HW# 3

1 $\phi = \dfrac{Q_{in}}{\varepsilon_0}$

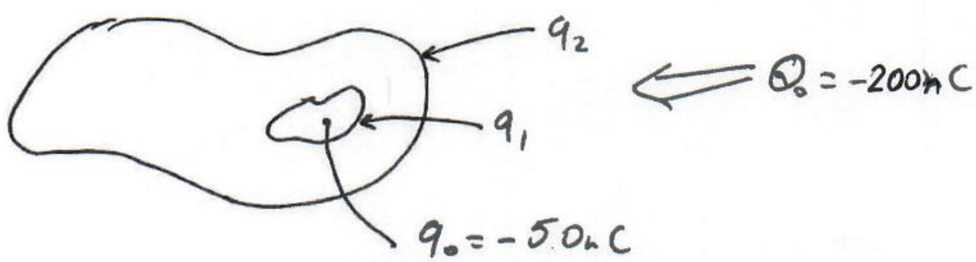
(a) $Q_{in} = (-15-4)\,nC = -19\cdot 10^{-9}\,C$, then

$$\phi = -\frac{19\cdot 10^{-9}}{8.85\cdot 10^{-12}} = \underline{-2.14\times 10^{3}}\left(\frac{N}{C}m^2\right)$$

(b) $Q_{in} = (-4-6+7)\,nC = -3\times 10^{-9}\,C$

$$\phi = \frac{-3\times 10^{-9}}{8.85\cdot 10^{-12}} = \underline{-0.33\times 10^{3}}\left(\frac{N}{C}\cdot m^2\right)$$

2



(a) $q_0 + q_1 = 0 \Rightarrow q_1 = -q_0 = \underline{50\,nC}$

(b) $q_1 + q_2 = Q_0 \Rightarrow q_2 = Q_0 - q_1 = -200 - 50 = \underline{-250\,nC}$

3 $\phi_{net} = 6\cdot\phi_1 = 6\cdot 500 = 3000\ \frac{N}{C}m^2$

through one face

$\phi_{net} = \dfrac{Q_{in}}{\varepsilon_0} \Rightarrow Q_{in} = \varepsilon_0\,\phi_{net} = 8.85\cdot 10^{-12}\cdot 3000 =$

$= 26.55\cdot 10^{-9}\,C = \underline{26.55\,nC}$

4 $V_p = V_1 + V_2$; $V_1 = k_e \frac{q_1}{r_1} = 9\cdot10^9 \cdot \frac{(+4\cdot10^{-9})}{0.01} =$

$= +3600\,(V)$

$V_2 = k_e \frac{q_2}{r_2} = 9\cdot10^9 \frac{(-6\cdot10^{-9})}{0.01} =$

$= -5400\,(V)$

$V_p = +3600 - 5400 = \underline{-1800\,(V)}$

5 $V_p = V_1 + V_2 + V_3 \Rightarrow \quad V_1 = k_e \frac{q_1}{r}$; $V_2 = k_e \frac{q_2}{r}$; $V_3 = k_e \frac{q_3}{r}$

$\underline{r = 0.02\ m}$

then:

$V_p = k_e \frac{(q_1+q_2+q_3)}{r} = 9\cdot10^9 \cdot \frac{(-10+8-4)\times10^{-9}}{0.02} = \underline{-2700\,(V)}$

6 $V = k_e \frac{Q}{R}$, $R = 2cm = 0.02\ m$; $V = 500\ V$, then:

$Q = \frac{VR}{k_e} = \frac{500\cdot0.02}{9\cdot10^9} = 1.1\cdot10^{-9}\ C = \underline{1.1\ nC}$

7 $U = U_{12} + U_{13} + U_{23} = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}}$

(a) $r_{12} = 0.05\ m$, $r_{13} = 0.03\ m$; $r_{23} = 0.04\ m$, then:

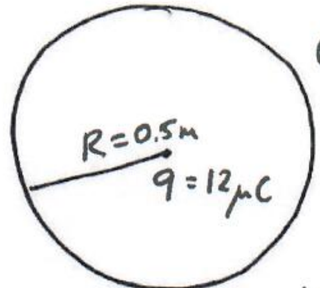
$U_a = 9\cdot10^9 \cdot \left(\frac{10^{-6}\cdot5\cdot10^{-6}}{0.05} - \frac{10^{-6}\cdot5\cdot10^{-6}}{0.03} - \frac{5\cdot10^{-6}\cdot5\cdot10^{-6}}{0.04} \right) = \underline{-6.23\,(J)}$

