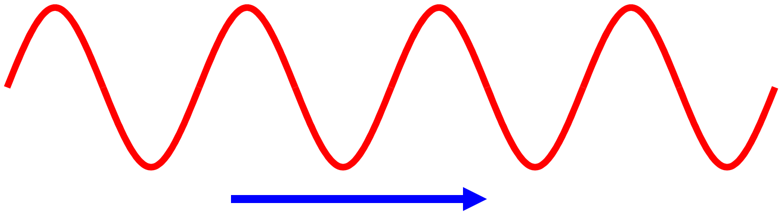
Chapter 17

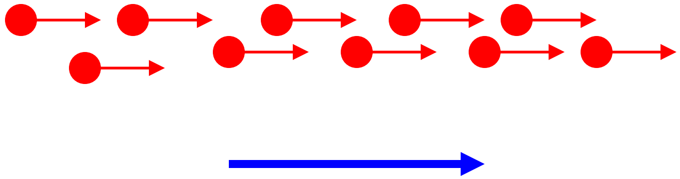
Light as a Wave:
Wave Optics

The Nature of Light – Particle or Waves?

WAVE?



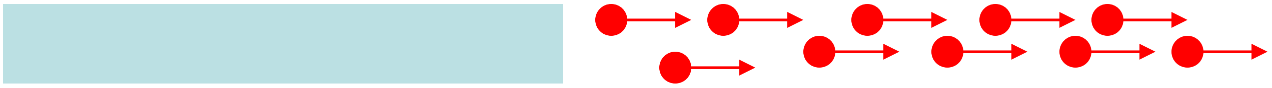
PARTICLES?



The Nature of Light – Particle or Waves?

- Before the beginning of the nineteenth century, light was considered to be a stream of particles
- Newton was the chief architect of the particle theory of light
  - He believed the particles left the object and stimulated the sense of sight upon entering the eyes

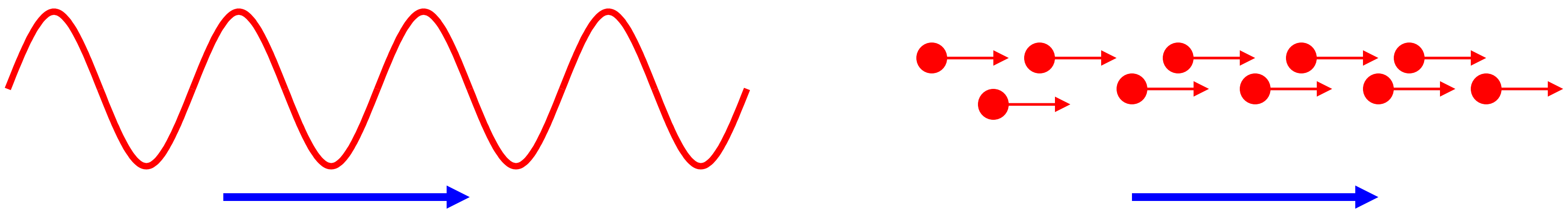
He was wrong (?) LIGHT IS A WAVE.



The Nature of Light – Particle or Waves?

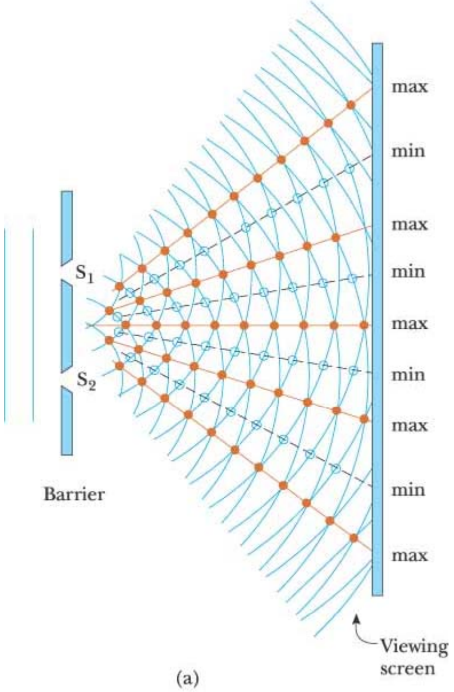
How can we distinguish between particles and waves?

For waves we have interference, for particles – not!



The Nature of Light – Wave Theory?

- Christian Huygens argued that light might be some sort of a wave motion
- Thomas Young (1801) provided the first clear demonstration of the wave nature of light
  - He showed that light rays interfere with each other
  - Such behavior could not be explained by particles



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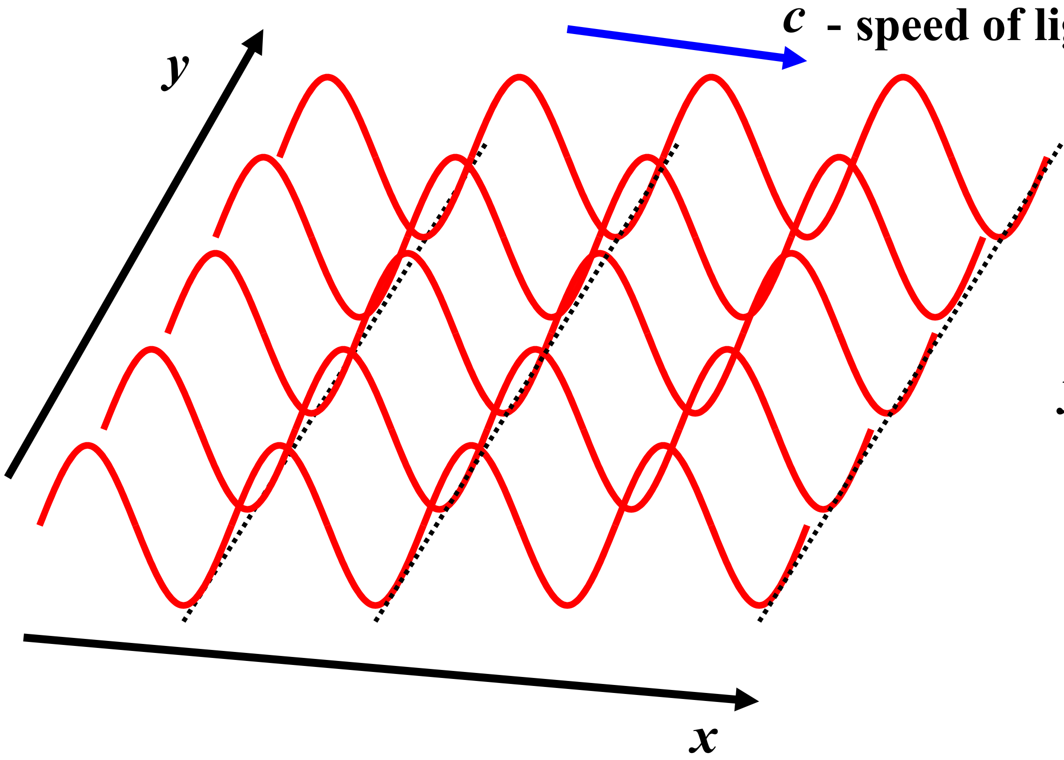
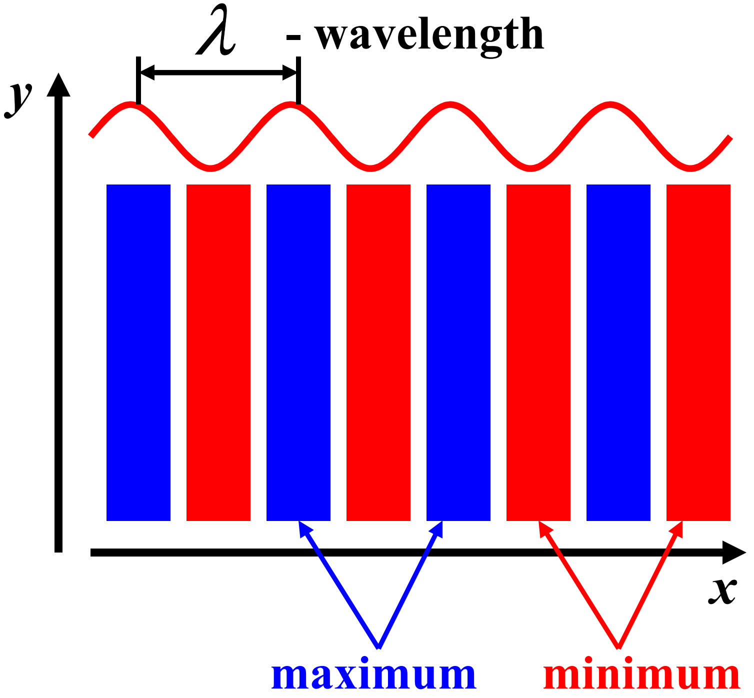
During the nineteenth century, other developments led to the general acceptance of the wave theory of light

Light as a Wave

Plane wave



changes only along one direction



Period of "oscillation" – $T = \dfrac{\lambda}{c}$
(time to travel distance of wavelength)

Frequency of light $f = \dfrac{1}{T} = \dfrac{c}{\lambda}$

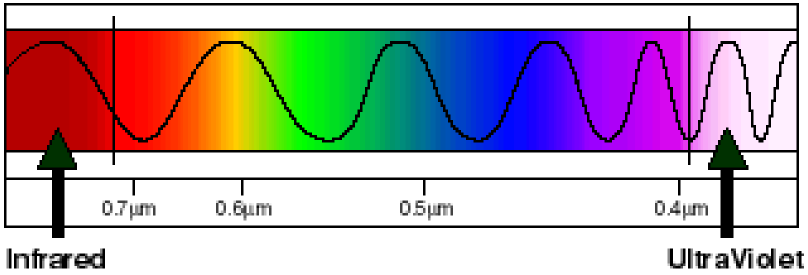
Light as a Wave

Light is characterized by

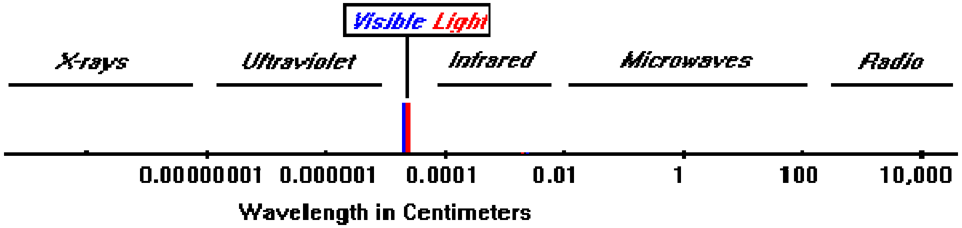
- **its speed c and**
- **wavelength λ (or frequency f)**

Different frequency (wavelength) – different color of light

Visible Light Region
of the Electromagnetic Spectrum



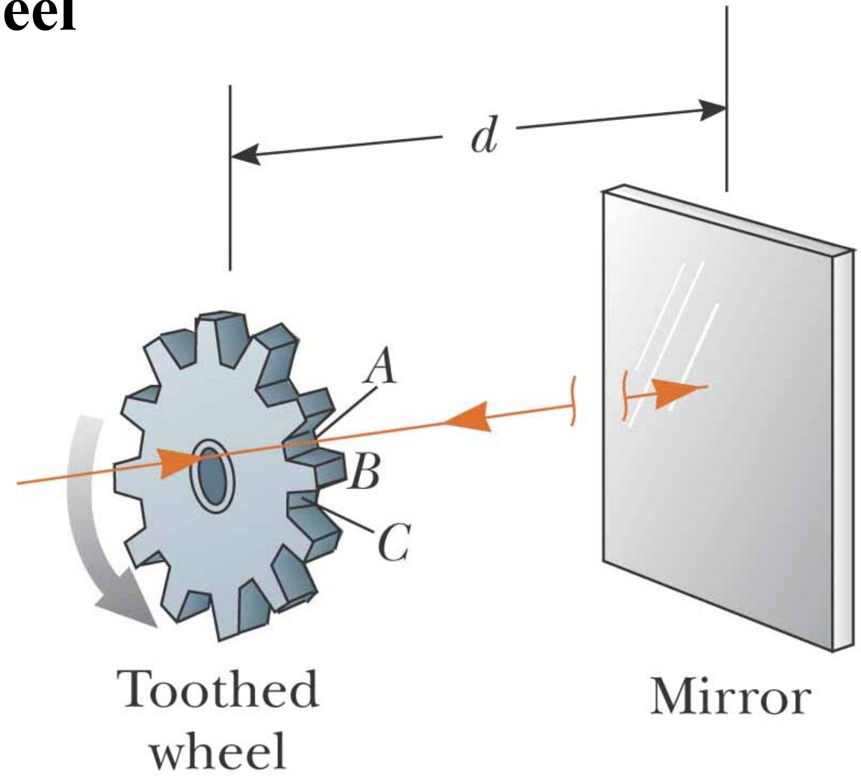
The Electromagnetic Spectrum



What is the speed of light?

Measurements of the Speed of Light – Fizeau's Method (1849)

- d is the distance between the wheel and the mirror
- Δt is the time for one round trip
- Then $c = 2d / \Delta t$
- Fizeau found a value of $c = 3.1 \times 10^8$ m/s



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$\boxed{c = 3.00 \times 10^8 \text{ m/s}}$ - Speed in Vacuum!

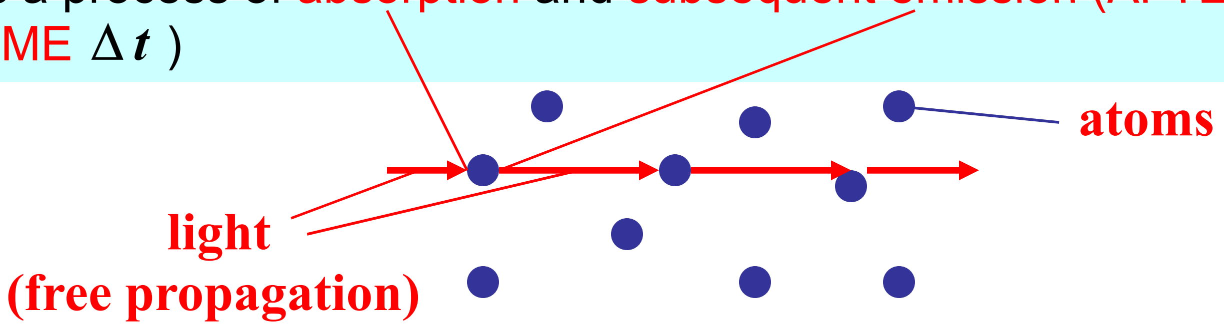
Speed of Light

What is the speed of light in a medium?

The speed of light in a medium is smaller than the speed in vacuum.

