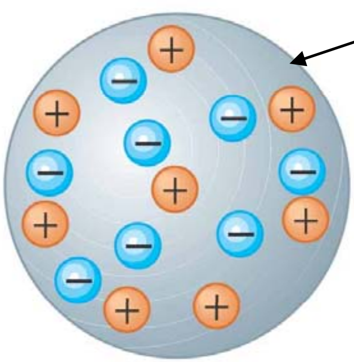
Electric Forces and Fields: Coulomb's Law

Reading: Chapter 20

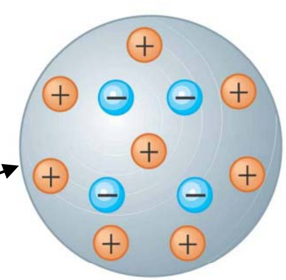
Electric Charges

- There are **two kinds** of electric charges
  - Called **positive** and **negative**
    - Negative charges are the type possessed by electrons
    - Positive charges are the type possessed by protons
- Charges of the same sign repel one another and charges with opposite signs attract one another
- Electric charge is always conserved in isolated system



(a)

Neutral – equal number of positive and negative charges



(e)

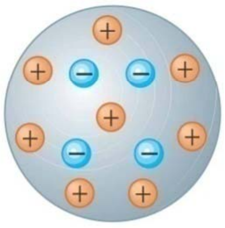
Positively charged

Electric Charges: Conductors and Isolators

- Electrical conductors are materials in which some of the electrons are free electrons
  - These electrons can move relatively freely through the material
  - Examples of good conductors include copper, aluminum and silver

- Electrical insulators are materials in which all of the electrons are bound to atoms
  - These electrons can not move relatively freely through the material
  - Examples of good insulators include glass, rubber and wood

- Semiconductors are somewhere between insulators and conductors



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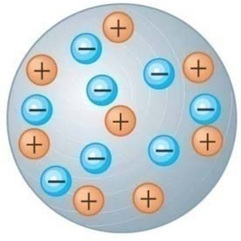
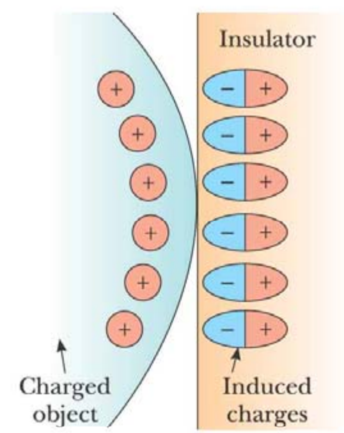
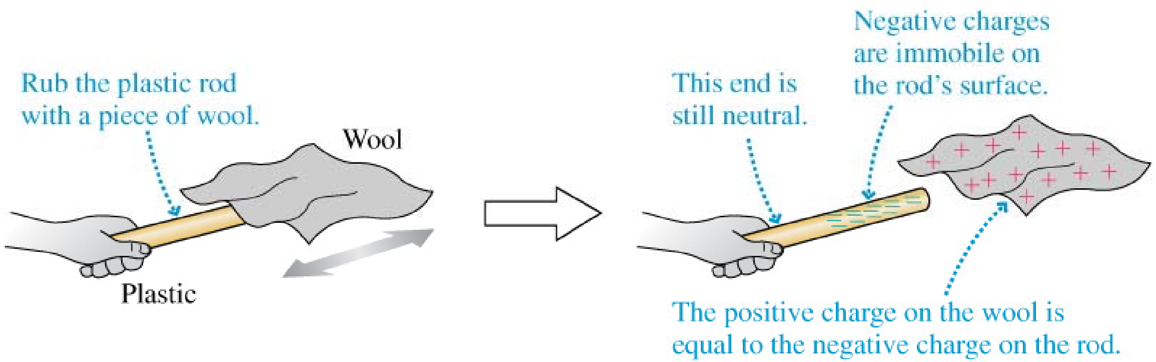


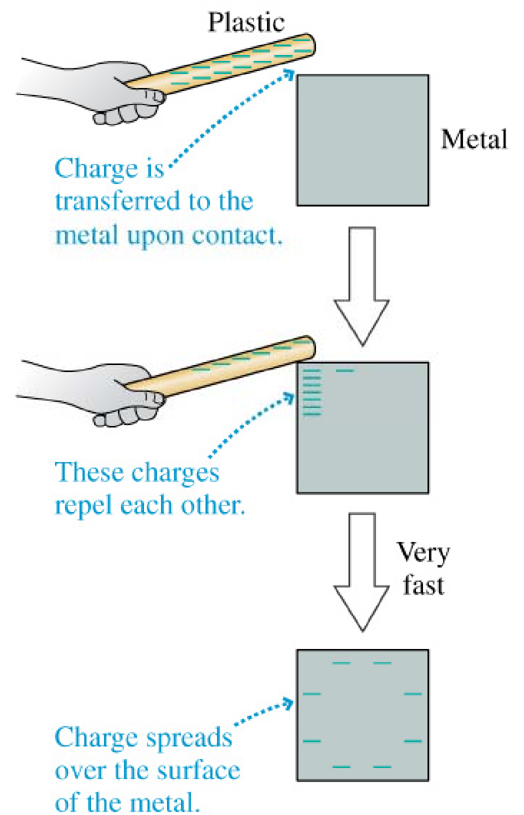
(a)



(a)

Electric Charges





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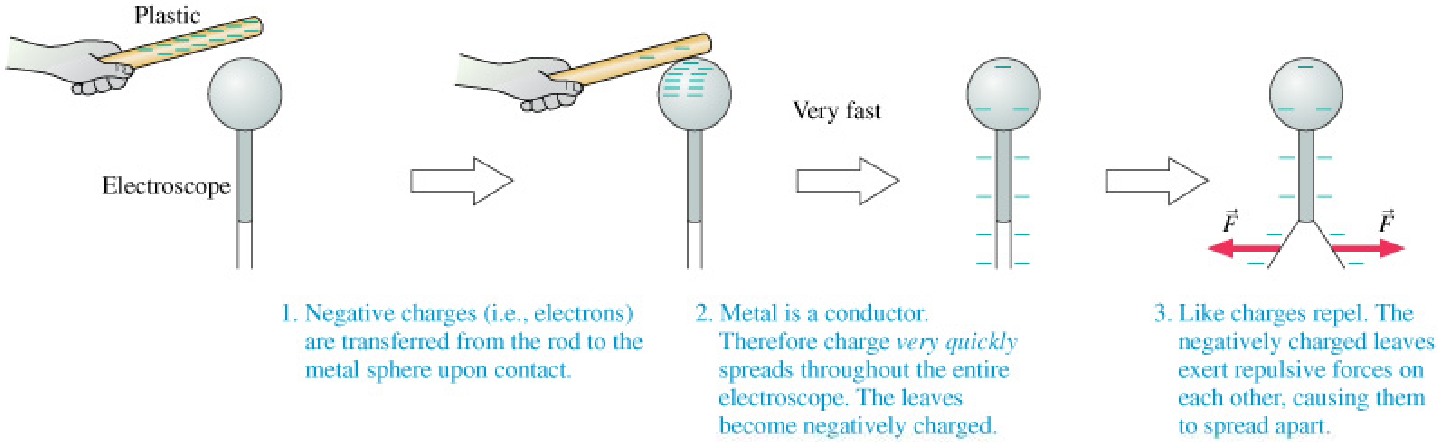
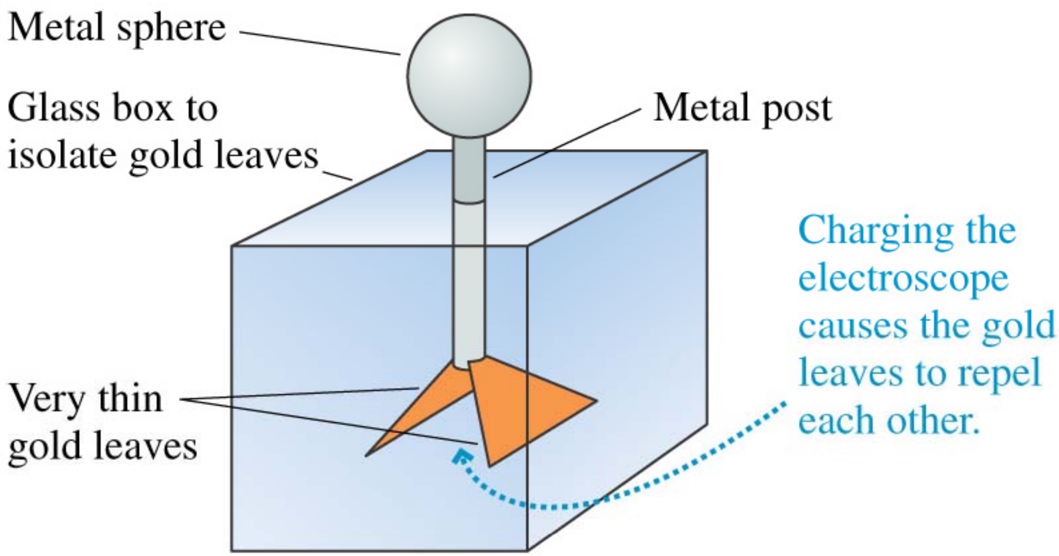
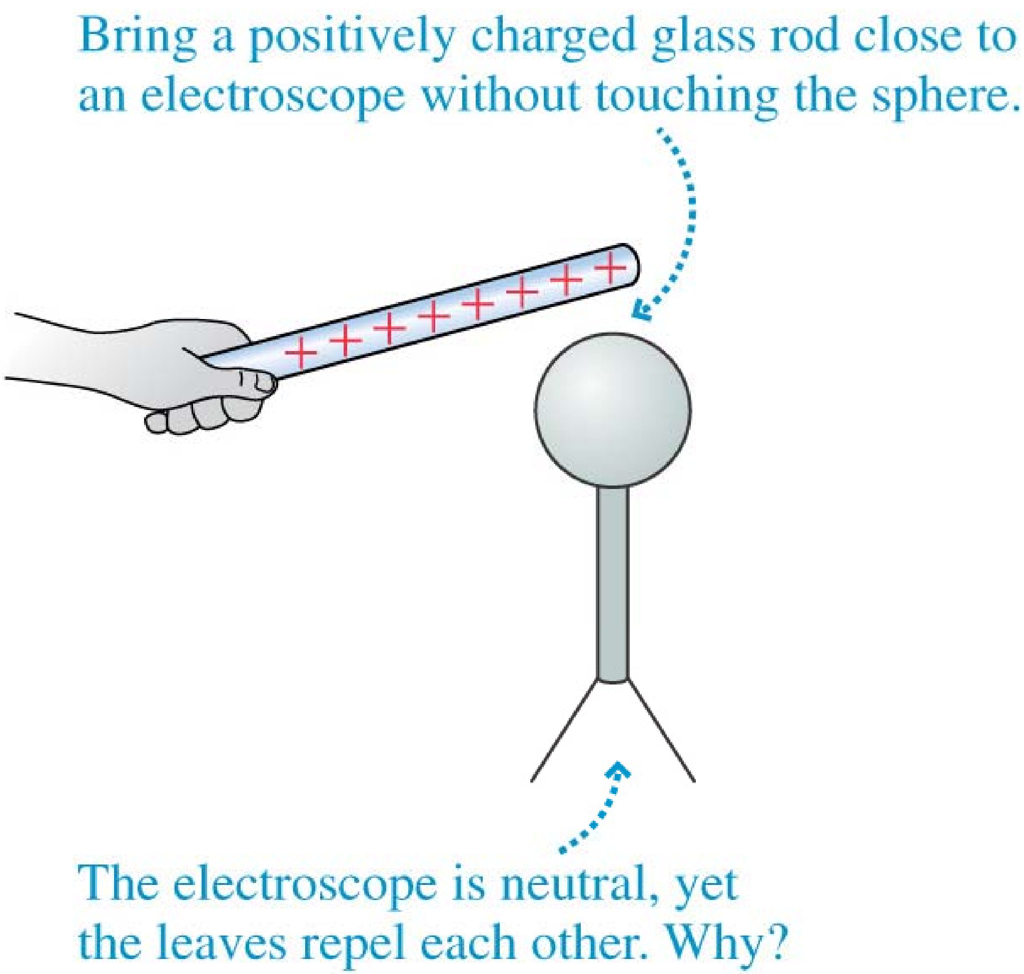
Electric Charges