(b) $r_{12} = 0.07\ m$ $r_{13} = 0.03\ m$, $r_{23} = 0.04\ m$ then:

$U_b = 9\cdot10^9 \left(\frac{10^{-6}\cdot5\cdot10^{-6}}{0.07} - \frac{10^{-6}\cdot5\cdot10^{-6}}{0.03} - \frac{5\cdot10^{-6}\cdot5\cdot10^{-6}}{0.04} \right) = \underline{-6.5\,(J)}$

$\boxed{U_b < U_a}$

(8)



$\underline{V_p \text{ at } r = 0.2\,m}$

$(r<R)$ $\quad V_p = V_{sphere} + V_q$

$$V_{sphere} = \frac{k_e Q}{2R}\left(3-\frac{r^2}{R^2}\right) = \frac{9\cdot10^9\cdot(-16)\cdot10^{-6}}{2\cdot0.5}\left(3-\left(\frac{0.2}{0.5}\right)^2\right) = -4.1\cdot10^5\,(V)$$

$$V_q = k_e\frac{q}{r} = 9\cdot10^9\cdot\frac{12\cdot10^{-6}}{0.2} = 5.4\cdot10^5\,(V)$$, then

$$V_p = (5.4-4.1)\cdot10^5 = \underline{1.3\cdot10^5\,(V)}$$

(9) $\quad V_p = 0$, $V_p = V_{sphere} + V_q = 0$,

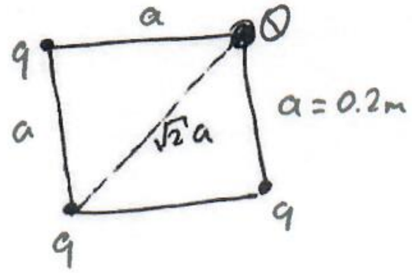
$V_{sphere} = k_e\frac{Q}{2R}\left(3-\frac{r^2}{R^2}\right)$, $V_q = k_e\frac{q}{r}$, then:

$$k_e\frac{Q}{2R}\left(3-\frac{r^2}{R^2}\right) + k_e\frac{q}{r} = 0 \quad \Rightarrow \quad q = -\frac{Qr}{2R}\left(3-\frac{r^2}{R^2}\right) =$$

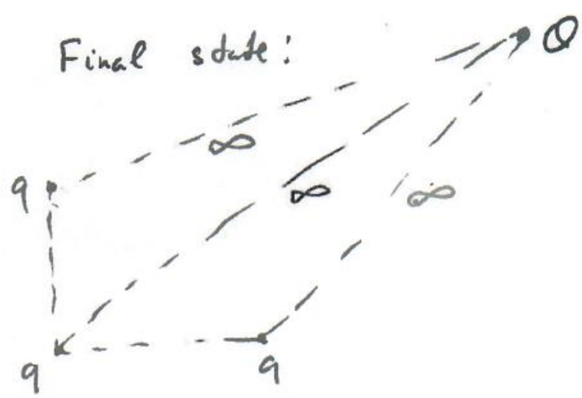
$$= -\frac{5\cdot10^{-6}\cdot0.1}{2\cdot0.2}\left(3-\frac{0.1^2}{0.2^2}\right) = \underline{-3.4\cdot10^{-6}C}$$

$$\underline{q = -3.4\,\mu C}$$

(10) Initial state:



Final state:



Initial energy:

$$E_i = k_e \frac{qQ}{a} + k_e \frac{qQ}{a} + k_e \frac{qQ}{\sqrt{2}\,a}$$

Final energy:

$$E_f = 0 + \frac{m v^2}{2}$$

Then:

$$k_e \frac{qQ}{a}\left(2 + \frac{1}{\sqrt{2}}\right) = \frac{m v^2}{2} \quad , m = 0.1\,kg$$

$$9 \cdot 10^9 \cdot \frac{10^{-6} \cdot 5 \cdot 10^{-6}}{0.2}\left(2 + \frac{1}{\sqrt{2}}\right) = \frac{0.1\, v^2}{2} \quad \Rightarrow \quad 0.609 = 0.05\, v^2$$

$$v = \sqrt{\frac{0.609}{0.05}} = \underline{3.5 \; m/s}$$