To understand this you can think about it in a following way:

- The medium consists of atoms (or molecules), which can absorb light and then emit it,
- so the propagation of light through the medium can be considered as a process of absorption and subsequent emission (AFTER SOME TIME Δt)



very schematic picture

Speed of Light

$$v = \frac{c}{n}$$ - The speed of light in the medium

The properties of the medium is characterized by one dimensionless constant – *n*, (it is called index of refraction, we will see later why)

- **which is equal to 1 for vacuum (and very close to 1 for air),**
- **greater then 1 for all other media**

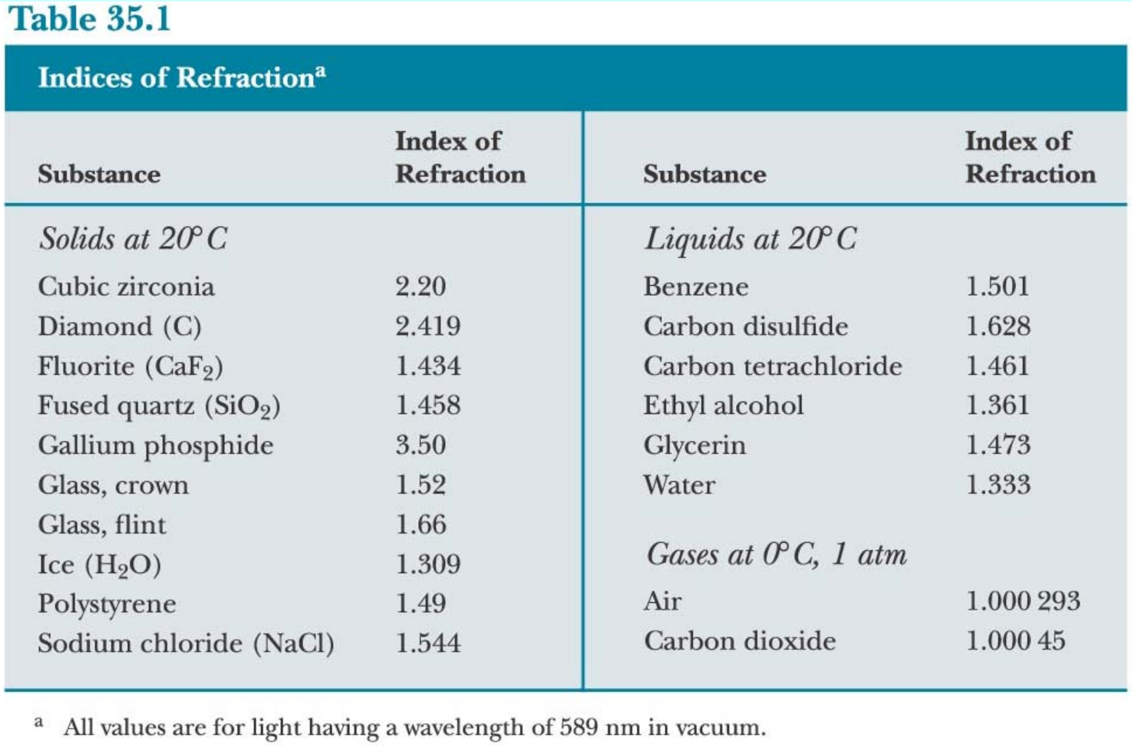


Table 35.1

Indices of Refraction[a]

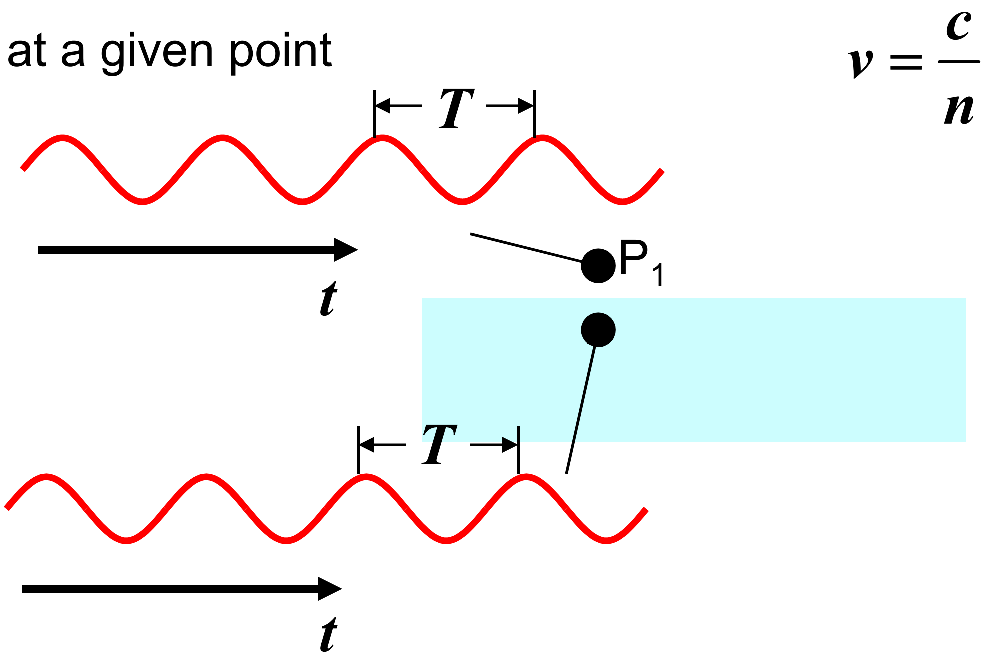
Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20° C		*Liquids at 20° C*	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF_2)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO_2)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H_2O)	1.309	*Gases at 0° C, 1 atm*	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

[a] All values are for light having a wavelength of 589 nm in vacuum.

Light in the Media

E at a given point

$$v = \frac{c}{n}$$ - The speed of light in the medium



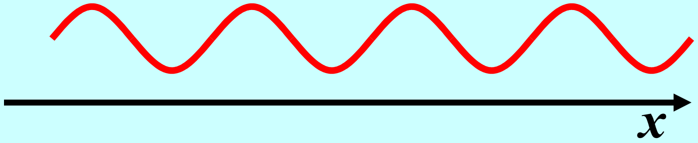
The same period (frequency) in all media, then

$$\lambda_n = \frac{c}{n} T = \frac{\lambda_{air}}{n}$$

Light as a Wave

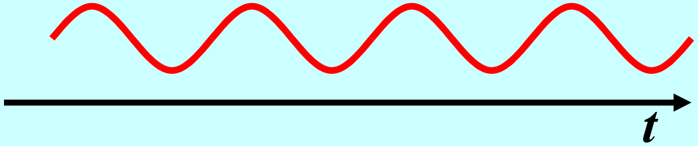
$$E(x,t) = E_0 \sin\left(2\pi\frac{x}{\lambda} + 2\pi ft\right) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \omega t\right)$$

Distribution of some Field inside the wave of frequency *f*



At a given time ***t*** we have sin-function of ***x*** with "initial" phase, depending on ***t***

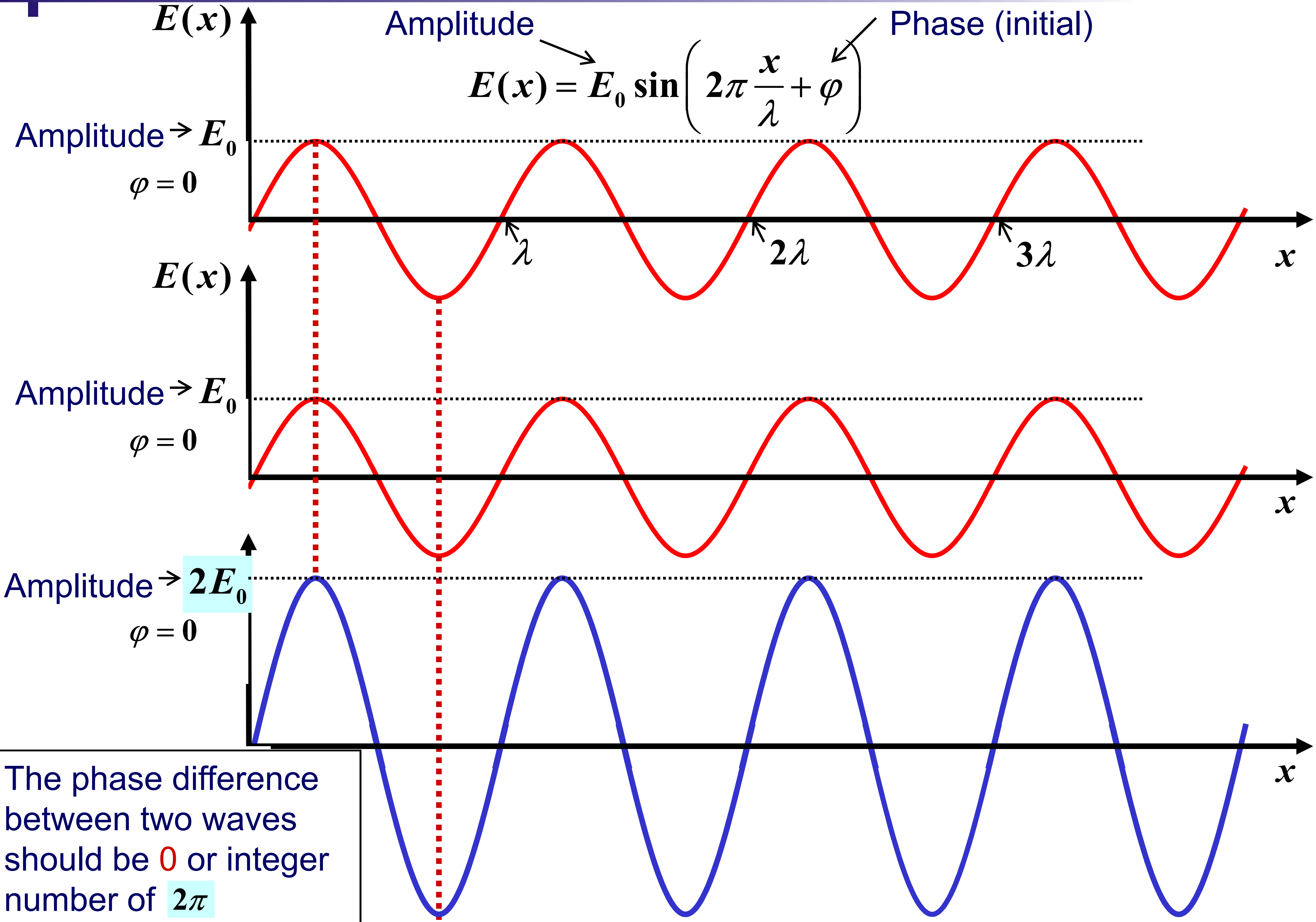
$$E(x) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \varphi_t\right) \qquad \varphi_t = 2\pi f t = \omega t$$



At a given space point ***x*** we have sin-function of ***t*** with "initial" phase, depending on ***x***

$$E(x) = E_0 \sin\left(\omega t + \varphi_x\right) \qquad \varphi_x = 2\pi\frac{x}{\lambda}$$

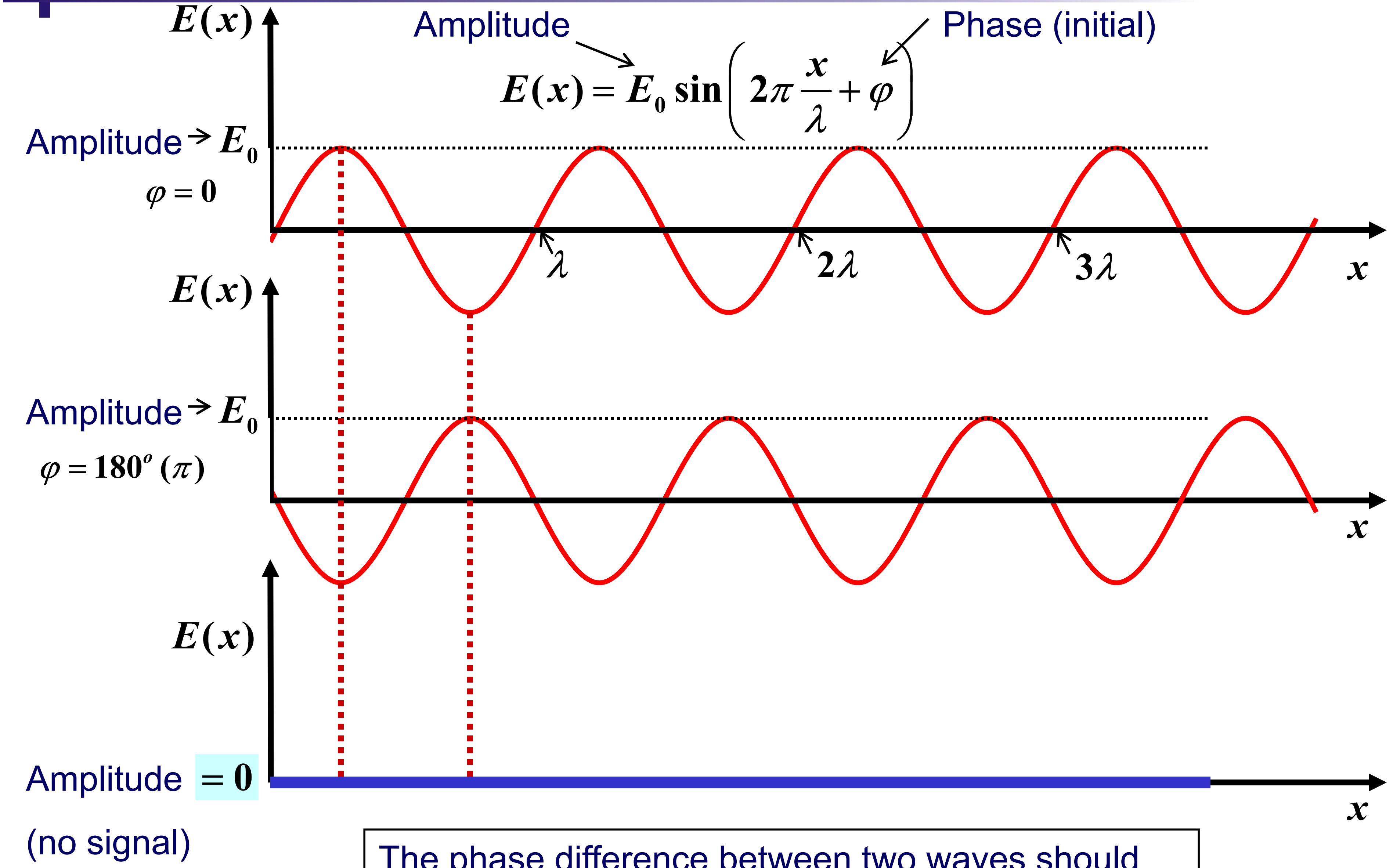
Sin-function: Constructive Interference



$$E(x) = E_0 \sin\left(2\pi\frac{x}{\lambda} + \varphi\right)$$

The phase difference between two waves should be 0 or integer number of 2π

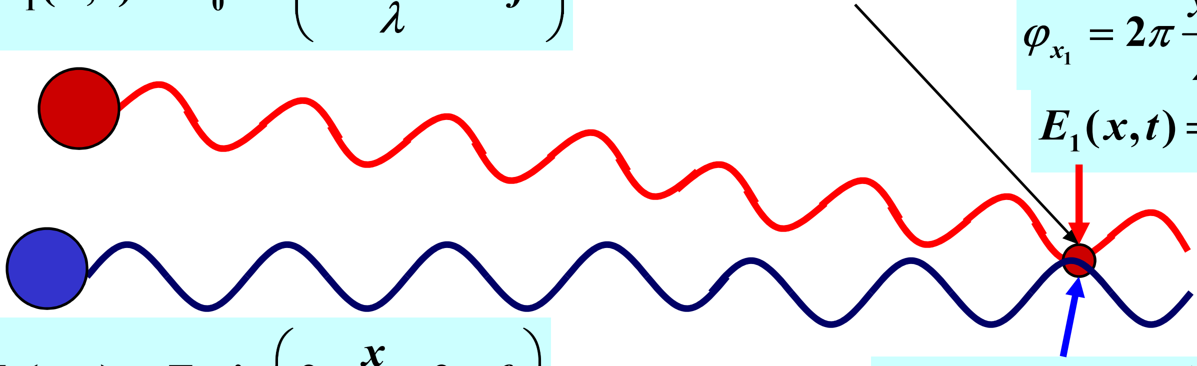
Sin-function: Destructive Interference



$$E(x) = E_0 \sin\left(2\pi \frac{x}{\lambda} + \varphi\right)$$

The phase difference between two waves should be π or π plus integer number of 2π

