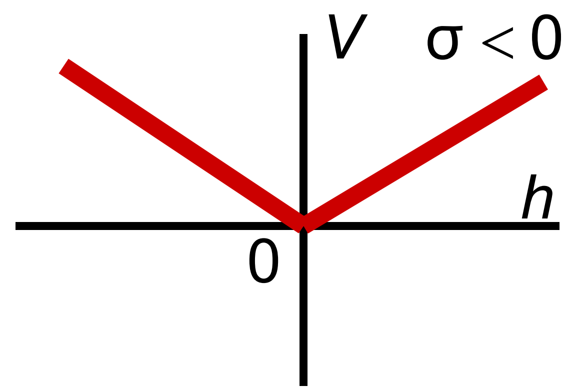
$$V_B = V_A - hE = -hE = -\frac{\sigma h}{2\varepsilon_0}$$

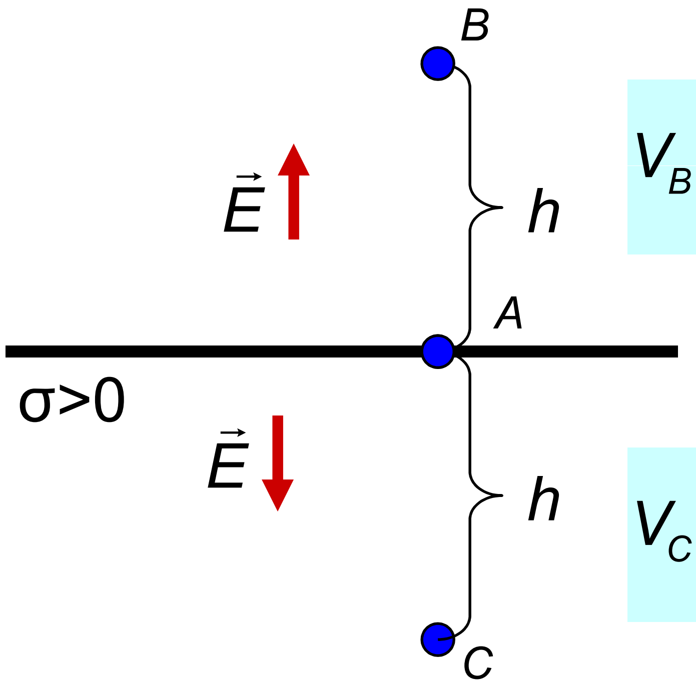
$$V_C = V_A - hE = -hE = -\frac{\sigma h}{2\varepsilon_0}$$

Electric Potential: Example

Plane: Uniform electric field







$$V_B = -\frac{\sigma h}{2\varepsilon_0}$$

$$V_C = -\frac{\sigma h}{2\varepsilon_0}$$

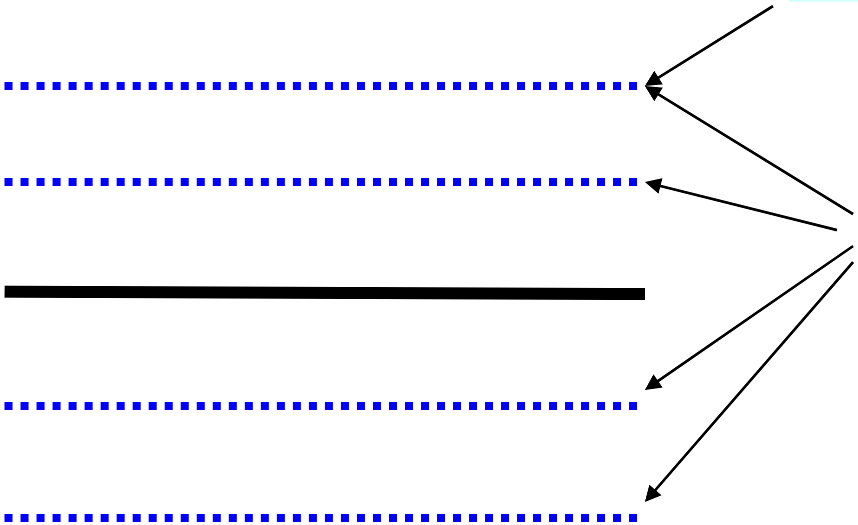
All points with the same *h* have the same potential

Electric Potential: Example

Plane: Uniform electric field

The same potential

$$V = -\frac{\sigma h}{2\varepsilon_0}$$



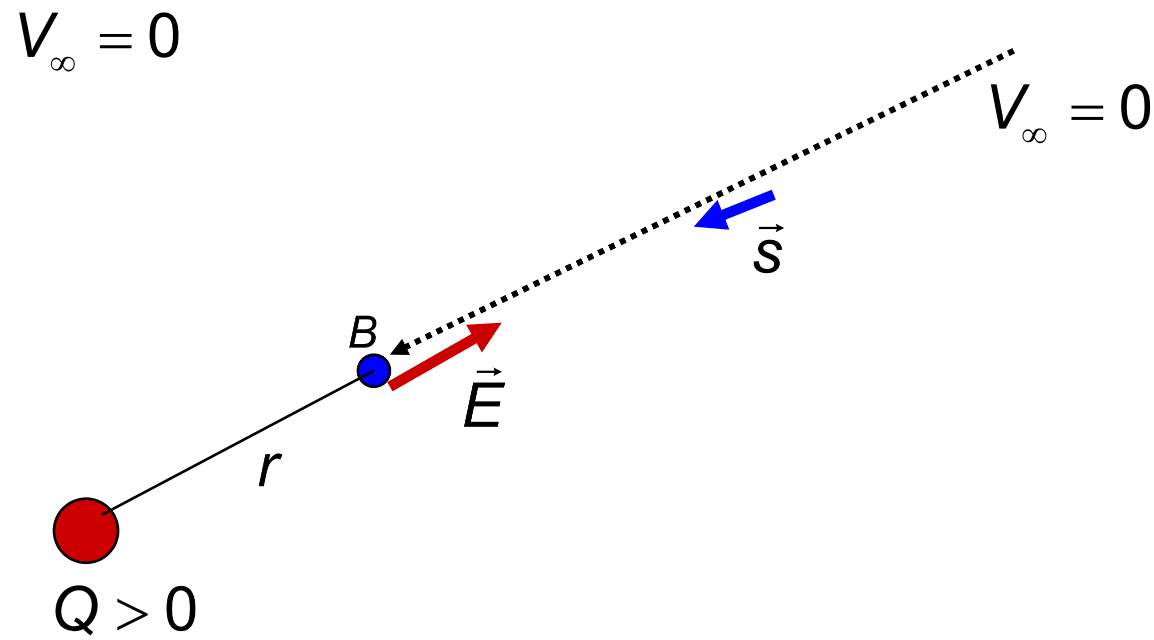
equipotential lines

$$V = -\frac{\sigma h}{2\varepsilon_0}$$

Electric Potential: Example

Point Charge

$$E_r = k_e \frac{|Q|}{r^2}$$



$$V_B = k_e \frac{Q}{r}$$

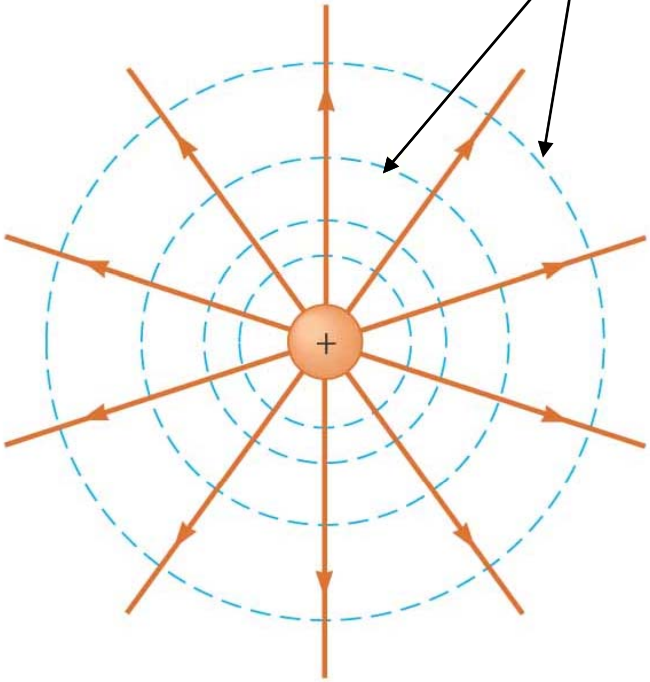
Electric Potential: Example

Point Charge

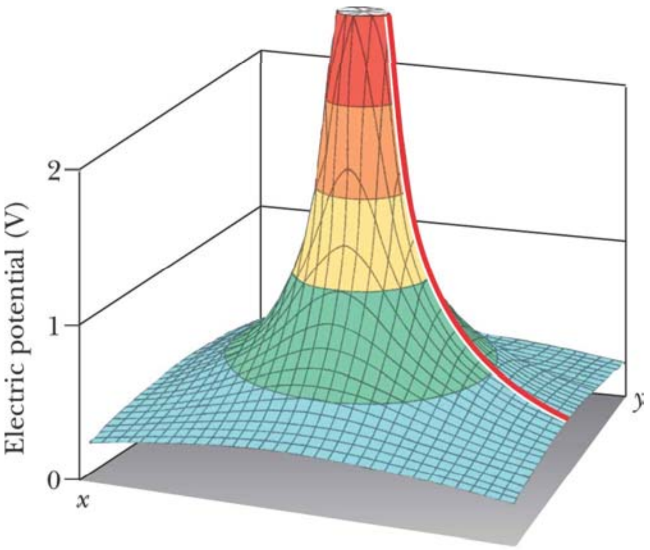
$$V_\infty = 0$$

$$V_r = k_e \frac{Q}{r}$$

equipotential lines



(b)



Units

- Units of potential: ***1 V = 1 J/C***
  - ***V*** is a volt
  - It takes one joule (***J***) of work to move a 1-coulomb (***C***) charge through a potential difference of 1 volt (***V***)

❑ Another unit of energy that is commonly used in atomic and nuclear physics is the ***electron-volt***

❑ One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of ***1 volt***

$$1\ eV = 1.60 \times 10^{-19}\ J$$

Potential and Potential Energy

- If we know potential then the potential energy of point charge ***q*** is

$$U = qV$$

(this is similar to the relation between electric force and electric field)

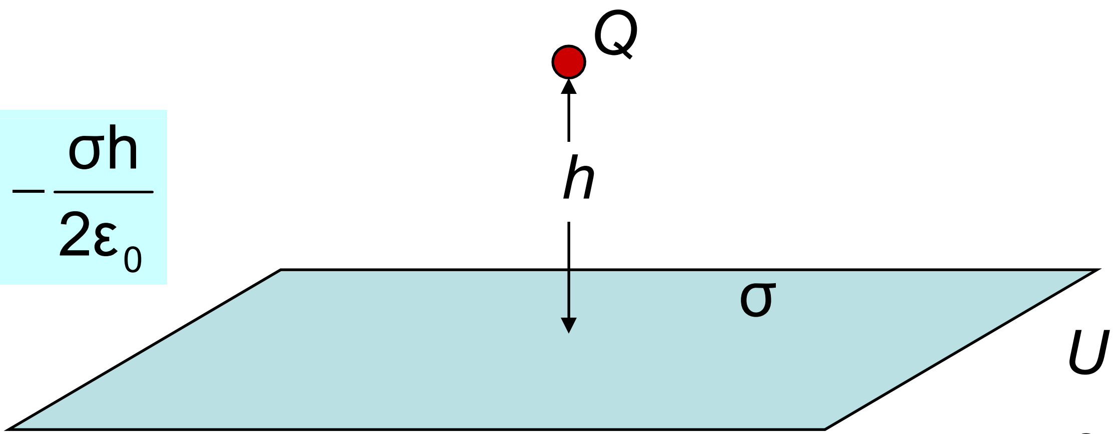
$$\vec{F} = q\vec{E}$$

Potential Energy: Example

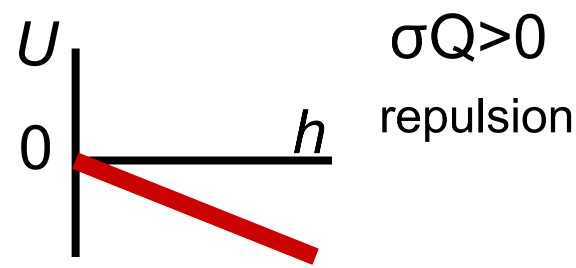
What is the potential energy of point charge ***q*** in the field of uniformly charged plane?