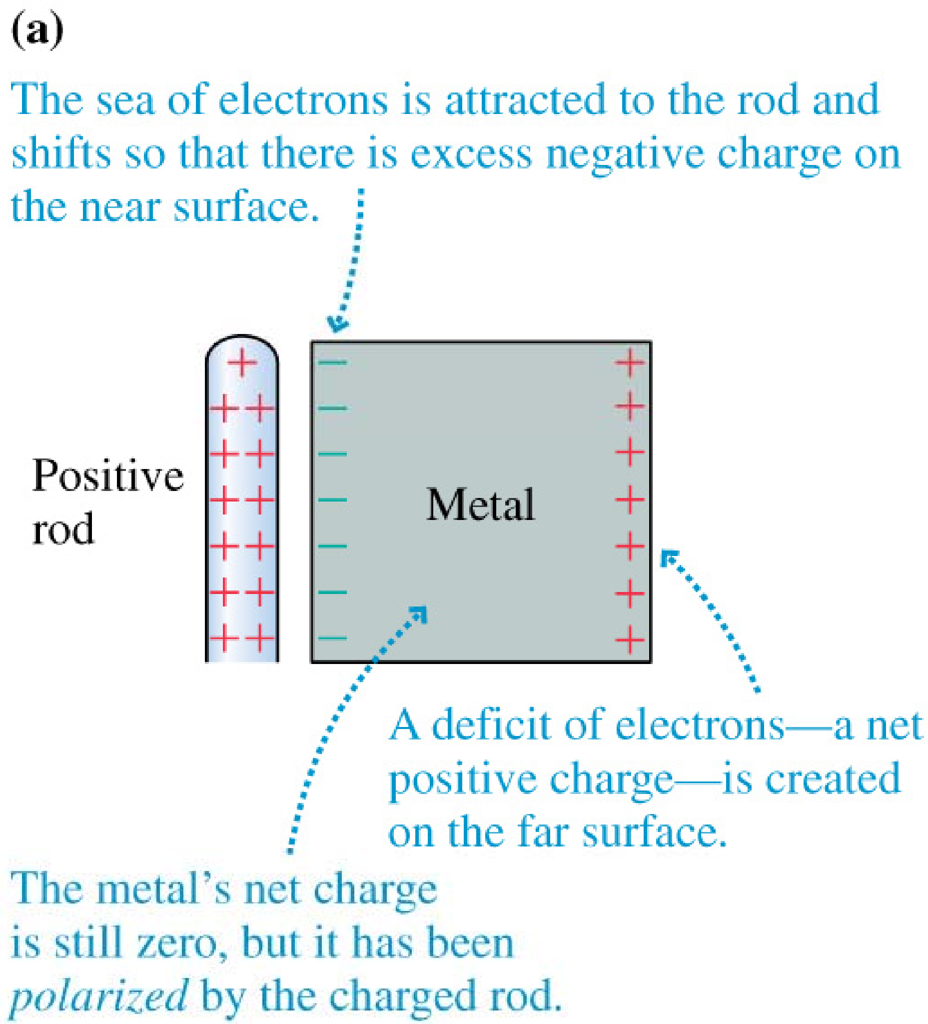




Electric Charges



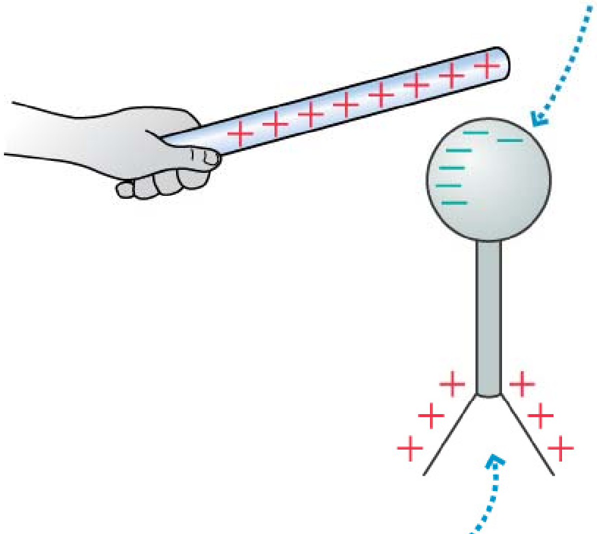
Electric Charges



Electric Charges

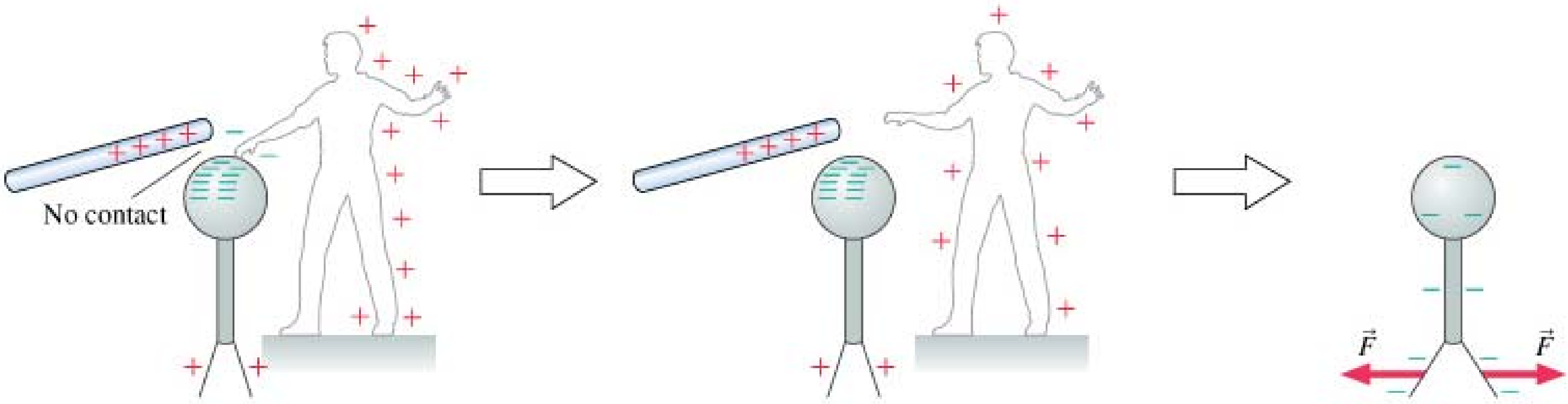
(b)

The electroscope is polarized by the charged rod. The sea of electrons shifts toward the rod.



Although the net charge on the electroscope is still zero, the leaves have excess positive charge and repel each other.

Electric Charges

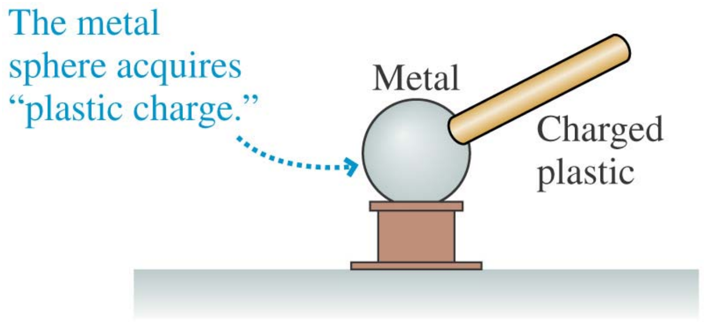


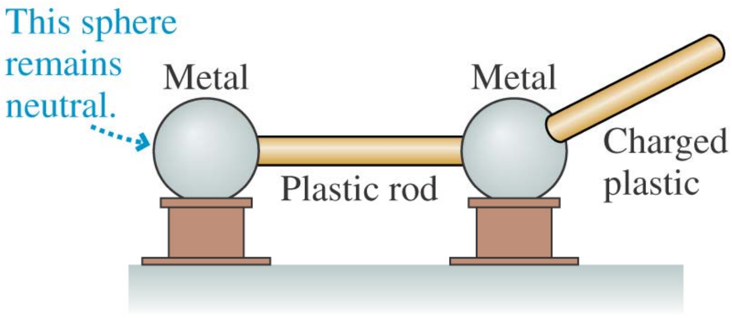
1. The charged rod polarizes the electroscope+person conductor. The leaves repel slightly, due to polarization within the electroscope, but overall the electroscope has an excess of electrons and the person has a deficit of electrons.

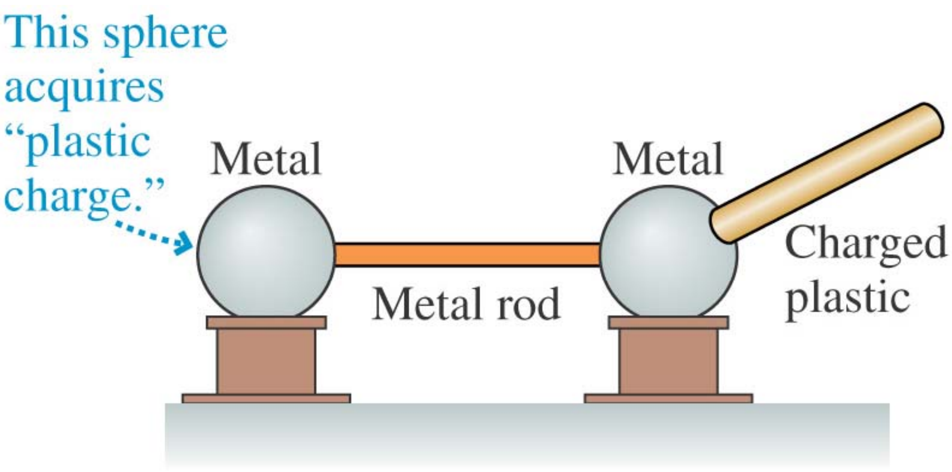
2. The negative charge on the electroscope is isolated when contact is broken.