Waves: Interference



$$E_1(x,t) = E_0 \sin\left(2\pi\frac{x}{\lambda} + 2\pi ft\right)$$

$$E_2(x,t) = E_0 \sin\left(2\pi\frac{x}{\lambda} + 2\pi ft\right)$$

Interference – sum of two waves

$$\varphi_{x_1} = 2\pi\frac{x_1}{\lambda}$$

$$E_1(x,t) = E_0 \sin\left(2\pi ft + \varphi_{x_1}\right)$$

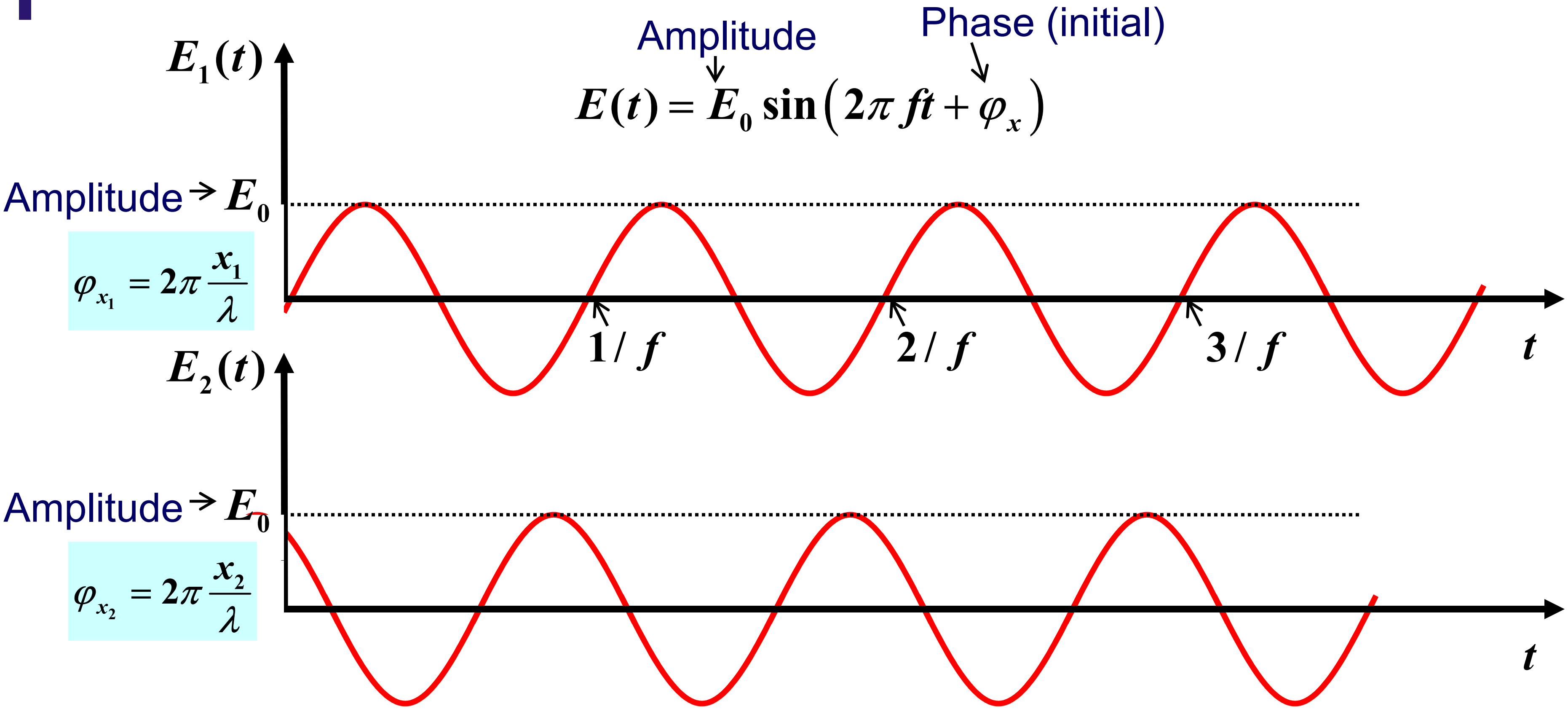
$$E_2(x,t) = E_0 \sin\left(2\pi ft + \varphi_{x_2}\right)$$

$$\varphi_{x_2} = 2\pi\frac{x_2}{\lambda}$$

- In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave

- In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave

Waves: Interference



Amplitude   Phase (initial)

$$E(t) = E_0 \sin\left(2\pi ft + \varphi_x\right)$$

$$\varphi_{x_1} = 2\pi \frac{x_1}{\lambda}$$

$$\varphi_{x_2} = 2\pi \frac{x_2}{\lambda}$$

Constructive Interference: The phase difference between two waves should be 0 or integer number of 2π

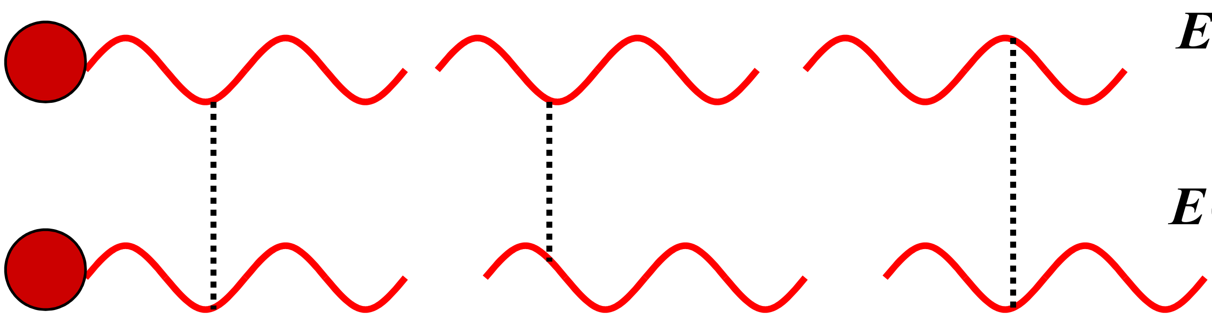
$$\varphi_{x_1} - \varphi_{x_2} = 2\pi m \qquad m = 0, \pm 1, \pm 2 ...$$

Destructive Interference: The phase difference between two waves should be π or π integer number of 2π

$$\varphi_{x_1} - \varphi_{x_2} = \pi + 2\pi m \qquad m = 0, \pm 1, \pm 2 ...$$

Conditions for Interference

coherent

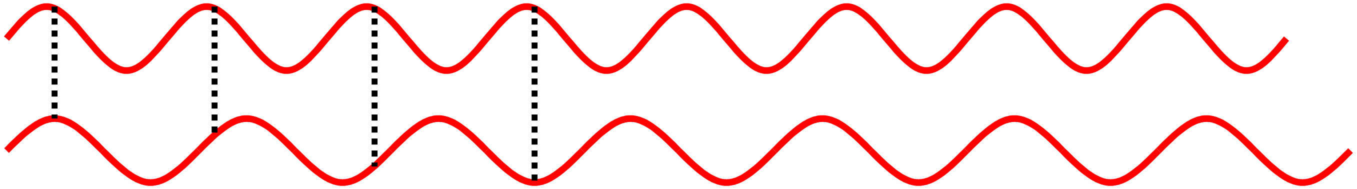


$$E(x) = E_0 \sin(\omega t + \varphi_1)$$

$$E(x) = E_0 \sin(\omega t + \varphi_2)$$

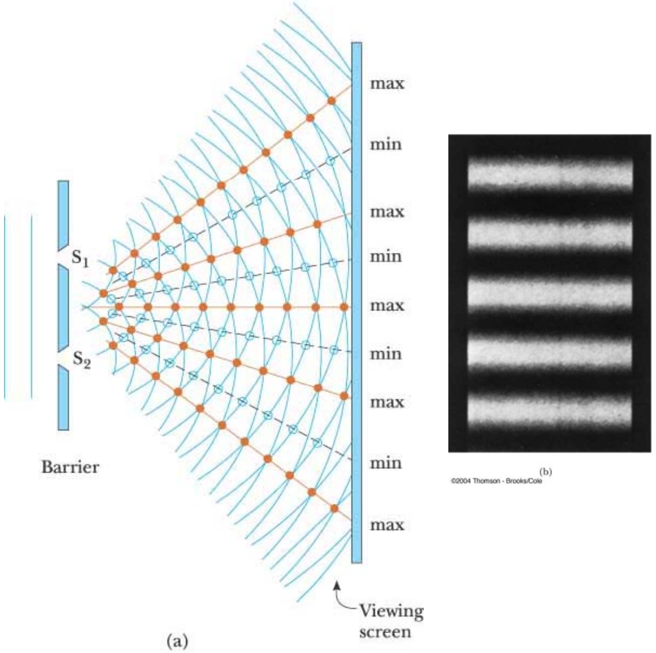
The sources should be monochromatic (have the same frequency)

$$E(x) = E_0 \sin(\omega_1 t + \varphi)$$



$$E(x) = E_0 \sin(\omega_2 t + \varphi)$$

1. Double-Slit Experiment (interference)



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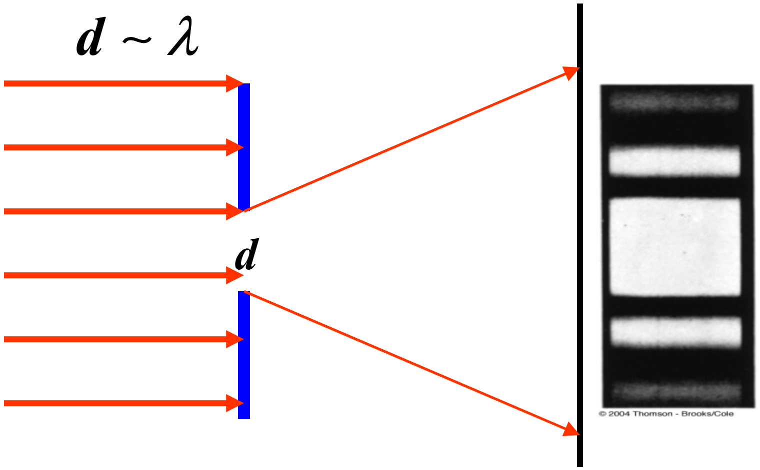
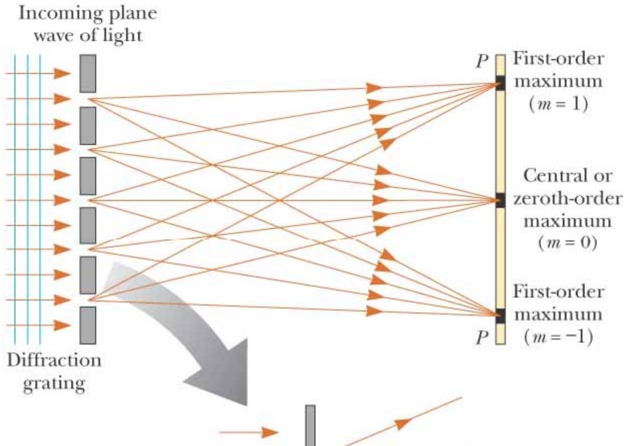
2. Single-Slit Diffraction

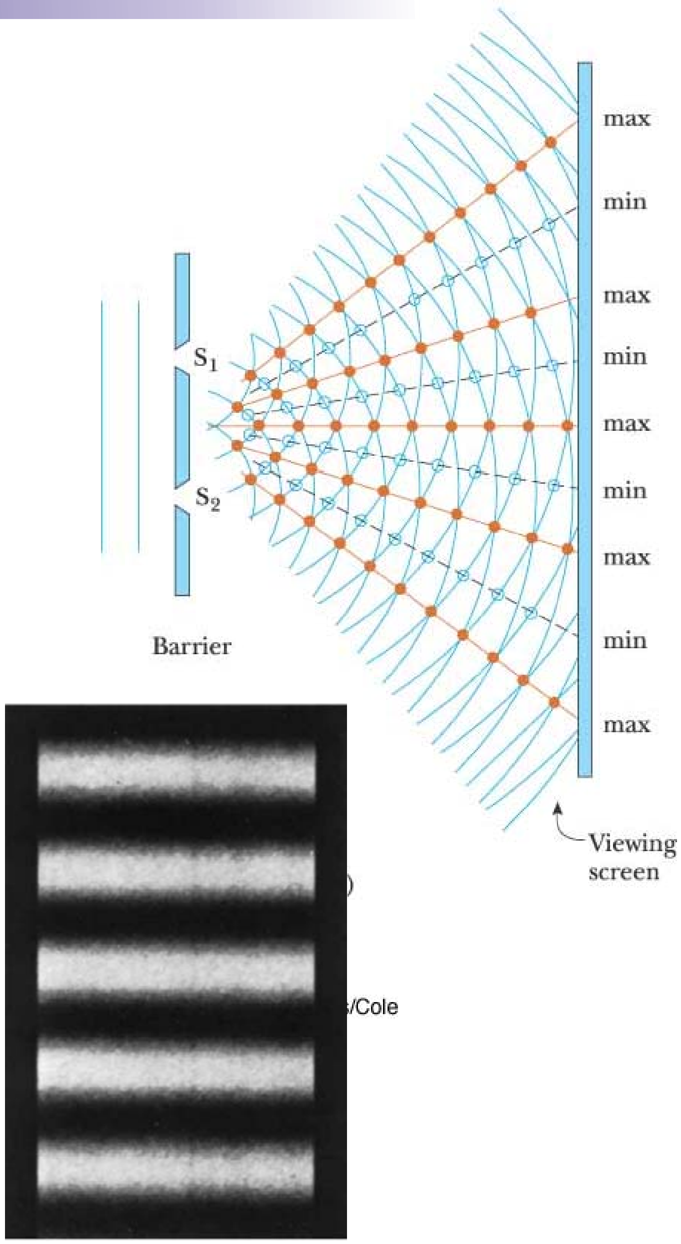


3. Diffraction Grading



Young’s Double-Slit Experiment

- Thomas Young first demonstrated interference in light waves from two sources in 1801
- The narrow slits S_1 and S_2 act as sources of waves
- The waves emerging from the slits originate from the *same wave front* and therefore *are always in phase*