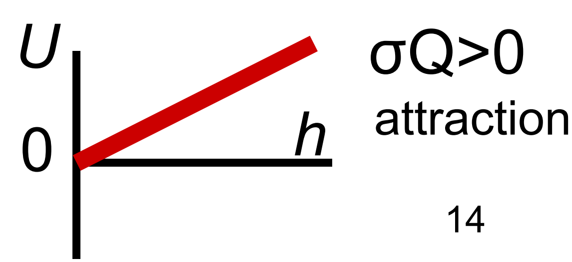
$$U = QV$$

$$V = -\frac{\sigma h}{2\varepsilon_0}$$



$$U = qV = -\frac{\sigma Q}{2\varepsilon_0}\mathrm{h}$$



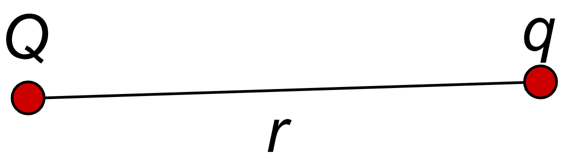


Potential Energy: Example

What is the potential energy of two point charges ***q*** and ***Q***?

This can be calculated by two methods:

A



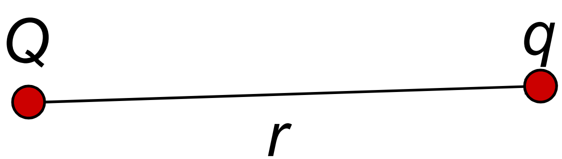
$$U_q = qV_Q$$

$$V_Q = k_e \frac{Q}{r}$$

The potential energy of point charge *q* in the field of point charge *Q*

$$U_q = qV_Q = k_e \frac{qQ}{r}$$

B



$$U_Q = QV_q$$

$$V_q = k_e \frac{q}{r}$$

The potential energy of point charge *Q* in the field of point charge *q*

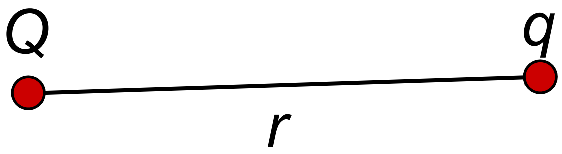
$$U_Q = QV_q = k_e \frac{qQ}{r}$$

In both cases we have the same expression for the energy. This expression gives us the energy of two point charges.

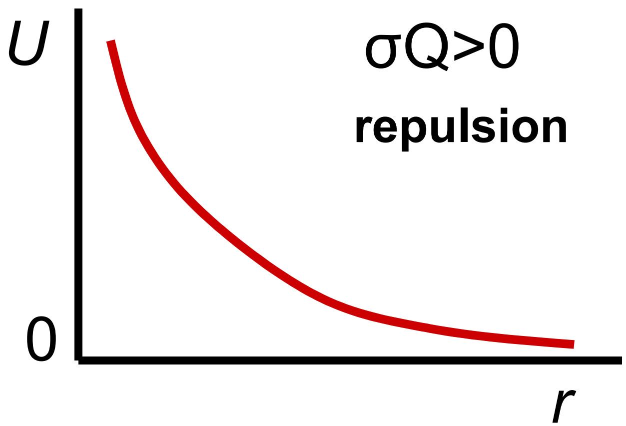
$$U = k_e \frac{qQ}{r}$$

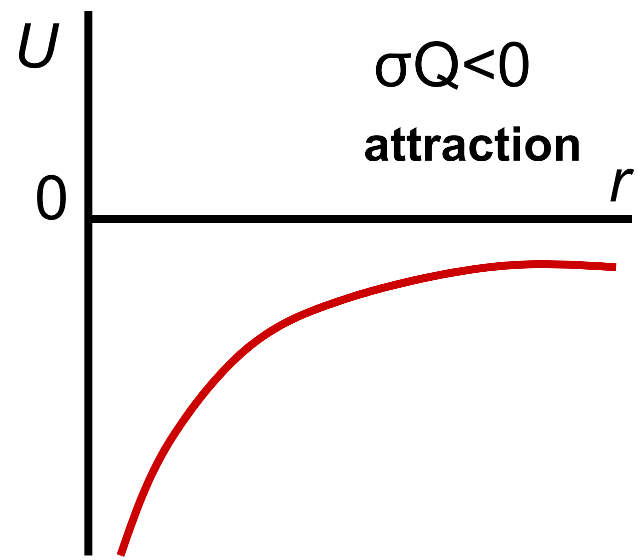
Potential Energy: Example

Potential energy of two point charges:



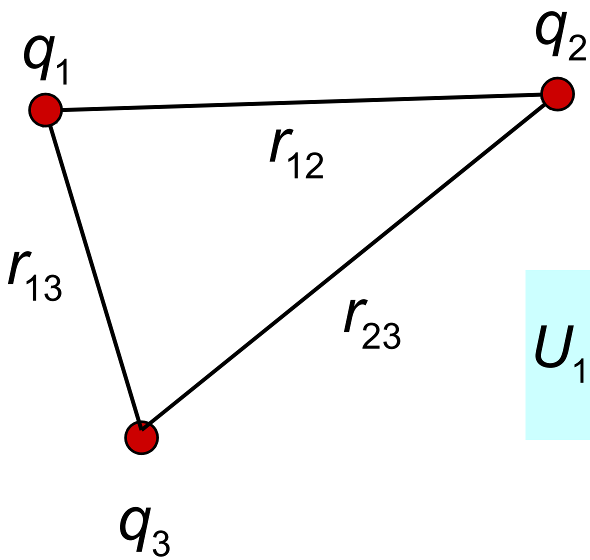
$$U = k_e \frac{qQ}{r}$$





Potential Energy: Example

Find potential energy of three point charges:



$$U = U_{12} + U_{13} + U_{23}$$

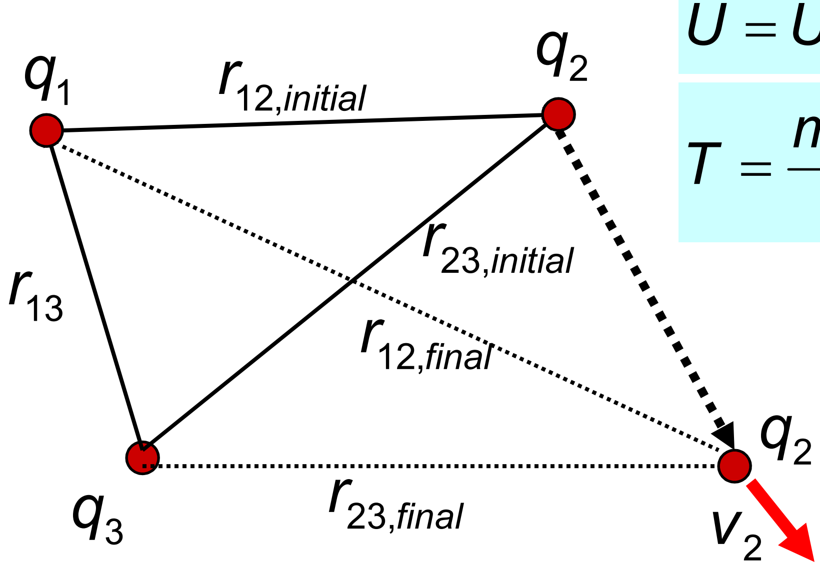
$$U_{12} = k_e \frac{q_1 q_2}{r_{12}} \qquad U_{13} = k_e \frac{q_1 q_3}{r_{13}} \qquad U_{23} = k_e \frac{q_2 q_3}{r_{23}}$$

$$U = U_{12} + U_{13} + U_{23} = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}}$$

Potential Energy: Applications: Energy Conservation

For a closed system: Energy Conservation:

The sum of potential energy and kinetic energy is constant



$$U = U_{12} + U_{13} + U_{23}$$ - ***Potential energy***

$$T = \frac{mv^2}{2}$$ - ***Kinetic energy***

Example: Particle 2 is released from the rest. Find the speed of the particle when it will reach point P.