3. When the rod is removed, the leaves first collapse, as the polarization vanishes, then repel as the excess negative charge spreads out. The electroscope has been *negatively* charged.

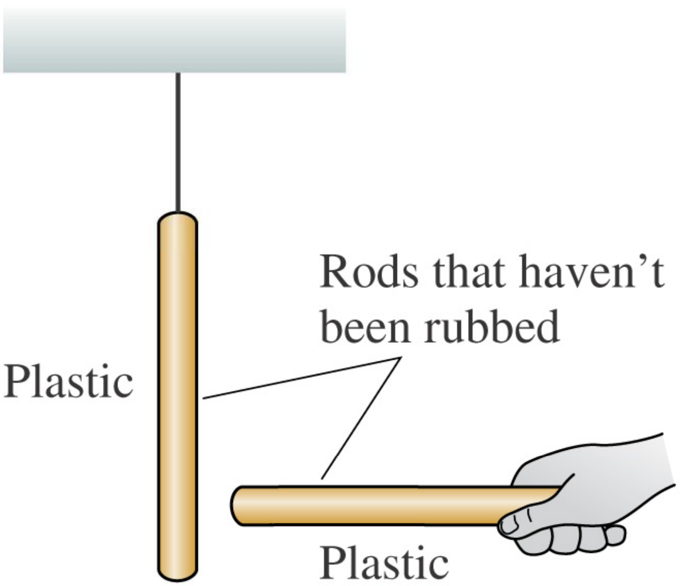
Electric Charges

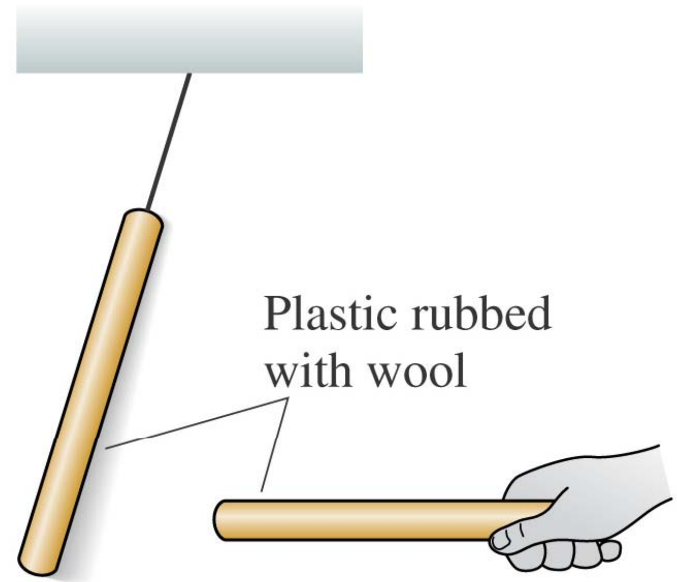


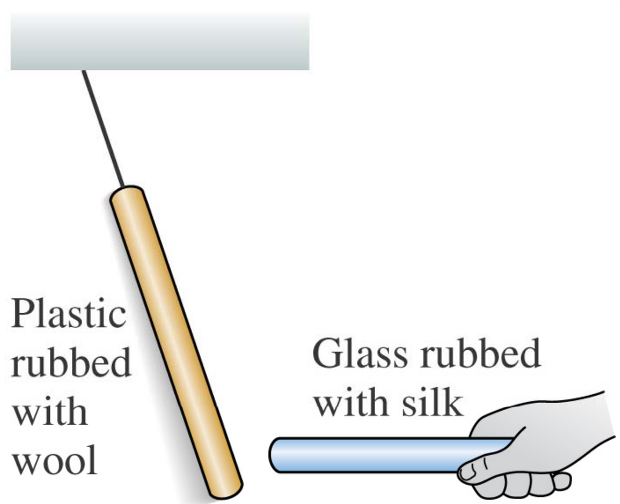




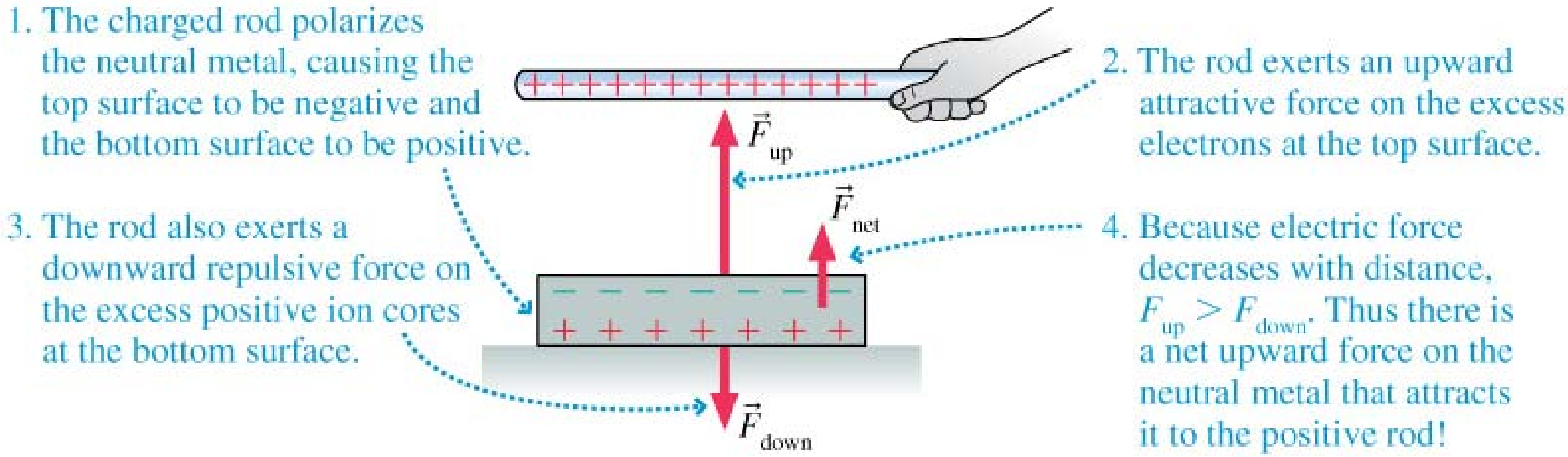
Electric Charges



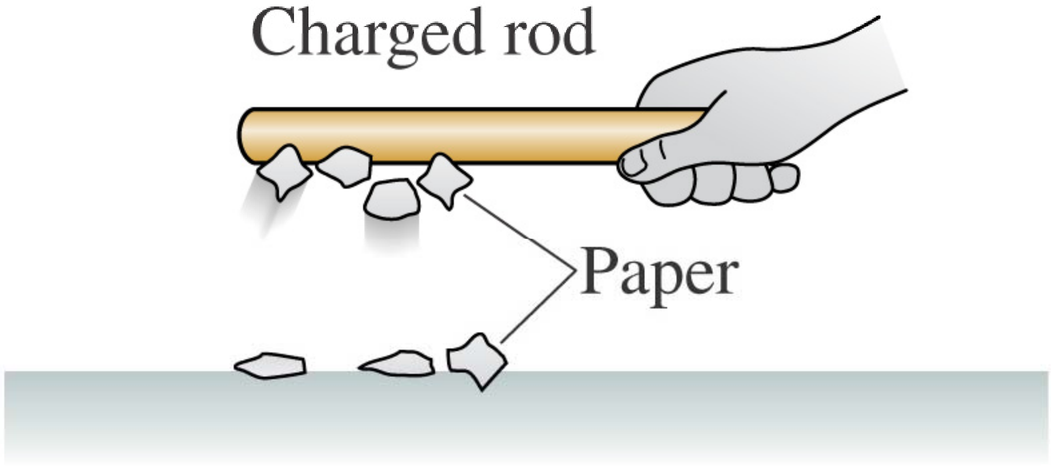




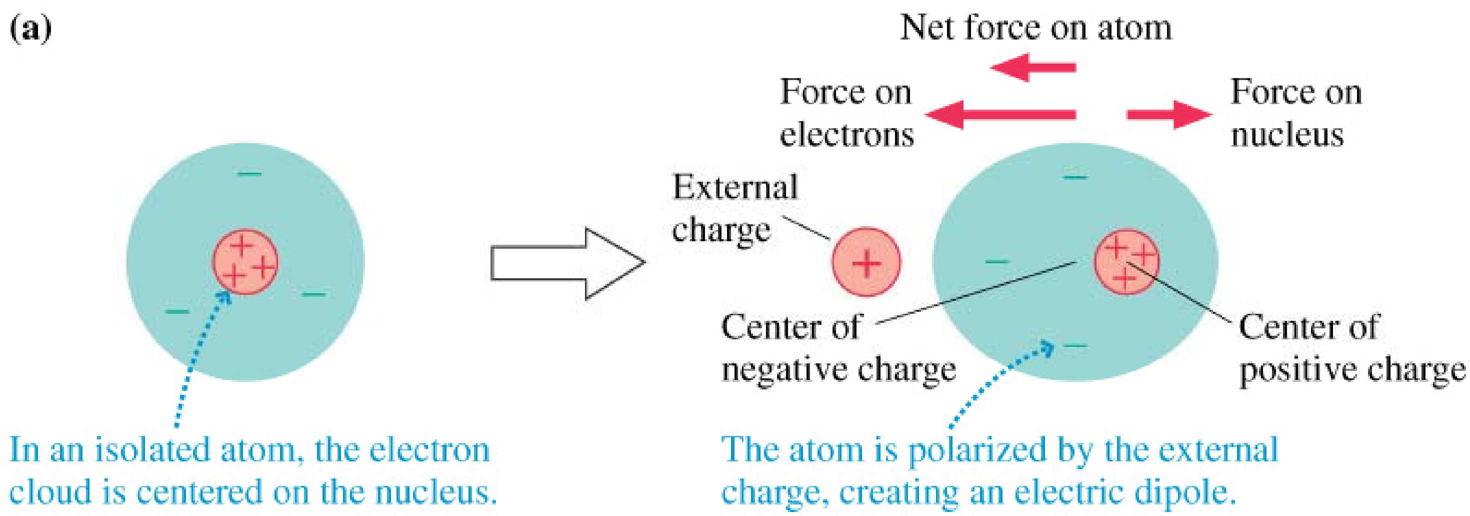
Electric Charges



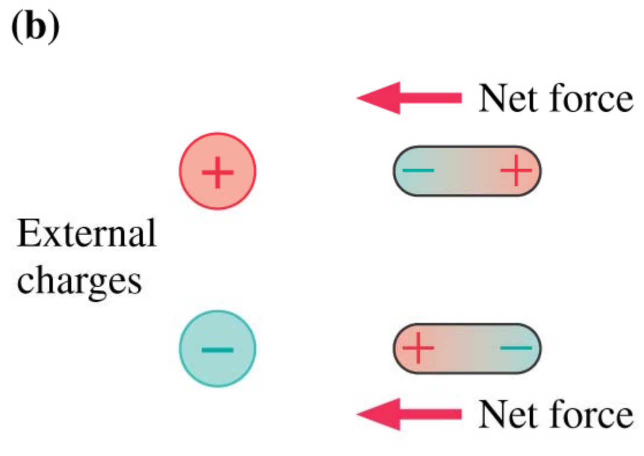
Electric Charges



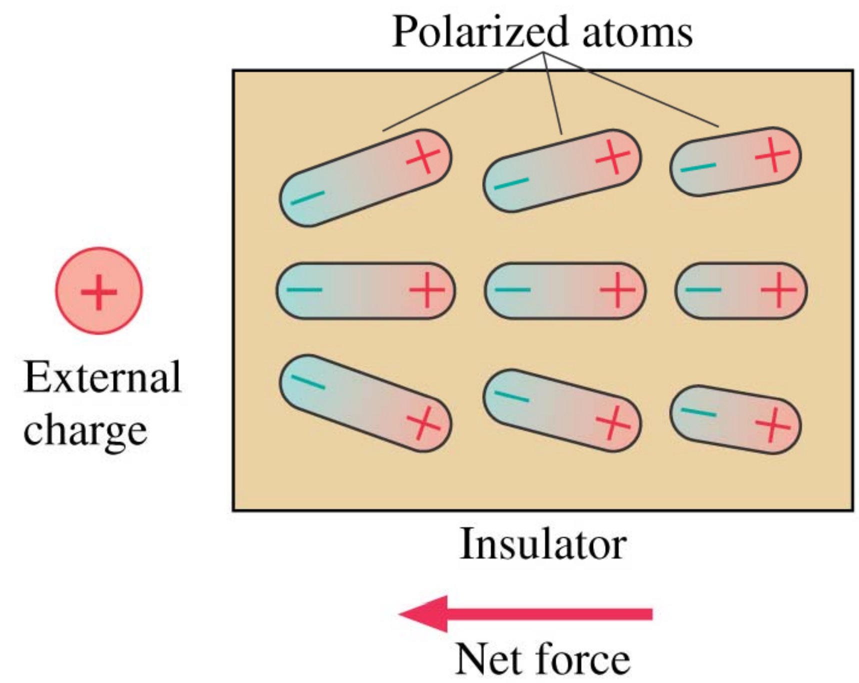
Electric Charges



Electric Charges



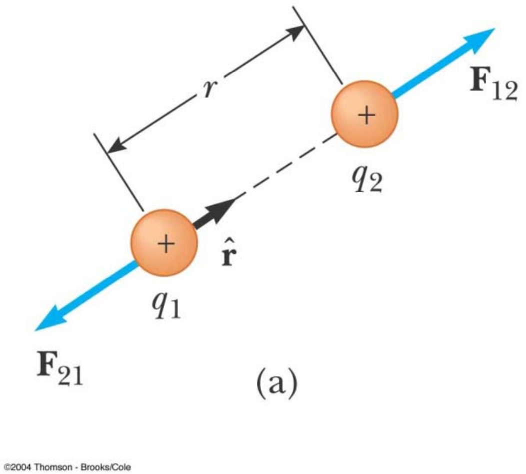
Electric dipoles can be created by either positive or negative charges. In both cases, there is an attractive net force toward the external charge.



Coulomb's Law

- Mathematically, the force between two electric charges:

$$F_{12} = k_e \frac{q_1 q_2}{r^2}$$



- The SI unit of charge is the **coulomb** (C)
- k_e is called the **Coulomb constant**
  - k_e = 8.9875 x 10^9 ***N·m²/C²*** = $1/(4\pi e_o)$
  - e_o is the **permittivity of free space**
  - e_o = 8.8542 x 10^{-12} C^2 / N·m^2

  Electric charge:

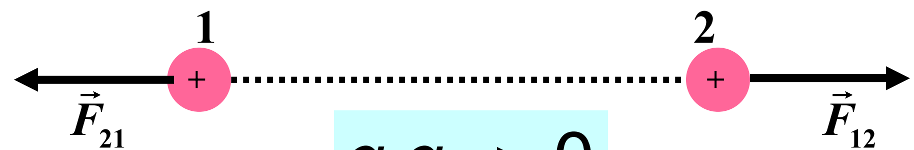
  - ❑ electron e = -1.6 x 10^{-19} C
  - ❑ proton e = 1.6 x 10^{-19} C

Coulomb's Law

$$F_{12} = F_{21} = k_e \frac{|q_1||q_2|}{r^2}$$

Direction depends on the sign of the product

$$q_1 q_2$$



$$q_1 q_2 > 0$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

opposite directions, the same magnitude

$$q_1 q_2 > 0$$

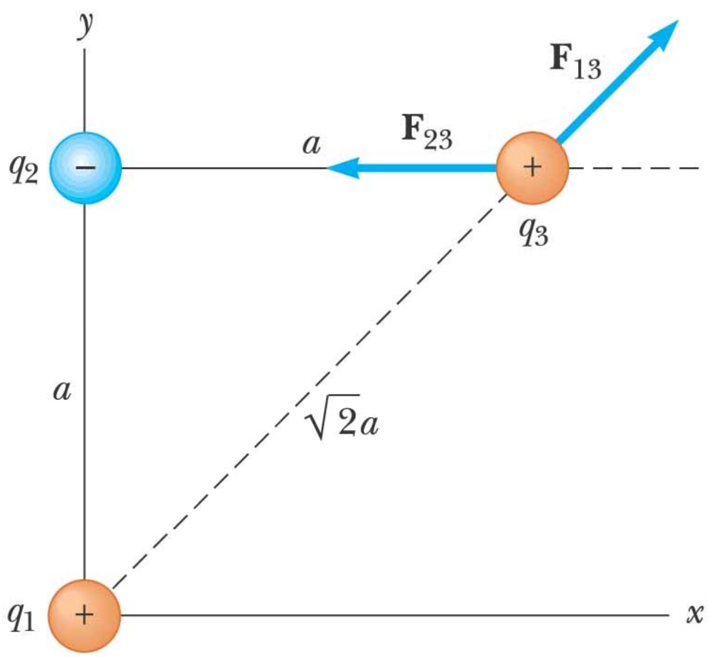
$$q_1 q_2 < 0$$

The force is attractive if the charges are of opposite sign
The force is repulsive if the charges are of like sign

Magnitude: $$F_{12} = F_{21} = k_e \frac{|q_1||q_2|}{r^2}$$

Coulomb's Law: Superposition Principle

- The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$
- The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$
- The *resultant force* exerted on q_3 is the vector sum of $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$