Double-Slit Experiment: Interference

$$E(x) = E_0 \sin\left(\omega t + \varphi_x\right)$$

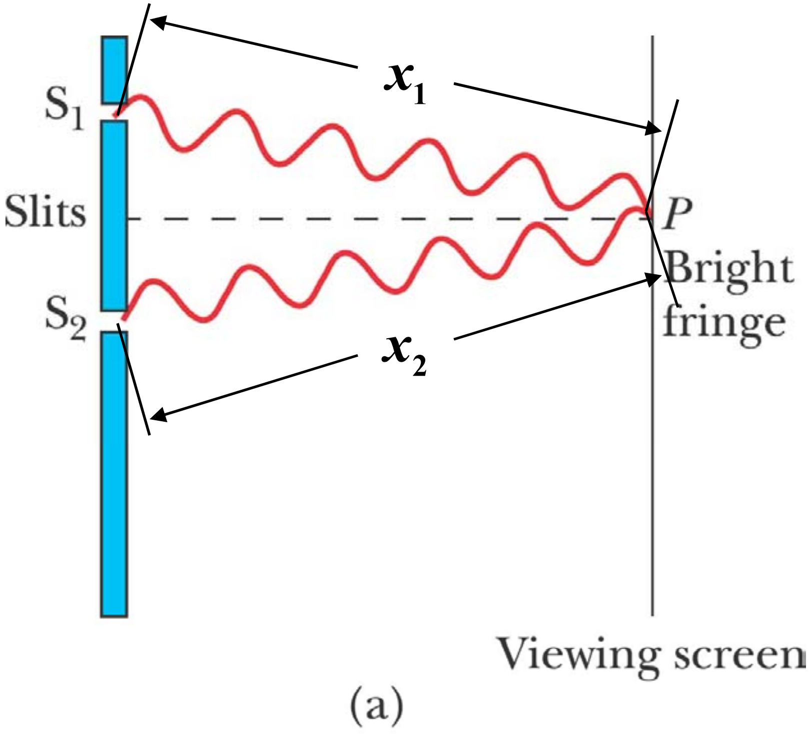
$$\varphi_x = 2\pi \frac{x}{\lambda}$$

The phase of wave 1:

$$\varphi_{x,1} = 2\pi \frac{x_1}{\lambda}$$

The phase of wave 2:

$$\varphi_{x,2} = 2\pi \frac{x_2}{\lambda}$$



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Constructive Interference: (bright fringe) $\varphi_{x,2} - \varphi_{x,1} = 2\pi n$ where ***n*** is integer

$$2\pi \frac{x_2}{\lambda} - 2\pi \frac{x_1}{\lambda} = 2\pi n \longrightarrow \boxed{x_2 - x_1 = n\lambda}$$

Destructive Interference: (dark fringe) $\varphi_{x,2} - \varphi_{x,1} = \pi + 2\pi n$ where ***n*** is integer

$$\boxed{x_2 - x_1 = \frac{\lambda}{2} + n\lambda}$$

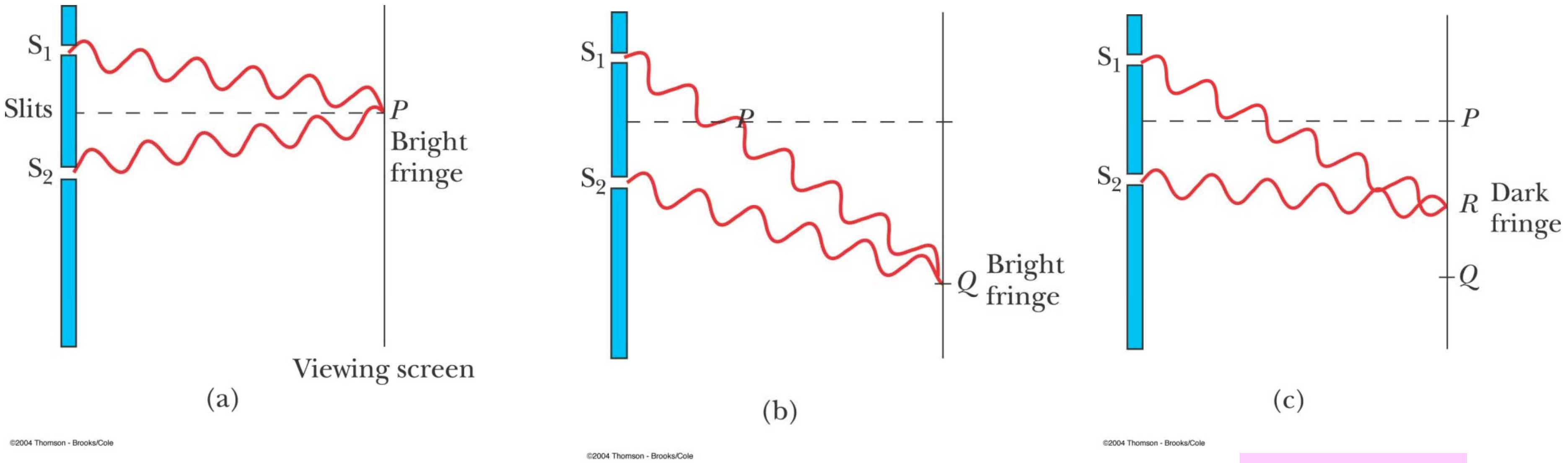
Double-Slit Experiment: Interference

Constructive Interference:
(bright fringe)

$$x_2 - x_1 = n\lambda$$

Destructive Interference:
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + n\lambda$$



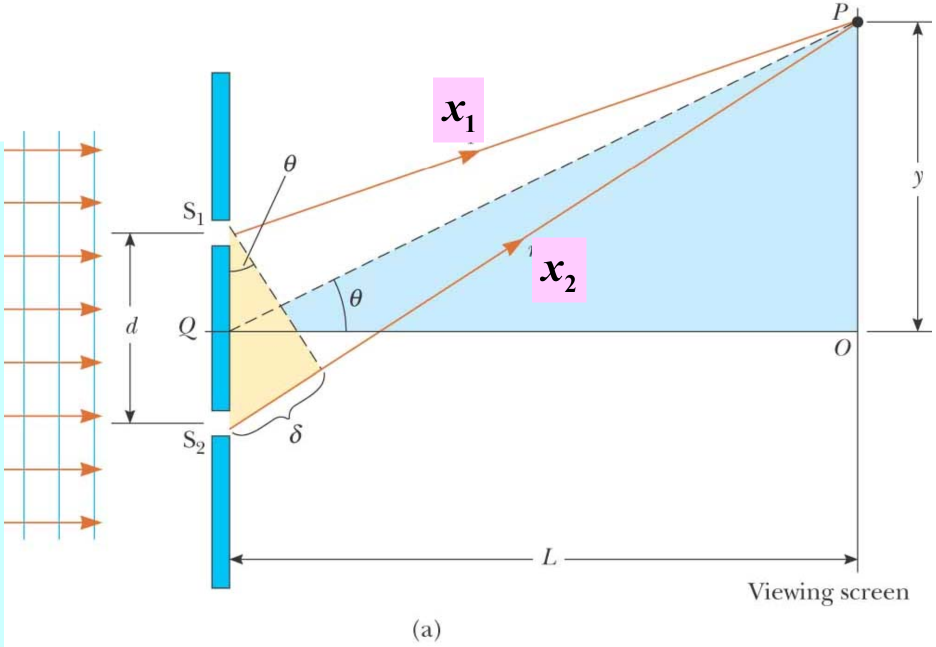
$$x_2 - x_1 = 0$$

$$x_2 - x_1 = \lambda$$

$$x_2 - x_1 = \frac{\lambda}{2}$$

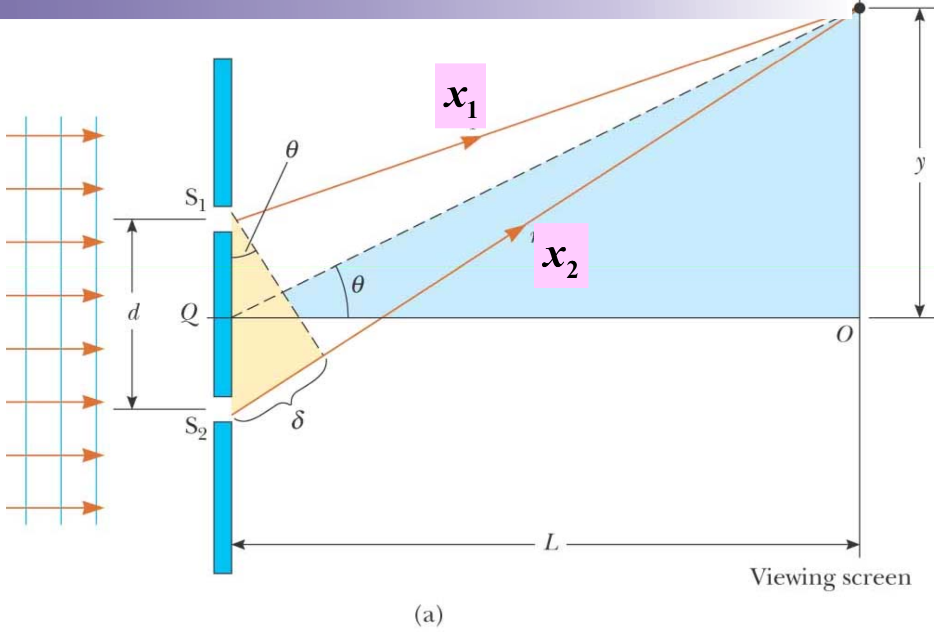
Double-Slit Experiment: Interference

- The path difference, δ, is found from the tan triangle
- $\delta = x_2 - x_1 = d \sin \theta$
  - This assumes the paths are parallel
  - Not exactly true, but a very good approximation if L is much greater than d



Double-Slit Experiment: Interference

$\delta = x_2 - x_1 = d \sin \theta$



(a)

Bright fringes (constructive interference):

$\delta = d \sin \theta = n\lambda$ $n = 0, \pm 1, \pm 2, \ldots$

n is called the **order number**

- when $n = 0$, it is the *zeroth-order maximum*
- when $n = \pm 1$, it is called the *first-order maximum*

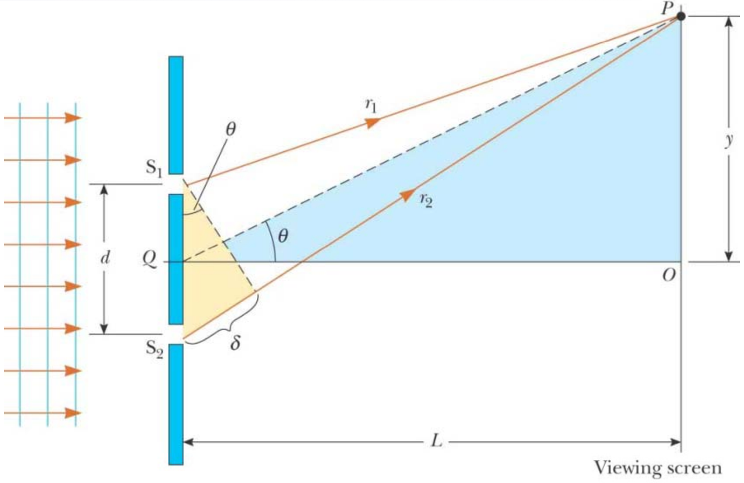
Dark fringes (destructive interference):

$\delta = d \sin \theta = (n + \frac{1}{2})\lambda$ $n = 0, \pm 1, \pm 2, \ldots$

Double-Slit Experiment: Interference

$$\delta = x_2 - x_1 = d \sin \theta$$

The positions of the fringes can be measured vertically from the zeroth-order maximum



(a)

θ is small and therefore the small angle approximation $\tan \theta \sim \sin \theta$ can be used

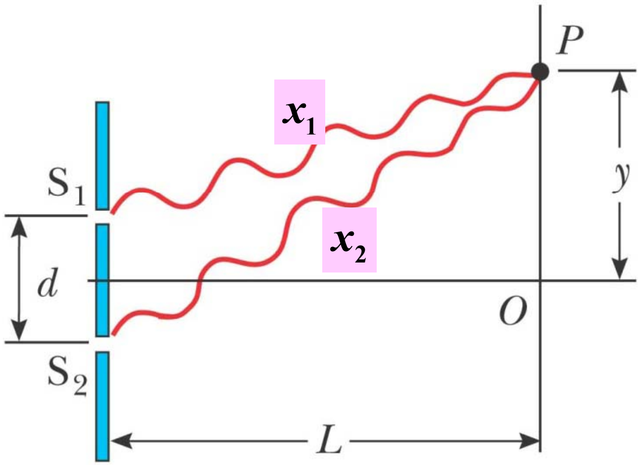
$y = L \tan \theta \approx L \sin \theta$

For bright fringes

$$y_{\text{bright}} = \frac{\lambda L}{d} n \quad (\text{n} = 0, \pm 1, \pm 2 \ldots)$$

For dark fringes

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(n + \frac{1}{2}\right) \quad (n = 0, \pm 1, \pm 2 \ldots)$$



Constructive Interference: $\varphi_{x,2} - \varphi_{x,1} = 2\pi n$ where ***n*** is integer $\boldsymbol{n = 0, \pm 1, \pm 2, ...}$

(bright fringe)

$$2\pi \frac{x_2}{\lambda} - 2\pi \frac{x_1}{\lambda} = 2\pi n \longrightarrow \boldsymbol{x_2 - x_1 = n\lambda}$$

$$y_{\text{bright}} = \frac{\lambda L}{d} n \quad (\text{n} = 0, \pm 1, \pm 2 \ldots)$$

Destructive Interference: $\varphi_{x,2} - \varphi_{x,1} = \pi + 2\pi n$ where ***n*** is integer $\boldsymbol{n = 0, \pm 1, \pm 2, ...}$

(dark fringe)

$$\boldsymbol{x_2 - x_1 = \frac{\lambda}{2} + n\lambda}$$

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(n + \frac{1}{2}\right) \quad (n = 0, \pm 1, \pm 2 \ldots)$$

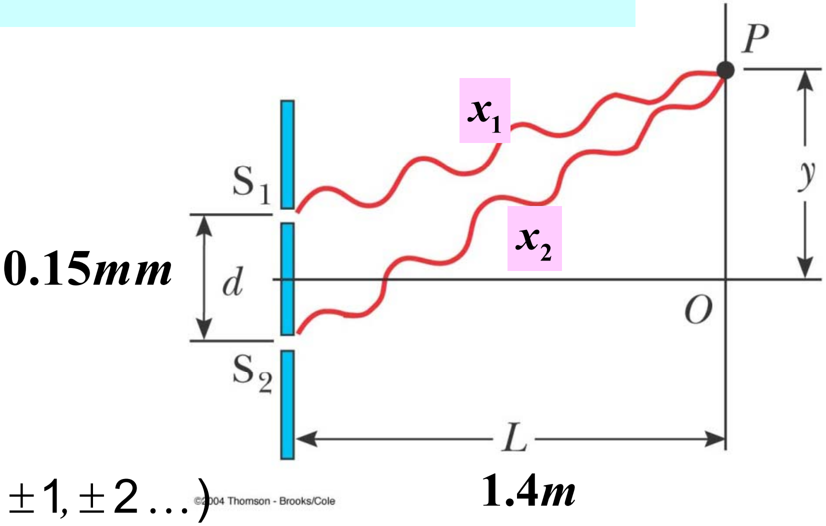
Double-Slit Experiment: Example

The two slits are separated by 0.150 mm, and the incident light includes light of wavelengths $\lambda_1 = 540nm$ and $\lambda_2 = 450nm$. At what minimal distance from the center of the screen the bright line of the λ_1 light coincides with a bright line of the λ_2 light