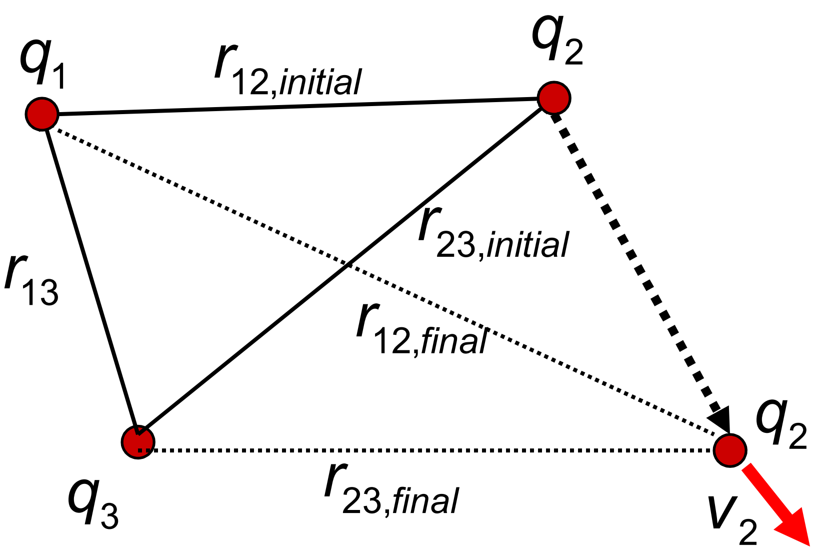
Initial Energy is the sum of kinetic energy and potential energy (velocity is zero – kinetic energy is zero)

$$E_{initial} = T + U = T + U_{12} + U_{13} + U_{23} = k_e \frac{q_1 q_2}{r_{12,initial}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,initial}}$$

Potential Energy: Applications: Energy Conservation

For a closed system: Energy Conservation:

The sum of potential energy and kinetic energy is constant



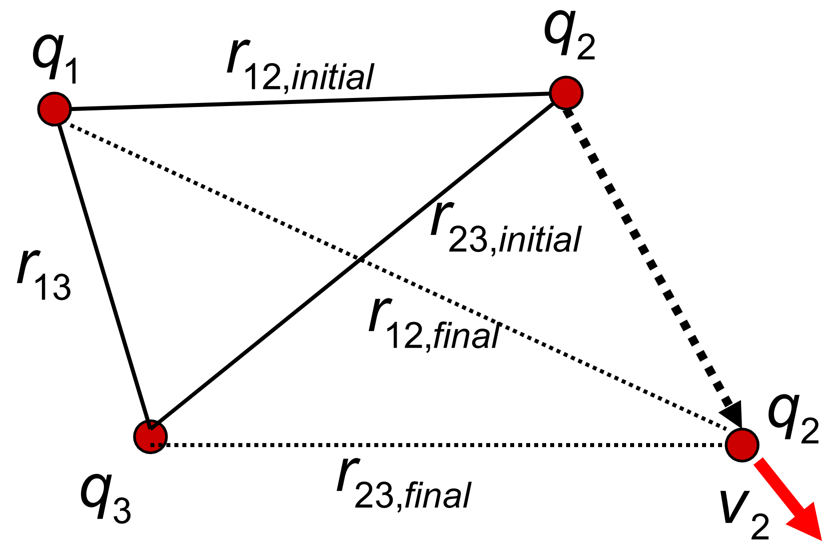
Final Energy is the sum of kinetic energy and potential energy (velocity of particle 2 is nonzero – kinetic energy)

$$E_{final} = T + U = T + U_{12} + U_{13} + U_{23} = \frac{m_2 v_2^2}{2} + k_e \frac{q_1 q_2}{r_{12,final}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,final}}$$

Potential Energy: Applications: Energy Conservation

For a closed system: Energy Conservation:

The sum of potential energy and kinetic energy is constant



Final Energy = Initial Energy

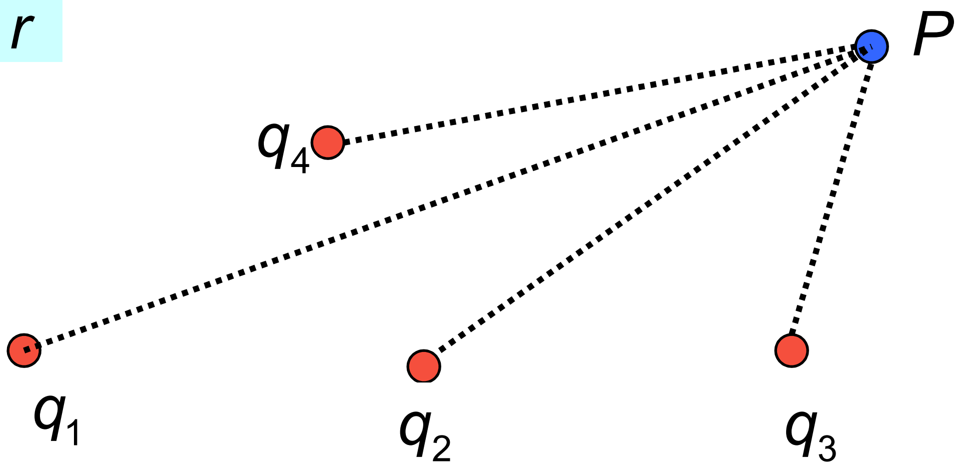
$$k_e \frac{q_1 q_2}{r_{12,initial}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,initial}} = \frac{m_2 v_2^2}{2} + k_e \frac{q_1 q_2}{r_{12,final}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23,final}}$$

$$\frac{m_2 v_2^2}{2} = k_e \frac{q_1 q_2}{r_{12,initial}} - k_e \frac{q_1 q_2}{r_{12,final}} + k_e \frac{q_2 q_3}{r_{23,initial}} - k_e \frac{q_2 q_3}{r_{23,final}}$$

$$v_2 = \sqrt{\frac{2}{m_2}\left[k_e q_1 q_2 \left(\frac{1}{r_{12,initial}} - \frac{1}{r_{12,final}}\right) + k_e q_2 q_3 \left(\frac{1}{r_{23,initial}} - \frac{1}{r_{23,final}}\right)\right]}$$

Electric Potential of Multiple Point Charge

$$V_r = k_e \frac{Q}{r}$$



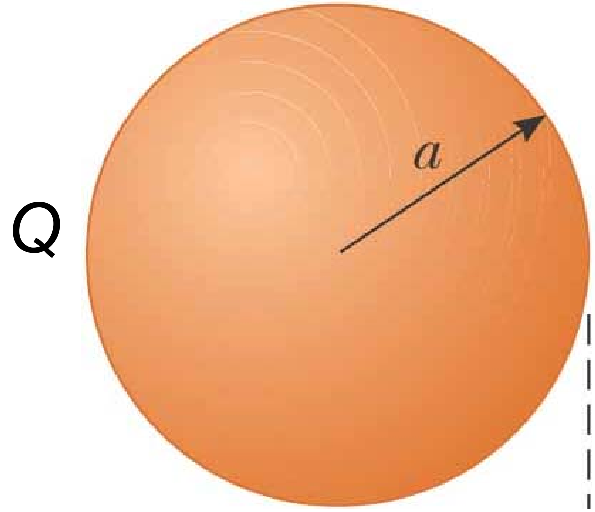
$$V_r = V_1 + V_2 + V_3 + V_4 = k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2} + k_e \frac{q_3}{r_3} + k_e \frac{q_4}{r_4}$$