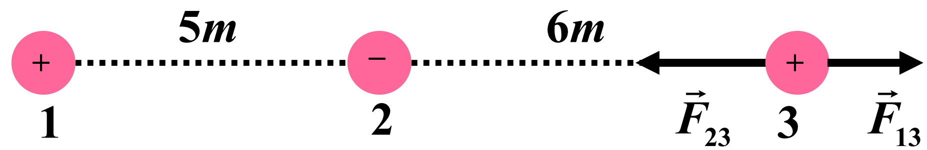


Coulomb's Law

$$F_{21} = k_e \frac{q_1 q_2}{r^2}$$

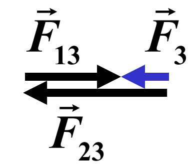
$q_1 = 1\,MC$ $q_2 = -2\,MC$

$q_3 = 3\,MC$



Resultant force:

$$\vec{F}_3 = \vec{F}_{21} + \vec{F}_{31}$$



Magnitude:

$$F_{23} = k_e \frac{|q_3||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6} \cdot 3 \cdot 10^{-6}}{6^2} N = 15 \cdot 10^{-4} N$$

$$F_{13} = k_e \frac{|q_3||q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 3 \cdot 10^{-6}}{11^2} N = 2.2 \cdot 10^{-4} N$$

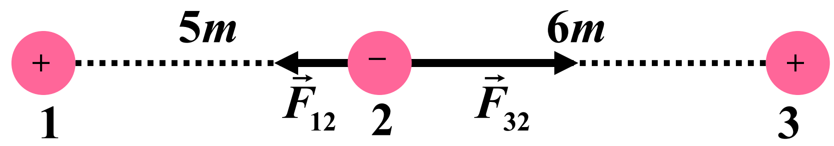
$$F_3 = F_{23} - F_{13} = (15 \cdot 10^{-4} - 2.2 \cdot 10^{-4}) N = 12.8 \cdot 10^{-4} N$$

Coulomb's Law

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

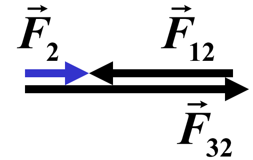
$q_1 = 1 \mu C$ $q_2 = -2 \mu C$

$q_3 = 3 \mu C$



Resultant force:

$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{32}$$



Magnitude:

$$F_{32} = k_e \frac{|q_3||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6} \cdot 3 \cdot 10^{-6}}{6^2} N = 15 \cdot 10^{-4} N$$

$$F_{12} = k_e \frac{|q_1||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 2 \cdot 10^{-6}}{5^2} N = 7.2 \cdot 10^{-4} N$$

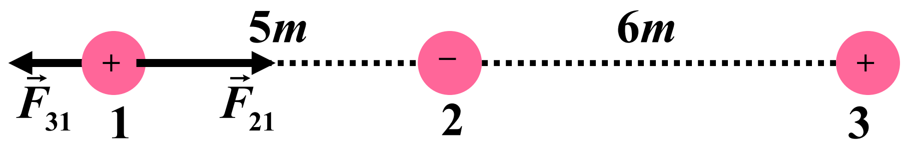
$$F_2 = F_{32} - F_{12} = (15 \cdot 10^{-4} - 7.2 \cdot 10^{-4}) N = 7.8 \cdot 10^{-4} N$$

Coulomb's Law

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

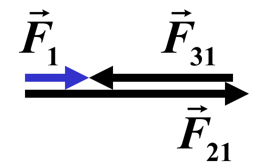
$q_1 = 1 \mu C$ $q_2 = -2 \mu C$

$q_3 = 3 \mu C$



Resultant force:

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$



Magnitude:

$$F_{31} = k_e \frac{|q_3||q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 3 \cdot 10^{-6}}{11^2} N = 2.2 \cdot 10^{-4} N$$

$$F_{21} = k_e \frac{|q_1||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 2 \cdot 10^{-6}}{5^2} N = 7.2 \cdot 10^{-4} N$$

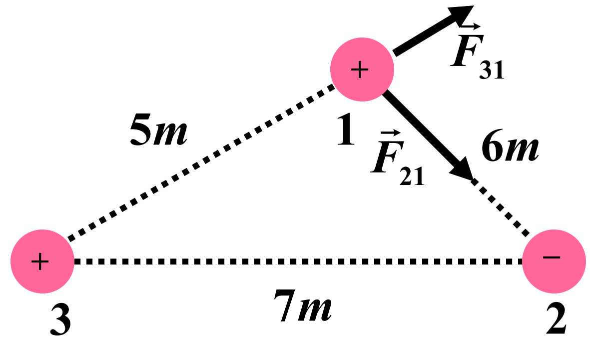
$$F_1 = F_{21} - F_{31} = (7.2 \cdot 10^{-4} - 2.2 \cdot 10^{-4}) N = 5 \cdot 10^{-4} N$$