Bright lines:

$$y_{\text{bright},1} = \frac{\lambda_1 L}{d} n_1 \quad (n_1 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{\text{bright},2} = \frac{\lambda_2 L}{d} n_2 \quad (n_2 = 0, \pm 1, \pm 2 \ldots)$$



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$$y_{\text{bright},1} = \frac{540 \cdot 10^{-9} \cdot 1.4}{0.15 \cdot 10^{-3}} n_1(\text{m}) = 5n_1(\text{mm}) \qquad (n_1 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{\text{bright},2} = \frac{450 \cdot 10^{-9} \cdot 1.4}{0.15 \cdot 10^{-3}} n_2(\text{m}) \approx 4n_2(\text{mm}) \qquad (n_2 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{\text{bright},1} = 0, 5, 10, 15, 20, 25\ldots(\text{mm})$$

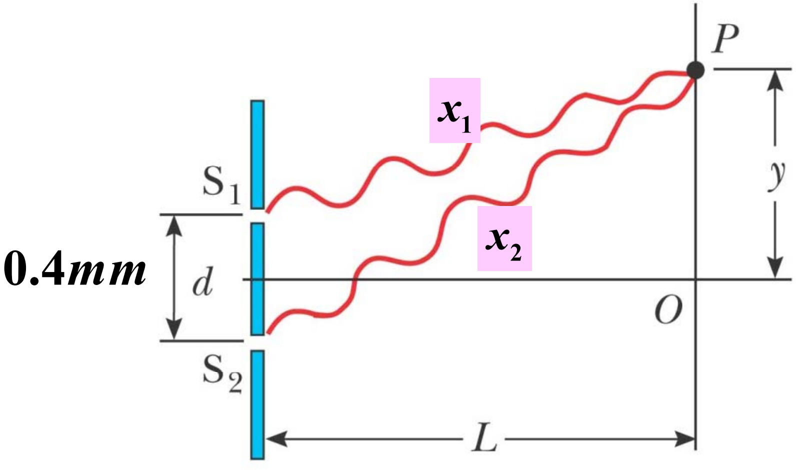
$$y_{\text{bright},1} = 0, 4, 8, 12, 16, 20\ldots(\text{mm})$$

Double-Slit Experiment: Example

Light with a wavelength of ***442 nm*** passes through a double-slip system that has a slip separation ***d=0.4 mm***. Determine ***L*** so that the first dark fringe appears directly opposite both slits.

Dark lines:

$$y_{\text{dark,n}} = \frac{\lambda L}{d}\left(n + \frac{1}{2}\right) \quad (n = 0, \pm 1, \pm 2 \ldots)$$



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$$y_{\text{dark,1}} = \frac{d}{2}$$

$$y_{\text{dark,1}} = \frac{1}{2}\frac{\lambda L}{d}$$

$$\frac{d}{2} = \frac{1}{2}\frac{\lambda L}{d}$$

$$L = \frac{d^2}{\lambda} = \frac{0.4^2 \cdot 10^{-6} m^2}{442 \cdot 10^{-9} m} = 0.36m = 36cm$$

Chapter 17

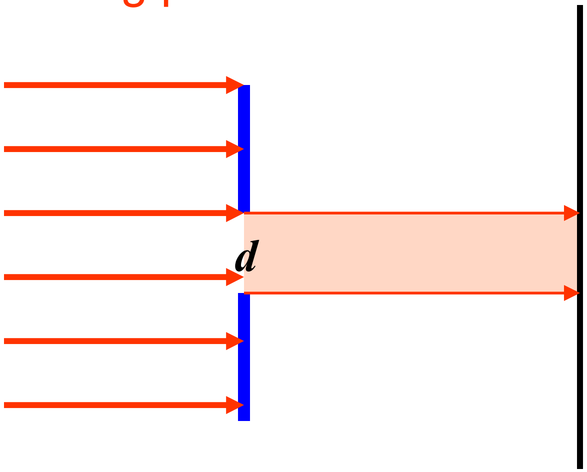
Diffraction Pattern and Interference

Diffraction

Diffraction:

Light spreads beyond the narrow path defined by the slit into regions that would be in shadow if light traveled in straight lines

Wrong picture if $d \simeq \lambda$

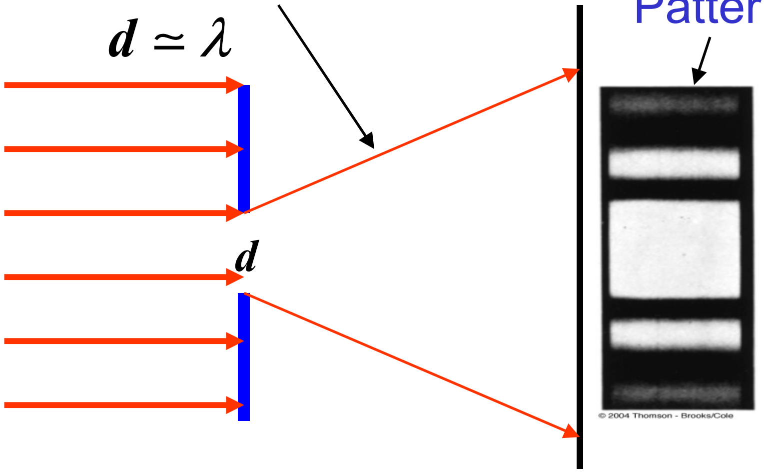


Geometric Optics - if $d >> \lambda$

Diffraction

$d \simeq \lambda$

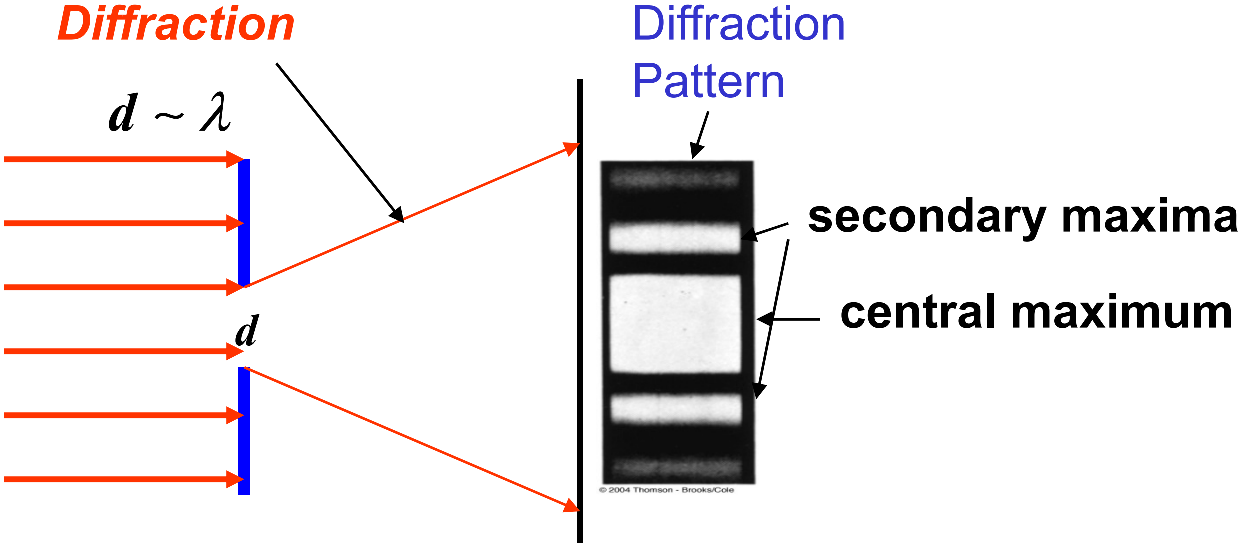
Diffraction Pattern



Diffraction and Interference are closely related;

Diffraction Patterns are due to Interference

Diffraction Pattern



Diffraction

$d \sim \lambda$

d

Diffraction Pattern

secondary maxima

central maximum

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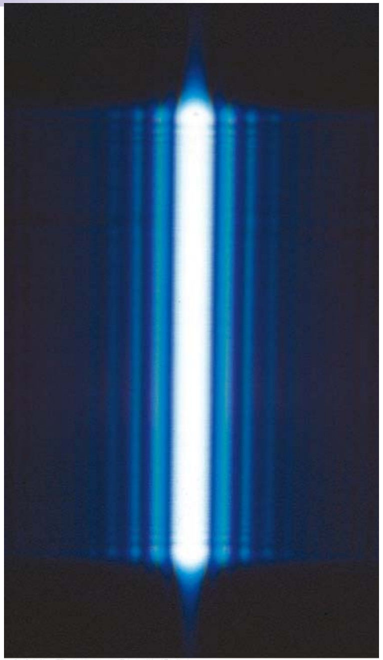

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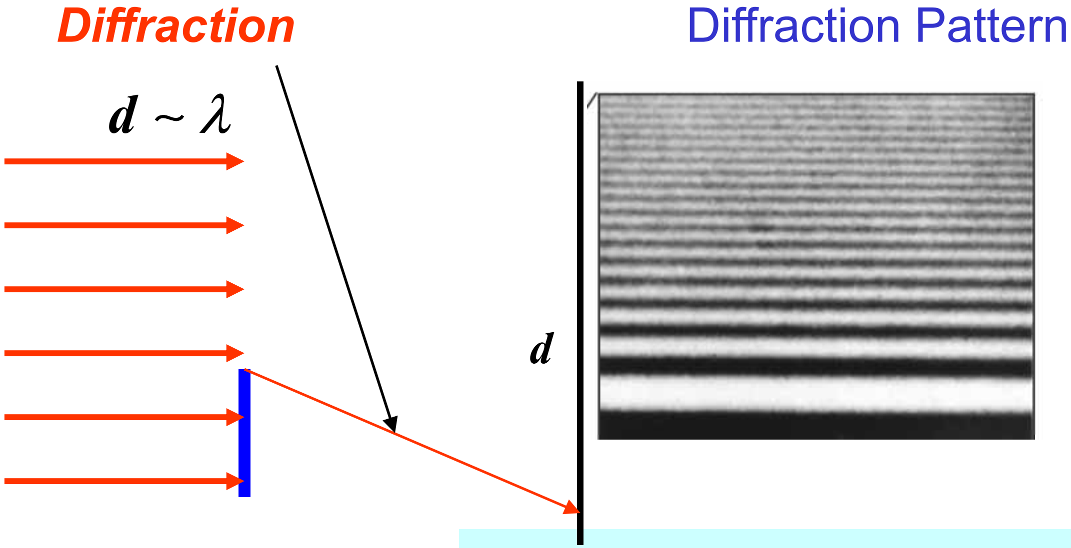


Diffraction

$d \sim \lambda$

d

Diffraction Pattern

Diffraction Pattern is similar to Interference Pattern

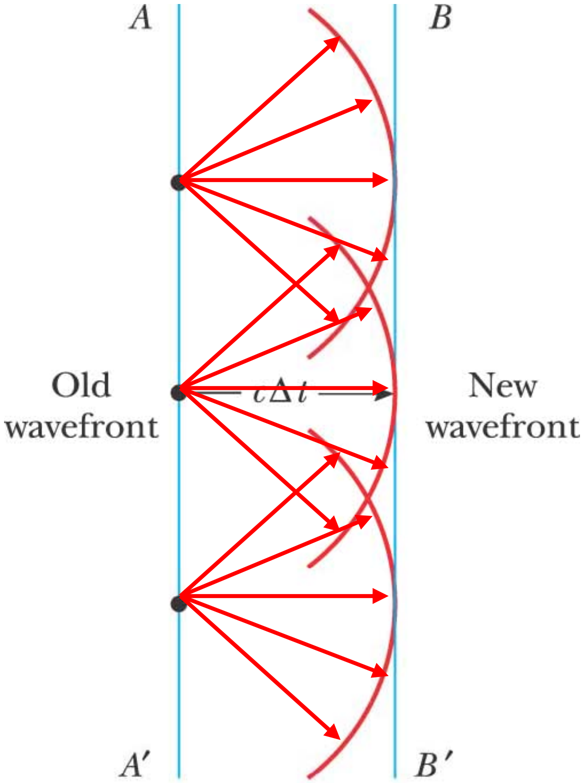
Huygens’s Principle

Huygens's Principle

Huygens's Principle is a geometric construction for determining the position of a new wave at some point based on the knowledge of the wave front that preceded it

- **All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outward through a medium with speeds characteristic of waves in that medium**

- **After some time has passed, the new position of the wave front is the surface tangent to the wavelets**

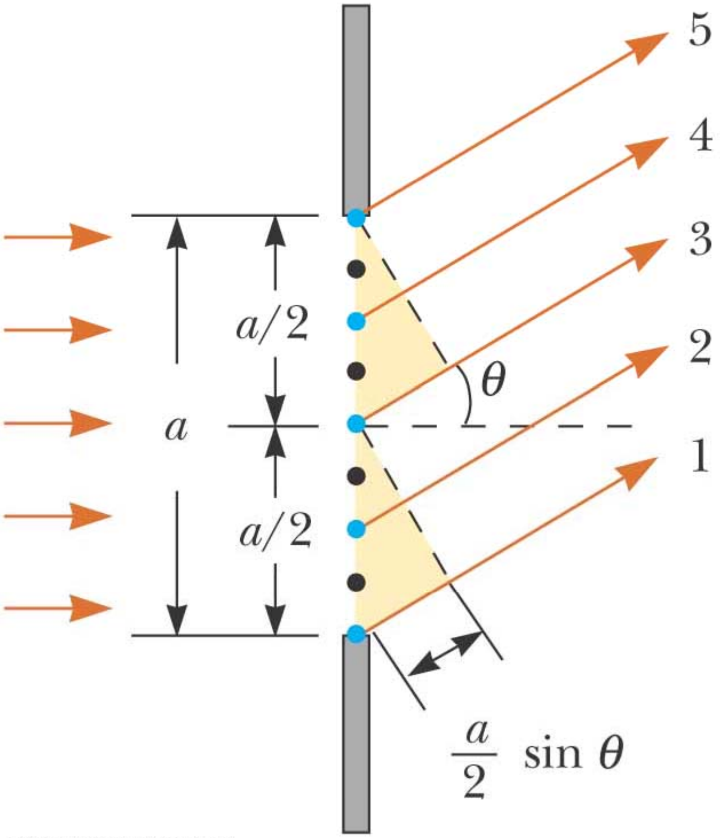


(a)