The potential is a **scalar sum.**

The electric field is a **vector sum.**

Spherically Symmetric Charge Distribution

Uniformly distributed charge *Q*



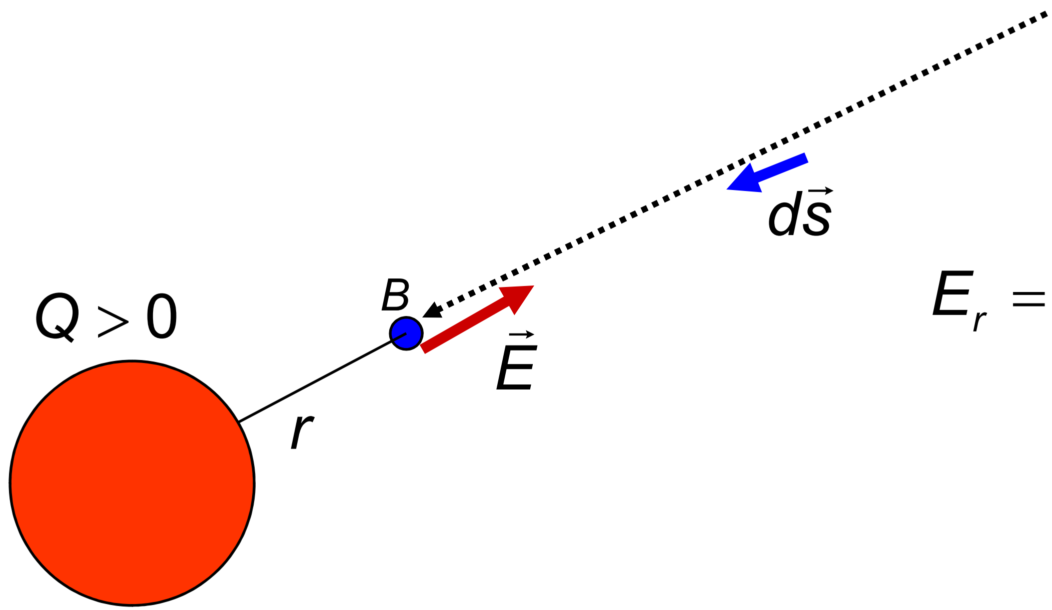
$$V = ?$$

Spherically Symmetric Charge Distribution

$$V_\infty = 0$$

$$V_\infty = 0$$

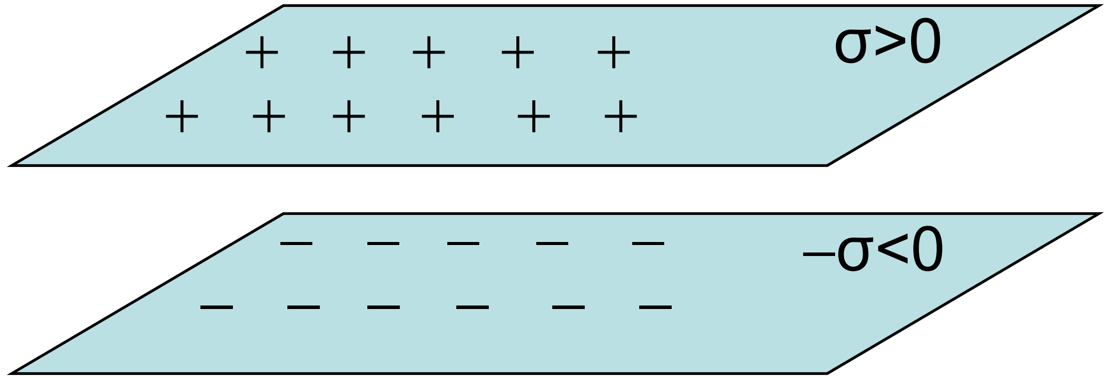
$$r > a$$

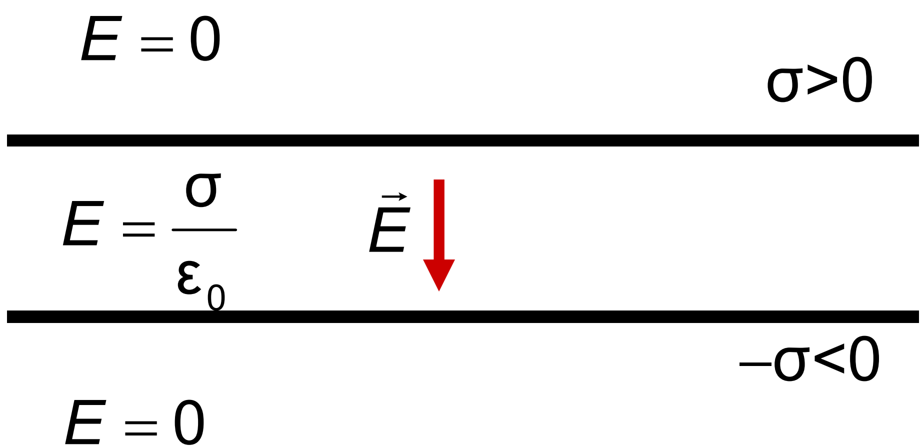


$$E_r = k_e \frac{Q}{r^2}$$

$$\boxed{V_B = k_e \frac{Q}{r}}$$

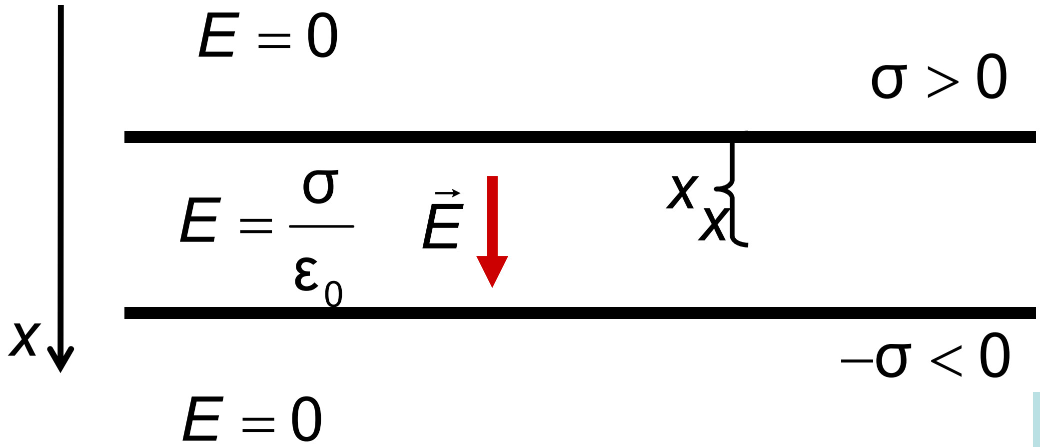
Important Example: Capacitor





$$V = ?$$

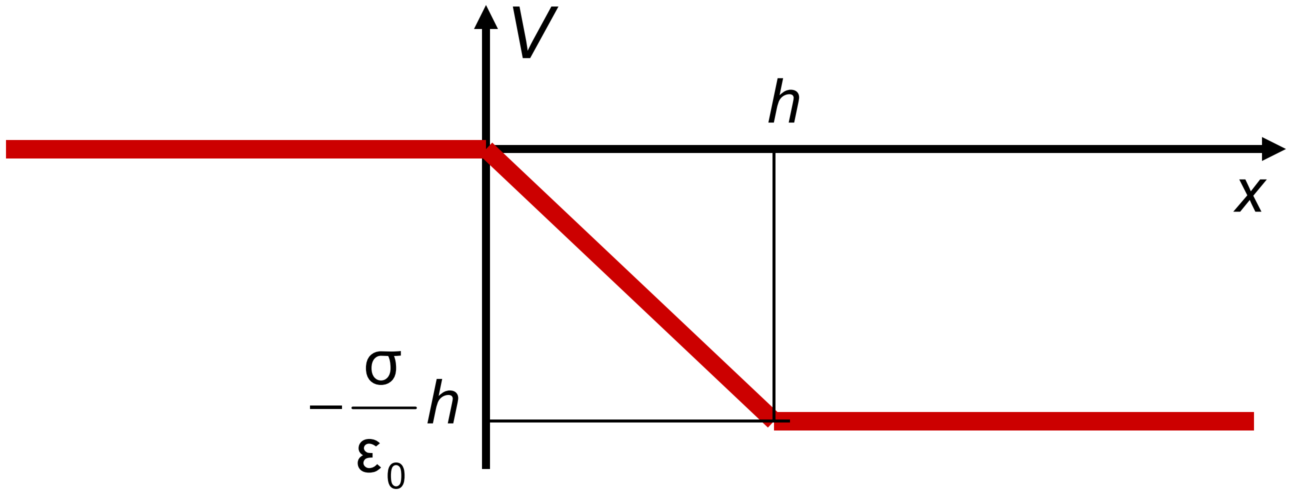
Important Example



$$V = 0$$

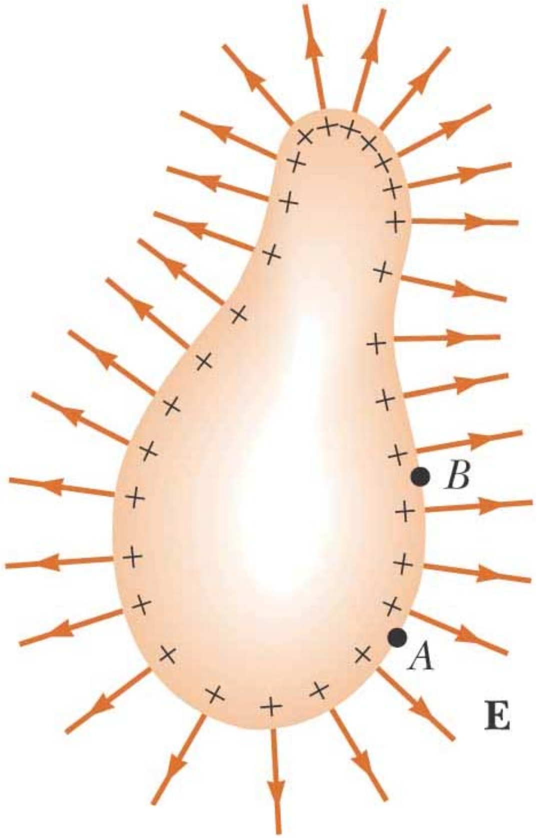
$$V = -\frac{\sigma}{\varepsilon_0} x$$

$$V = -Eh = -\frac{\sigma}{\varepsilon_0} h$$



Electric Potential: Charged Conductor

- The potential difference between ***A*** and ***B*** is **zero!!!!**
- Therefore ***V*** is constant everywhere on the surface of a charged conductor in equilibrium
  - **$\Delta V = 0$** between any two points on the surface
- The surface of any charged conductor is an equipotential surface
- Because the electric field is zero inside the conductor, the electric potential is **constant everywhere inside the conductor** and equal to the value at the surface

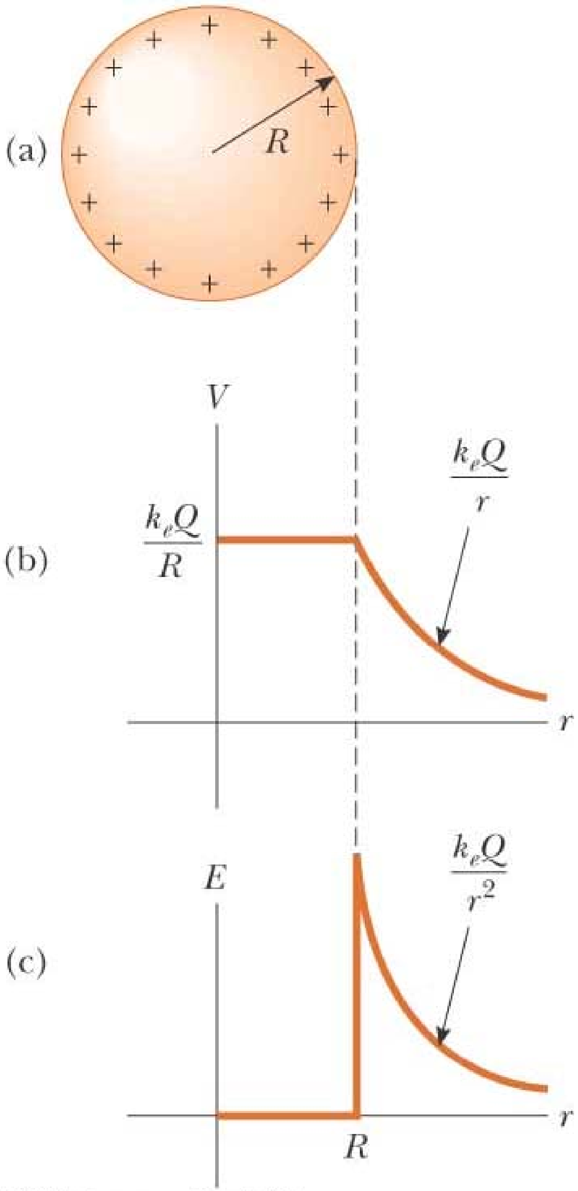


Electric Potential: Conducting Sphere: Example

$$V_r = k_e \frac{Q}{r}$$ for $\boldsymbol{r > R}$

$$V_{sphere} = k_e \frac{Q}{R}$$ for $\boldsymbol{r < R}$

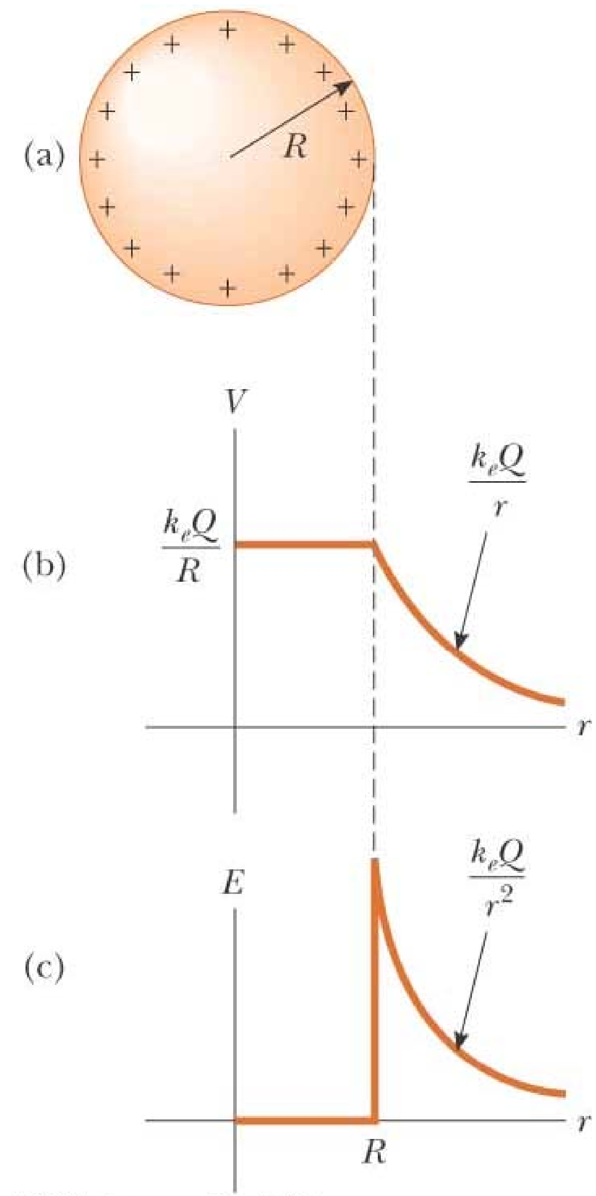
The potential of conducting sphere!!