Coulomb's Law

$$F_{21} = k_e \frac{q_1 q_2}{r^2}$$

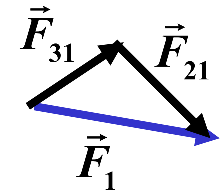
$q_1 = 1\mu C$ $\quad q_2 = -2\mu C$

$q_3 = 3\mu C$



Resultant force:

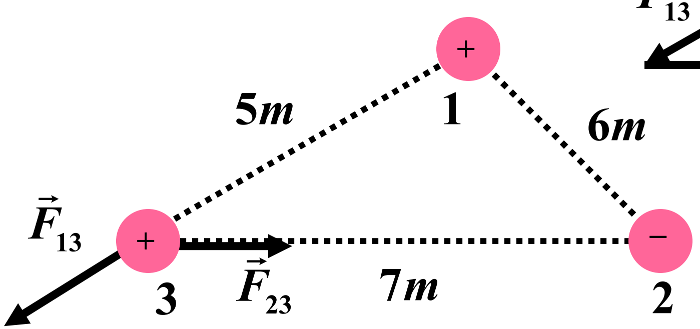
$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$

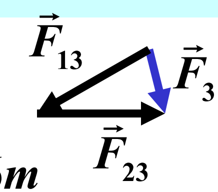


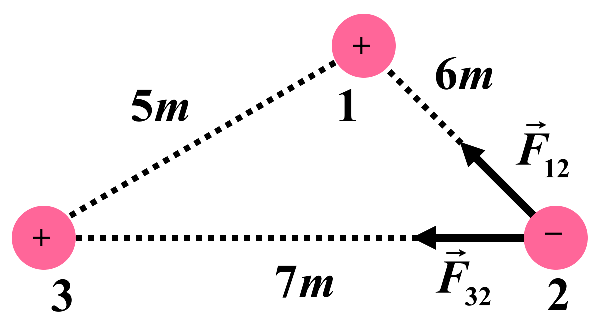
Magnitude:

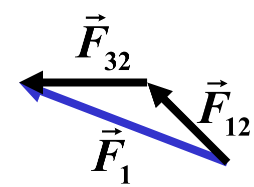
$$F_{21} = k_e \frac{|q_1||q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 2 \cdot 10^{-6}}{6^2} N = 5 \cdot 10^{-4} N$$

$$F_{31} = k_e \frac{|q_3||q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6} \cdot 3 \cdot 10^{-6}}{5^2} N = 1.1 \cdot 10^{-3} N$$







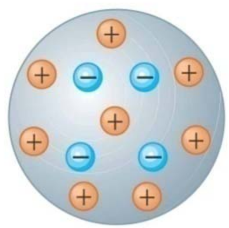


Conservation of Charge

Electric charge is always conserved in isolated system

Two identical sphere

$q_1 = 1 MC$



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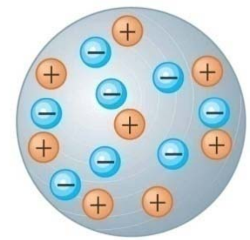
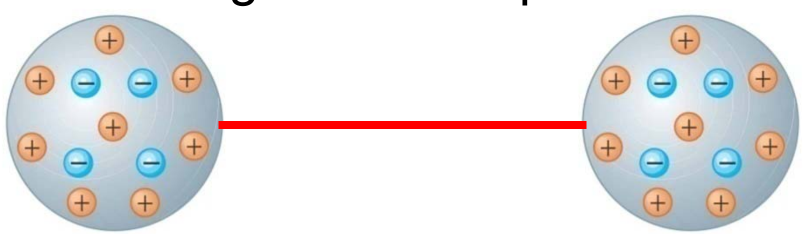


(a)

$q_2 = -2 MC$

They are connected by conducting wire. What is the electric charge of each sphere?



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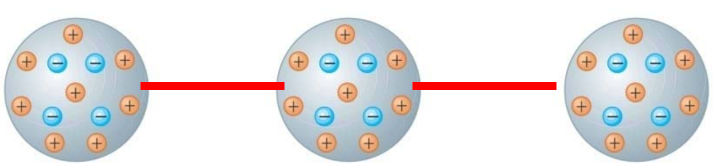
The same charge ***q***. Then the conservation of charge means that :

$$2q = q_1 + q_2$$

$$q = \frac{q_1 + q_2}{2} = \frac{1-2}{2} MC = -0.5 MC$$

For three spheres: $q_1 = 1 MC$ $q_2 = -2 MC$ $q_3 = 3 MC$



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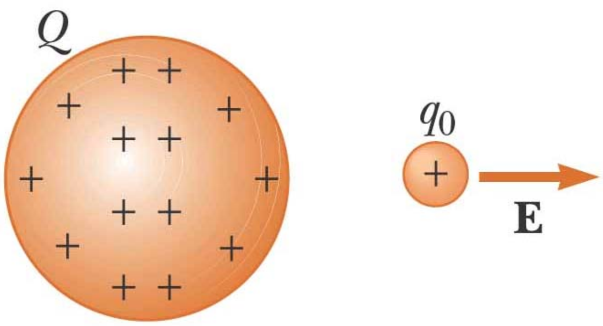
$$3q = q_1 + q_2 + q_3$$

$$q = \frac{q_1 + q_2 + q_3}{3} = \frac{1-2+3}{3} MC = 1 MC$$

Electric Field

Electric Field

- An **electric field** is said to exist in the region of space around a charged object
  - This charged object is the **source charge**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it.
- The electric field is defined as the electric force on the test charge per unit charge



$$\vec{E} = \frac{\vec{F}}{q_0}$$

- If you know the electric field you can find the force

$$\vec{F} = q\vec{E}$$

If *q* is positive, F and E are in the same direction
If *q* is negative, F and E are in opposite directions

Electric Field

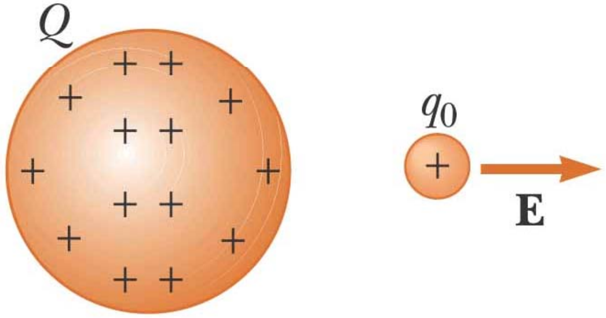
- The direction of **E** is that of the force on a positive test charge
- The SI units of **E** are N/C

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Coulomb's Law: $$F = k_e \frac{q q_0}{r^2}$$

Then

$$E = \frac{F}{q_0} = k_e \frac{q}{r^2}$$

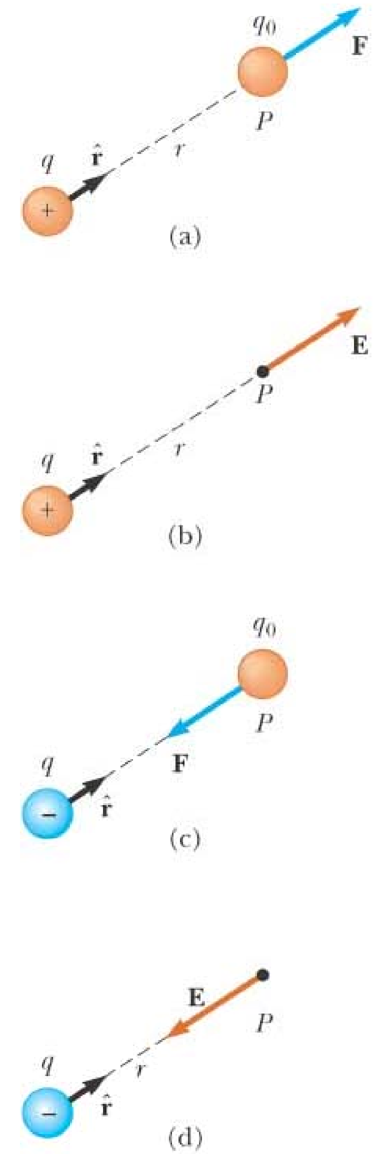


Electric Field

- q is positive, **F** is directed away from q
- The direction of **E** is also away from the positive source charge
- q is negative, **F** is directed toward q
- **E** is also toward the negative source charge

$$F = k_e \frac{qq_0}{r^2}$$

$$E = \frac{F}{q_0} = k_e \frac{q}{r^2}$$



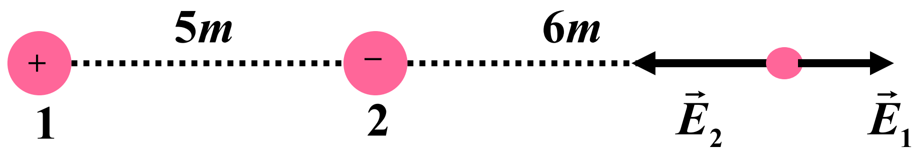
Electric Field: Superposition Principle

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of electric fields of all the charges

$$\vec{E} = \sum_i \vec{E}_i$$

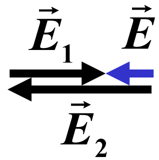
Electric Field

$$E = k_e \frac{q}{r^2}$$



Electric Field:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



$q_1 = 1\mu C$ $q_2 = -2\mu C$

Magnitude:

$$E_2 = k_e \frac{|q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6}}{6^2} N/C = 5 \cdot 10^2 N/C$$

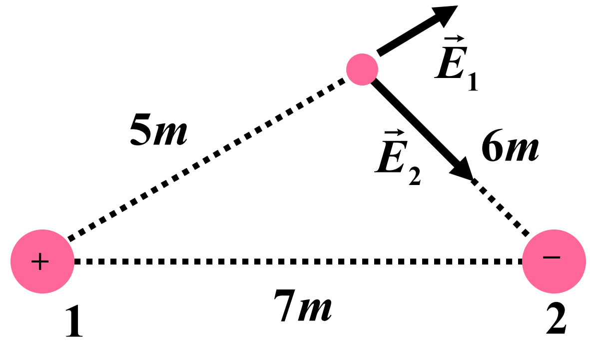
$$E_1 = k_e \frac{|q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6}}{11^2} N/C = 0.7 \cdot 10^2 N/C$$

$$E = E_2 - E_1 = (5 \cdot 10^2 - 0.7 \cdot 10^2) N/C = 4.3 \cdot 10^2 N/C$$

Electric Field

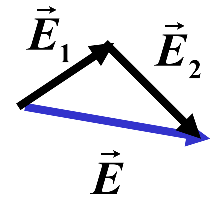
$$E = k_e \frac{q}{r^2}$$

$q_1 = 1\mu C$ $q_2 = -2\mu C$



Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



Magnitude:

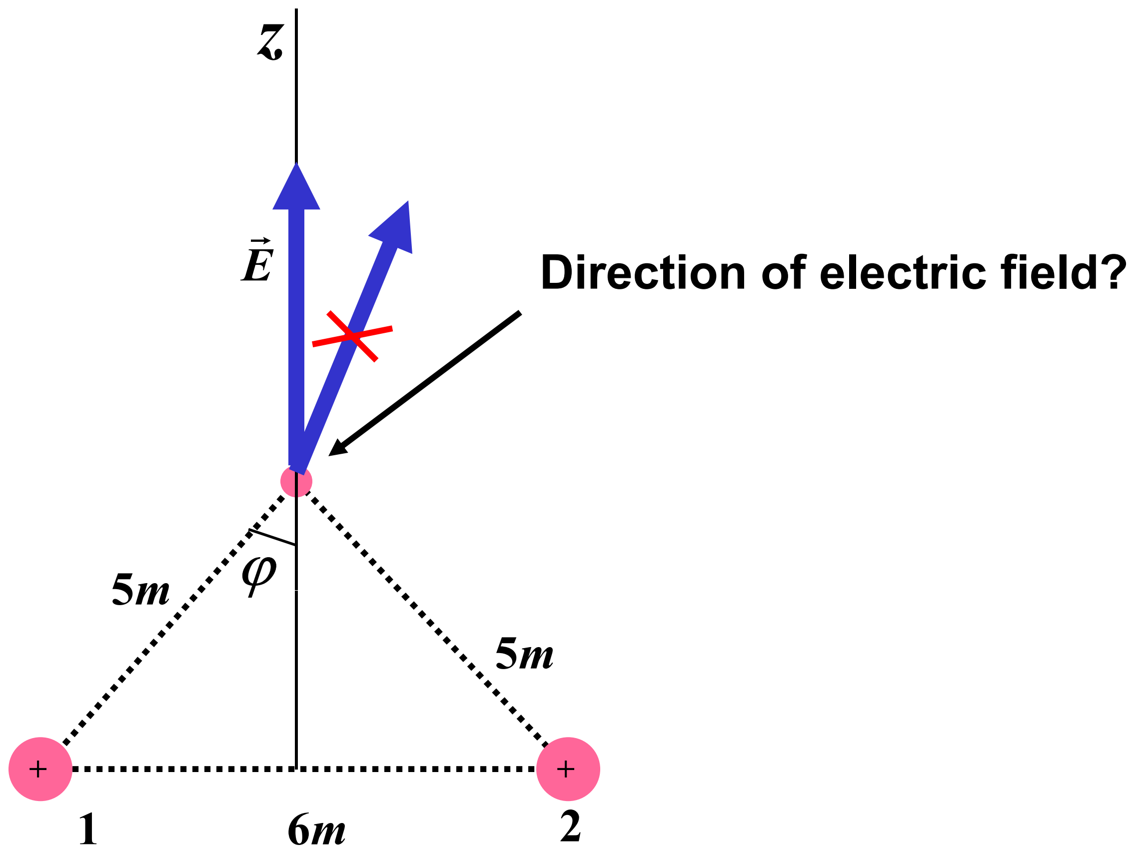
$$E_2 = k_e \frac{|q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{2 \cdot 10^{-6}}{6^2} N/C = 5 \cdot 10^2 N/C$$

$$E_1 = k_e \frac{|q_1|}{r^2} = 8.9875 \cdot 10^9 \frac{10^{-6}}{5^2} N/C = 0.37 \cdot 10^2 N/C$$

Electric Field

$$E = k_e \frac{q}{r^2}$$

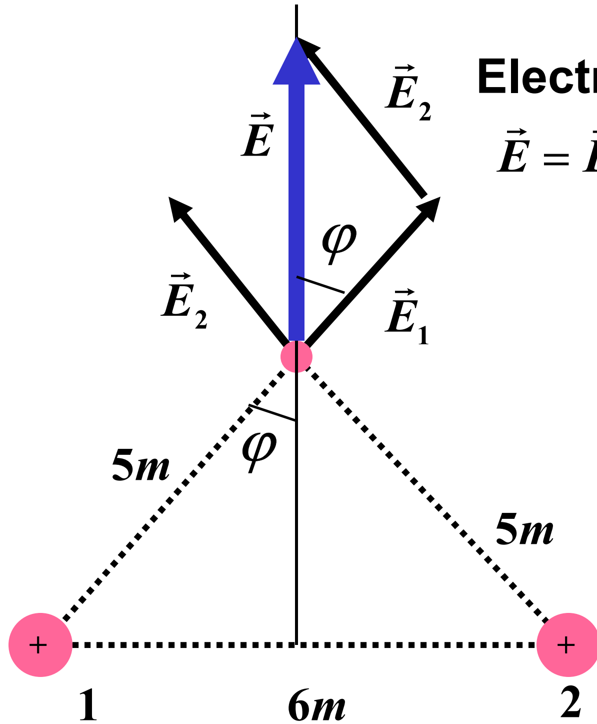
$q_1 = 10\ \mu C$ $q_2 = 10\ \mu C$



Electric Field

$$E = k_e \frac{q}{r^2}$$

$q_1 = 10\,\mu C$ $q_2 = 10\,\mu C$



Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Magnitude:

$$E_2 = E_1 = k_e \frac{|q_2|}{r^2} = 8.9875 \cdot 10^9 \frac{10 \cdot 10^{-6}}{5^2} N/C = 3.6 \cdot 10^3 N/C$$

$$E = 2E_1 \cos\varphi$$

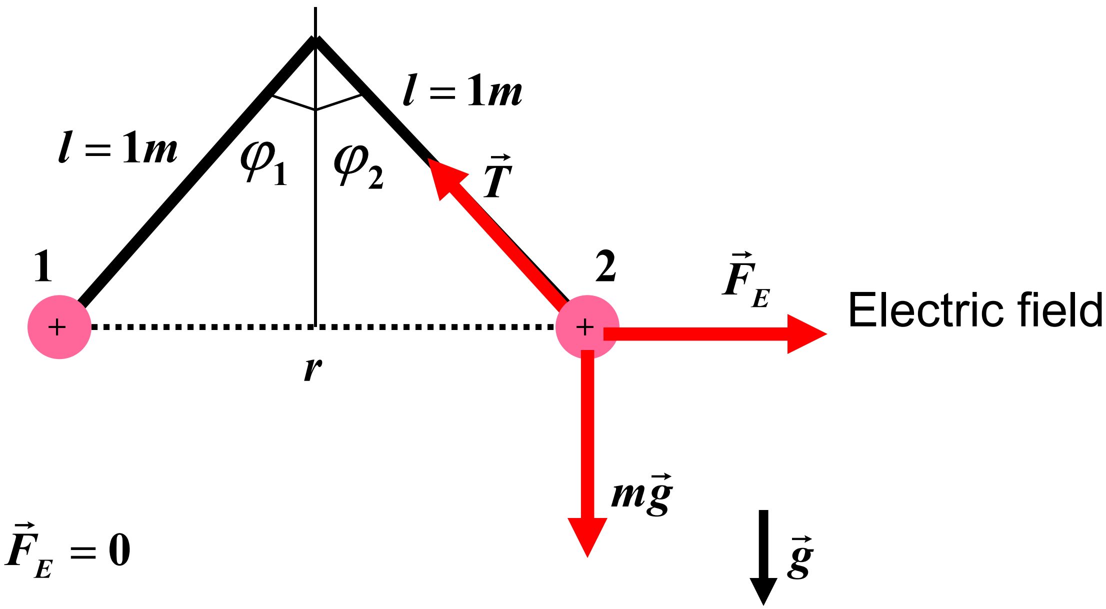
$$\cos\varphi = \frac{\sqrt{5^2 - 3^2}}{5} = \frac{4}{5}$$

$$E = \frac{8}{5} E_1$$

Example

$q_1 = 10\,\mu C$ $q_2 = 10\,\mu C$

$$m_1 = m_2 = m$$



$$\vec{T} + m\vec{g} + \vec{F}_E = 0$$

$$F_E = k_e \frac{q_1 q_2}{r^2}$$

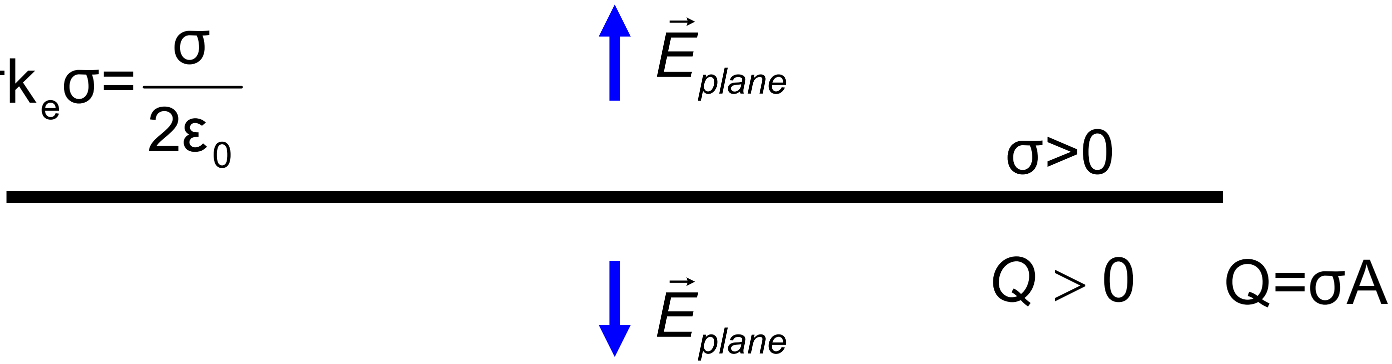
$$T\cos\varphi_2 = mg \qquad T\sin\varphi_2 = F_E$$

$$\tan\varphi_2 = \frac{F_E}{mg} = k_e \frac{q_1 q_2}{r^2 mg}$$

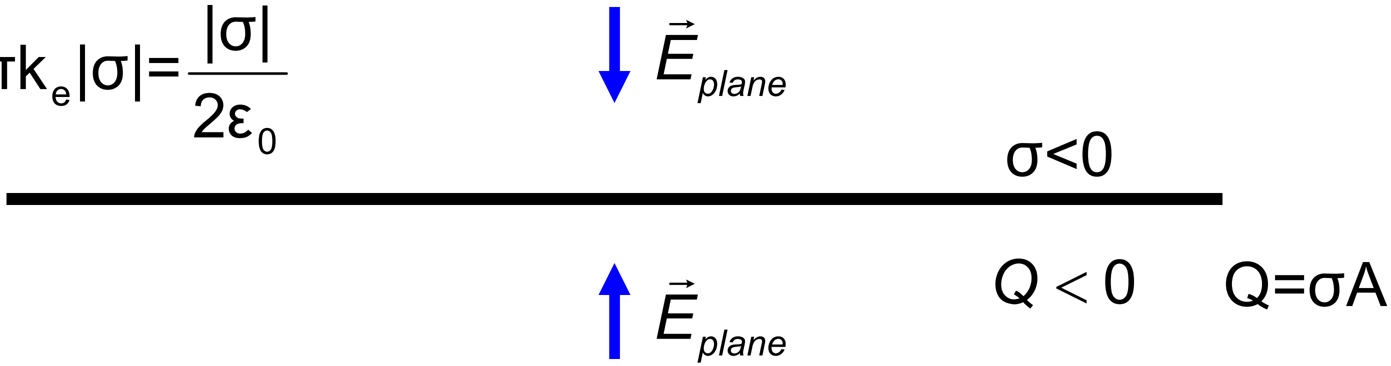
$$\tan\varphi_1 = \frac{F_E}{mg} = k_e \frac{q_1 q_2}{r^2 mg}$$