Single-Slip Diffraction

Single Slit Diffraction

- Each portion of the slit acts as a source of light waves
- Therefore, light from one portion of the slit can *interfere* with light from another portion



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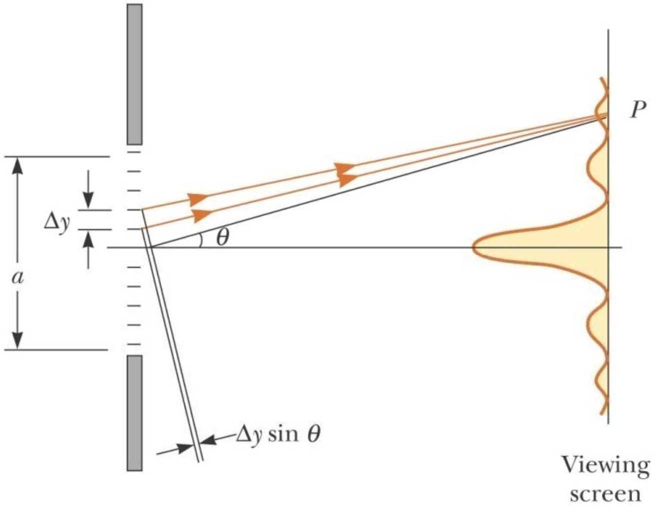
Intensity of Single-Slit Diffraction Pattern

$$I(\varphi) = I_{\max}\left[\frac{\sin(\varphi/2)}{\varphi/2}\right]^2$$

$$\varphi = \frac{2\pi a \sin\theta}{\lambda}$$

$$I(\theta) = I_{\max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$



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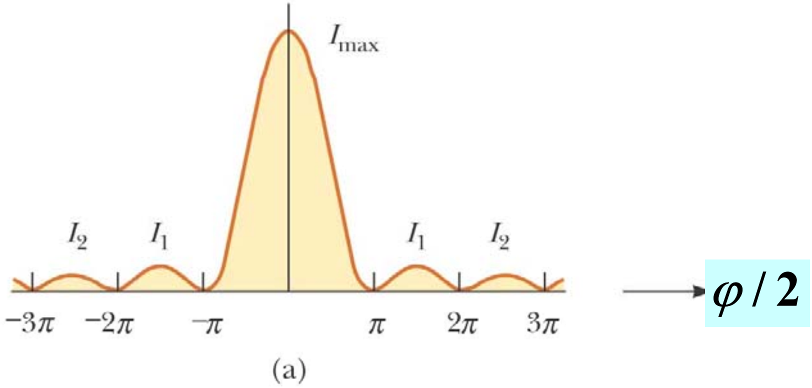
The first minimum occurs at

$\sin(\varphi/2) = 0$ or $\varphi = 2\pi$ or

$\sin\theta_{dark} = \lambda / a$

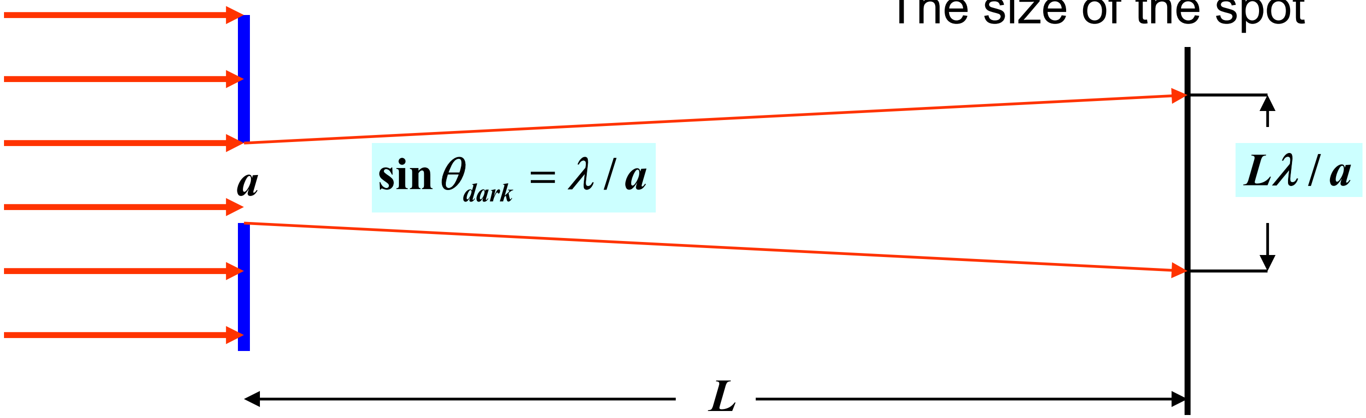


(a)

(b)

Diffraction

$$I(\theta) = I_{\max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$



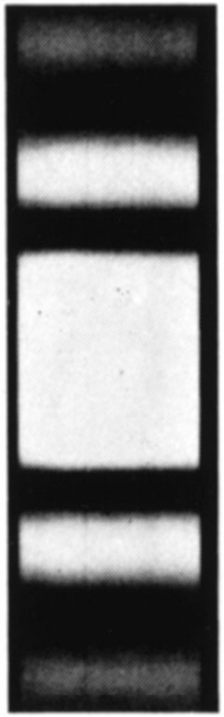
The size of the spot

a

$\sin\theta_{dark} = \lambda / a$

$L\lambda / a$

L



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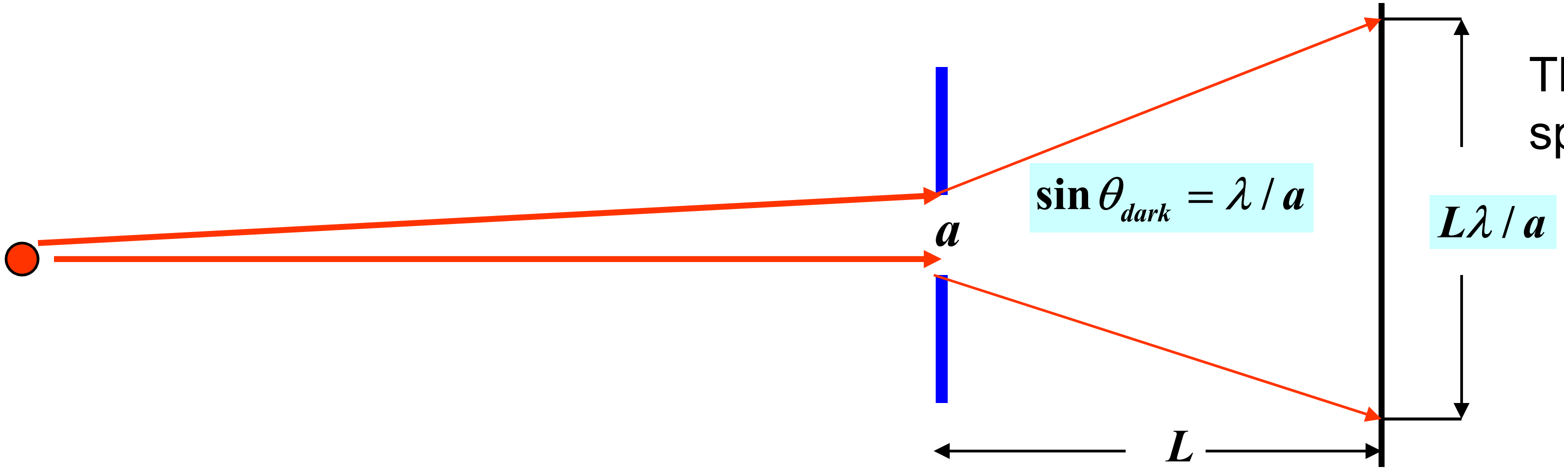


a

$\sin\theta_{dark} = \lambda / a$

$L\lambda / a$

L

The size of the spot (image)

Diffraction: Example

The source of the light emits the light with wavelength $\lambda = 540nm$.

The diffraction pattern is observed in the water, $n = 1.33$.

L = 10m, a=0.5 mm

What is the size of the spot, *D* ?

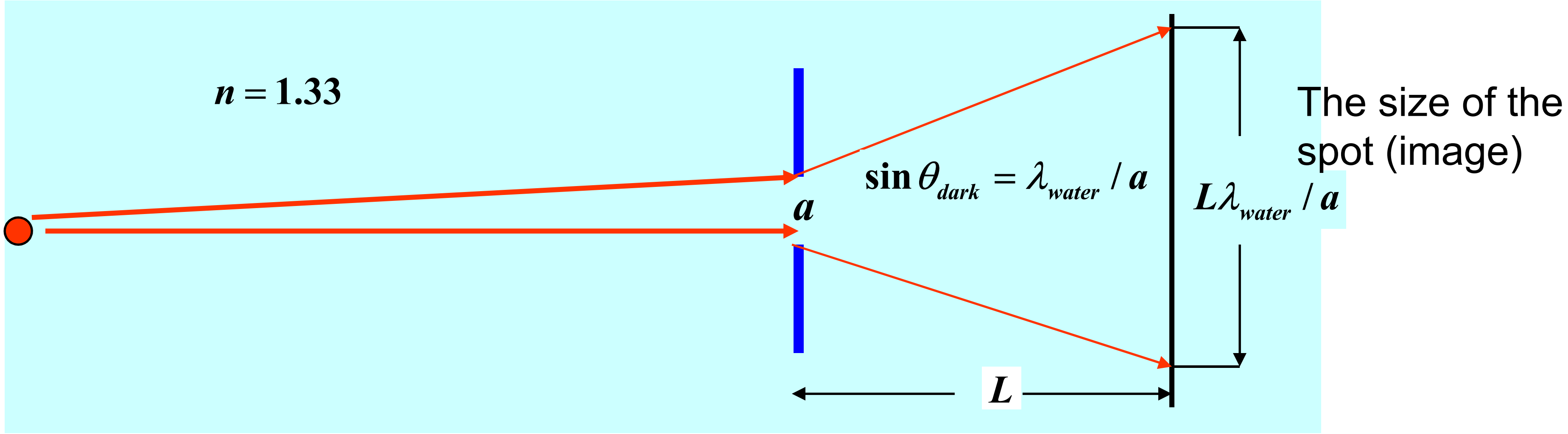
wavelength in the water

$\sin\theta_{dark} = \lambda_{water} / a$

$D = L\lambda_{water} / a$

$\lambda_{water} = \lambda / n$

$$D = \frac{L\lambda}{na} = \frac{10 \cdot 540 \cdot 10^{-9}}{1.33 \cdot 0.5 \cdot 10^{-3}} m = 8 \cdot 10^{-3} m = 8mm$$

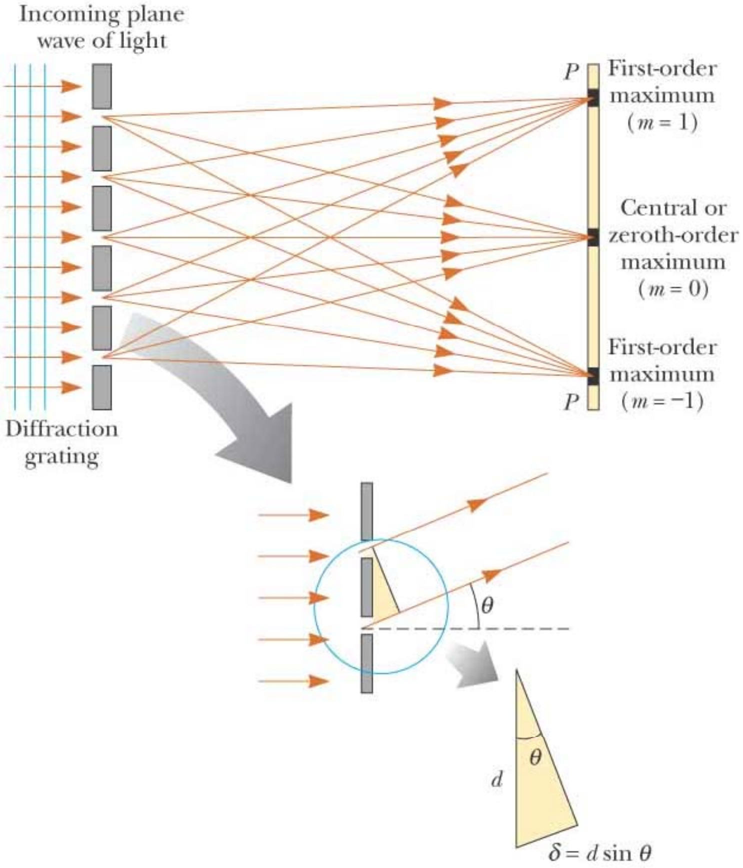


Chapter 17.3

Diffraction Grading

Diffraction Grading

- The ***diffraction grating*** consists of a ***large number*** of ***equally spaced*** parallel slits
  - A typical grating contains several thousand lines per centimeter
- The intensity of the pattern on the screen is the result of the ***combined effects of interference and diffraction***
  - Each slit produces diffraction, and the diffracted beams interfere with one another to form the final pattern



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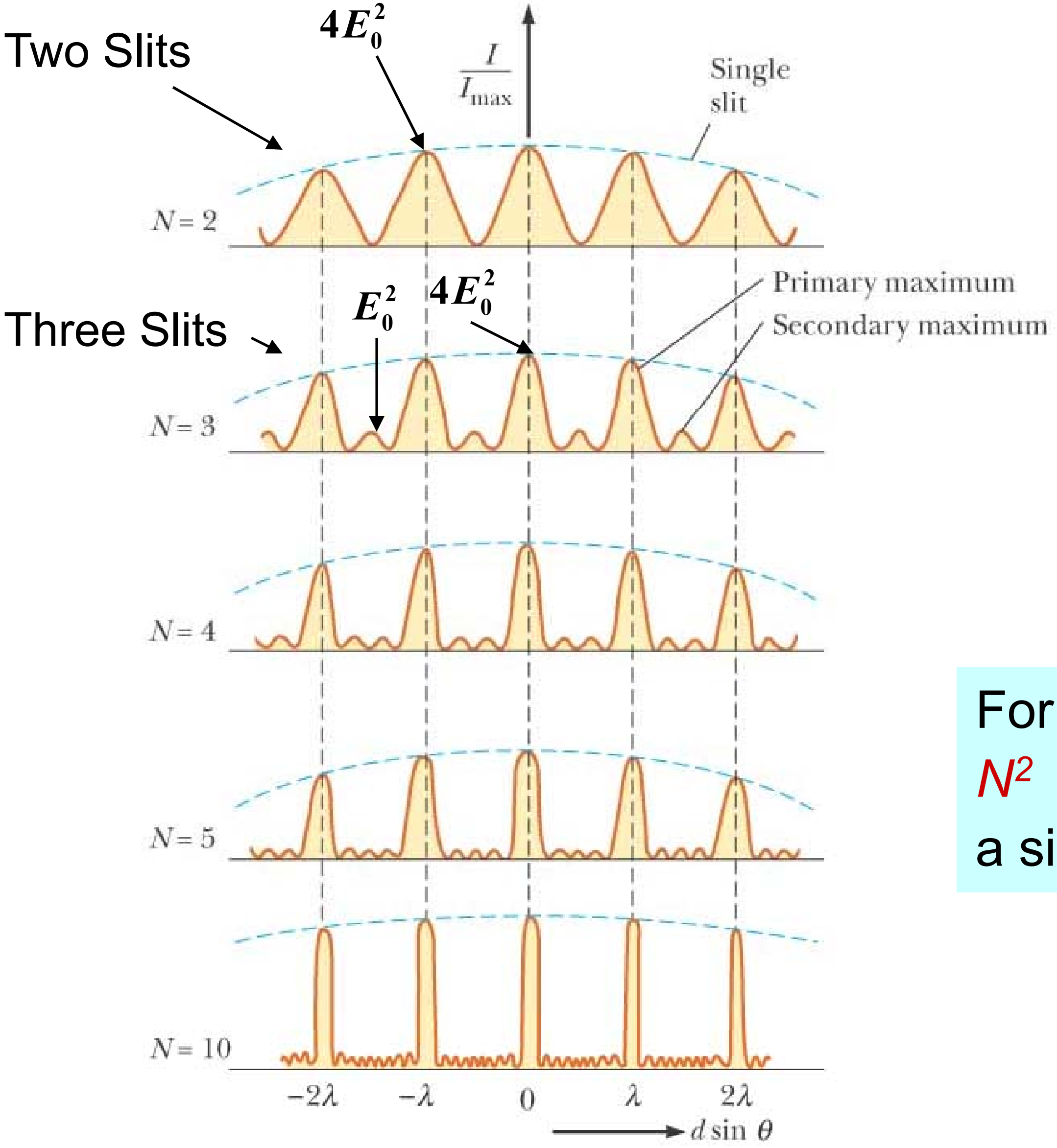
N-Slit Interference: Intensity Graph