Conducting Sphere: Example

What is the potential of conducting sphere with radius 0.1 m and charge $1\mu C$?

$$V_{sphere} = k_e \frac{Q}{R} = 9 \cdot 10^9 \frac{10^{-6} C}{0.1 m} = 90000 V$$

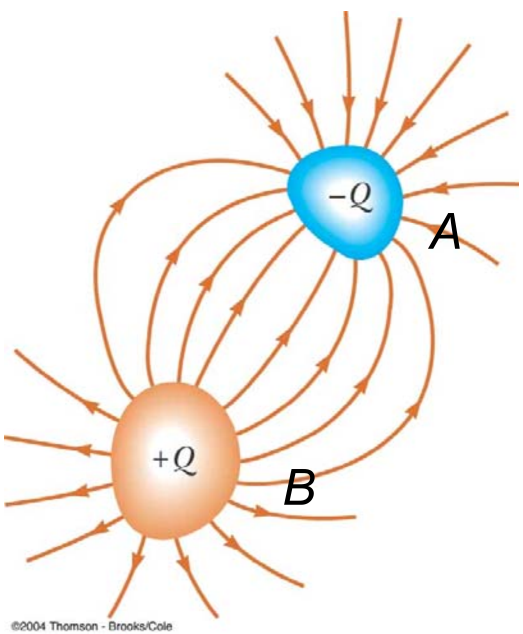


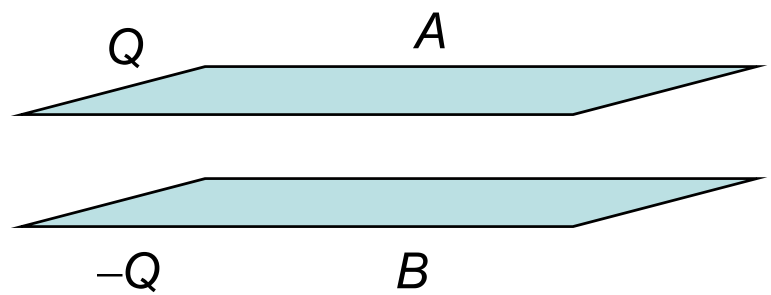
Capacitance

Capacitors

- **Capacitors are devices that store electric charge**

- A capacitor consists of two conductors
  - These conductors are called ***plates***
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions
- A potential difference exists between the plates due to the charge





Q - the charge of capacitor

$\Delta V = V_A - V_B$ - a potential difference of capacitor

Capacitors

- A capacitor consists of two conductors

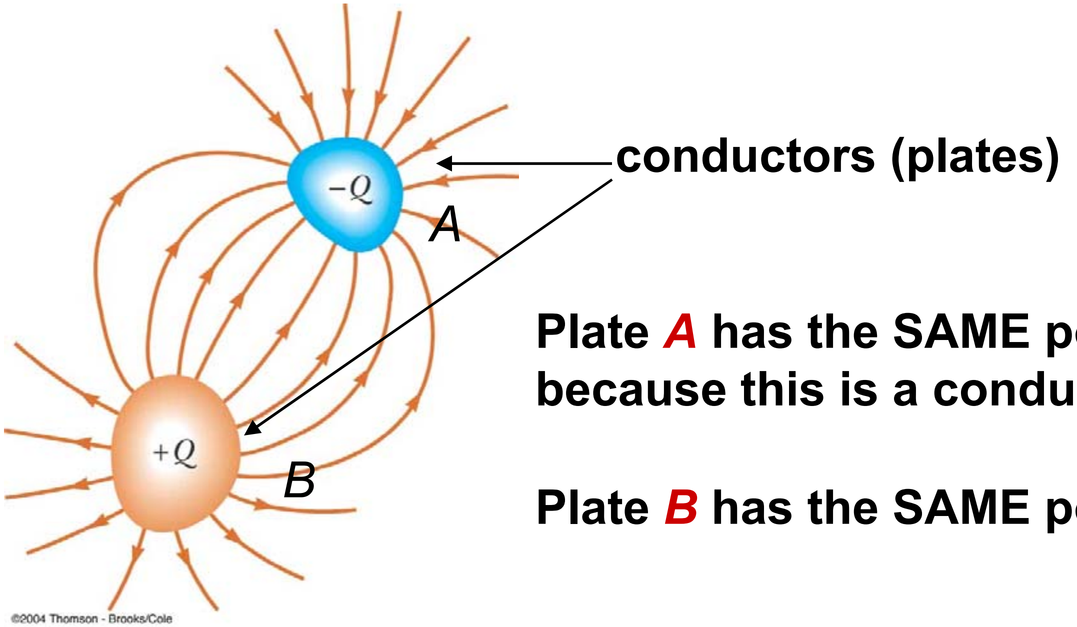


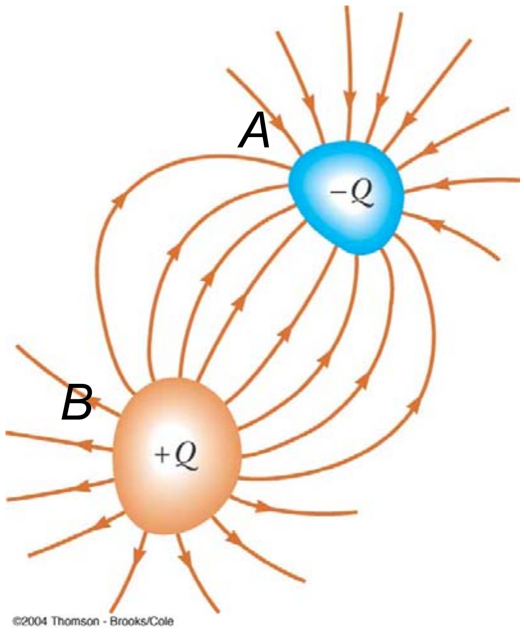
Plate *A* has the SAME potential at all points because this is a conductor .

Plate *B* has the SAME potential at all points.

So we can define the potential difference between the plates:

$$\Delta V = V_A - V_B$$

Capacitance of Capacitor



$$C = \frac{Q}{\Delta V}$$

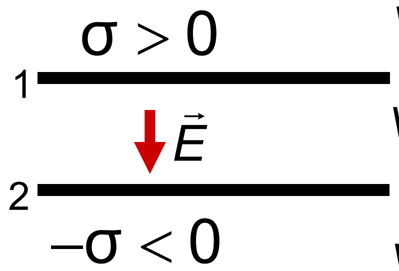
- The SI unit of capacitance is the ***farad*** (F) = C/V.
- Capacitance is always a positive quantity
- The capacitance of a given capacitor is constant and determined only by geometry of capacitor
- The farad is a large unit, typically you will see microfarads (μF) and picofarads (***pF***)

Capacitor: Parallel Plates

$E = 0$

$E = \dfrac{\sigma}{\varepsilon_0}$

$E = 0$

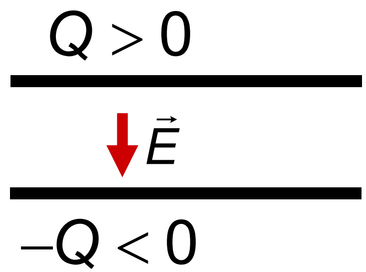


$V = 0$

$V = -\dfrac{\sigma}{\varepsilon_0} x$

$V = -\dfrac{\sigma}{\varepsilon_0} h$

$y = \dfrac{Q}{S}$



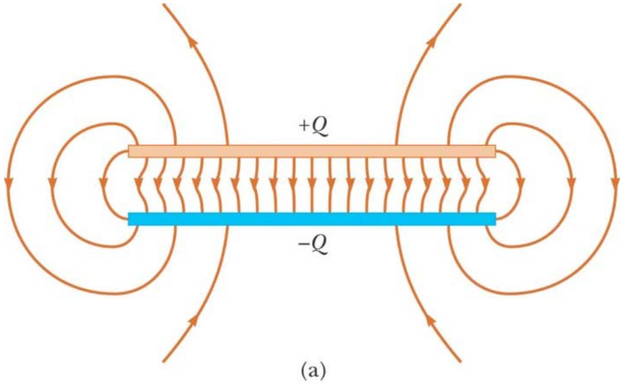
The potential difference:

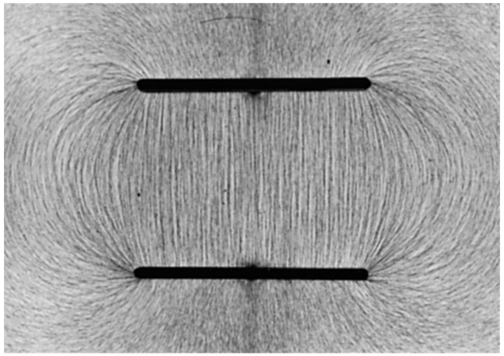
$$\Delta V = \frac{\sigma}{\varepsilon_0} h = \frac{Q}{\varepsilon_0 S} h$$

The capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 S}{h}$$

Capacitor: Parallel Plates: Assumptions

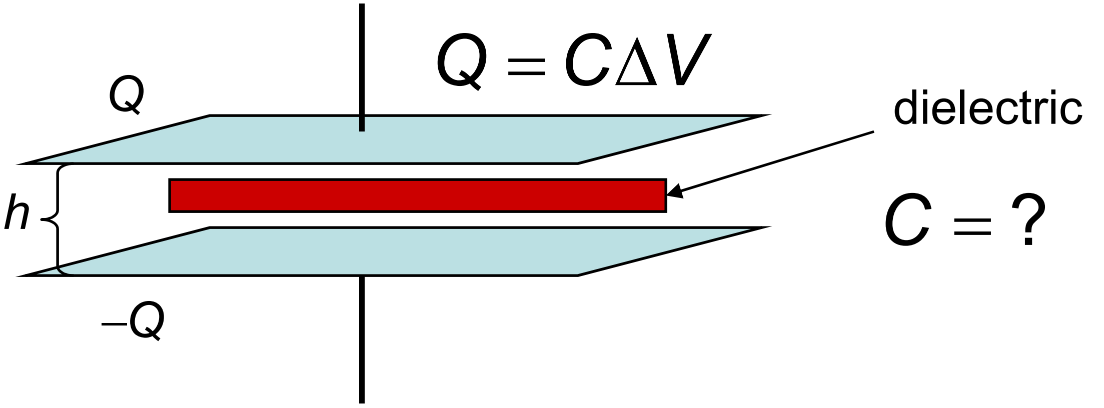




- Main assumption - the electric field is uniform
- This is valid in the central region, but not at the ends of the plates
- If the separation between the plates is small compared to the length of the plates, the effect of the non-uniform field can be ignored

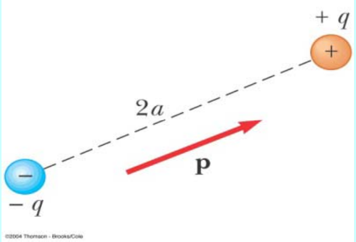
Capacitors with Dielectric (Insulator)

Dielectric (insulator) inside capacitor. Capacitance = ?