$$\varphi_2 = \varphi_1$$

$$E_{plane} = 2\pi k_e \sigma = \frac{\sigma}{2\varepsilon_0}$$

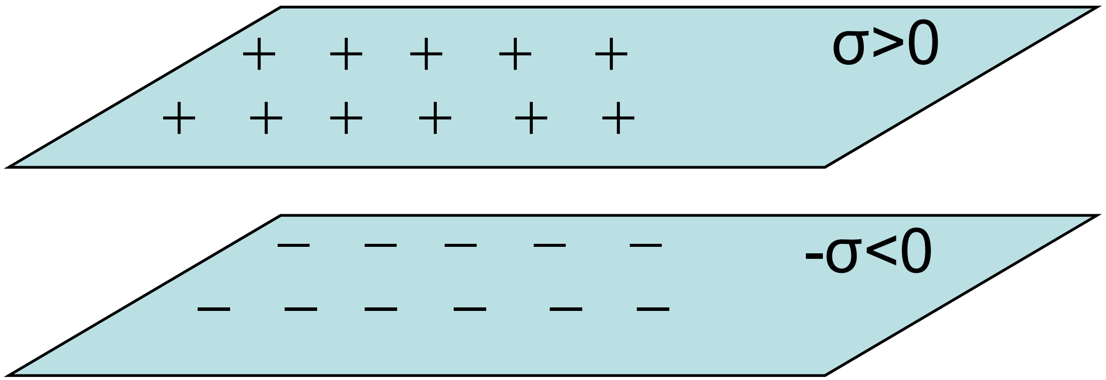


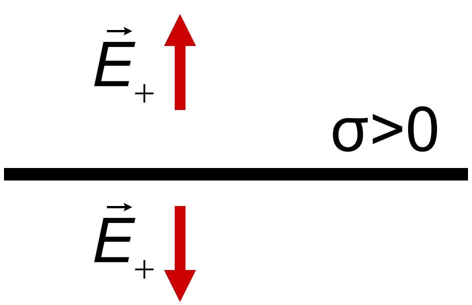
$$E_{plane} = 2\pi k_e |\sigma| = \frac{|\sigma|}{2\varepsilon_0}$$

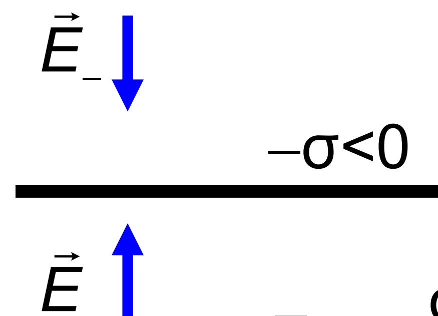


Important Example

Find electric field





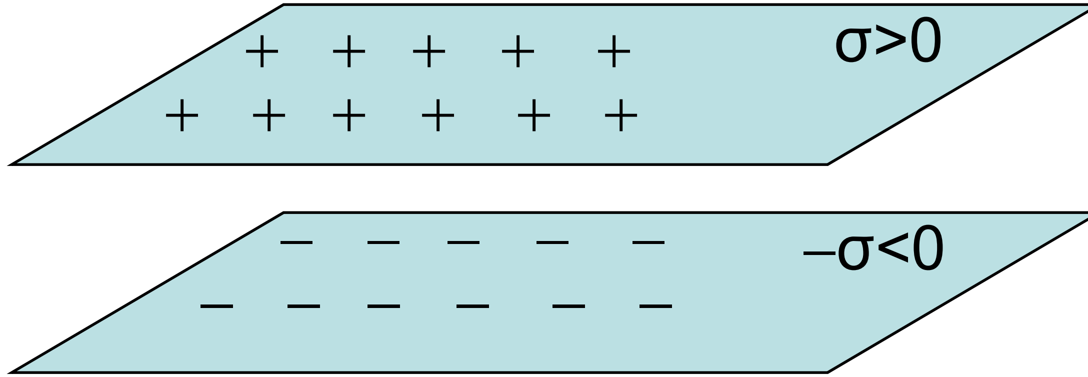


$$E_+ = \frac{\sigma}{2\varepsilon_0}$$

$$E_- = \frac{\sigma}{2\varepsilon_0}$$

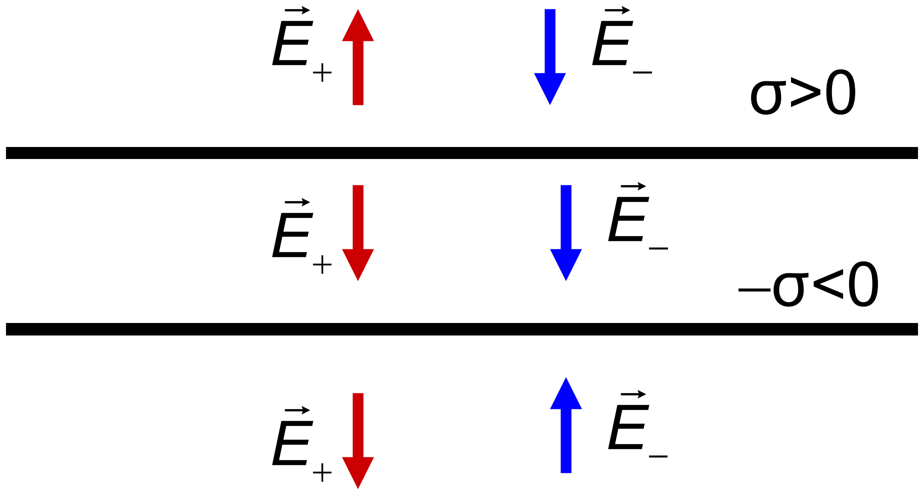
Important Example

Find electric field



$$E_+ = \frac{\sigma}{2\varepsilon_0} \qquad E_- = \frac{\sigma}{2\varepsilon_0}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

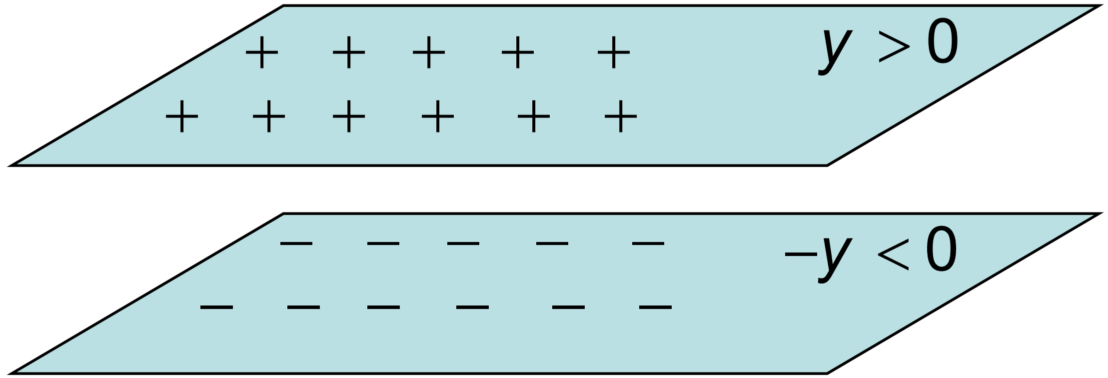


$$E = E_+ - E_- = 0$$

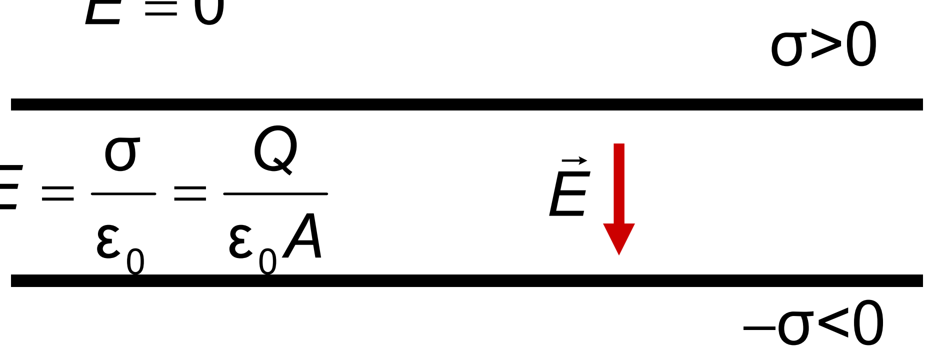
$$E = E_+ + E_- = \frac{\sigma}{\varepsilon_0}$$

$$E = E_+ - E_- = 0$$

Important Example



$$E = 0$$

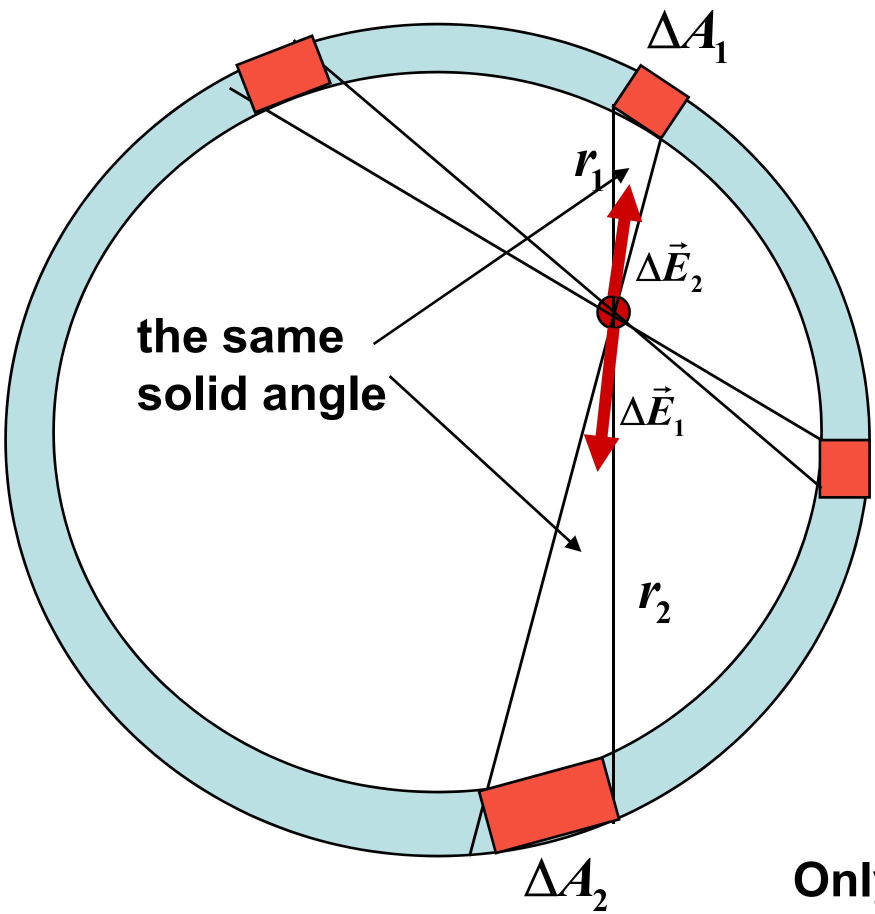


$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$E = 0$$

Electric field due to a thin uniformly charged spherical shell

- When ***r* < *a***, electric field is ZERO: ***E* = 0**



$$\Delta q_1 = \sigma \Delta A_1 \qquad \Delta q_2 = \sigma \Delta A_2$$

$$\Delta A_1 = r_1^2 \Delta\Omega \qquad \Delta A_2 = r_2^2 \Delta\Omega$$

$$\Delta E_1 = k_e \frac{\Delta q_1}{r_1^2} = k_e \frac{\sigma \Delta A_1}{r_1^2} = k_e \frac{\sigma r_1^2 \Delta\Omega}{r_1^2} = k_e \sigma \Delta\Omega$$