For *N* slits, the primary maxima is N^2 times greater than that due to a single slit

Diffraction Grading

The condition for *maxima* is

$$\Delta\varphi = 2\pi m \quad , \qquad m = 0, \pm 1, \pm 2, \ldots$$

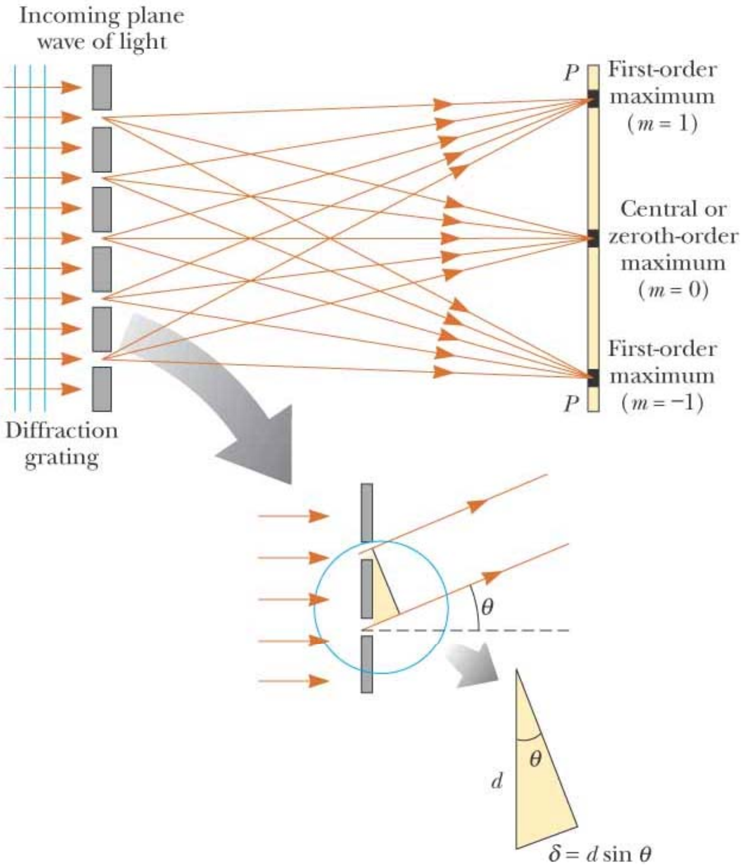
$$\Delta\varphi = 2\pi \frac{\delta}{\lambda} = 2\pi \frac{d \sin\theta_{bright}}{\lambda}$$

then

$$d \sin\theta_{bright} = m\lambda$$

The integer ***m*** is the *order number* of the diffraction pattern

$$\Delta\varphi = 2\pi m$$



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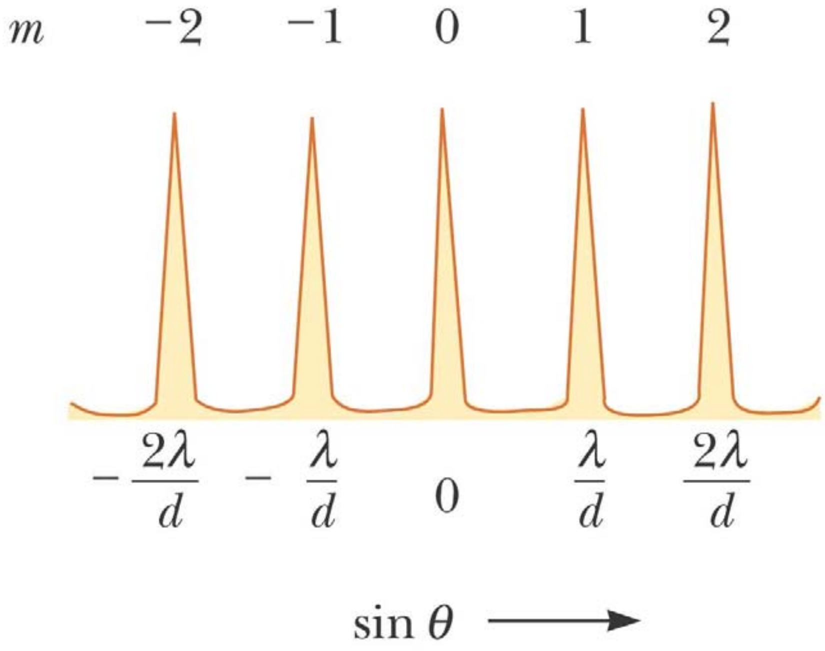
Diffraction Grading

- All the wavelengths are seen at ***m* = 0**
  - This is called the zeroth-order maximum
- The first-order maximum corresponds to *m* = 1
- Note the sharpness of the principle maxima and the broad range of the dark areas

$$d \sin \theta_{bright} = m\lambda$$



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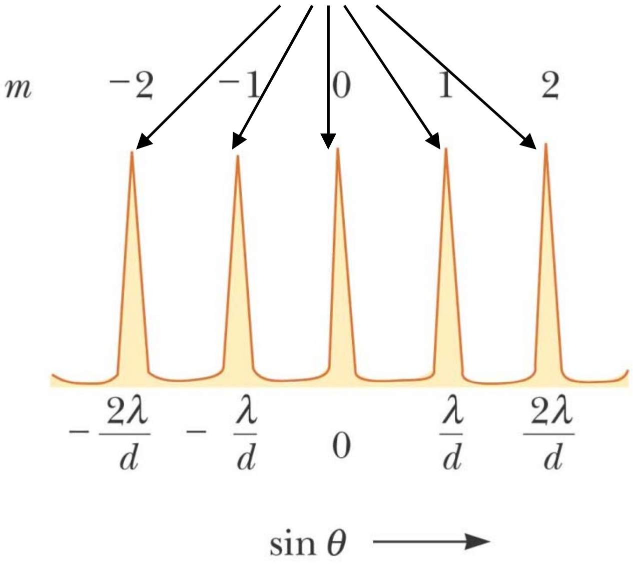
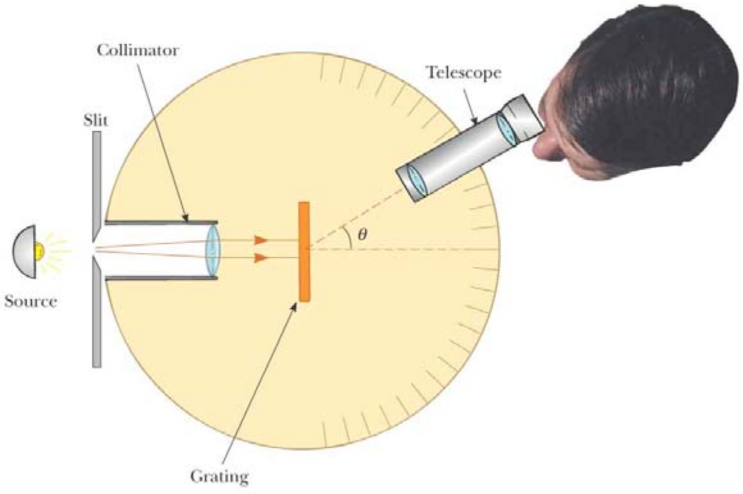
Diffraction Grading Spectrometer

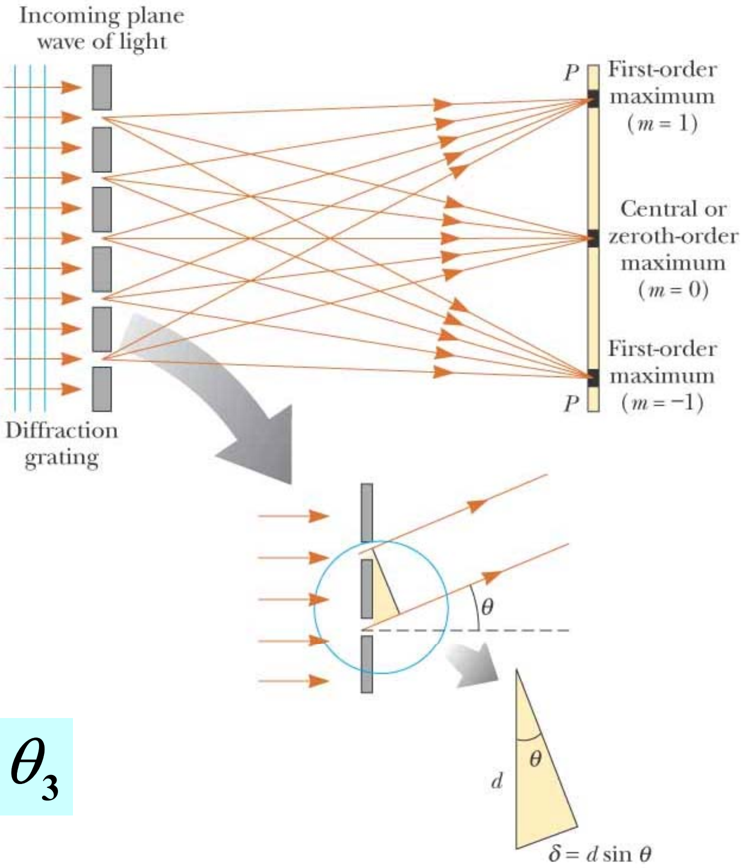
- The collimated beam is incident on the grating
- The diffracted light leaves the gratings and the telescope is used to view the image
- The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders


sharp peaks



Diffraction Grading: Example

Three discrete spectral lines occur at angles 10.09^0, 13.71^0, and 14.77^0 in the first order spectrum of a grading spectrometer. If the grading has ***N*=**3600 slits per centimeter, what are the wavelength of the light?



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$$d\sin\theta_{bright}=m\lambda$$

First order means that ***m=1,*** then

$$\lambda_1=d\sin\theta_1 \qquad \lambda_2=d\sin\theta_2 \qquad \lambda_3=d\sin\theta_3$$

$$d=\frac{1cm}{N}$$

Then

$$\lambda_1=\frac{1}{N}\sin\theta_1=\frac{\sin 10.09}{3600}cm=480nm \qquad \lambda_2=\frac{\sin 13.71}{3600}cm=658nm \qquad \lambda_3=\frac{\sin 14.77}{3600}cm=708nm$$

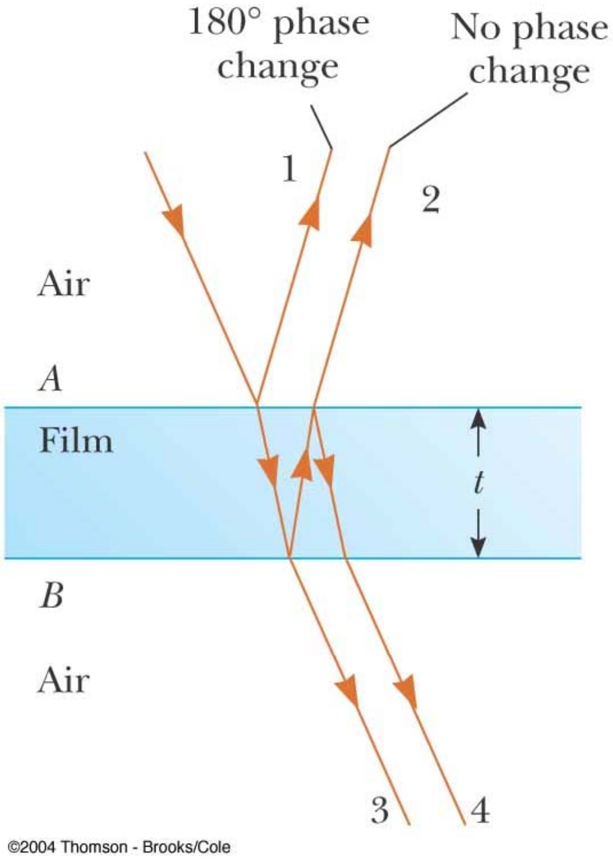
Chapter 17.4

Interference in Thin Films

Interference in Thin Films

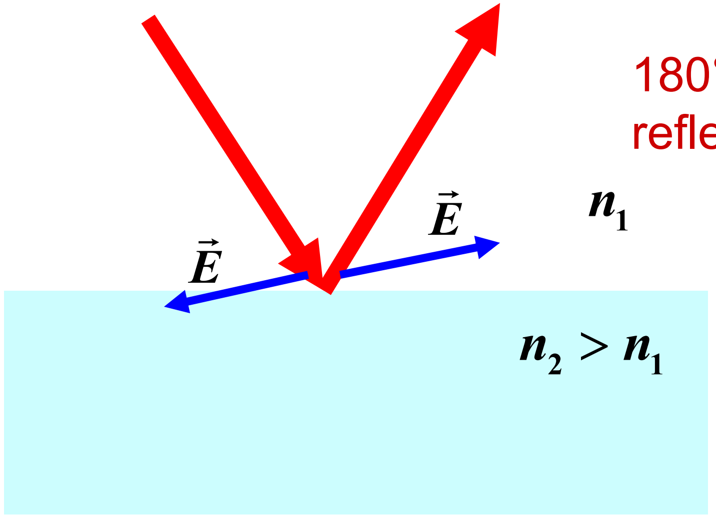
Interference:

1. **Rays 1 and 2**
2. **Rays 3 and 4**



No phase change

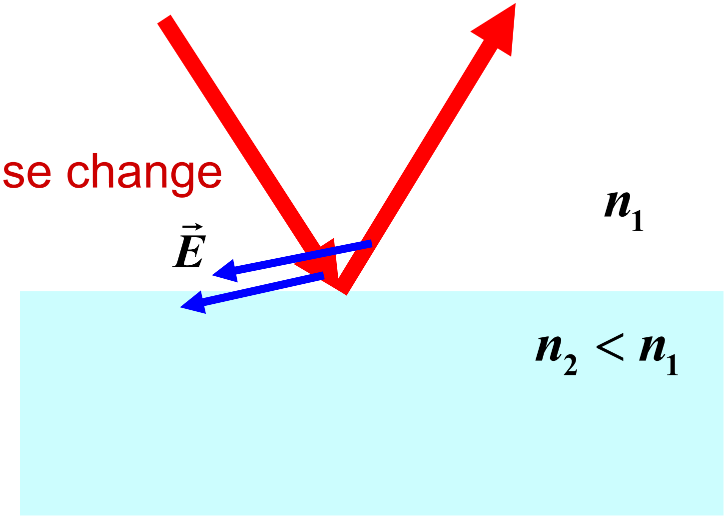
Phase Change due to Reflection



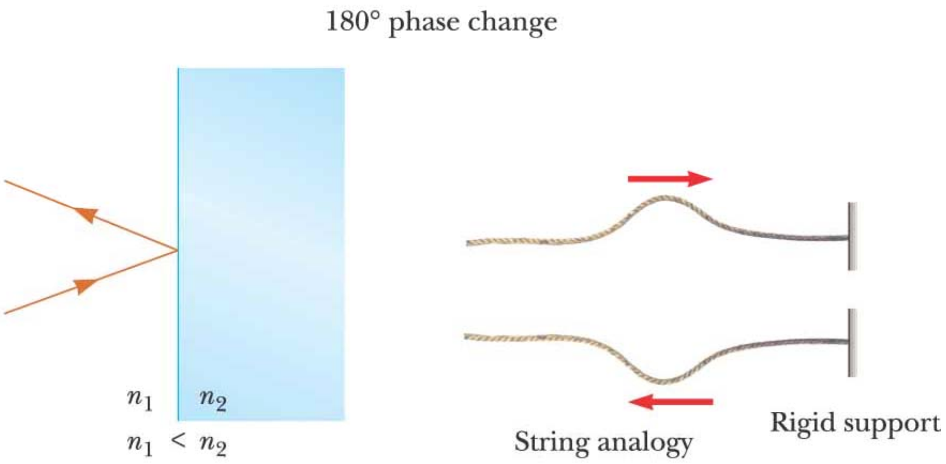
180° phase change produced by reflection

Only upon reflection from a medium of higher index of refraction than the one in which a wave was traveling

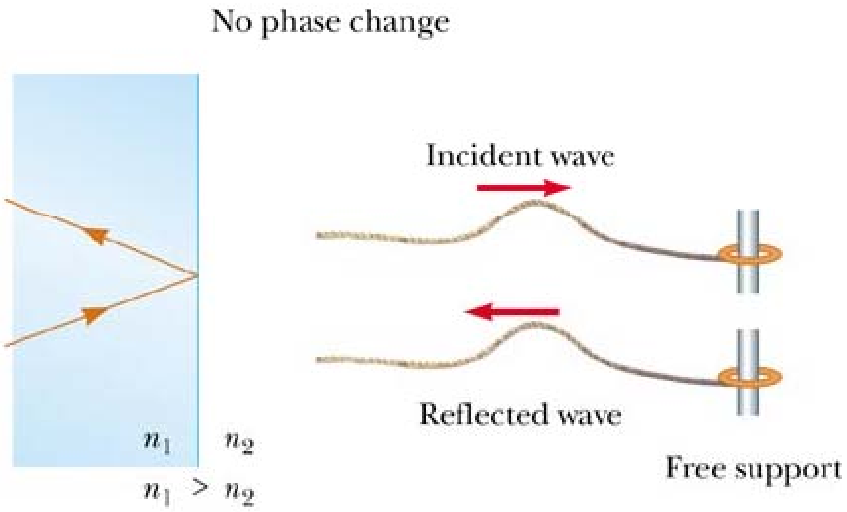
No phase change



Phase Change due to Reflection



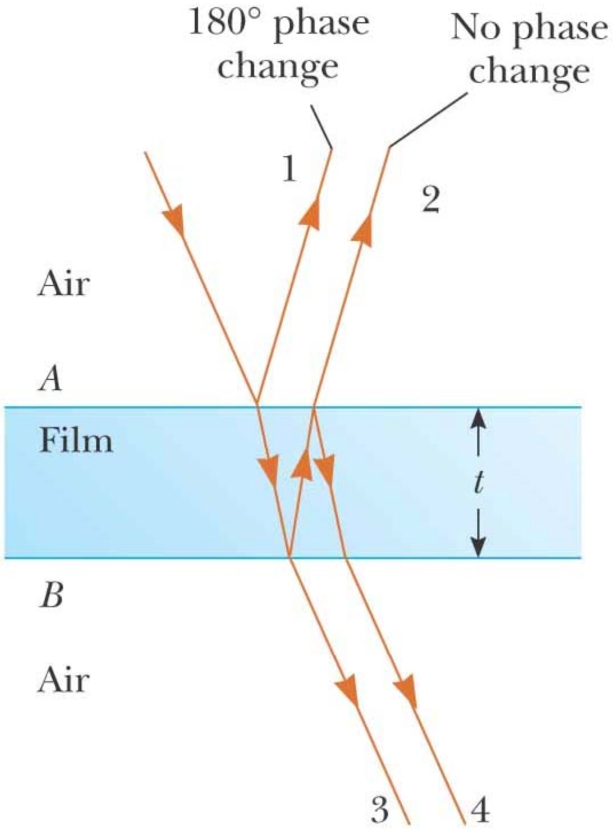
(a)



Interference in Thin Films

Interference:

1. **Rays 1 and 2**
2. **Rays 3 and 4**



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No phase change

Interference in Thin Films

Interference:

1. Rays 1 and 2
2. Rays 3 and 4

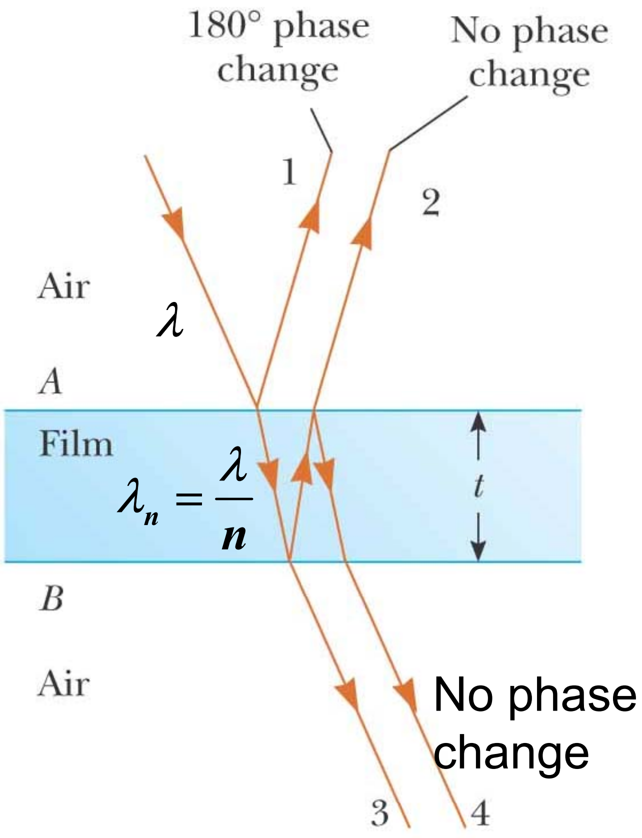
1. Rays 1 and 2

Phase difference:

$$\Delta\varphi = \pi + 2\pi\frac{2t}{\lambda_n} = \pi + 2\pi\frac{2nt}{\lambda}$$

Due to phase change

Ray *2* travels an additional distance *2t* compared to Ray *1*



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Constructive interference:

$$\Delta\varphi = \pi + 2\pi\frac{2nt}{\lambda} = 2\pi m$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

Destructive interference:

$$\Delta\varphi = \pi + 2\pi\frac{2nt}{\lambda} = \pi + 2\pi m$$

$$2nt = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

Interference in Thin Films

Interference:

1. Rays 1 and 2
2. Rays 3 and 4

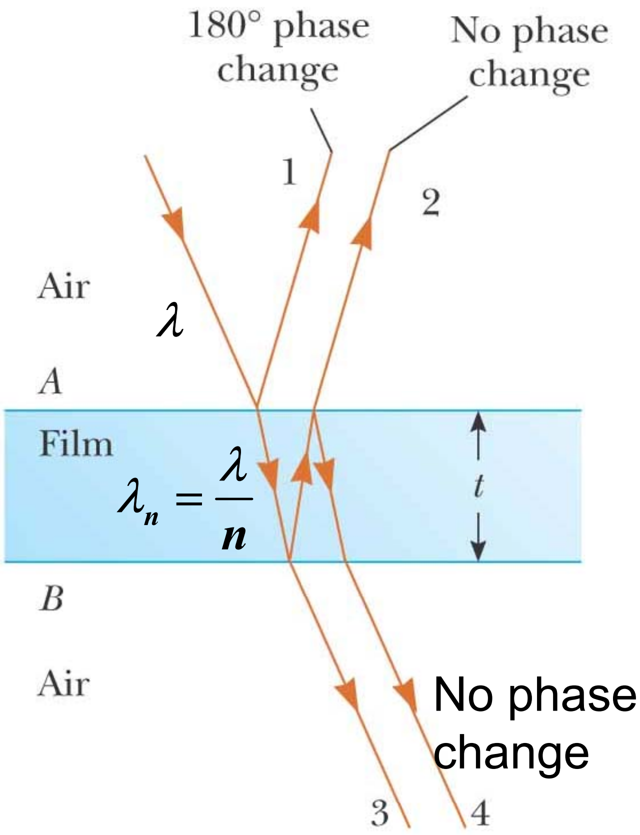
2. Rays 3 and 4

Phase difference:

$$\Delta\varphi = 0 + 2\pi\frac{2t}{\lambda_n} = 2\pi\frac{2nt}{\lambda}$$

no phase change

Ray *4* travels an additional distance *2t* compared to Ray *3*



No phase change

Constructive interference:

$$\Delta\varphi = 2\pi\frac{2nt}{\lambda} = 2\pi m$$

$$2nt = m\lambda \qquad m = 0, \pm1, \pm2, \ldots$$

Destructive interference:

$$\Delta\varphi = 2\pi\frac{2nt}{\lambda} = \pi + 2\pi m$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, \pm1, \pm2, \ldots$$

Soap Bubble Example

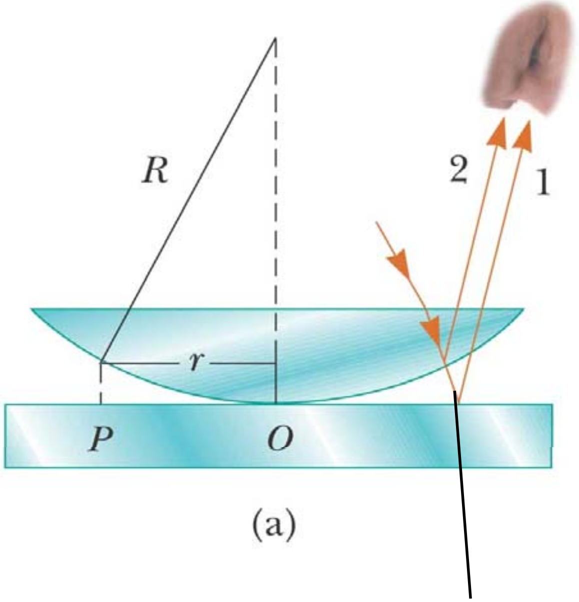
Constructive interference:

$$2nt = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

Destructive interference:

$$2nt = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

Newton’s Rings



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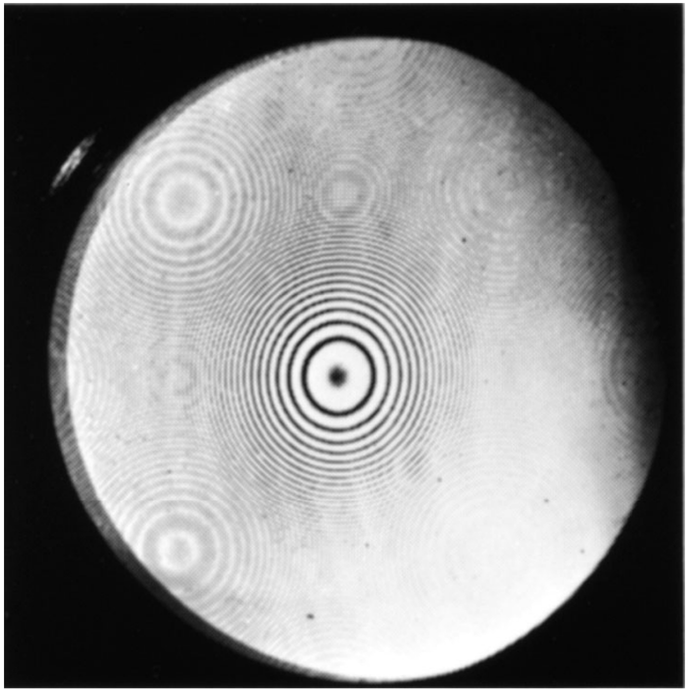
phase change

Constructive interference:

$$2nt = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

Destructive interference:

$$2nt = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$



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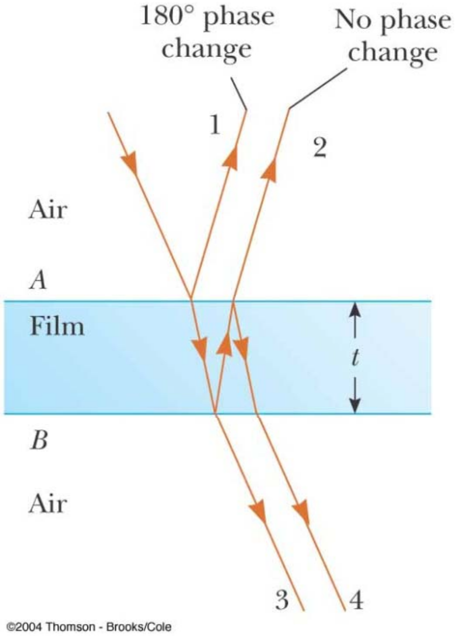
Thin Films: Example

A soap bubble ***(n=1.33)*** is floating in air: If the thickness of the bubble wall is ***115 nm***, what is the wavelength of the light that is most strongly reflected?



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Constructive interference of “1” and “2”

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

(remember phase shift)

For m = 0

$$\lambda = 4nt = 4 \cdot 1.33 \cdot 115nm = 612nm$$

Orange

The same for the soap film on the glycerin ***(n=1.47)***

Constructive interference of “1” and “2”, but no phase shift (because $n_{glycerin} > n_{soap}$)

$$2nt = m\lambda$$

For m = 1

$$\lambda = 2nt = 2 \cdot 1.33 \cdot 115nm = 306nm$$

Ultraviolet