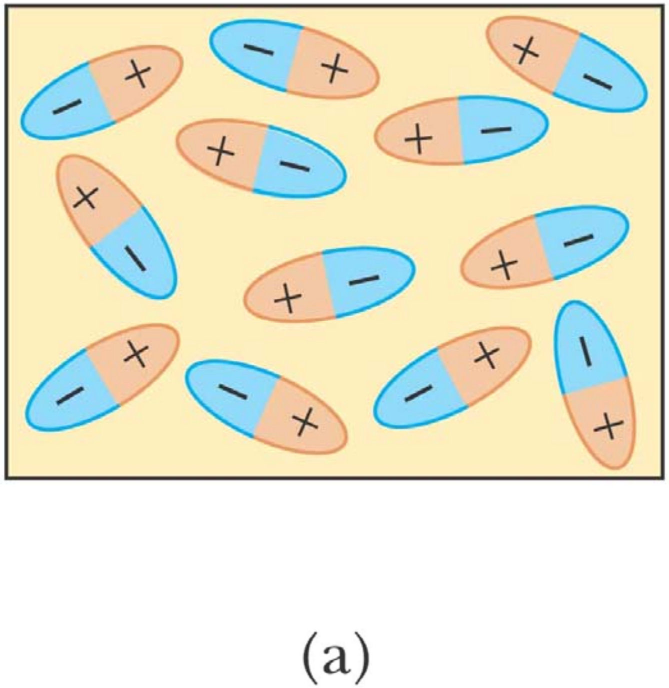


Dielectric (insulator)

- The molecules that make up the dielectric are modeled as **dipoles**
- An electric **dipole** consists of two charges of equal magnitude and opposite signs

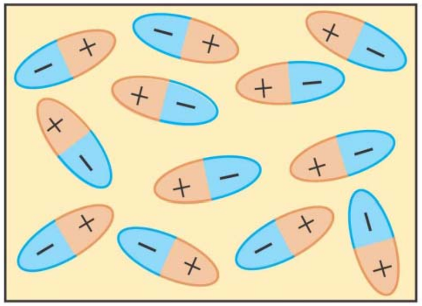


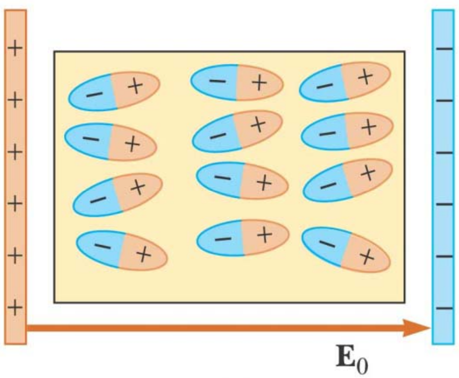
- The molecules are randomly oriented in the absence of an electric field



Dielectric in Electric Field

- Dielectric in External Electric Field
- The molecules partially align with the electric field
- The degree of alignment of the molecules with the field depends on temperature and the magnitude of the field



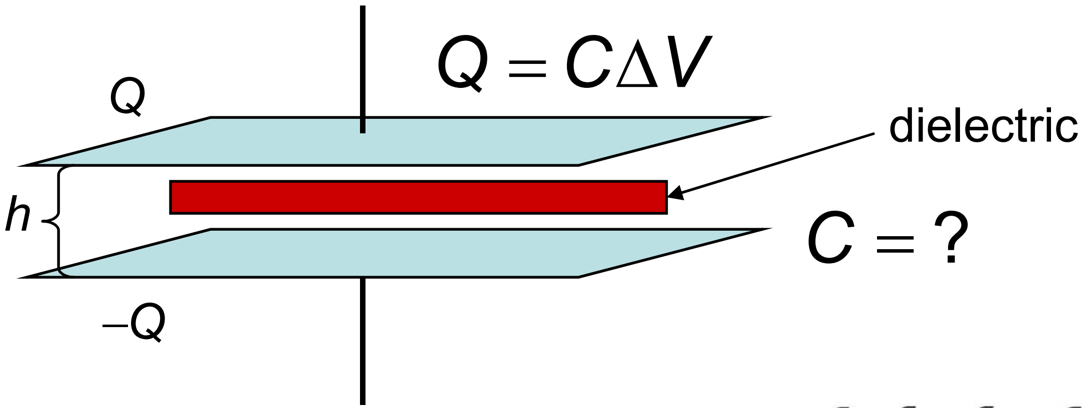


(b)

Polarization

Capacitors with Dielectric

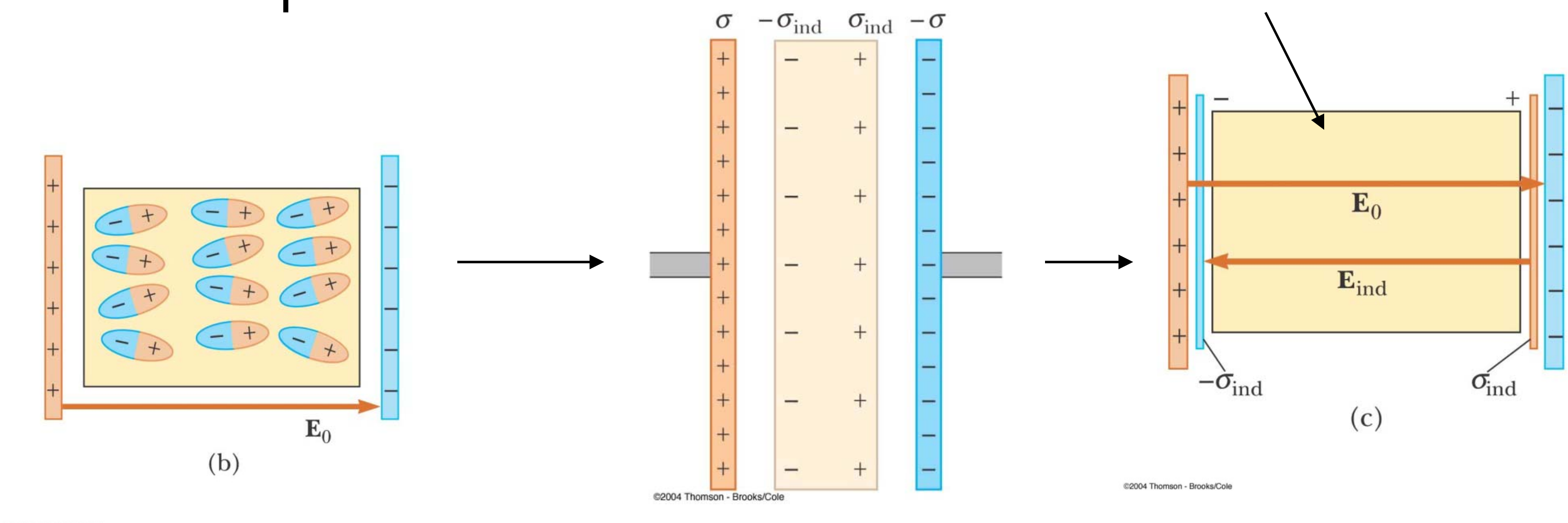
Dielectric inside capacitor. Capacitance = ?



$$Q = C\Delta V$$

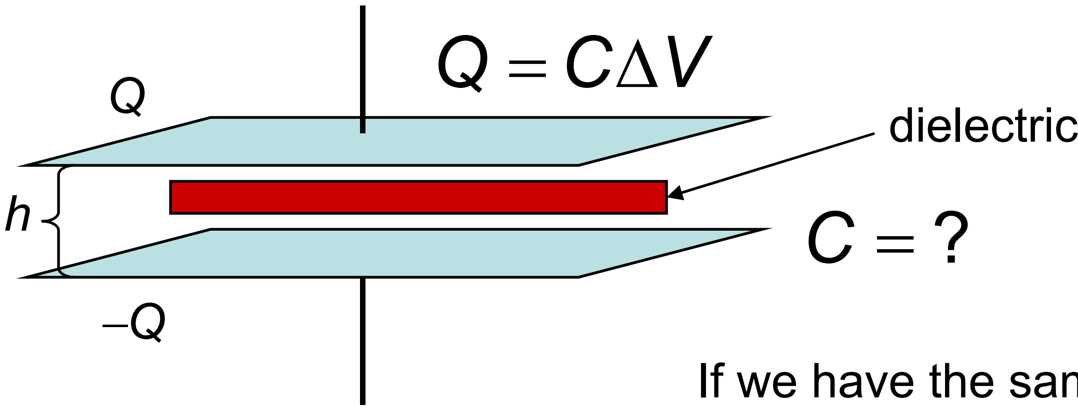
$$C = ?$$

The electric field inside dielectric $E_0 - E_{ind}$



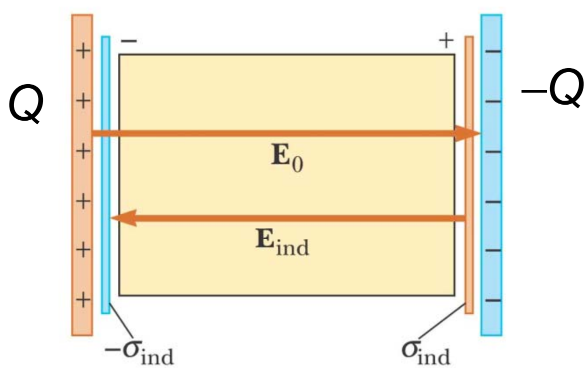
Capacitors with Dielectric

Dielectric inside capacitor. Capacitance = ?



$$Q = C\Delta V$$

$$C = ?$$



(c)

If we have the same charge as without dielectric then the potential difference is increased, since

$$\Delta V = h(E_0 - E_{ind})$$

without dielectric it was

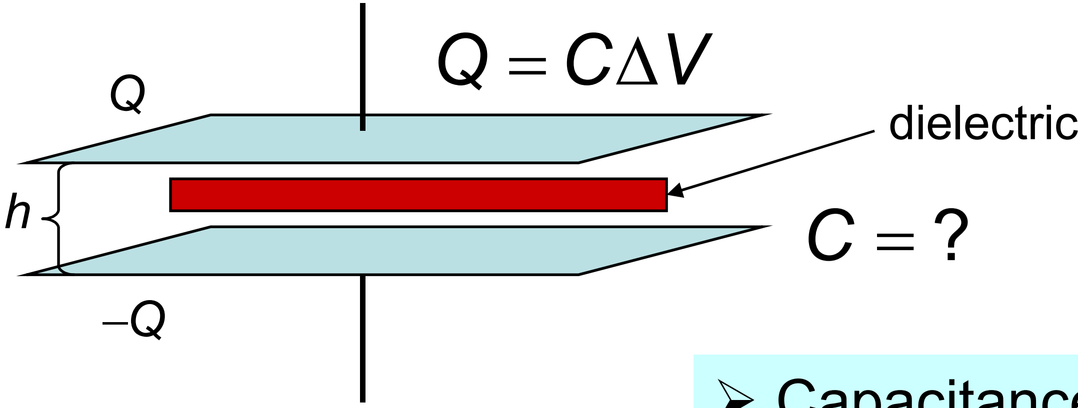
$$\Delta V_0 = hE_0$$

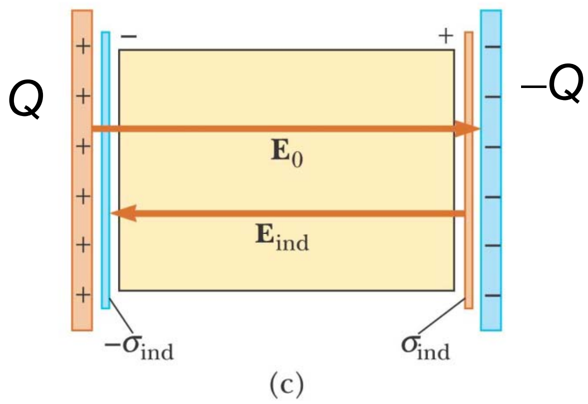
$\Delta V < \Delta V_0$ then $C = \dfrac{Q}{\Delta V} > C_0 = \dfrac{Q}{\Delta V_0}$

Capacitance is increased.