$$\Delta E_2 = k_e \frac{\Delta q_2}{r_2^2} = k_e \frac{\sigma \Delta A_2}{r_2^2} = k_e \frac{\sigma r_2^2 \Delta\Omega}{r_2^2} = k_e \sigma \Delta\Omega$$

$$\Delta E_1 = \Delta E_2$$

$$\Delta\vec{E}_1 + \Delta\vec{E}_2 = 0$$

Only because in Coulomb law → $E \propto \frac{1}{r^2}$

Motion of Charged Particle

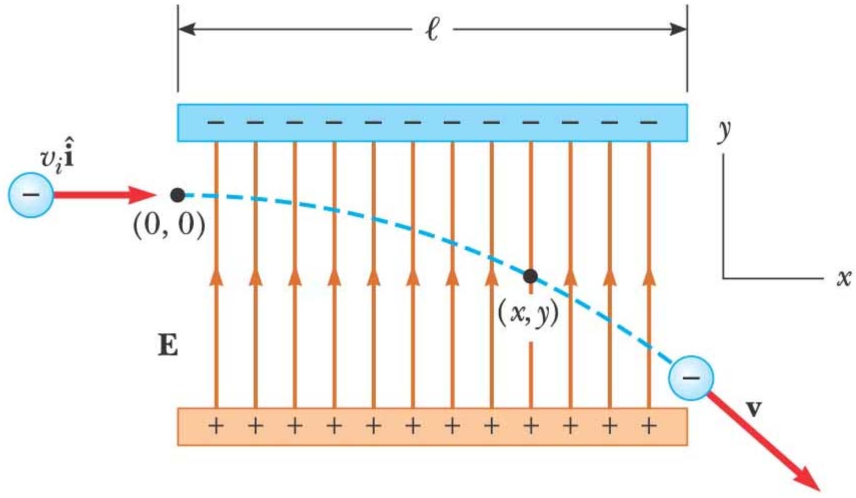
Motion of Charged Particle

- When a charged particle is placed in an electric field, it experiences an electrical force
- If this is the only force on the particle, it must be the net force
- The net force will cause the particle to accelerate according to Newton's second law

$\vec{F} = q\vec{E}$ - Coulomb's law

$\vec{F} = m\vec{a}$ - Newton's second law

$$\vec{a} = \frac{q}{m}\vec{E}$$



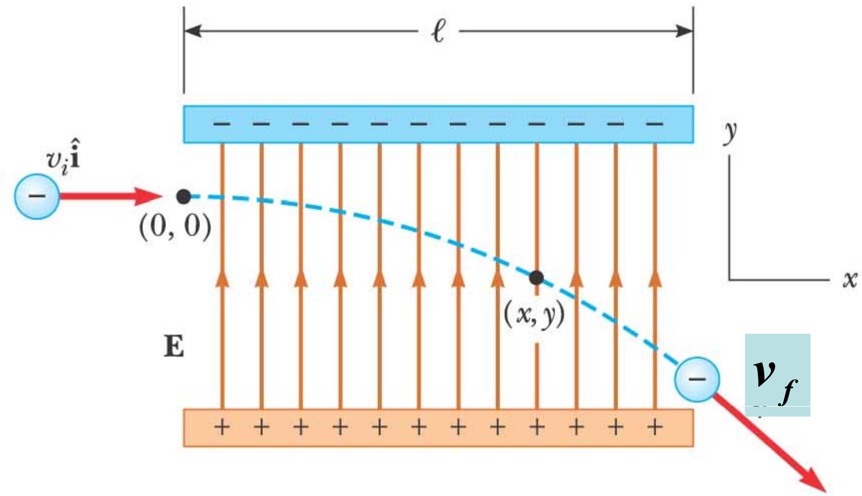
Motion of Charged Particle

What is the final velocity?

$$\vec{F} = q\vec{E}$$ - Coulomb's law

$$\vec{F} = m\vec{a}$$ - Newton's second law

$$a_y = -\frac{|q|}{m}E$$



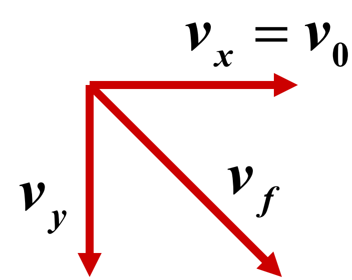
Motion in ***x*** – with constant velocity v_0

Motion in ***x*** – with constant acceleration $a_y = -\frac{|q|}{m}E$

$$t = \frac{l}{v_0}$$ - travel time

After time ***t*** the velocity in ***y*** direction becomes

$$v_y = a_y t = -\frac{|q|}{m}Et$$ then $$v_f = \sqrt{v_0^2 + \left(\frac{q}{m}Et\right)^2}$$



Conductors in Electric Field

Electric Charges: Conductors and Isolators

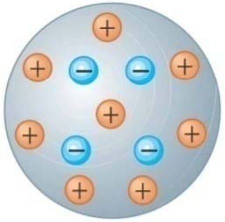
➢ Electrical conductors are materials in which some of the electrons are free electrons

- These electrons can move relatively freely through the material
- Examples of good conductors include copper, aluminum and silver

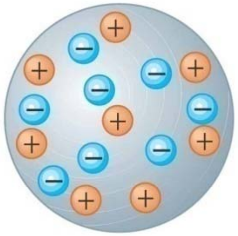
➢ Electrical insulators are materials in which all of the electrons are bound to atoms

- These electrons can not move relatively freely through the material
- Examples of good insulators include glass, rubber and wood

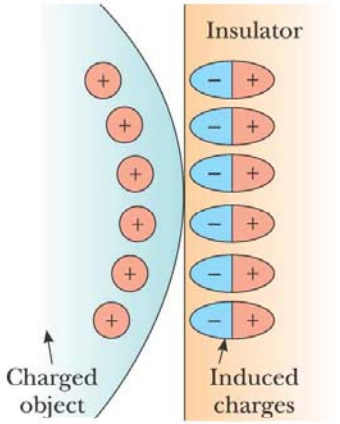
➢ Semiconductors are somewhere between insulators and conductors



(e)



(a)



(a)

Electrostatic Equilibrium

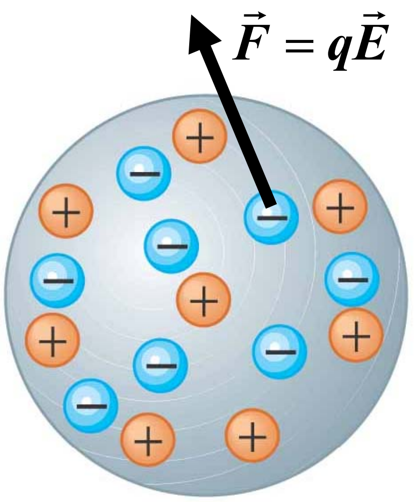
Definition:

when there is no net motion of charges within a conductor, the conductor is said to be in **electrostatic equilibrium**

Because the electrons can move freely through the material

- **no motion** means that there are **no electric forces**
- **no electric forces** **means that the electric field inside the conductor is 0**

If electric field inside the conductor is not **0**, $\vec{E} \neq 0$ then there is an electric force $\vec{F} = q\vec{E}$ and, from the second Newton's law, there is a motion of free electrons.

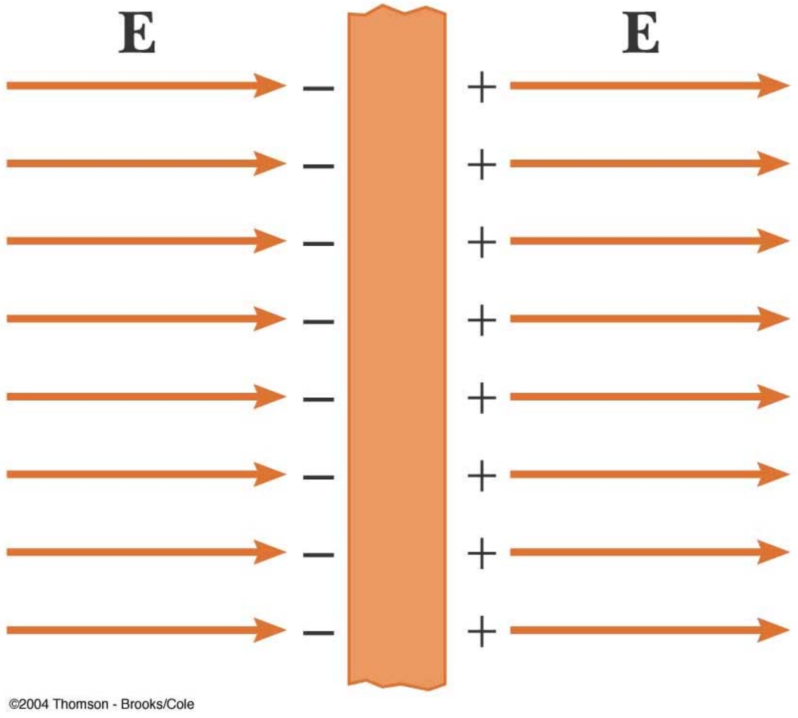


(a)

Conductor in Electrostatic Equilibrium

- **The electric field is zero everywhere inside the conductor**

- Before the external field is applied, free electrons are distributed throughout the conductor
- When the external field is applied, the electrons redistribute until the magnitude of the internal field equals the magnitude of the external field
- There is a net field of zero inside the conductor



Conductor in Electrostatic Equilibrium

- **If an isolated conductor carries a charge, the charge resides on its surface**