Capacitors with Dielectric

Dielectric inside capacitor. Capacitance = ?





- Capacitance is increased
- To characterize this increase the coefficient (dielectric constant of material) is introduced, so

$$C = \kappa C_0$$

(this is true only if dielectric completely fills the region between the plates)

Table 26.1

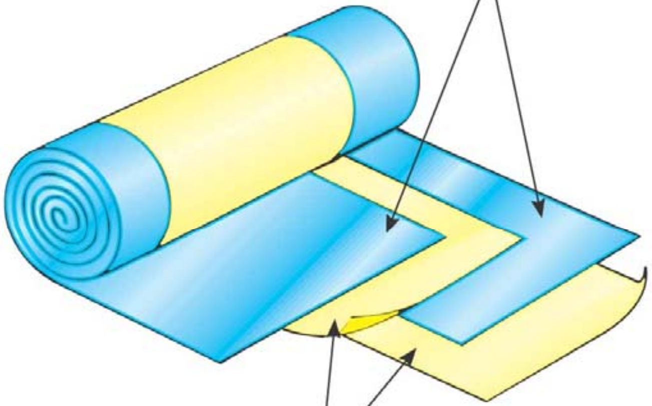
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

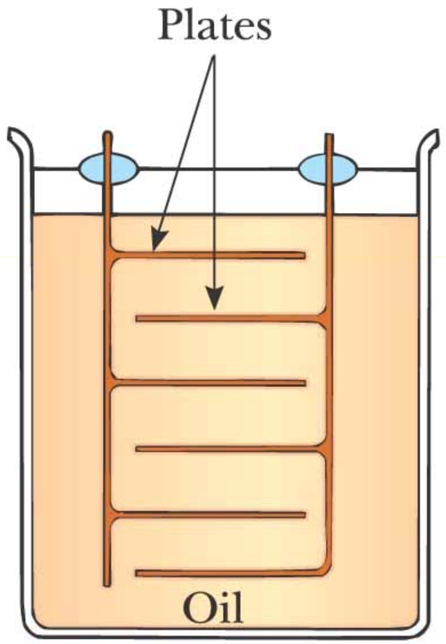
Type of Capacitors: Tubular

- Metallic foil may be interlaced with thin sheets of paper
- The layers are rolled into a cylinder to form a small package for the capacitor



Type of Capacitors: Oil Filled

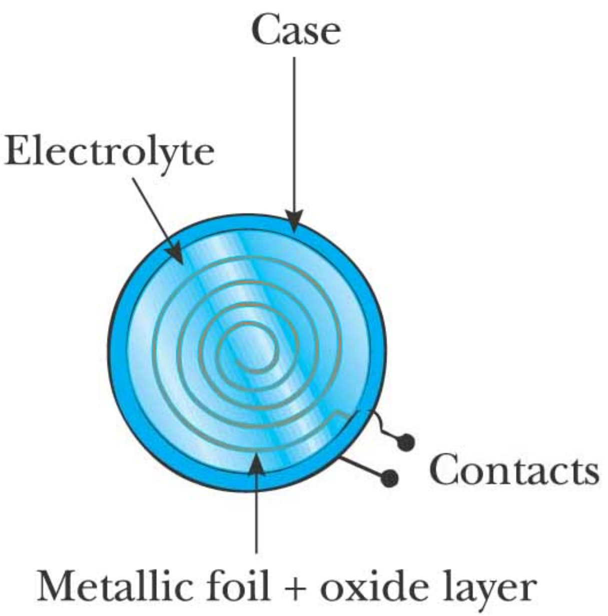
- Common for high- voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil



(b)

Type of Capacitors: Electrolytes

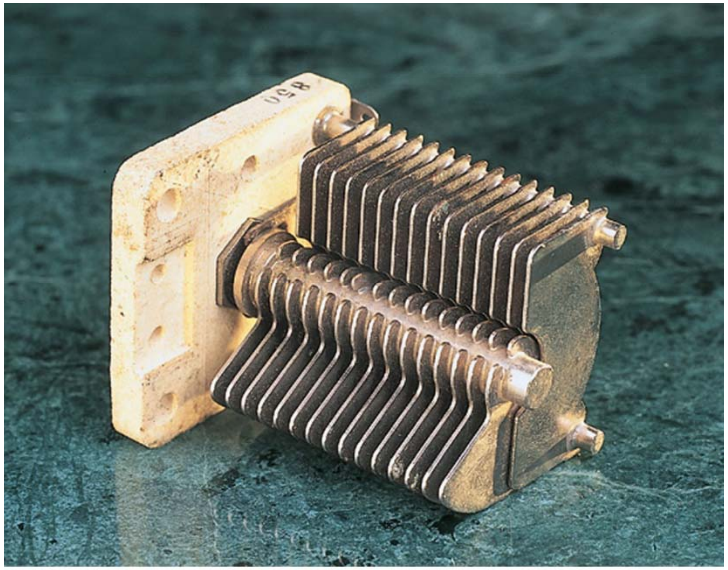
- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution



(c)

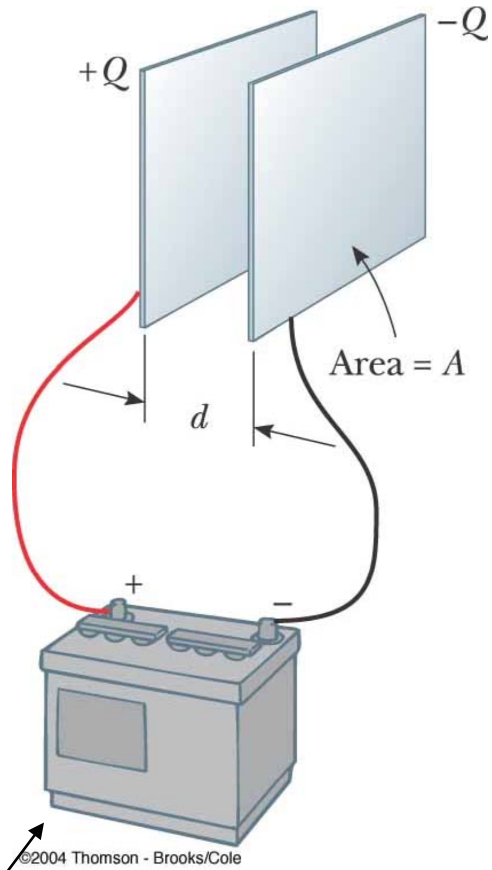
Type of Capacitors: Variable

- Variable capacitors consist of two interwoven sets of metallic plates
- One plate is fixed and the other is movable
- These capacitors generally vary between 10 and 500 pF
- Used in radio tuning circuits



Capacitor: Charging

- Each plate is connected to a terminal of the battery
- The battery establishes an electric field in the connecting wires
- This field applies a force on electrons in the wire just outside of the plates
- The force causes the electrons to move onto the negative plate
- This continues until equilibrium is achieved
  - The plate, the wire and the terminal are all at the same potential
- At this point, there is no field present in the wire and there is no motion of electrons

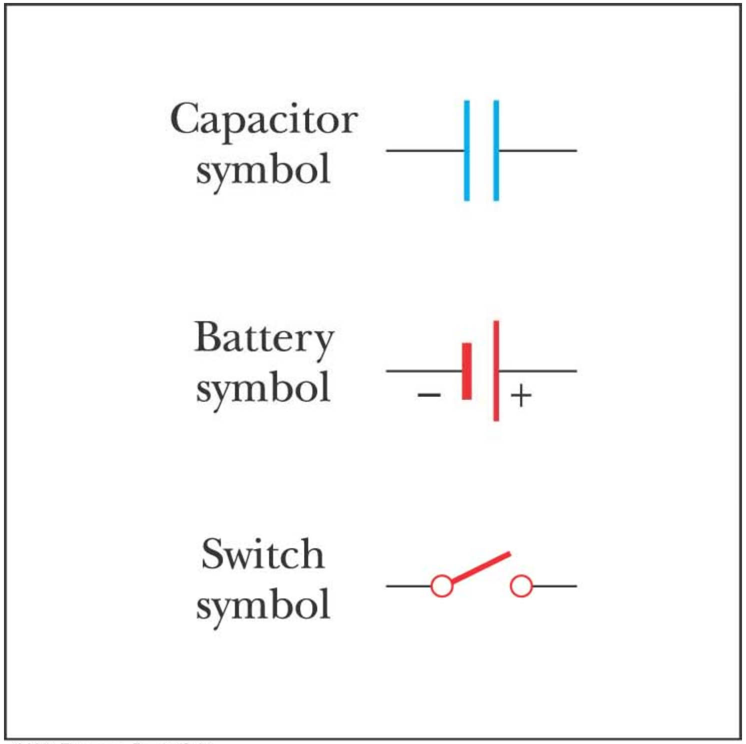


Battery- produces the fixed voltage – the fixed potential difference

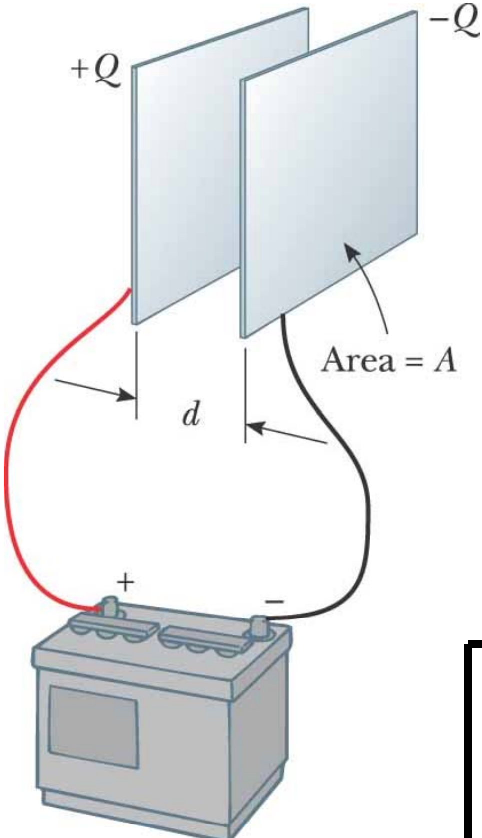
Capacitance and Electrical Circuit

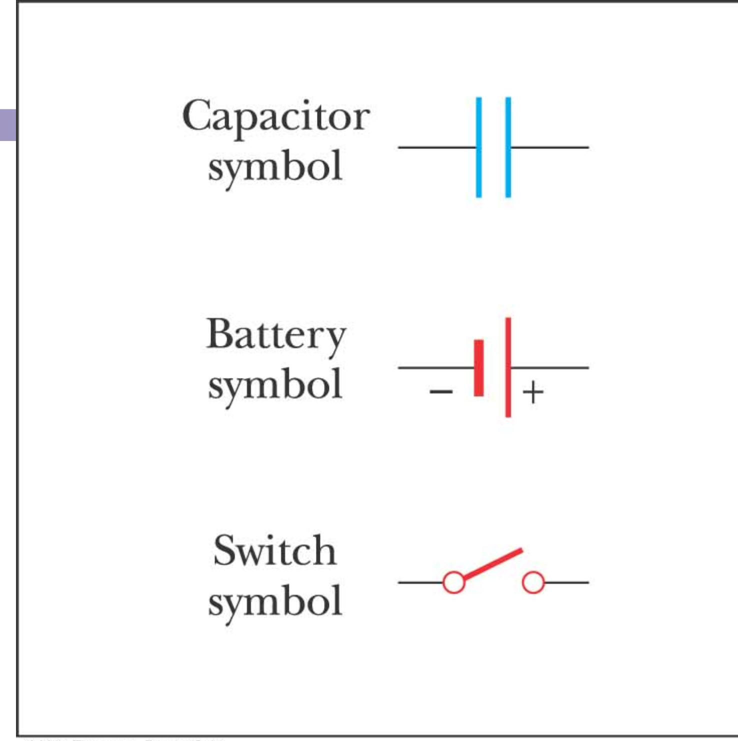
Electrical Circuit

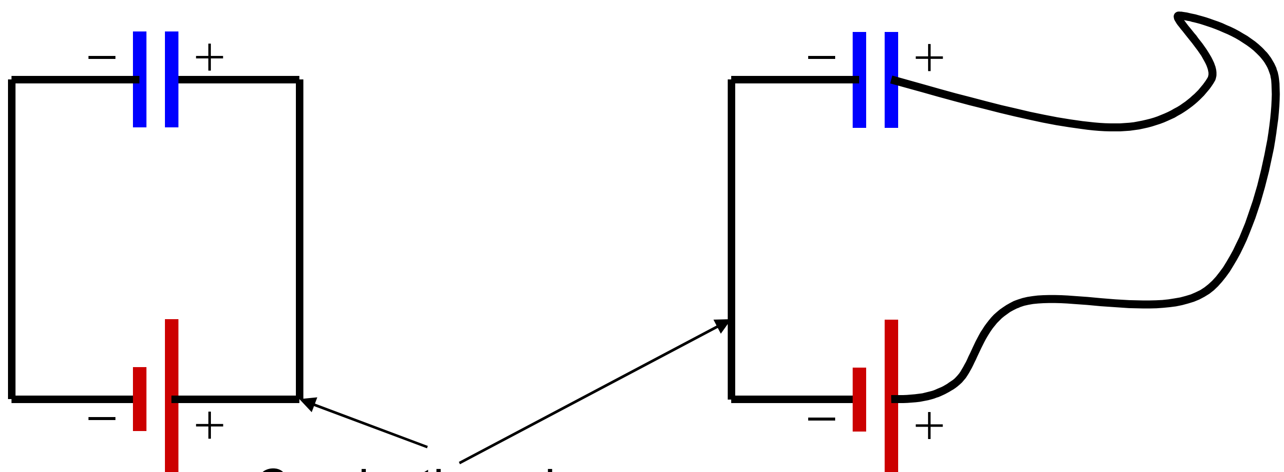
- A circuit diagram is a simplified representation of an actual circuit
- Circuit symbols are used to represent the various elements
- Lines are used to represent wires
- The battery's positive terminal is indicated by the longer line



Electrical Circuit

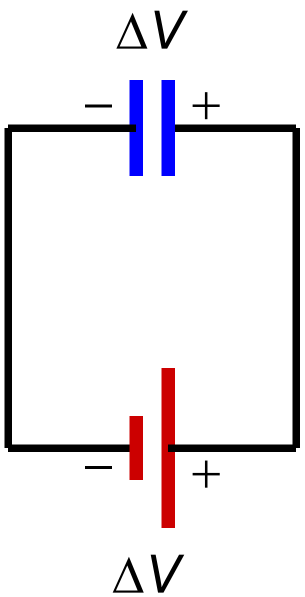






Conducting wires.
In equilibrium all the points of the wires have the same potential

Electrical Circuit

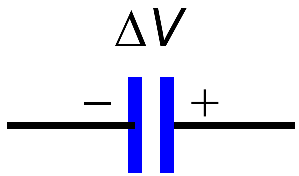


The battery is characterized by the voltage – the potential difference between the contacts of the battery

In equilibrium this potential difference is equal to the potential difference between the plates of the capacitor.

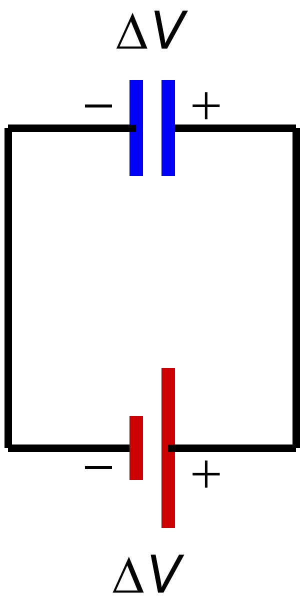
Then the charge of the capacitor is

$$Q = C\Delta V$$

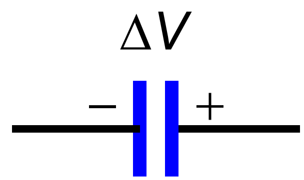


If we disconnect the capacitor from the battery the capacitor will still have the charge ***Q*** and potential difference ΔV

Electrical Circuit

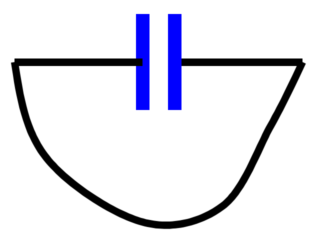


$$Q = C\Delta V$$



If we connect the wires the charge will disappear and there will be no potential difference

$$\Delta V = 0$$

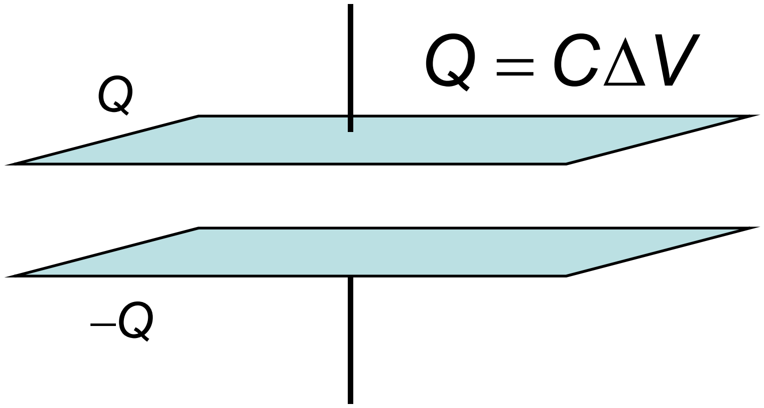


Energy Stored in a Capacitor: Application

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

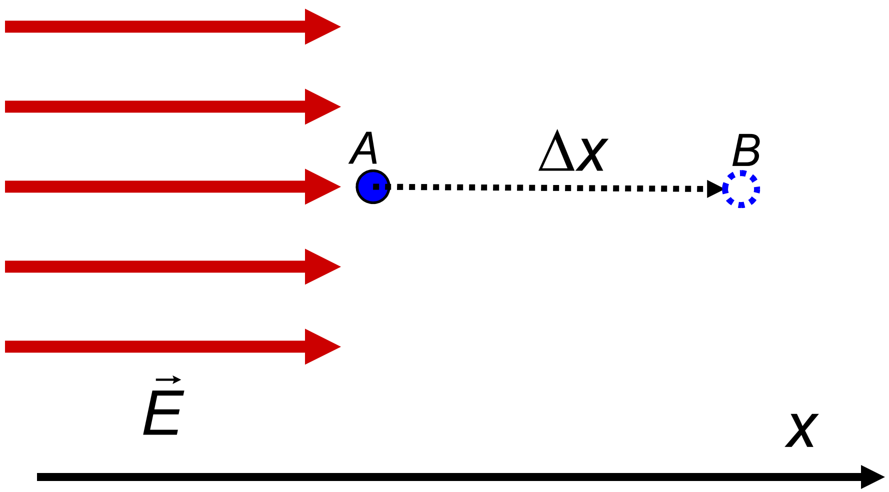
One of the main application of capacitor:

- capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse



Electric Potential and Electric Field

➢ **Can we find electric field if we know electric potential?**



$$V_B - V_A = -\Delta x\, E$$

$$E = -\frac{V_B - V_A}{\Delta x} = -\frac{\Delta V}{\Delta x}$$

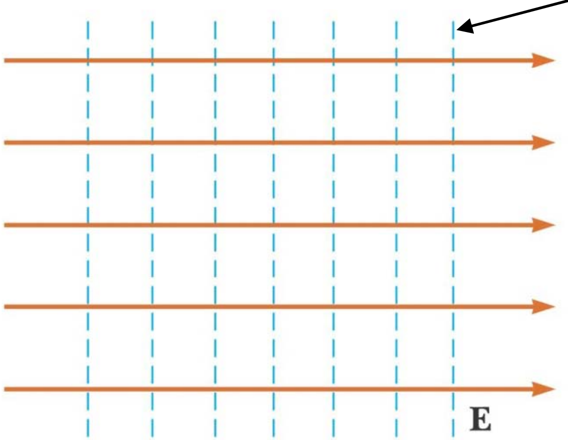
$$\boxed{E = -\frac{\Delta V}{\Delta x}}$$

Electric Potential and Electric Field

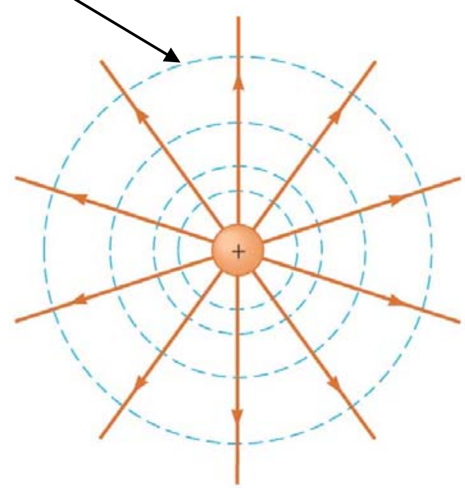
$$E = -\frac{\Delta V}{\Delta s}$$

- Equipotential lines are everywhere perpendicular to the electric field.

Equipotential lines



(a)



(b)